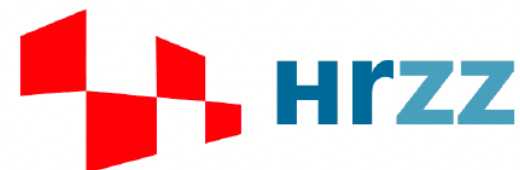


# Lifetimes of heavy baryons

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# Setting the stage

- Up-to-date predictions for the lifetimes of weakly decaying heavy baryons in Heavy Quark Expansion (HQE) framework
- Baryons names and their quark contents:

## Single charm

$\Lambda_c^+$  (*cud*)

$\Xi_c^+$  (*cus*)

$\Xi_c^0$  (*cds*)

.....

$\Omega_c^0$  (*css*)

## Single bottom

$\Lambda_b^0$  (*bud*)

$\Xi_b^0$  (*bus*)

$\Xi_b^-$  (*bds*)

.....

$\Omega_b^-$  (*bss*)

## Double charm

$\Xi_{cc}^{++}$  (*ccu*)

$\Xi_{cc}^+$  (*ccd*)

$\Omega_{cc}^0$  (*ccs*)

## Based on:

Charm mesons and single charm baryons:

[\[2204.11935\]](#) with J. Gratrex and B. Melić

Bottom baryons:

[\[2301.07698\]](#) with J. Gratrex, A. Lenz, B. Melić, M. L. Piscopo and A. V. Rusov

Double charm baryons:

[\[In preparation\]](#) with L. Dulibić, J. Gratrex and B. Melić

Related analyses for heavy mesons can be found in:

[\[2109.13219\]](#) by D. King, A. Lenz, M.L. Piscopo, Th. Rauh, A.V.Rusov, C.Vlahos

[\[2208.02643\]](#) by A. Lenz, M.L. Piscopo, A.V. Rusov

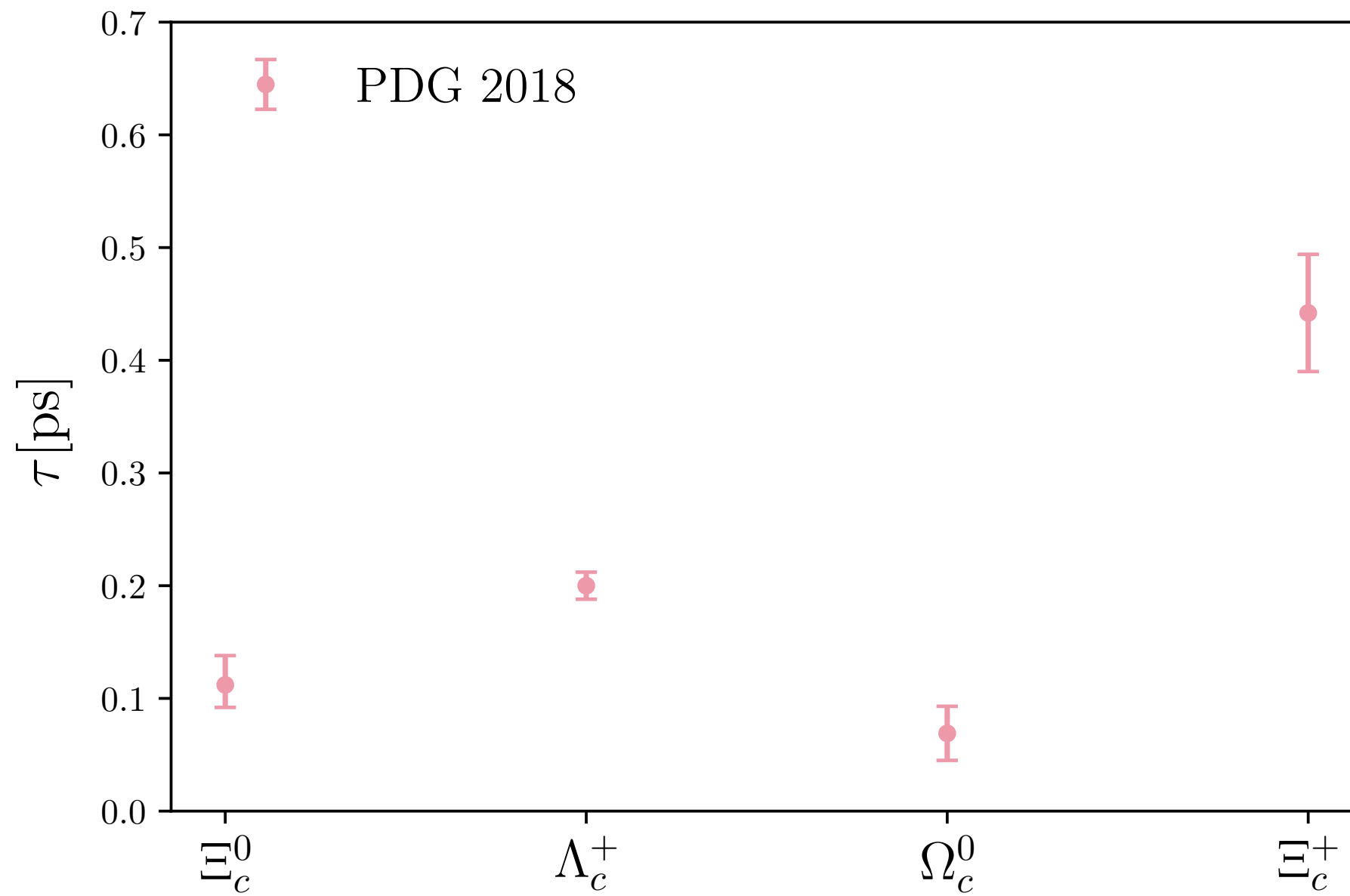
## Our analyses involve:

- Implementing new **Wilson** coefficient for the **Darwin** operator
- Including **radiative QCD** corrections to the Wilson coefficients, where available
- Updating numerical inputs, including new **estimates of non-perturbative matrix elements**

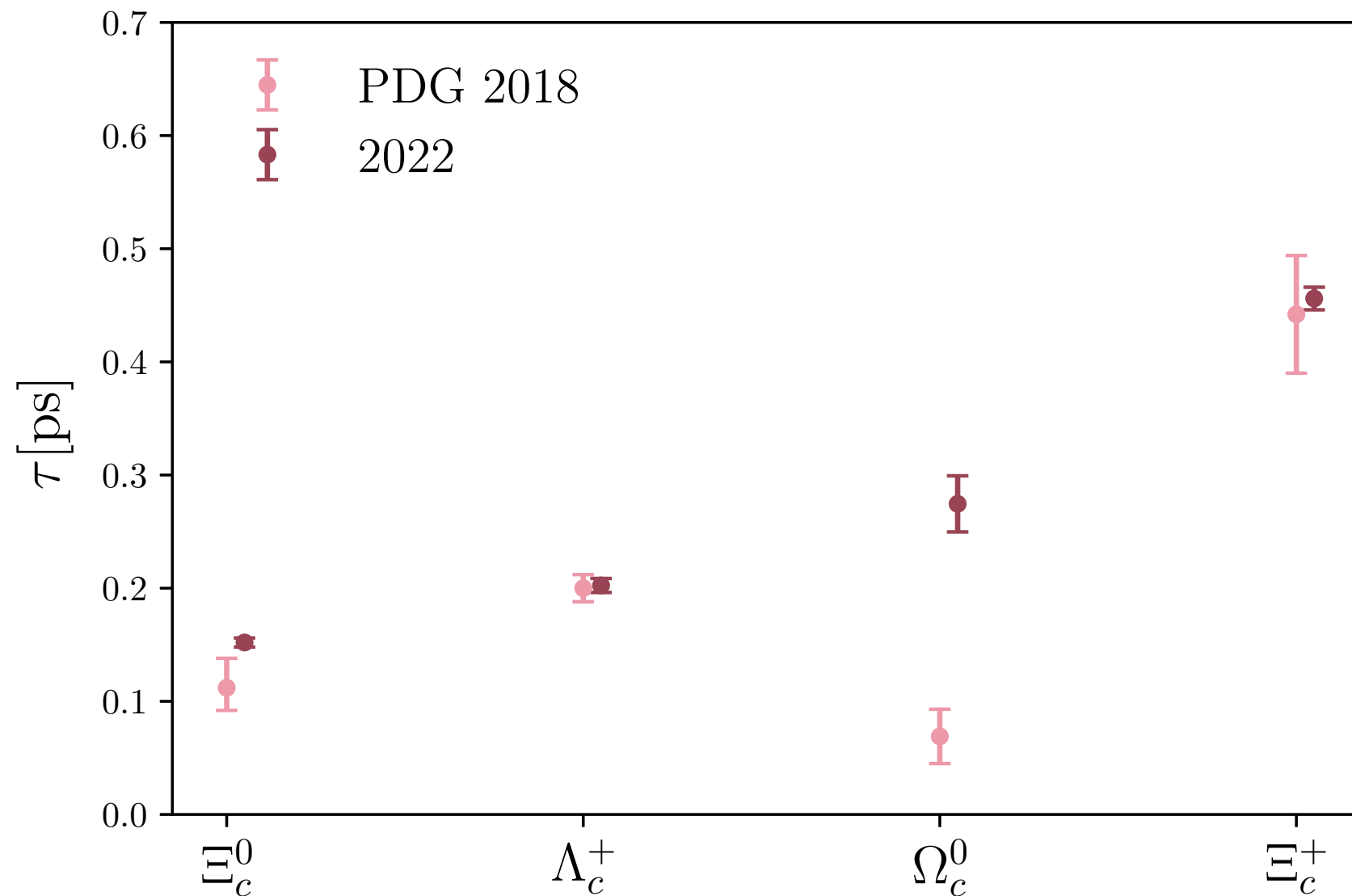
# Experimental status

# Experimental status

## c-baryon lifetimes

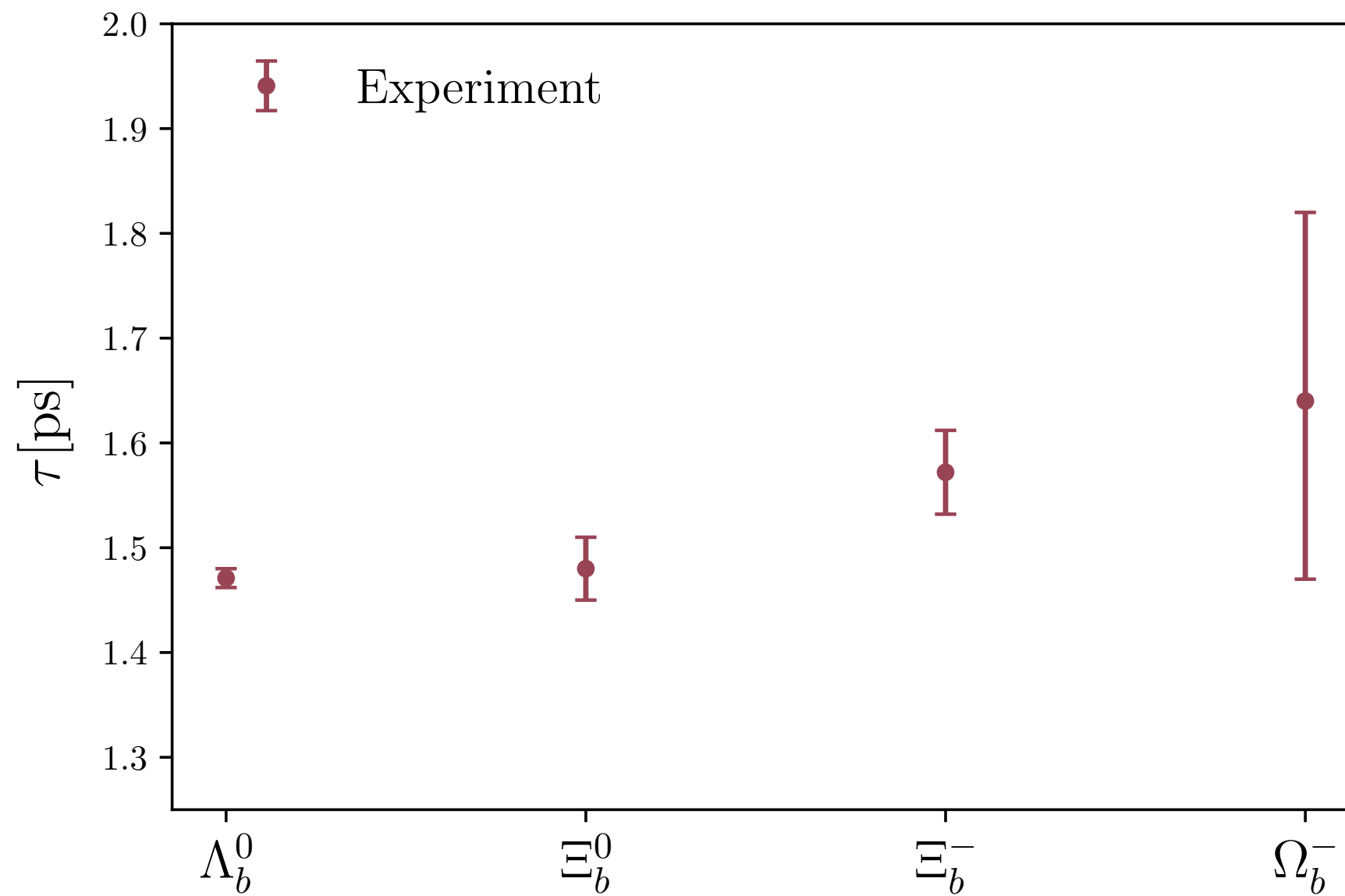


## LHCb Update



- The latest LHCb measurements [\[LHCb, 1906.08350, 2109.01334\]](#)  $\tau(\Omega_c)$  are **four times larger** than, and inconsistent with earlier findings

## b-baryon lifetimes





# Theoretical background

# Theoretical Background (Heavy Quark Expansion)

- According to optical theorem the **total decay width** is given by the **imaginary part of the forward matrix element (ME) of the transition operator**

$$\Gamma_H = \frac{1}{2m_H} \Im \langle H | T | H \rangle, \quad T = i \int d^4x T[\mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0)]$$

$\mathcal{H}_{eff}$  denotes  $\Delta C (\Delta B) = 1$  **weak effective Hamiltonian**.

- Given  $m_Q \gg \Lambda_{QCD}$ , applying the Heavy Quark Expansion (HQE) in powers of  $\sim \Lambda_{QCD}/m_Q$ , **expanding the T in a series of local operators**
- The **LO term arises from the decay of a free heavy quark**, leading to the expectation of equal lifetimes, e.g.  $\tau(\Lambda_b)/\tau(B_d) \simeq 1$ .

- Decay width in a **schematic form**:

$$\Gamma(H) = \frac{G_F^2 m_Q^5}{192 \pi^3} \left[ c_3 + \frac{c_\pi \mu_\pi^2 + c_G \mu_G^2}{m_Q^2} + \frac{c_\rho \rho_D^3}{m_Q^3} + \dots \right. \\ \left. + \frac{16\pi^2}{2m_H} \left( \sum_{i,q} \frac{c_{6,i}^q \langle H | O_i^q | H \rangle}{m_Q^3} + \frac{c_{7,i}^q \langle H | P_i^q | H \rangle}{m_Q^4} + \dots \right) \right]$$

- Wilson coefficients  $c_i$  have individual perturbative  $\alpha_s$ -expansions
- The hadronic MEs:  $\mu_\pi^2$ ,  $\mu_G^2$ ,  $\rho_D^3$ ,  $\langle H | O_i^q | H \rangle$  are nonperturbative and depend on the hadron  $H$
- First row in the expansion: non-spectator two-quark contributions.**  
Universal up to small differences in values of  $\mu_\pi^2, \mu_G^2, \rho_D^3$ :

$$2M_{\mathcal{B}}\mu_\pi^2(\mathcal{B}) \equiv - \langle \mathcal{B} | \bar{Q}_\nu (iD_\mu) (iD^\mu) Q_\nu | \mathcal{B} \rangle ,$$

$$2M_{\mathcal{B}}\mu_G^2(\mathcal{B}) \equiv \langle \mathcal{B} | \bar{Q}_\nu (iD_\mu) (iD_\nu) (-i\sigma^{\mu\nu}) Q_\nu | \mathcal{B} \rangle ,$$

$$2M_{\mathcal{B}}\rho_D^3(\mathcal{B}) \equiv \langle \mathcal{B} | \bar{Q}_\nu (iD_\mu) (i\nu \cdot D) (iD^\mu) Q_\nu | \mathcal{B} \rangle .$$

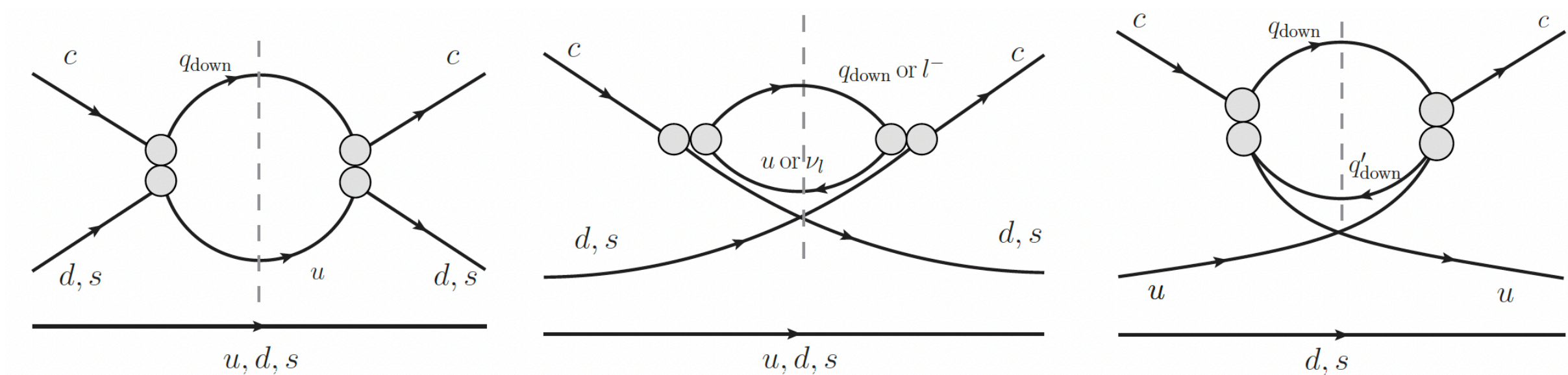
## Second row: spectator contributions

- Second row: MEs of four-quark operators. Dimension-6 basis:

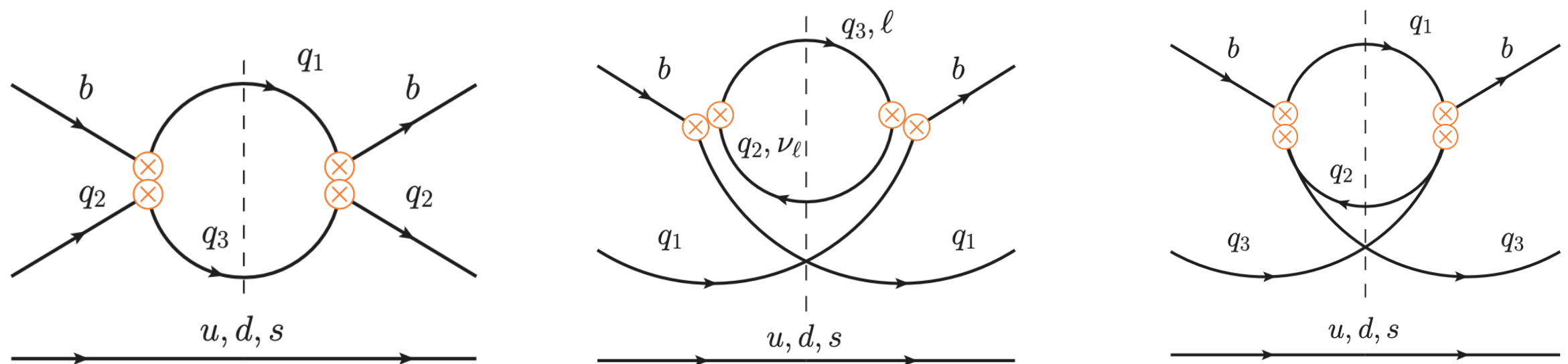
$$O_1^q = (\bar{c}_i q_i)_{V-A} (\bar{q}_j c_j)_{V-A}, \quad O_2^q = (\bar{c}_i q_i)_{S-P} (\bar{q}_j c_j)_{S+P},$$

$$\tilde{O}_1^q = (\bar{c}_i q_j)_{V-A} (\bar{q}_j c_i)_{V-A}, \quad \tilde{O}_2^q = (\bar{c}_i q_j)_{S-P} (\bar{q}_j c_j)_{S+P}.$$

- Sensitive to light quark flavours. **The primary drivers of lifetime splittings.**
- Three topologies relevant for baryons: **Weak Exchange, Constructive/ Destructive Pauli interference**



## Second row: spectator contributions



- $16\pi^2$  factor (loop enhancement) compared the two-quark contributions.
- For the charm, the spectator dimension-6 can even dominate over the ‘leading’ dimension-3. Compare  $16\pi^2(\Lambda_{QCD}/m_c)^3 \sim 6$  to  $16\pi^2(\Lambda_{QCD}/m_b)^3 \sim 0.3$
- Unlike in the case of mesons, no expectation of helicity suppression (all topologies relevant)

# Matrix elements

- For  $\mu_\pi^2, \mu_G^2$  follow the **standard approach** by exploiting the **HQE** for a **heavy hadron mass**
- Chromomagnetic term for  $\Omega_{c(b)}$

$$\mu_G^2(\Omega_Q) = \frac{2}{3}(m_{\Omega_Q^*}^2 - m_{\Omega_Q}^2) + \mathcal{O}(1/m_Q), \quad Q = c \text{ or } b$$

- **The spin structure** of the light constituents in single-heavy baryon triplet:  $\mu_G^2(\mathcal{T}_Q) = 0$
- **Extracting  $\mu_\pi^2$**  involves combining b- and c baryons and mesons and employing **more assumptions**, e.g. for  $\Lambda_b$

$$(\overline{M}_D - M_{\Lambda_c}) - (\overline{M}_B - M_{\Lambda_b}) = \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) (\mu_\pi^2(B) - \mu_\pi^2(\Lambda_b)) + \mathcal{O}\left(\frac{1}{m_c}, \frac{1}{m_b}\right)$$

$\overline{M}_D$  - spin averaged mass.  $\mu_\pi^2(B)$  from the fit **[Bordone, Capdevila, Gambino, 2107.00604]**

- Recently, the **Darwin contribution** has been evaluated for **nonleptonic decays** [Mannel, Moreno, Pivovarov, 2004.09485], [Lenz, Piscopo, Rusov, 2004.09527]
- **Sizeable contribution for charm**,  $\sim 20\%$  of the dimension-three and five
- Relate the Darwin ME to those of dimension-6 via gluon field equation of motion

$$2M_{\mathcal{B}}\rho_D^3(\mathcal{B}) = g_s^2 \sum_{q=u,d,s} \langle \mathcal{B} | -\frac{1}{8}O_1^q + \frac{1}{24}\tilde{O}_1^q + \frac{1}{4}O_2^q - \frac{1}{12}\tilde{O}_2^q | \mathcal{B} \rangle + \mathcal{O}(1/m_Q)$$



- The vacuum insertion approximation (VIA) is not available for baryons
- Treat baryon four-quark matrix elements in non-relativistic constituent quark model (NRCQM) - somewhat similar in spirit to the VIA
- The constituent model provides good fit of heavy hadron masses [Karliner, Rosner, 1408.5877]
- Applying such an approach to B mesons, e.g. for the estimate of the decay constant, would lead to the values not inconsistent with measurements/lattice
- The model relates the MEs to baryon wave functions at origin, e.g.

$$\frac{\langle \mathcal{T}_b | \mathcal{O}_1^q | \mathcal{T}_b \rangle}{2M_{\mathcal{T}_b}} = - |\psi^{\mathcal{T}_b}(0)|^2, \quad \frac{\langle \Omega_b | \mathcal{O}_1^q | \Omega_b \rangle}{2M_{\Omega_b}} = - 6 |\psi^{\Omega_b}(0)|^2$$

- Use constituent model to **express baryon masses** in terms of **wave functions** [Rujula, Georgi, Glashow PRD12 147 (1975)]

$$M_{\mathcal{B}} = \sum_i m_i^{\mathcal{B}} + \dots + \sum_{i>j} \frac{16\pi\alpha_s}{9} \frac{\langle \vec{s}_i \vec{s}_j \rangle}{m_i^{\mathcal{B}} m_j^{\mathcal{B}}} |\psi^{\mathcal{B}}(0)|^2$$

Similar equations for mesons, with different spin-term prefactor,  $m_i^{\mathcal{B}}$  is constituent quark masses in a baryon - values from mass spectra fits [Karliner, Rosner, 1408.5877]

- **Combining these relations** for the **hyperfine partners** (e.g.  $1/2^+$  and  $3/2^+$ ) results in the wave functions in terms of the mass splittings, e.g.

$$\frac{\langle \Xi_b^0 | \mathcal{O}_1^u | \Xi_b^0 \rangle}{2M_{\Xi_b}} = \frac{\langle \Xi_b^- | \mathcal{O}_1^d | \Xi_b^- \rangle}{2M_{\Xi_b}} = -y_{\tilde{q}} \frac{4}{3} \frac{M_{\Xi_b^*} - M_{\Xi_b'}}{M_{B^*} - M_B} |\psi^B(0)|^2$$

with  $|\psi^{B_q}(0)|^2 = 1/12 F_{B_q}^2(\mu_0)$ .

- Going one step further in  $1/m_Q \rightarrow$  **dimension-7 four-quark** operators

$$P_1^q = m_q (\bar{Q}_i (1 - \gamma_5) q_i) (\bar{q}_j (1 - \gamma_5) Q_j), \quad P_2^q = \frac{1}{m_Q} (\bar{Q}_i \overleftarrow{D}_\rho \gamma_\mu (1 - \gamma_5) D^\rho q_i) (\bar{q}_j \gamma^\mu (1 - \gamma_5) Q_j),$$

e.t.c

- Initially given in terms of QCD field  $Q$
- Expressing them in terms of HQET field  $h_v$  introduces a proliferation of local and nonlocal terms  $\rightarrow$  unclear on how to estimate the MEs
- For b-baryons, the  $1/m_b$  under control, dimension-7 as an addition to uncertainty
- For charm baryons, dimension seven is up to **50 % of the dimension-6**
- Constituent model relations/scaling arguments within the QCD basis, e.g.

$$\langle \mathcal{T}_c | P_1^q | \mathcal{T}_c \rangle \simeq \frac{1}{2} m_q |\psi_{cq}^{\mathcal{T}_c}(0)|^2, \quad \langle \mathcal{T}_c | P_2^q | \mathcal{T}_c \rangle \simeq -\Lambda_{QCD} |\psi_{cq}^{\mathcal{T}_c}(0)|^2$$

## Some specifics for double-charm baryons

- A system with **two c-quarks** - cc-pair viewed as diquark ( $\mathcal{D}$ )
- **Two** contributions to a given ME, in the example of the mass expansion

$$M_{\mathcal{B}_{cc}} = 2m_c + \bar{\Lambda} + \frac{\mu_\pi^2(\mathcal{B}_{cc})}{2m_c} - \frac{\mu_{G\mathcal{D}-q}^2(\mathcal{B}_{cc})}{2m_c} - \frac{\mu_{Gc-c}^2(\mathcal{B}_{cc})}{2m_c} + \dots$$

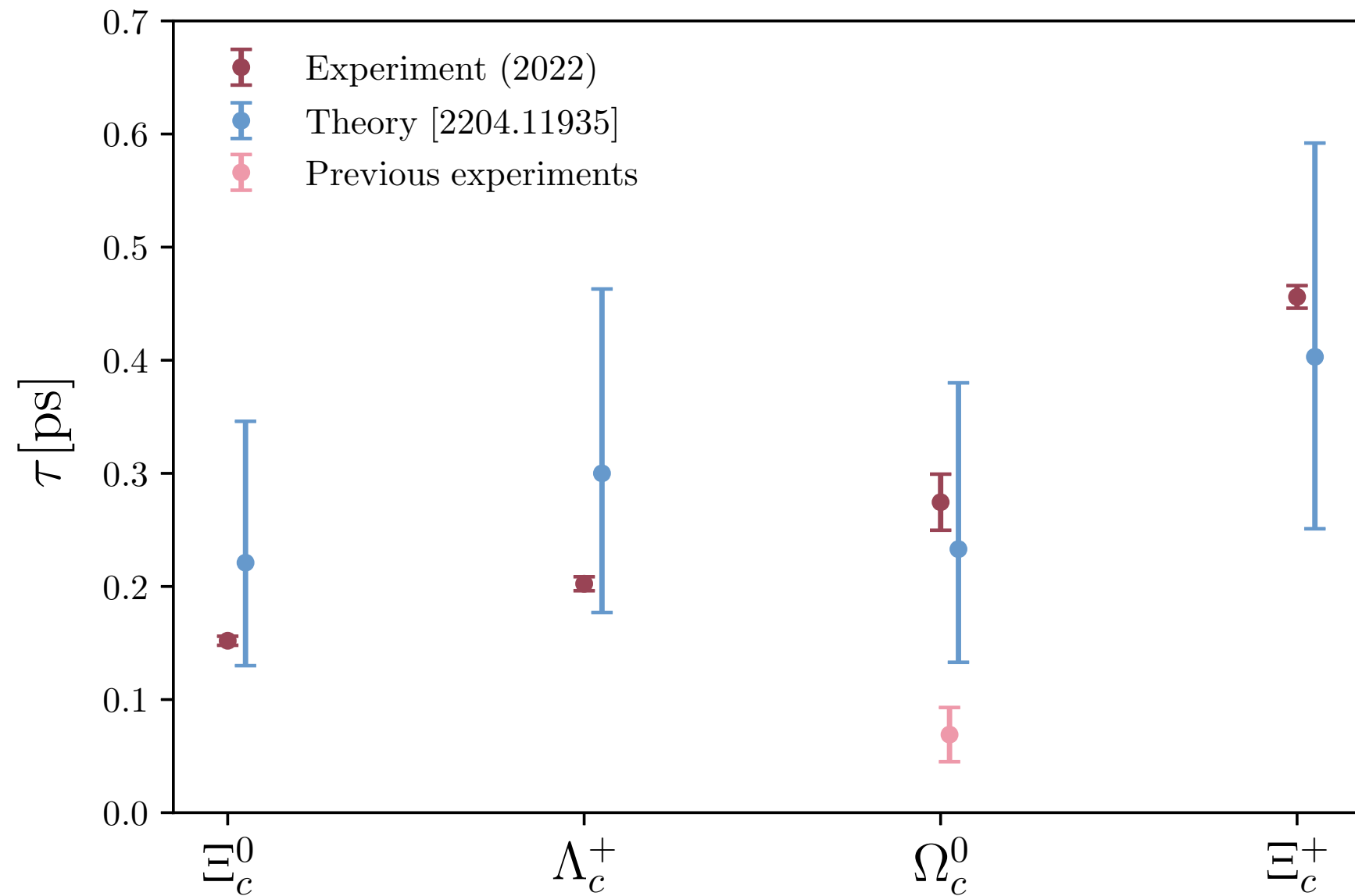
- The **‘additional’** c-c term **accessed by NRQCD expansions**, e.g. up to  $\mathcal{O}(v^7)$

$$\begin{aligned} \bar{Q}Q = & \psi_Q^\dagger \psi_Q - \frac{1}{2m_c^2} \psi_Q^\dagger (i\vec{D})^2 \psi_Q - \frac{1}{2m_c^2} \psi_Q^\dagger (g_s \vec{\sigma} \cdot \vec{B}) \psi_Q \\ & - \frac{1}{4m_c^3} \psi_Q^\dagger (g_s \vec{D} \cdot \vec{E}) \psi_Q + \frac{3}{8} \psi_Q^\dagger (i\vec{D})^4 \psi_Q + \dots \end{aligned}$$

in terms of non relativistic 2-component Pauli spinor field  $\psi_Q$ .

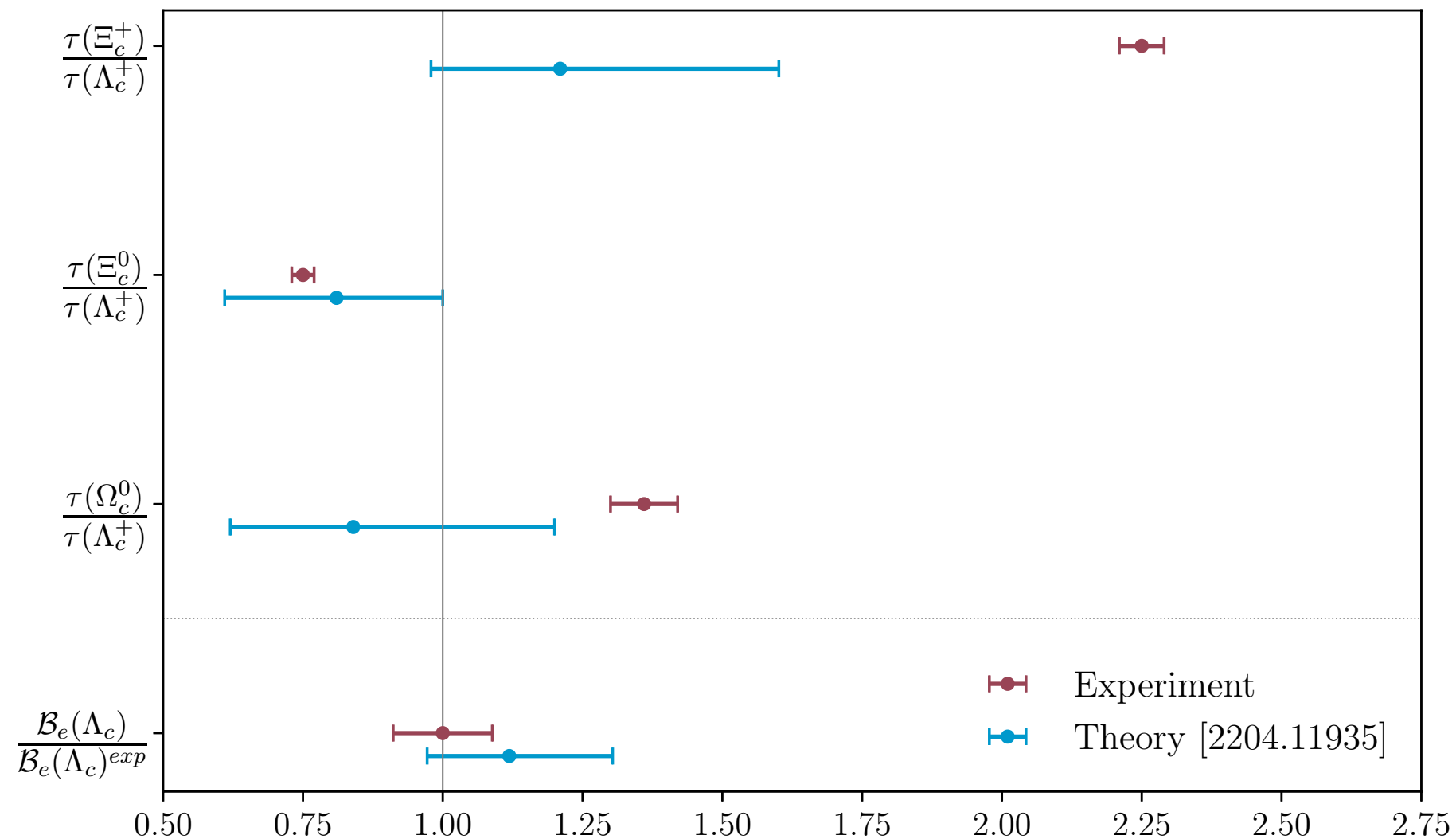
# Results

## c-baryon lifetimes



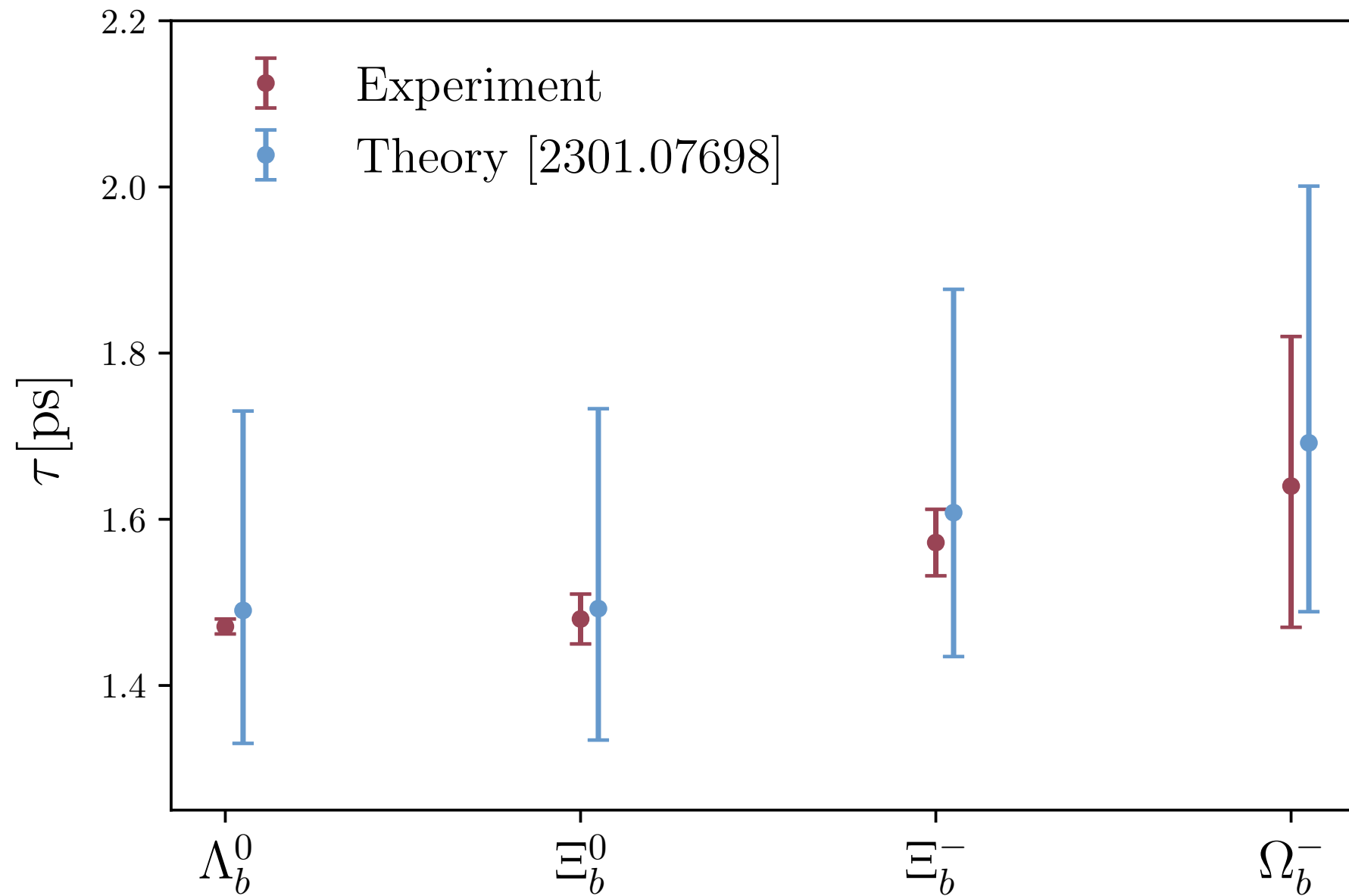
- New  $\Omega_c$ -lifetime result accommodated well. Sizeable theoretical uncertainties

## c-baryons ratios



- Improved predictability of lifetime ratios **due to cancellation of leading non-spectator terms**
- Some tensions remain, particularly  $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$ .

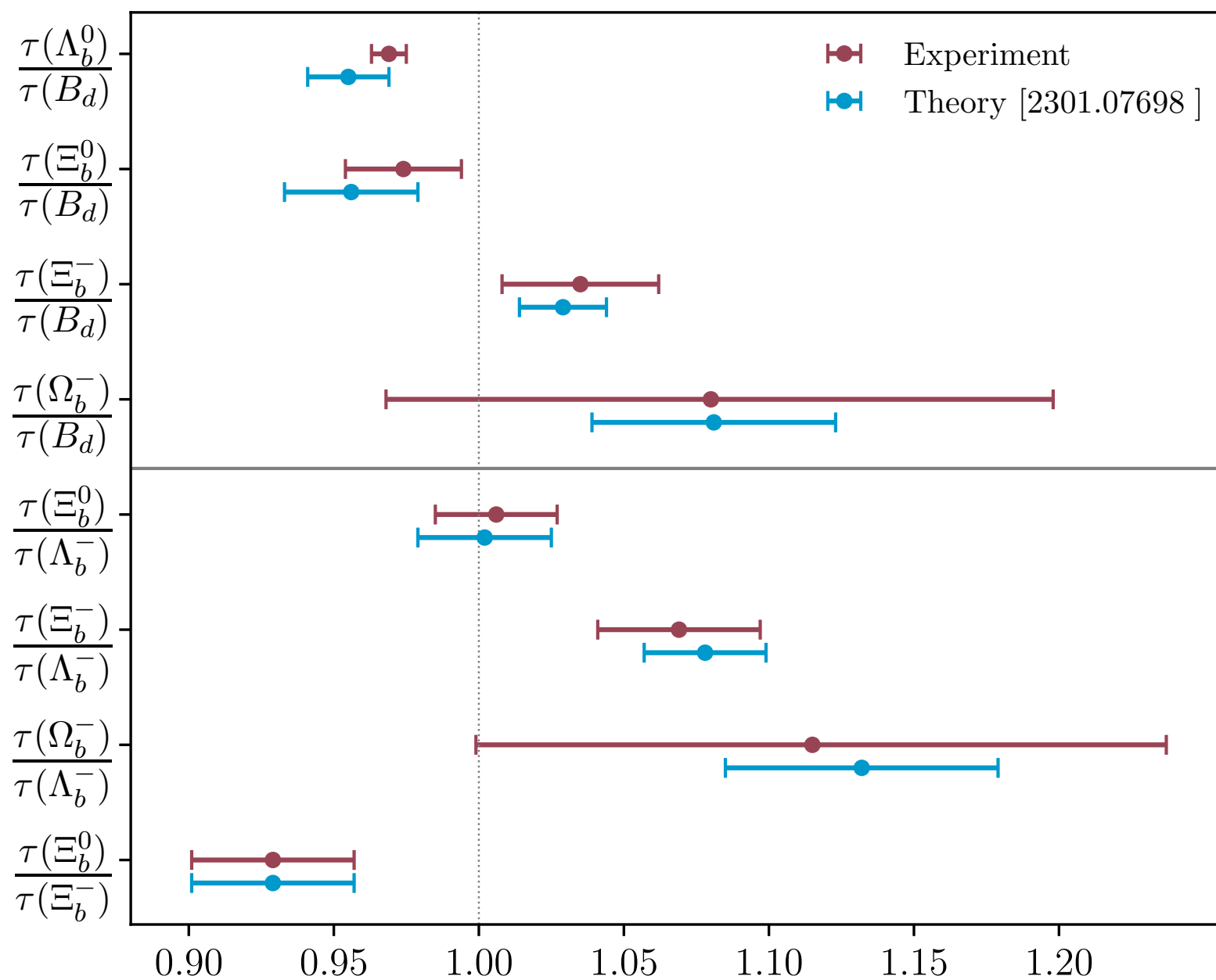
## b-baryons lifetimes



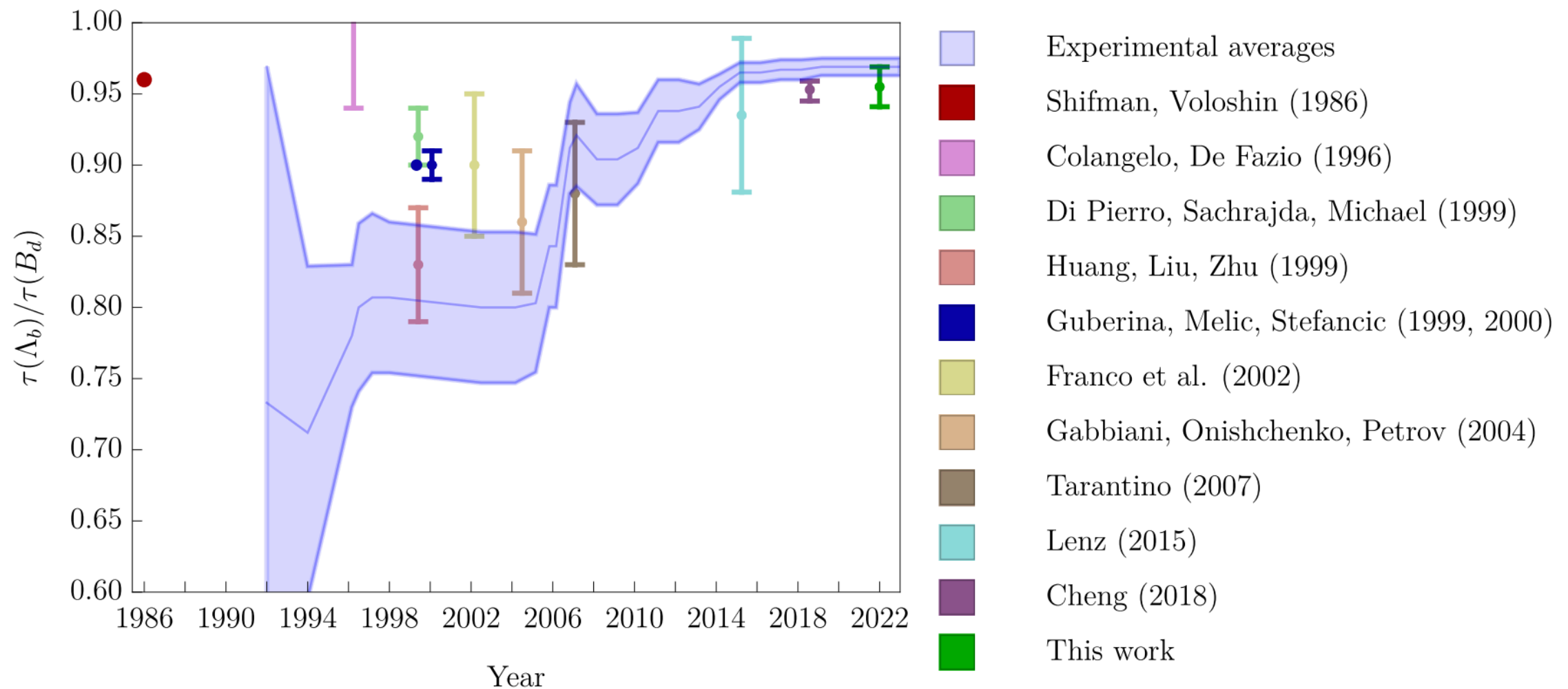
- Agreement with experiments for individual lifetimes



## b-baryons ratios



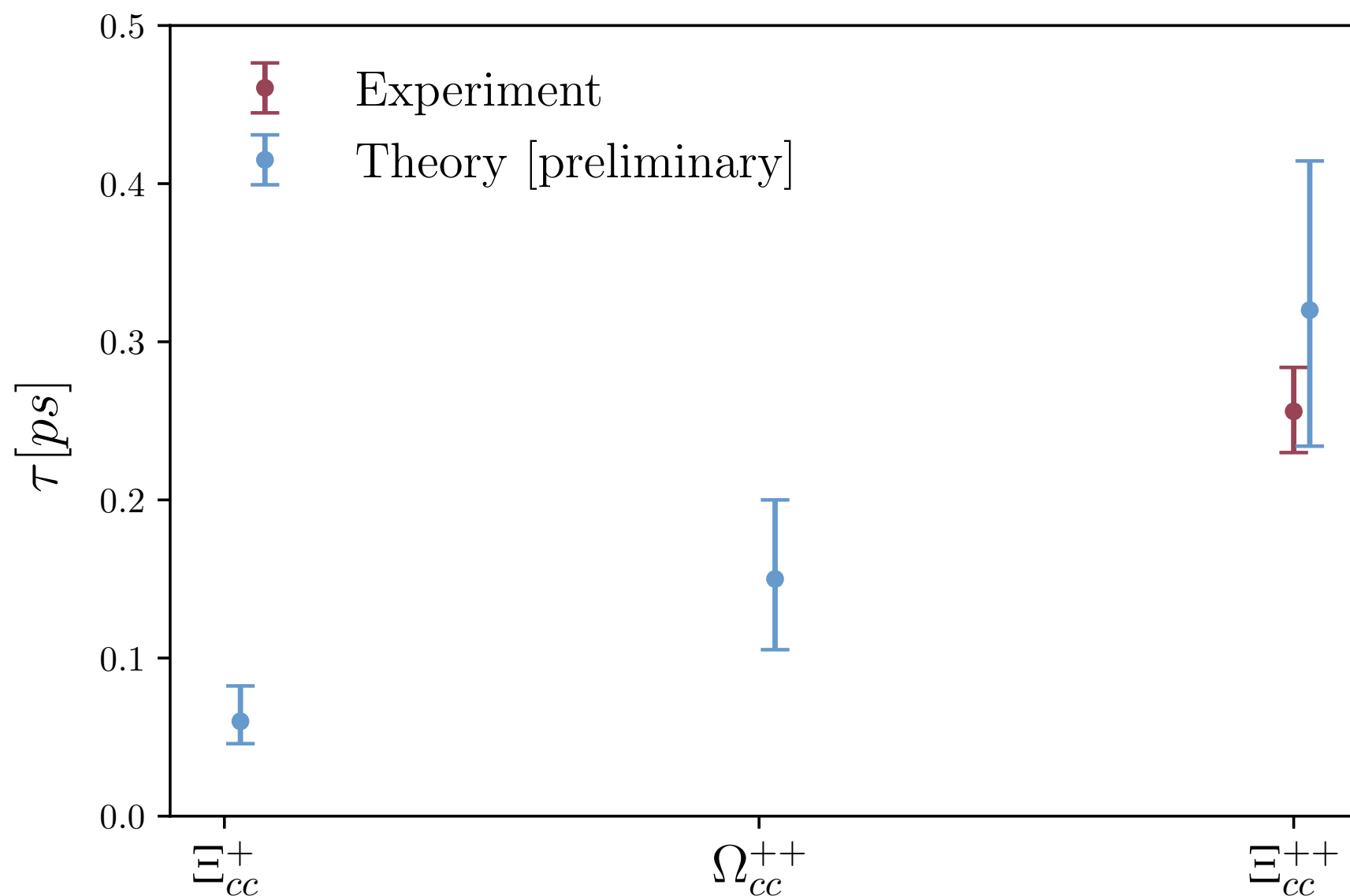
- Excellent agreement in all lifetime ratios



- Gradual historical convergence of both experimental and theoretical values
- Present day measurements compared our prediction:

$$\frac{\tau(\Lambda_b^0)^{HQE}}{\tau(B_d^0)} = 1 - (0.045 \pm 0.014), \quad \frac{\tau(\Lambda_b^0)^{Exp}}{\tau(B_d^0)} = 1 - (0.031 \pm 0.006)$$

## cc-baryons



- The measurement available for  $\tau(\Xi_{cc}^{++})$  [LHCb, 1806.02744]
- Measuring  $\Xi_{cc}^+$  and  $\Omega_{cc}^+$  with the expected LHCb Run-3 data was suggested as feasible

- Results for charm baryons broadly compatible with experiment, favouring recent LHCb result for the  $\Omega_c$ -lifetime
- The question of the applicability of  $1/m_c$  expansion due to slow  $1/m_c$ -convergence remains open
- Question of optimal mass scheme for charm quark. E.g., the pole mass is problematic

$$m_c^{pole} = \bar{m}_c(\bar{m}_c)(1 + 0.16 + 0.15 + 0.21 + \dots)$$

- For charm, we tested four mass-schemes: pole,  $\overline{MS}$ , kinetic, MSR. Effect: rearranging the  $\alpha_s$ -expansions.
- At the *present* scale of numerical accuracy, the four schemes are not distinguishable – still important technically.

- Theoretical values for lifetime ratios of b-baryons show an excellent consistency of HQE with the experimental data.
- Apart from evaluating further in  $(\alpha_s, 1/m_b)$ -expansions, which would be welcome,
- it is perhaps timely to revisit lattice QCD or the sum rules evaluations of four-quark MEs.
- Apart from the exploratory study in [\[UKQCD \(Di Pierro, Sachrajda, Michael\), 9906031\]](#), no lattice determinations for the four-quark baryonic matrix elements are available. Perhaps feasible.
- An alternative approach using HQET sum rules for the four-quark baryonic matrix elements, exploratory study in [\[Colangelo, De Fazio, 9604425\]](#), could be revisited in the future.

**Thank you!**