Hilbert space of de Sitter quantum gravity

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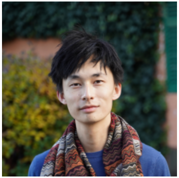
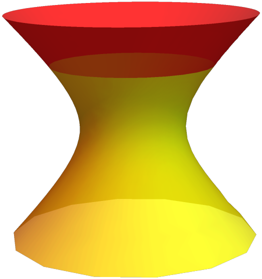


Victor Godet

- `2303.16316` and `2303.16315`, Joydeep Chakravarty, Tuneer Chakraborty, Victor Godet, Priyadarshi Paul, S.R.

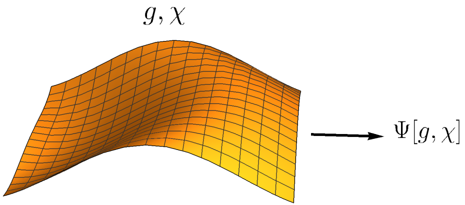
Overview: Two questions.



Focus on asymptotically dS spacetime from a global perspective.

- What is the Hilbert space for gravity in such a spacetime?
- Gravity localizes information unusually. How does holography of information work in such a spacetime?

Wavefunctionals



States can be represented as wavefunctionals on the late-time slice.

$\Psi[g, \chi]$ assigns an amplitude to a configuration of

metric on a spacelike slice $\quad g$

and

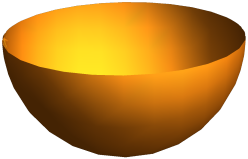
matter fields $\quad \chi$

Vacuum wavefunctional

- We understand the Euclidean vacuum state well.

$$|0\rangle \leftrightarrow \Psi_0[g, \chi]$$

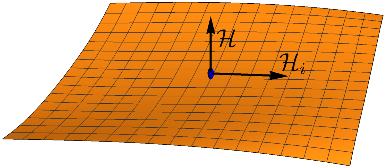
- Computed using the Hartle-Hawking proposal



[Hartle,Hawking, 1983]

- What about other states?

Constraints of gravity



Wavefunctionals in quantum gravity obey

$$\mathcal{H}\Psi[g,\phi] = 0; \qquad \mathcal{H}_i\Psi[g,\phi] = 0.$$

Proposed Procedure: Solve for the Hilbert space by finding a complete basis of solutions to the WDW equation.

WDW equation

Explicitly,

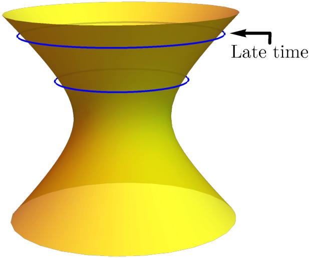
$$\mathcal{H} = 2\kappa^2 g^{-1}\Big(g_{ik}g_{jl}\pi^{kl}\pi^{ij} - \frac{1}{d-1}(g_{ij}\pi^{ij})^2\Big) - \frac{1}{2\kappa^2}(R - 2\Lambda)$$

$$+ \mathcal{H}_{\text{matter}} + \mathcal{H}_{\text{int}},$$

$$\mathcal{H}_i = -2g_{ij}D_k\frac{\pi^{jk}}{\sqrt{g}} + \mathcal{H}_i^{\text{matter}},$$

Simplifying the WDW equation

- In the regime
$$\Lambda \gg R; \qquad \Lambda \gg V_{\text{matter}}$$
the WDW equation turns out to be tractable.



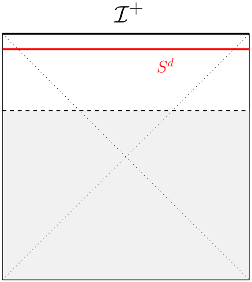
The limit $\Lambda \gg R$ focuses us on the late-time slice.

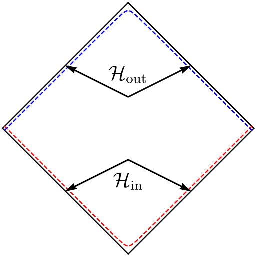
Late-time limit

- Late-time solution sufficient to understand Hilbert space. (cf. asymptotic quantization).

[Ashtekar, 1981]

- Insufficient for bulk dynamics/"earlier-time physics".





Solution

At large volume all solutions of the WDW equation take the form

$$\Psi \longrightarrow e^{iS[g,\chi]} Z[g,\chi]$$

see AdS solutions by Freidel (2008),Regado, Khan, Wall (2022)

1. S is a divergent universal phase factor.
2. $Z[g,\chi]$ is diff invariant and almost Weyl invariant

$$\Omega \frac{\delta Z[g,\chi]}{\delta \Omega(x)} = \mathcal{A}_d[g] Z[g,\chi].$$

   $\mathcal{A}_d$ is an imaginary local function of g in even d for dS_{d+1}.
3.

$$|Z[g,\chi]|^2$$

   is Weyl invariant.

Phase factor

The phase factor S contains terms familiar from holographic renormalization in AdS.

$$S = \frac{(d-1)}{\kappa^2}\int \sqrt{g}\, d^d x - \frac{1}{2\kappa^2(d-2)}\int \sqrt{g} R\, d^d x + \dots$$

[Papadimitriou, Skenderis, 2004]

It comprises integrals of local densities.

It doesn't depend on details of state.

Cancels out in $|\Psi[g,\chi]|^2$.

Expansion of $Z[g, \chi]$

After Weyl transformation to frame

$$g_{ij} = \delta_{ij} + \kappa h_{ij},$$

Expand

$$Z[g, \chi] = \exp[\sum_{n,m} \kappa^n \mathcal{G}_{n,m}]$$

with

$$\mathcal{G}_{n,m} = \int d\vec{y}\, d\vec{z}\, G^{\vec{i}\vec{j}}_{n,m}(\vec{y}, \vec{z}) h_{i_1 j_1}(z_1) \ldots h_{i_n j_n}(z_n) \chi(y_1) \ldots \chi(y_m),$$

Coefficient fns obey same Ward identities as CFT correlators.

$$G^{\vec{i}\vec{j}}_{n,m}(\vec{y}, \vec{z}) \sim \langle T^{i_1 j_1}(y_1) \ldots T^{i_n j_n}(y_n) \phi(z_1) \ldots \phi(z_m) \rangle^{\text{connected}}_{\text{CFT}} ,$$

"CFT" is not unitary; not even necessarily local.

Hartle-Hawking state and other states

$$\Psi_0 = e^{iS} \exp[\sum_{n,m} \kappa^n \mathcal{G}_{n,m}]$$

[Pimentel, 2013]

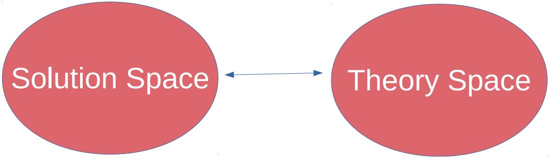
Not just the Hartle-Hawking state but all states have this form.

Interactions do not constrain precise form of $\mathcal{G}_{n,m}$ beyond conformal invariance of coefficient fns.

State space as theory space

List of correlators$\{G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z})\} \longrightarrow$ WDW solution

But list of correlators can be thought of as defining a “theory”.



(**Caution**: Additional constraints on allowed states come from normalizability.)

Small fluctuations basis for states

Starting with $\mathcal{G}_{n,m}$ for H.H. state,

$$\mathcal{G}^{\lambda}_{n,m} = (1-\lambda)\mathcal{G}_{n,m} + \lambda\widetilde{\mathcal{G}}_{n,m}$$

Then

$$\frac{\partial\Psi_\lambda[g,\chi]}{\partial\lambda} = \sum_{n,m}\kappa^n\delta\mathcal{G}_{n,m}\Psi_0[g,\chi]$$

$$= \sum_{n,m}\kappa^n\int d\vec{x}\, G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z})h_{i_1j_1}(z_1)\ldots h_{i_nj_n}(z_n)\chi(y_1)\ldots\chi(y_m)\Psi_0[g,\chi]$$

Summary: solution space

$$|\Psi\rangle = \sum_{n,m} \kappa^n \int d\vec{y} d\vec{z}\, \delta G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z}) h_{i_1 j_1}(z_1) \dots h_{i_n j_n}(z_n) \chi(y_1) \dots \chi(y_m) |0\rangle$$

Matter fluctuation

Smearing functions satisfy CFT Ward identities

Metric fluctuation

Euclidean vacuum

Higuchi states

- When $\kappa \to 0$ Ward identifies cease to couple $\delta\mathcal{G}_{n,m}$ and $\delta\mathcal{G}_{n+1,m}$.

$$|\Psi_{\mathrm{ng}}\rangle = \int d\vec{x} f(x_1, \ldots x_n)\chi(x_1) \ldots \chi(x_n)|0\rangle$$

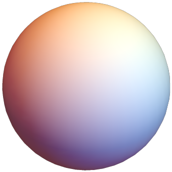
where f has the symmetries of a conformal correlator.

- These states are invariant under SO(d+1,1)!

$$U|\Psi_{\mathrm{ng}}\rangle = |\Psi_{\mathrm{ng}}\rangle$$

Matches a previous construction of the Hilbert space by Higuchi when $\kappa \to 0$.

Higuchi's argument: dS invariance



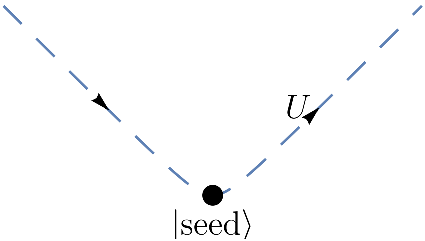
In gravity, charges can be measured at the boundary. But dS spatial slice has no boundaries.

Gauss law: Even as $\kappa \to 0$, all states must have zero charges,

$$U|\Psi\rangle = |\Psi\rangle, \qquad \forall\, U \in SO(d+1,1)$$

In original Hilbert space, the only such state is $|0\rangle$.

Higuchi's solution in the nongravitational limit



Starting with a "seed state", construct

$$|\Psi_{\mathrm{ng}}\rangle = \int [dU]\, U|\mathrm{seed}\rangle$$

[Higuchi, 1991]

[Moncrief, 1975]

Norm: $$(\Psi_{\mathrm{ng}}, \Psi_{\mathrm{ng}}) = \frac{1}{\mathrm{vol}(SO(d+1,1))}\langle\Psi_{\mathrm{ng}}|\Psi_{\mathrm{ng}}\rangle_{\mathrm{QFT}}$$

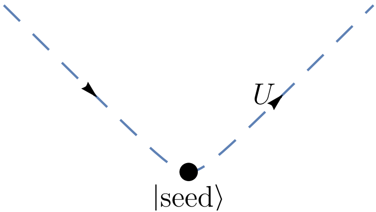
This can also be understood in terms of an equivalence relation on the original Hilbert space

$$|\Psi_{\mathrm{ng}}\rangle \sim U|\Psi_{\mathrm{ng}}\rangle$$

[Chandrasekaran,Longo,Penington,Witten, 2022]

Correction to Higuchi states

At $\kappa \to 0$ our states can be cast in Higuchi's form:



$$|\Psi_{\text{ng}}\rangle = \int dU\, U|\Psi_{\text{seed}}\rangle;$$

$$|\Psi_{\text{seed}}\rangle \propto \int d^d x_4 \ldots d^d x_m f(\hat{x}_1, \hat{x}_2, \hat{x}_3, x_4, \ldots, x_m) \\ \chi(\hat{x}_1)\chi(\hat{x}_2)\chi(\hat{x}_3)\chi(x_4)\ldots\chi(x_m)|0\rangle$$

Away from $\kappa \to 0$,

$$|\Psi\rangle = \sum \kappa^n \delta\mathcal{G}_{n,m}|0\rangle$$

Lowest order term is Higuchi's construction.

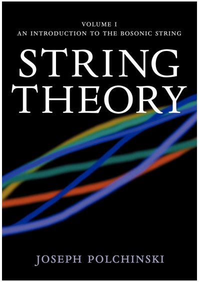
Our solution matches with Higuchi's construction as $\kappa \to 0$ and provides gravitational corrections to it at nonzero κ.

Proposal for norm

We propose

$$(\Psi, \Psi) = \frac{1}{\text{vol}(\text{diff}\times\text{Weyl})} \int DgD\chi \sum_{n,m,n',m'} \kappa^{n+n'} \delta\mathcal{G}^*_{n,m} \delta\mathcal{G}_{n',m'} |Z_0[g,\chi]|^2$$

Proposal is not unique. But natural and simple.



Fixing gauge

$$\text{Fix gauge}: \qquad \sum_i \partial_i g_{ij} = 0; \qquad \delta^{ij} g_{ij} = d$$

Gauge choice leaves behind residual global transformations.

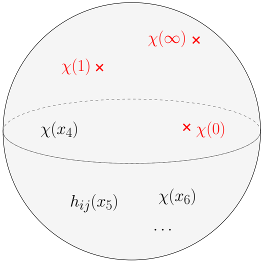
$$\begin{aligned} \text{translations}: &\quad \xi^i = \alpha^i; \\ \text{rotations}: &\quad \xi^i = M^{ij} x^j \\ \text{dilatations}: &\quad \xi^i = \lambda x^i \\ \text{SCTs}: &\quad \xi^i = (2(\beta \cdot x) x^i - x^2 \beta^i) + v^i_j \beta^j \end{aligned}$$

SCTs are corrected by a metric-dependent term.

[Hinterbichler, Hui, Khoury, 2013]

[Ghosh, Kundu, S.R., Trivedi, 2014]

Fixing residual gauge freedom



Fix residual transformations by fixing positions of "vertex operators" in $\delta\mathcal{G}_{n,m}$.

$$x_1 = 0, \qquad x_2 = 1 \qquad x_3 = \infty$$

Finally

$$(\Psi, \Psi) =\propto \sum_{n,m,n',m'} \kappa^{n+n'} \int DgD\chi\, \delta(\text{g.f})\Delta'_{\text{FP}}|Z_0[g,\chi]|^2 \overline{\delta\mathcal{G}^*_{n,m}\,\delta\mathcal{G}_{n',m'}}$$

Normalizable states require at least two insertions (2 + 2 > 3).
H.H. state is not naively normalizable.

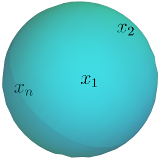
Higuchi's norm

In nongravitational limit, instead of fixing three points → divide by the volume of the conformal group.

$$(\Psi_{\mathrm{ng}}, \Psi_{\mathrm{ng}}) \propto \frac{1}{\mathrm{vol}(SO(d+1,1))} \langle \Psi_{\mathrm{ng}} | \Psi_{\mathrm{ng}} \rangle_{\mathrm{QFT}}$$

Our method supports Higuchi's proposal for the norm at $\kappa \to 0$ and provides gravitational corrections to it.

Cosmological correlators



We wish to understand "cosmological correlators" on the late-time slice.

$$\langle \chi(x_1) \dots \chi(x_n) \rangle$$

As written, expression does not commute with the constraints.

We propose interpretation as gauge-fixed operators

$$\langle\!\langle \Psi | \chi(x_1) \dots \chi(x_n) | \Psi \rangle\!\rangle_{CC} = \int |\Psi|^2 \chi(x_1) \dots \chi(x_n) \delta(\mathrm{g.f}) \Delta'_{FP} Dg D\chi$$

Cosmological correlators

$$\langle\!\langle \Psi_1 | \chi(x_1) \dots \chi(x_n) | \Psi_2 \rangle\!\rangle_{CC} = \int \Psi_1^* \Psi_2 \chi(x_1) \dots \chi(x_n) \delta(\mathrm{g.f}) \Delta'_{FP} Dg D\chi$$

gives unambiguous prescription for the matrix elements.

$\exists$ gauge invariant operator with the same matrix elements.

When $\kappa \to 0$,

$$\hat{C} = \int [dU] U^\dagger \chi(x_1) \dots \chi(x_n) U$$

At nonzero κ, gauge-fixing $\longrightarrow$ setting our reference frame as observers.

Symmetries of cosmological correlators

Residual gauge transformations turn into symmetries of cosmological correlators.

Translations/Dilatations:

$$\langle\!\langle \Psi|\chi(\lambda x_1 + v)\dots\chi(\lambda x_n + v)|\Psi\rangle\!\rangle_{\mathrm{CC}} = \lambda^{-n\Delta}\langle\!\langle \Psi|\chi(x_1)\dots\chi(x_n)|\Psi\rangle\!\rangle_{\mathrm{CC}}$$

Rotations:

$$\langle\!\langle \Psi|\chi(M\cdot x_1)\dots\chi(M\cdot x_n)|\Psi\rangle\!\rangle_{\mathrm{CC}} = \langle\!\langle \Psi|\chi(x_1)\dots\chi(x_n)|\Psi\rangle\!\rangle_{\mathrm{CC}}$$

SCTs relate cosmological correlators of different orders.

Symmetries of cosmological correlators

All states display the symmetries of the H.H. state although the precise values of cosmological correlators

$$\langle\!\langle \Psi | \chi(x_1) \ldots \chi(x_n) | \Psi \rangle\!\rangle_{\mathrm{CC}}$$

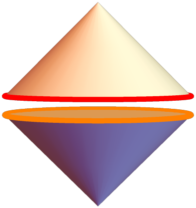
are different in different states.

Holography of information

Careful analysis of constraints shows gravity localizes information unusually.

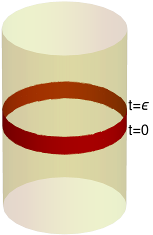
[Laddha, Prabhu, S.R., Shrivastava, 2020]

Asymptotically flat space



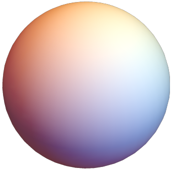
All operators on $\mathcal{I}^+$ can be represented near $\mathcal{I}^\pm$ when we include constrained components in the algebra.

Asymptotic AdS



All boundary operators can be represented on an infinitesimal time band at the boundary (Does not assume AdS/CFT)

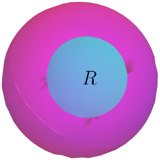
Holography of information in dS



How does holography of information work in dS?

Possible step towards: What is the holographic dual of gravity in dS?

Holography of information in dS

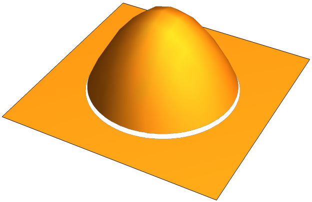


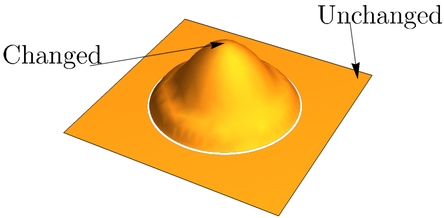
In dS, cosmological correlators in an arbitrarily small region on the asymptotic time slice are sufficient to determine them everywhere.

$$\langle\!\langle \Psi | \chi(\lambda x_1 + v) \ldots \chi(\lambda x_n + v) \rangle\!\rangle_{\mathrm{CC}} = \lambda^{-n\Delta} \langle\!\langle \langle \Psi | \chi(x_1) \ldots \chi(x_n) | \Psi \rangle\!\rangle_{\mathrm{CC}}$$

Holography of information and cosmological correlators

Cannot have split states in sharp contrast to QFT.





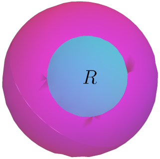
Holography of information

Holography of information is a consequence of the gravitational constraints

$$\Psi[g,\chi] = \sum_{n,m} \kappa^n \delta\mathcal{G}_{n,m} \Psi_0[g,\chi],$$

State is built up of conformally covariant ingredients, $G^{\vec{i},\vec{j}}_{n,m}$.

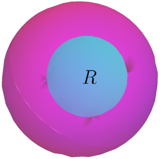
Therefore, providing $G^{\vec{i},\vec{j}}_{n,m}$ in any small region, specifies them everywhere.



Nongravitational limit

Holography of information persists in the nongravitational limit.

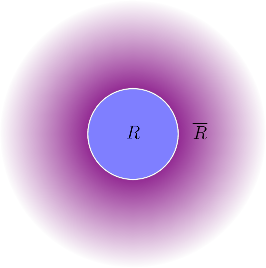
$$|\Psi_{\text{ng}}\rangle = \int dx_i f(x_1, \ldots x_n)\chi(x_1)\ldots\chi(x_n)|0\rangle$$



$$\langle\!\langle \Psi_{\text{ng},1}|A_{\mathcal{R}}|\Psi_{\text{ng},1}\rangle\!\rangle_{\text{CC}} = \langle\!\langle \Psi_{\text{ng},2}|A_{\mathcal{R}}|\Psi_{\text{ng},2}\rangle\!\rangle_{\text{CC}}$$
$$\Rightarrow \langle\!\langle \Psi_{\text{ng},1}|\mathcal{A}_{\overline{R}}|\Psi_{\text{ng},1}\rangle\!\rangle_{\text{CC}} = \langle\!\langle \Psi_{\text{ng},2}|\mathcal{A}_{\overline{R}}|\Psi_{\text{ng},2}\rangle\!\rangle_{\text{CC}},$$

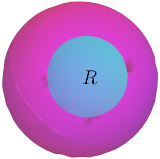
Holography of information

AdS and flat space



Whenever the complement of a region surrounds the region, it has information about the region.

dS



In dS, the complement of every region surrounds the region and vice versa!

Cautionary remarks

Holography of information $\Rightarrow$ mathematical difference between QFT and QG.

Caution:

- "cosmological correlators" are secretly nonlocal since they are gauge fixed.
- points at finite coordinate separation on the late-time slice are infinitely separated in physical distance.
- Identifying the state requires all-point correlators.

Conclusion

- **Hilbert space:** Solutions of WDW-eqn (in the large-volume limit) are of the form $e^{iS}Z[g, \chi]$, where $|Z[g, \chi]|^2$ is a diff and Weyl-invariant functional.

- All allowed states are of this form, not just the vacuum. (Vacuum itself does not appear normalizable.)

- **Symmetries.** Cosmological correlators, after gauge-fixing, are covariant under scaling, rotations, translations in all states. SCTs relate different cosmological correlators.

- **Holography of information:** Cosmological correlators in an arbitrarily small region suffice to determine the state. Sharp contrast with QFT.