

# APPLICATIONS OF THE TUNNELING POTENTIAL FORMALISM



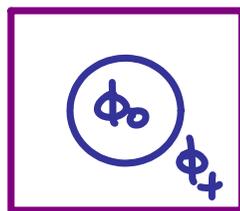
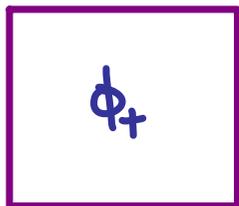
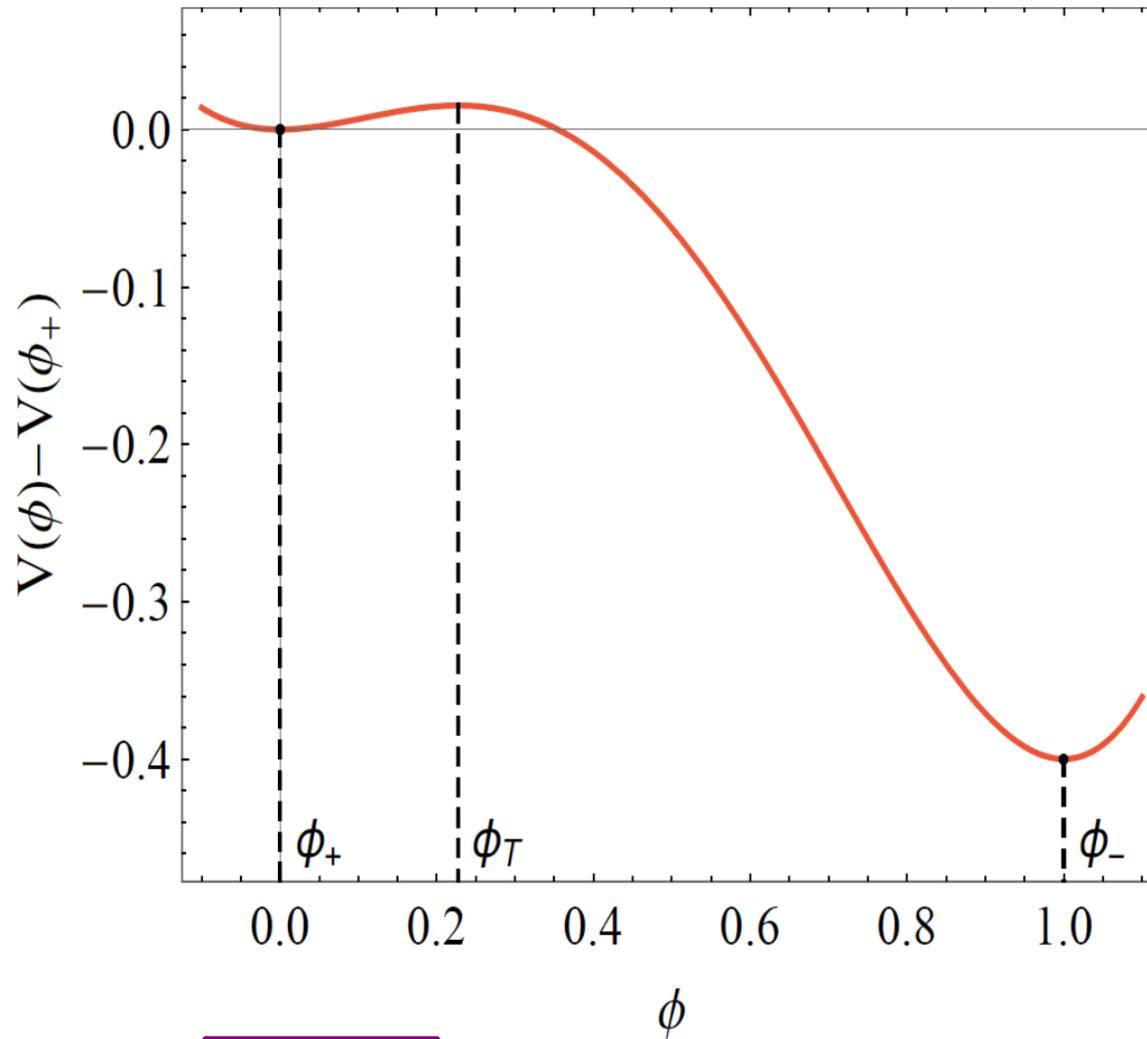
Portorož 2023

13/4/2023

J.R.Espindosa



# THE PROBLEM



$$I/N = A e^{-S}$$

calculate this

# EUCLIDEAN FORMALISM

Coleman '77

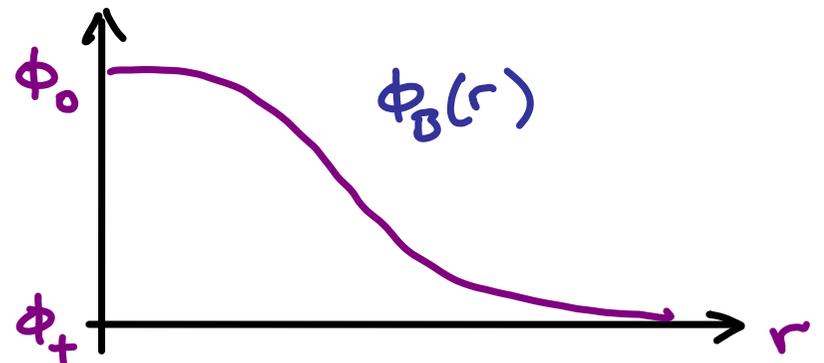
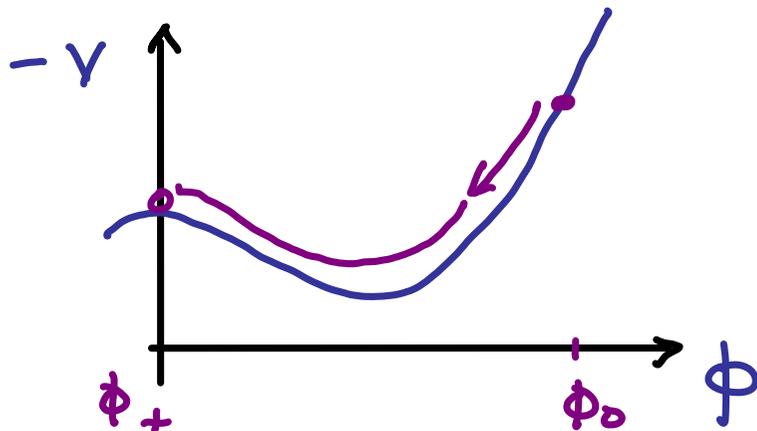
Euclidean bounce  $\phi_B(r)$  extremizes

$$S_E = 2\pi^2 \int_0^\infty dr r^3 \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) - V_+ \right]$$



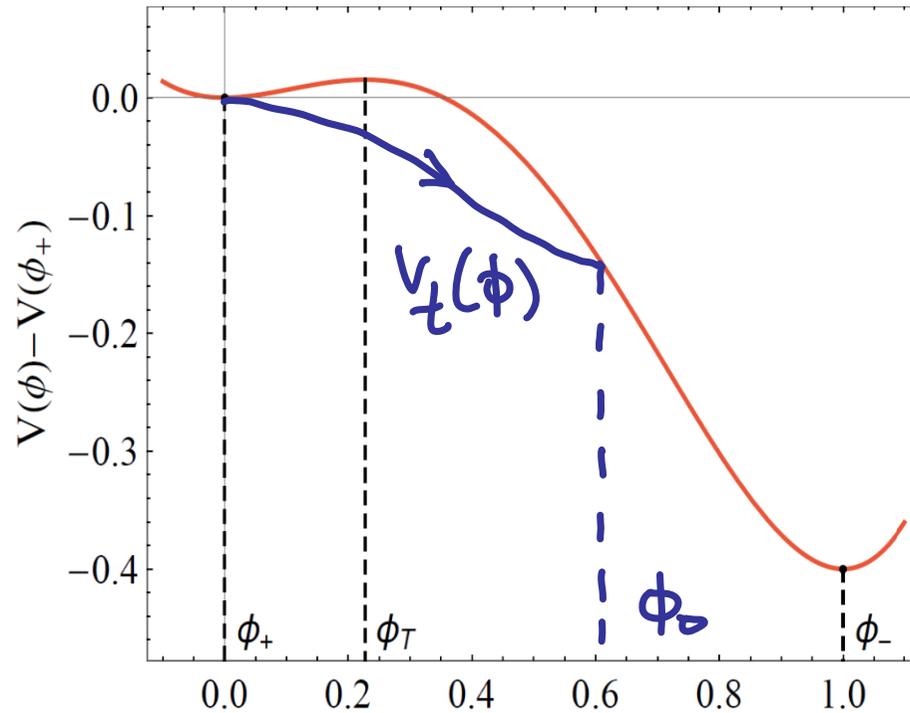
$$\ddot{\phi} + \frac{3}{r} \dot{\phi} = \frac{\partial V}{\partial \phi}$$

with  $\phi(0) = \phi_0$   $\dot{\phi}(0) = 0$   $\phi(\infty) = \phi_+$



# TUNNELING POTENTIAL FORMALISM

JRE '1805



$$S[V_t] = 54\pi^2 \int_{\phi_+}^{\phi_0} \frac{(V - V_t)^2}{(-V_t')^3} d\phi$$

$$S = \text{Min}_{V_t} S[V_t] \quad \phi_0 \text{ as in Euclidean}$$

# PROPERTIES & APPS OF $V_t$ -FORM.

★  $V_t$  monotonic  $\Leftrightarrow$  Easy to approximate JRE'1805

★  $S[V_t]$  minimized ( $\phi_0(r)$  saddle-point)

$\Leftrightarrow$  Good for numerics

$\Leftrightarrow$  Useful for multi-field case JRE, Konstandin'1811

★ Generalizes to any dimension

$d=3$  applicable to finite T phase transitions JRE'1805

★ New way of finding exact examples

$$V(\phi) = V_t + \frac{(V_t')^2}{3} \int_{\phi_0}^{\phi} \frac{d\bar{\phi}}{V_t'(\bar{\phi})}$$

JRE'1805

JRE, Fortin'2211

★ Related to Euclidean form. by a canonical transf.

JRE, Jinno, Konstandin'2209

# GRAVITY CORRECTIONS

Gravity relevant if tunneling involves

$$\Delta\phi \lesssim m_p \quad \text{or} \quad \Delta V \lesssim m_p^4$$

Need to include reaction of the metric.

## EUCLIDEAN FORMALISM W/GRAVITY

Coleman, De Luccia '80

Euclidean bounce  $\phi_B(\xi)$  and metric function  $f(\xi)$

$$O(4) \quad ds^2 = d\xi^2 + f^2(\xi) d\Omega_3^2$$

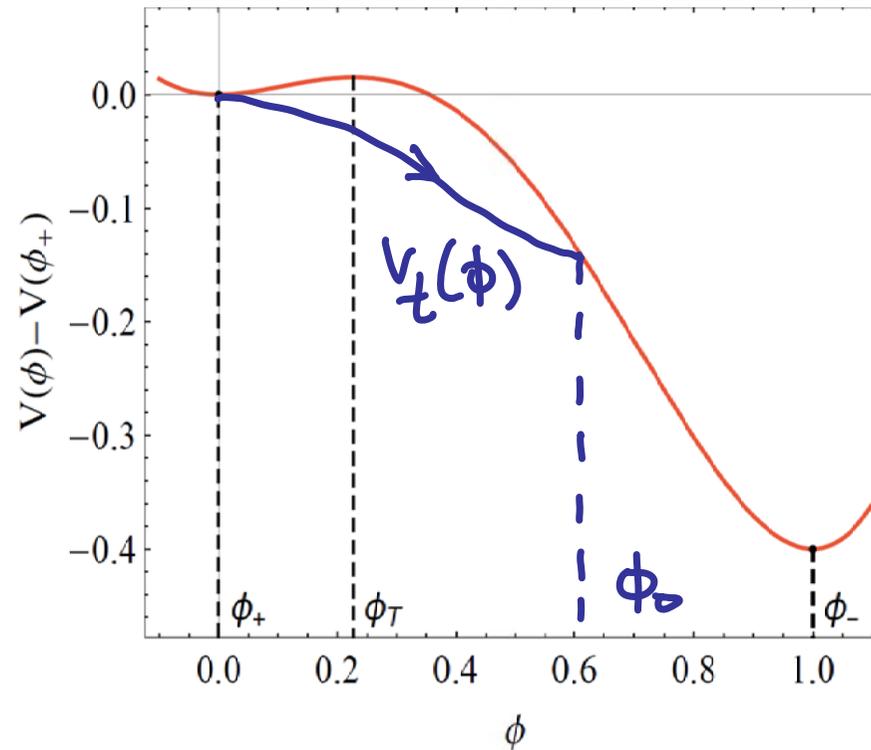
$$\Rightarrow \ddot{\phi} + \frac{3\dot{f}}{f} \dot{\phi} = \frac{\partial V}{\partial \phi} \quad \dot{f}^2 = 1 + \frac{\kappa f^2}{3} \left( \frac{1}{2} \dot{\phi}^2 - V \right)$$

$$S = \Delta S_E = S_E[\phi_B] - S_E[\phi_+]$$

$$\kappa \equiv 1/m_p^2$$

# $V_t$ FORMALISM w/ GRAVITY

JRE'1808



$$S[V_t] = \frac{6\pi^2}{k^2} \int_{\phi_+}^{\phi_0} \frac{(D + V_t')^2}{D V_t^2} d\phi$$

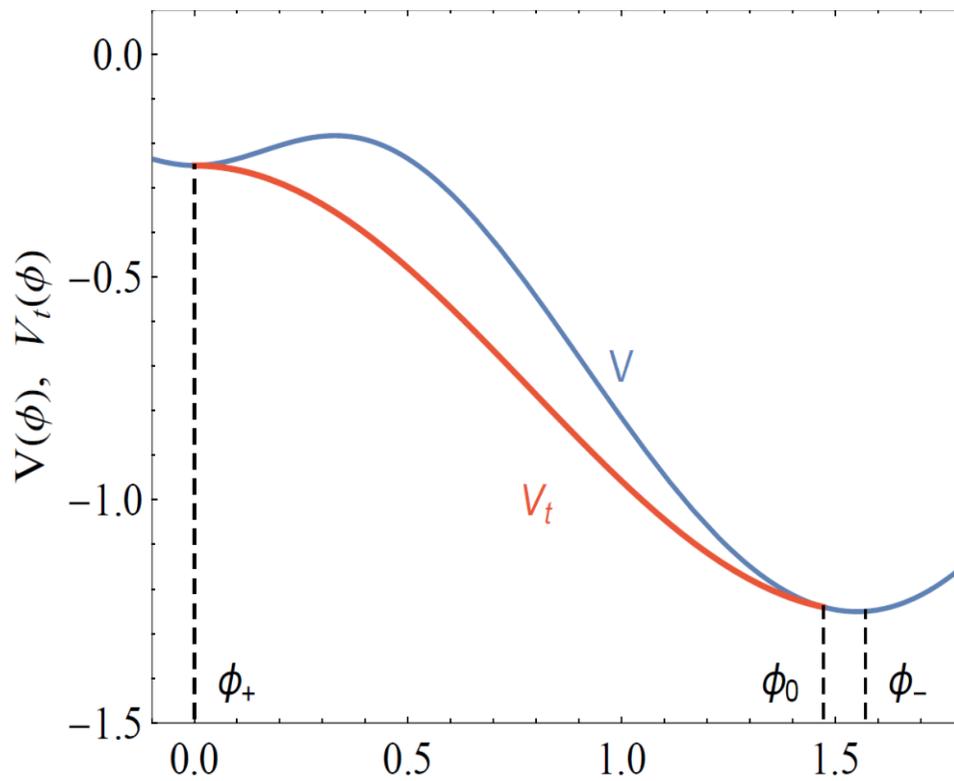
with

$$D = \sqrt{(V_t')^2 + 6k(V - V_t)V_t}$$

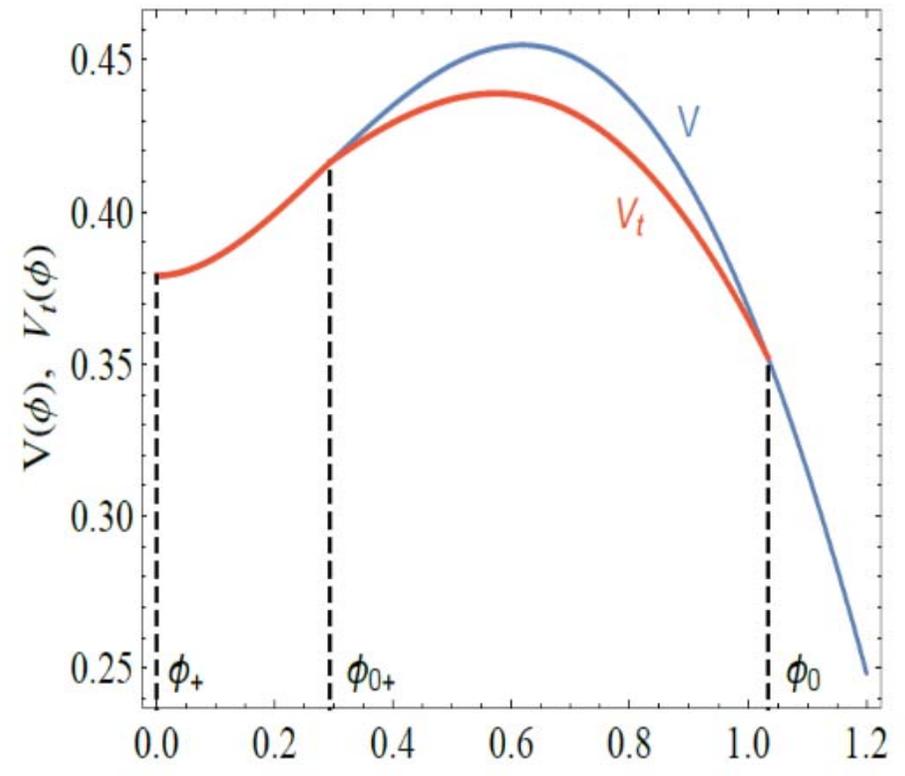
$$k \equiv 1/m_p^2$$

# PROPERTIES & APPS OF $V_t$ -FORM.

- ★ Universal formula, valid for AdS, Minkowski or dS



Minkowski or AdS



dS

$$S[V_t] = \Delta S_E$$

# PROPERTIES & APPS OF $V_t$ -FORM.

★ Universal formula, valid for AdS, Minkowski or dS

★ For dS,  $V_+ \uparrow \Leftrightarrow$  no CdL bounce

Hawking-Moss decay with rate

$$S_{HM} = \frac{24\pi^2}{\kappa^2} \left( \frac{1}{V_+} - \frac{1}{V_{top}} \right)$$

$S[V_t]$  reproduces this ✓

# PROPERTIES & APPS OF $V_t$ -FORM.

★ Universal formula, valid for AdS, Minkowski or dS

★ For dS,  $V_+ \uparrow \Leftrightarrow$  no CdL bounce

Hawking-Moss decay rate ✓

★ For AdS, Minkowski ( $V_+ \leq 0$ )

gravitational quenching of decay possible

Coleman, De Luccia '80

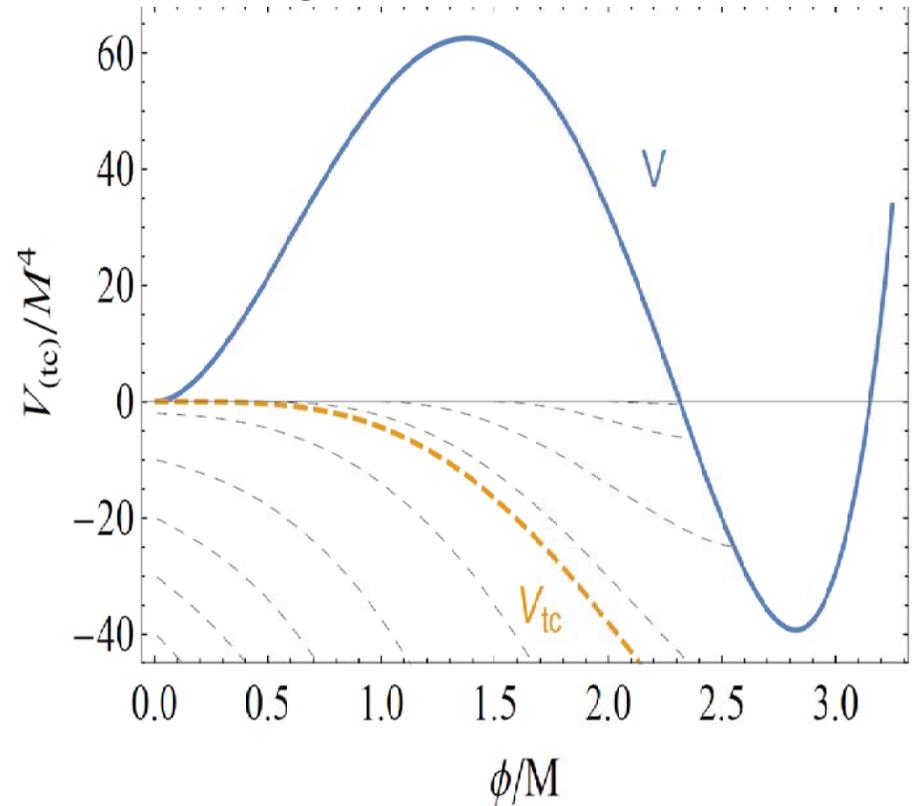
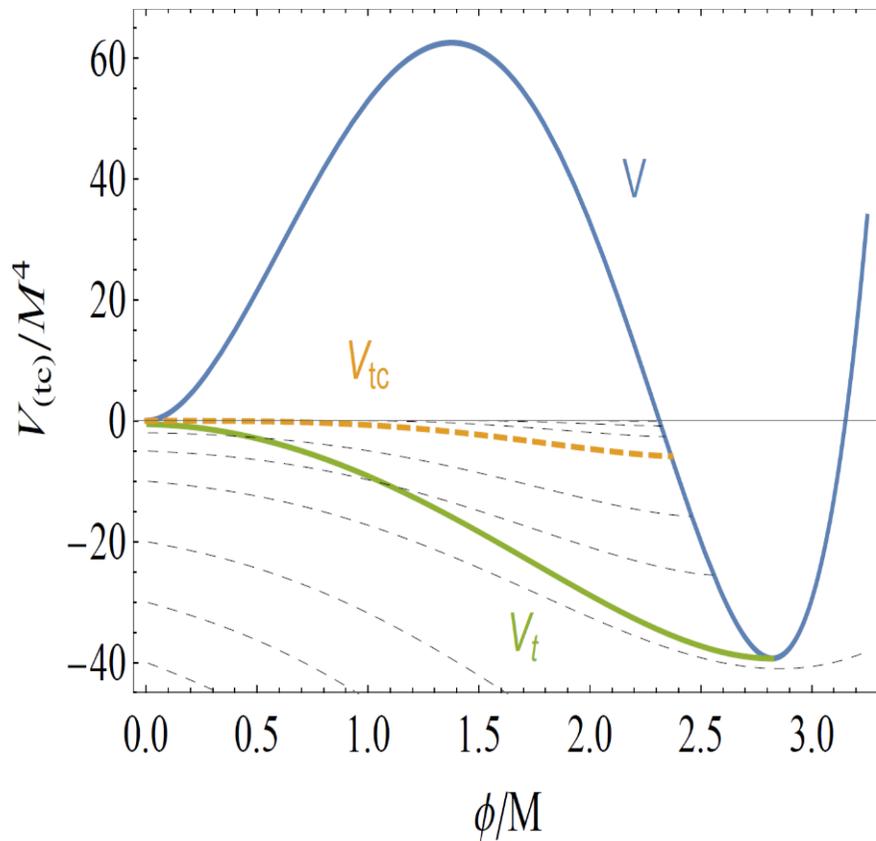
$$D = \sqrt{(V_t')^2 + 6\kappa(V - V_t)V_t} \quad \text{must be real}$$

$$|V_t'| > |V_{tc}'| \equiv \sqrt{6\kappa(V - V_t)(-V_t)}$$

# GRAVITATIONAL QUENCHING

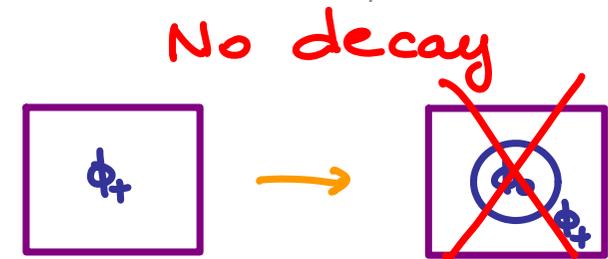
Weak grav. effects  $\longrightarrow$

Strong grav. effects



Decays

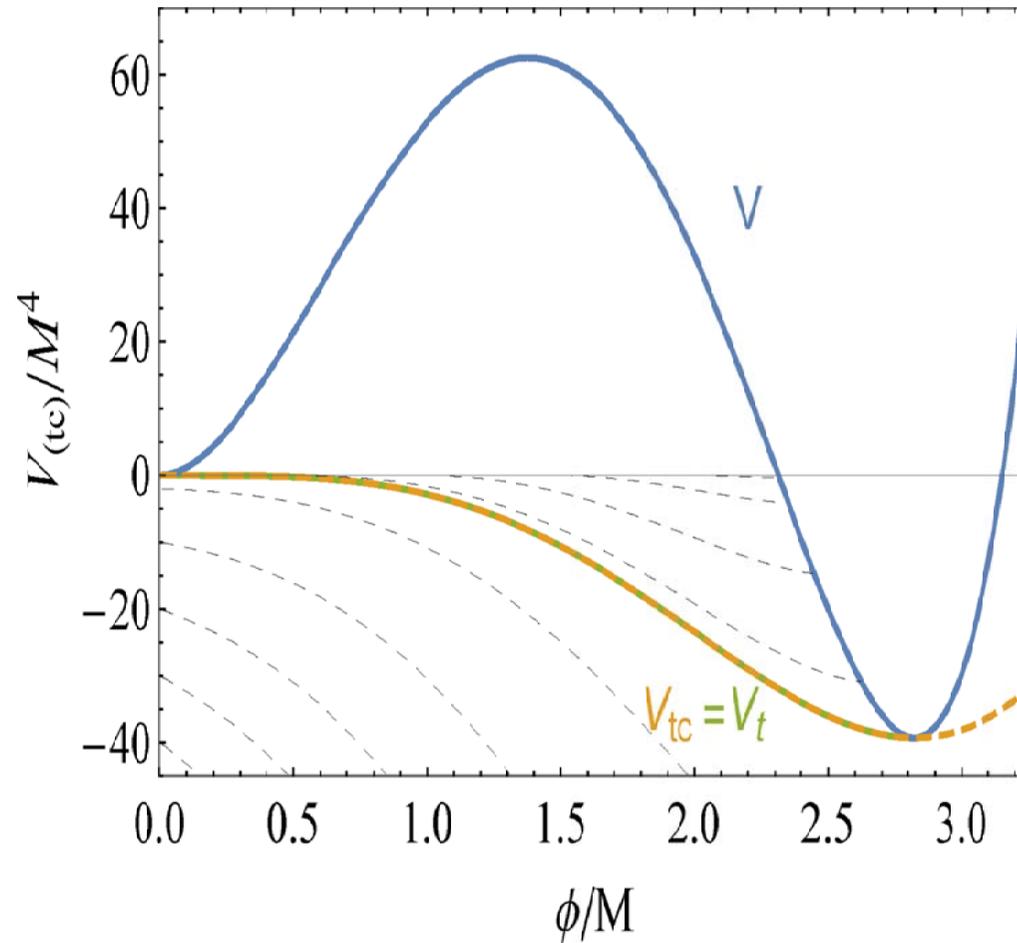
JRE '2005



Positive Energy Theorem

# GRAVITATIONAL QUENCHING

Critical case



Domain wall

$$D = 0$$

↓

$$V = V_t' - \frac{(V_t')^2}{6k V_t}$$

Critical Potentials

# PROPERTIES & APPS OF $V_t$ -FORM.

★ Universal formula, valid for AdS, Minkowski or dS

★ For dS,  $V_+ \uparrow \Leftrightarrow$  no CdL bounce

Hawking-Moss decay rate ✓

★ For AdS, Minkowski ( $V_+ \leq 0$ )

gravitational quenching of decay ✓

★ New way of finding exact examples

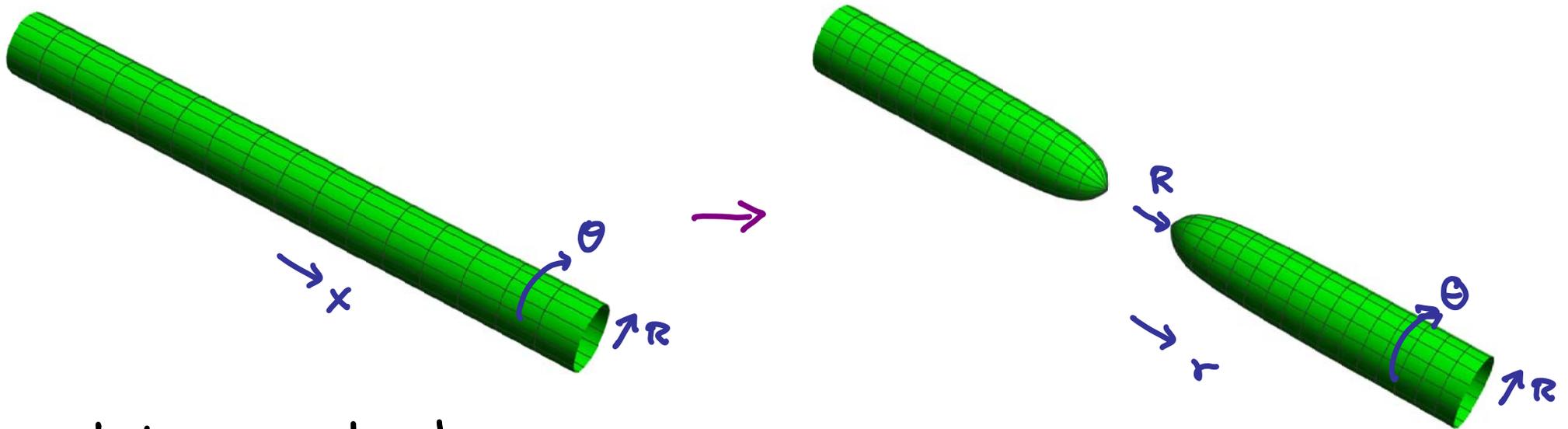
JRE, Fortin, Huertas '2106

# BUBBLE OF NOTHING DECAYS

Decays of spacetimes with compactified dim. like

5d KK ( $M^4 \times S^1$ )

Witten '82



Euclidean instanton:

$$ds^2 = \frac{dr^2}{1 - R^2/r^2} + r^2 d\Omega_3^2 + \underbrace{R^2 \left(1 - \frac{R^2}{r^2}\right)}_{R(r)^2} d\theta^2$$

with  $S = (\pi R m_p)^2$

$R(r)^2 \rightarrow 0$  at  $r \rightarrow R$

# 4d VIEW

Dine, Fox, Gorbatov '0405

5d  $\rightarrow$  4d + Scalar  $\phi$

$$R^2 \left(1 - \frac{R^2}{r^2}\right) = R^2 e^{-2\sqrt{2\kappa/3} \phi} \quad \left\{ \begin{array}{ll} r \rightarrow R & \phi \rightarrow \infty \\ r \rightarrow \infty & \phi \rightarrow 0 \end{array} \right.$$

and

$$ds_4^2 = d\xi^2 + g(\xi)^2 d\Omega_3^2 \quad \frac{d\xi}{dr} = \frac{1}{(1-R^2/r^2)^{1/4}}$$

BoN reduces to a CdL problem, with

$$\phi(0) = \infty \quad \dot{\phi}(0) = -\infty \quad \phi(\infty) = \phi_+$$

Action reproduced (paying attention to boundary terms)

★ 4d approach useful to study effect of  $V(\phi)$

Draper, García-García, Lillard '2105

# $V_t$ FOR BONs

Blanco-Pillado, JRE, Huertas, Sousa

Witten's BoN  $V_t = -6m_p^2 R^2 \sinh^3(\sqrt{2\kappa/3}\phi)$ ,  $V=0$

- ★  $S[V_t]$  ✓ without additional boundary terms
- ★ Allows bottom-up approach to study which  $V(\phi)$  admits BoN decays
- ★ Universal asymptotic behaviours uncovered
  - which  $V(\phi)$  can be obtained from extra-d theories?
  - IS BoN always the dominant decay?

# Q-BALLS

Coleman'85

Non-topological solitons stabilized by a (global) charge

They minimize energy for a fixed  $Q \gg 1$ .

- ★ Can be DM candidates
- ★ Natural in SUSY models (scalars with  $B, L$ )
- ★ Can play a role in baryogenesis and early universe phase transitions
- ★ Can produce GW signatures

Kusenko, Steinhardt, Shaposhnikov, Enquist, Rubakov, Dvali, ...

# Q-BALLS

★ Can feature in a movie



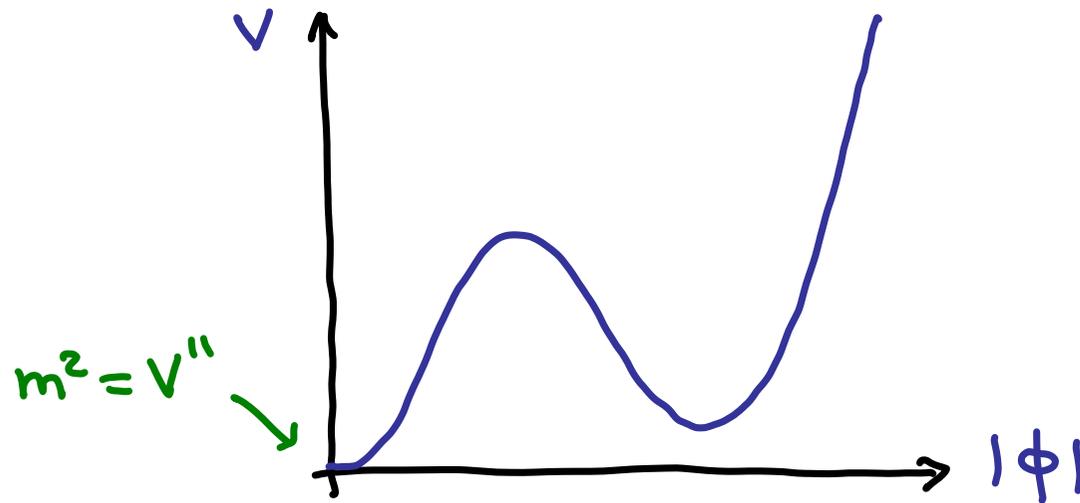
a SUSY Q-ball eating the sun...

# Q-BALLS

Coleman '85

Example

Complex  $\phi$  with global  $U(1)$  and  $V(|\phi|)$  like



$$\omega_0^2 \equiv \min \frac{2V}{\phi^2} < m^2$$

admits Q-ball solutions  $\phi(r) = \frac{1}{\sqrt{2}} f(r) e^{i\omega t}$

with  $0 < \omega_0^2 < \omega^2 < m^2$

# Q-BALLS

Q-ball profile  $\phi(r)$  extremal of (Lorentzian) action

$$S = 4\pi \int_0^{\infty} dr r^2 \left[ -\frac{1}{2} \dot{f}^2 + \frac{1}{2} f^2 \omega^2 - v(f) \right]$$

$$\Rightarrow \ddot{f} + \frac{2}{r} \dot{f} = \frac{\partial v}{\partial f} - \omega^2 f^2$$

with charge

$$Q = 4\pi \omega \int_0^{\infty} dr r^2 f^2$$

and energy

$$E = 4\pi \int_0^{\infty} dr r^2 \left[ \frac{1}{2} \dot{f}^2 + \frac{1}{2} f^2 \omega^2 + v(f) \right]$$
$$= \omega Q + 4\pi \int_0^{\infty} dr r^2 \left[ \frac{1}{2} \dot{f}^2 + v(f) - \frac{1}{2} \omega^2 f^2 \right]$$

# Q-BALLS

Q-ball profile  $\phi(r)$  extremal of (Lorentzian) action

$$S = 4\pi \int_0^\infty dr r^2 \left[ -\frac{1}{2} \dot{f}^2 + \frac{1}{2} f^2 \omega^2 - v(f) \right]$$

$$\Rightarrow \ddot{f} + \frac{2}{r} \dot{f} = \frac{\partial v}{\partial f} - \omega^2 f^2 = \frac{\partial}{\partial f} \underbrace{\left( v - \frac{1}{2} \omega^2 f^2 \right)}_{\tilde{v}(f)}$$

with charge

$$Q = 4\pi \omega \int_0^\infty dr r^2 f^2$$

and energy

$$E = 4\pi \int_0^\infty dr r^2 \left[ \frac{1}{2} \dot{f}^2 + \frac{1}{2} f^2 \omega^2 + v(f) \right]$$
$$= \omega Q + 4\pi \int_0^\infty dr r^2 \left[ \frac{1}{2} \dot{f}^2 + \underbrace{v(f) - \frac{1}{2} \omega^2 f^2}_{\tilde{v}} \right]$$

# $V_t$ FOR Q-BALLS

JRE, Heck, So khashvili

$$\ddot{f} + \frac{2}{r} \dot{f} = \frac{\partial \tilde{V}}{\partial f} \quad d=3 \text{ bounce eqn. !}$$

$$\delta E = 4\pi \int dr r^2 \left( \frac{1}{2} \dot{f}^2 + \tilde{V} \right) \quad d=3 \text{ action} \equiv \text{energy}$$

Tunneling Potential approach:

$$E[V_t] = \omega Q + \frac{16\pi}{3} \int_{\phi_0}^{\phi_+} \frac{[2(\tilde{V} - v_t)]^{3/2}}{(v_t')^2} d\phi$$

with all the good properties of the  $V_t$  - formalism

⇒ Alternative to Euclidean analyses

Heck, Rajaraman, Riley, Verhaaren '2009

# CONCLUSIONS

Tunneling potential formalism simple and useful in

- Vacuum decay
- Finite  $T$  transitions
- Gravity effects on vacuum decays

and beyond

- Bubble-of-nothing decays
- Q-balls
- Positive Energy theorems in AdS maxima

More? Euclidean solutions  $\Rightarrow$   $V_t$  solutions

# BACK-UP SLIDES

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# EUCLIDEAN - $V_t$ LINK

JRE '1805

$$V_t = V - \frac{1}{2} \dot{\Phi}^2$$

$$\Rightarrow \dot{\Phi} = -\sqrt{2(V - V_t)}$$

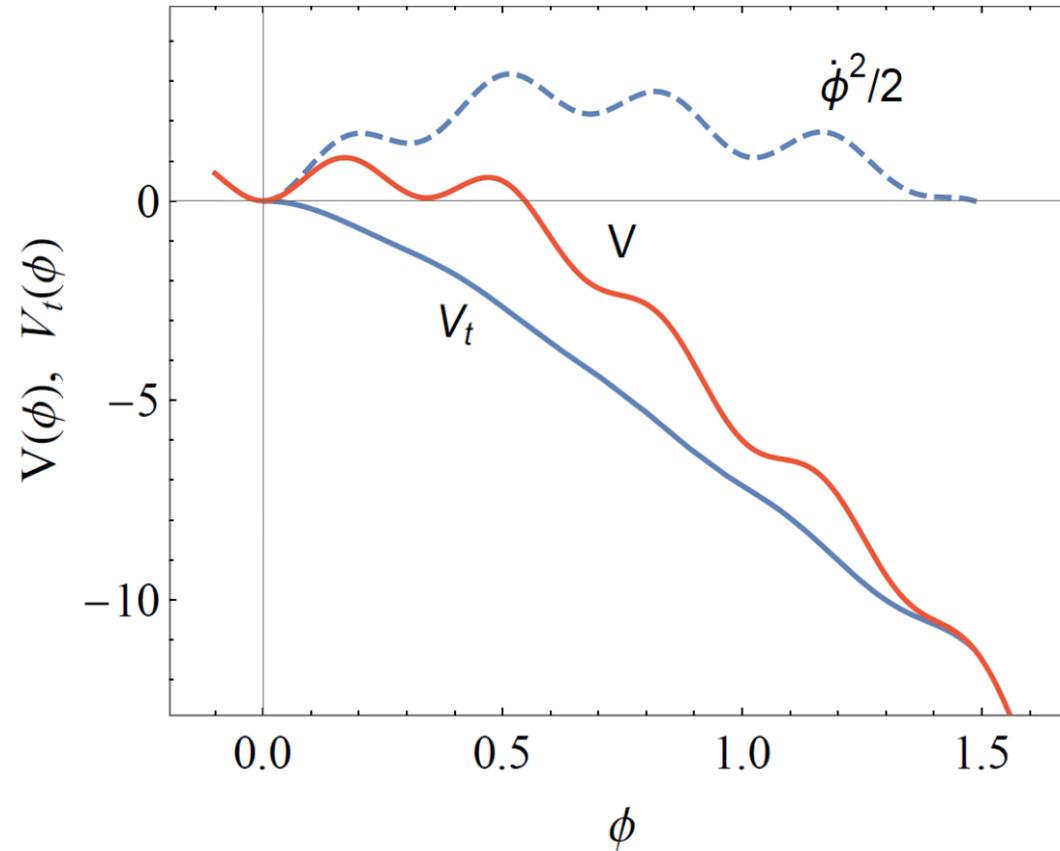
$$\Rightarrow \ddot{\Phi} = \frac{d}{d\Phi} \left( -\sqrt{2(V - V_t)} \right) \dot{\Phi} = V' - V_t'$$

$$\text{EOM: } \ddot{\Phi} + \frac{3}{r} \dot{\Phi} = V' \quad \Rightarrow \quad r = \frac{3\sqrt{2(V - V_t)}}{-V_t'}$$

$\Rightarrow$  Get rid of Euclidean quantities in terms of  $V$  &  $V_t$  ones

# APPLICATIONS

$V_t$  is smooth :



Bumpy  $V$  and  $\frac{1}{2} \dot{\phi}^2$  cancel each other out

⇒  $V_t$  is easy to approximate.

# ACTION

$S[v_t]$  agrees with the standard Euclidean action:

ds Then

$$S[v_t] = \underbrace{\frac{24\pi^2}{k^2} \int_{\phi_+}^{\phi_{0+}} \frac{v_t'}{v_t^2} d\phi}_{\frac{24\pi^2}{k^2} \left( \frac{1}{v_+} - \frac{1}{v_{0+}} \right)} + \underbrace{\frac{6\pi^2}{k^2} \int_{\phi_{0+}}^{\phi_0} \frac{(D + v_t')^2}{v_t^2 D} d\phi}_{\text{bounce}}$$



Hawking-Moss action  
when bounce disappears

and  $S[v_t]$  above agrees with Euclidean result

# METHODS COMPARED

	Manifest Covariance	$O(4)$ sym.	Negative mode
Hamiltonian/ Minkowskian	X	Looks Accidental	no
Euclidean	✓	Natural	yes
$V_L$ potential	✓	Built-in	no

# ASYMPTOTICS FOR BON DECAYS

Type	$V_t(\phi \rightarrow \infty)$	Constraints	$\beta$	$D(\phi \rightarrow \infty)$
+	$V_A/(1-a^2)e^{a\sqrt{6\kappa}\phi}$	$V_A > 0, a > 1$	$1/(3a^2)$	$e^{\sqrt{\kappa/6}(3a+1/a)\phi}$
0	$V_{tA}e^{\sqrt{6\kappa}\phi}$	$V_{tA} < 0, a < 1$	$1/3$	$e^{\sqrt{8\kappa/3}\phi}$
-	$V_A/(1-a^2)e^{a\sqrt{6\kappa}\phi}$	$V_A < 0, 1/\sqrt{3} < a < 1$	$1/(3a^2)$	$e^{\sqrt{\kappa/6}(3a+1/a)\phi}$
-*	$(3V_A/2)e^{a\sqrt{6\kappa}\phi}$	$V_A < 0, a > 1/\sqrt{3}$	$1$	$e^{a\sqrt{6\kappa}\phi}$

Table 1: Taking  $V(\phi \rightarrow \infty) = V_A e^{a\sqrt{6\kappa}\phi}$  we show, for the four different types discussed in the text: the asymptotic behaviours at  $\phi \rightarrow \infty$  of the tunneling potential,  $V_t(\phi)$ , and the quantity  $D(\phi)$ ; several constraints on their parameters; and the exponent  $\beta$  entering  $\rho(\xi \rightarrow 0) \sim \xi^\beta$ .