sl2cfoam-next: new developments and applications

Pietro Donà

Centre de Physique Théorique Marseille

LOOPs'24 Fort Lauderdale FLORIDA

Centre de Physique Théorique

CSTQ Center for SpaceTime and the Quantum

QISS

```julia
# import the library
using SL2Cfoam

# Barbero-Immirzi parameter
Immirzi = 1.0

# initializing sl2cfoam-next
sl2c_data_folder = "/home/pdona/data_sl2cfoam"
sl2c_config = SL2Cfoam.Config(VerbosityOff, VeryHighAccuracy, 100, 0)
SL2Cfoam.cinit(sl2c_data_folder,Immirzi, sl2c_config)

# truncation parameter
Dl = 5;

# set the 10 vertex boundary spins
spins = [1,1,1,1,1,1,1,1,1,1]
@time Av = vertex_compute(spins, Dl);
# Av.a is the 5-dimensional array with the data
Av.a[2,2,2,2,2]
```

Why should we do numerics?

TOOL (complementary to analytic calculations, asymptotic analysis and geometric interpretation)

Confirm analytical results

Help developing new results

Suggest new challenges

Rich numerical landscape

Semi-classical insights

Complex critical points
[M. Han, **D. Qu,** H. Liu, Z. Huang]
Right before me
2110.10670, 2301.02930, 2404.10563

Hybrid representation of spinfoam models
[**S. Steinhaus**, S. Asante, J. D. Simao]
Today 3PM FTL Room 312
2206.13540

Restricted spinfoam models
[S. Steinhaus, B. Bahr]
2007.01315

Effective models

Effective spinfoam models
[B. Dittrich, H. Haggard, J. Padua-Argüelles, **S. Asante**]
Thursday 9:40 AM here
2104.00485, 2011.14468, 2004.07013

Direct calculation

sl2cfoam-next : EPRL model and BF SU(2)
[**P.D.,** G. Sarno, F. Gozzini, P. Frisoni]
In this talk
2107.13952, 2302.00072, 2202.04360

Monte Carlo eval coherent BF SU(2) amplitude
[**S. Steinhaus**, S. Asante, J. D. Simao]
Today 3PM FTL Room 312
2403.04836

Covariant LQG & EPRL spinfoam model

Dynamics to Kinematical LQG states

Background independent

Lorentzian, Path Integral formulation

Quantum evolution (simplicial)

Regularized on a 2-complex

Local transition amplitude in the spin network basis

$$A_\Delta = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v(j_f, i_e)$$

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EPRL model

Sum over all possible lorentzian parallel transports

$$A_e(i_e) = 2i_e + 1$$

$$A_f(j_f) = 2j_f + 1$$

$$A_v \approx \int_{SL(2,\mathbb{C})^4} \prod_e \mathrm{d}g_e \bigotimes_f D^{\gamma j j}(g_{f_t}^{-1} g_{f_s})$$

SL(2,C) BF theory +
Quantum implementation of
linear simplicity constraints

Numerical challenge

$$A_v \approx \int_{SL(2,\mathbb{C})^4} \prod_e \mathrm{d}g_e \bigotimes_f D^{\gamma j j}(g_{f_t}^{-1} g_{f_s})$$

Problem - Standard methods fail

Integral over 4 copies of a non compact group

Gamma simple irreps are fast oscillating functions

No probability distribution for MC importance sampling

Numerical challenge

$$A_v \approx \int_{SL(2,\mathbb{C})^4} \prod_e \mathrm{d}g_e \bigotimes_f D^{\gamma j j}(g_{f_t}^{-1} g_{f_s})$$

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Solution - Divide and conquer!

$$A_v(j_f, i_e) = \sum_{l_f = j_f}^{\infty} \sum_{k_e} \prod_e (2k_e+1) B_4(j_f, l_f, i_e, k_e) 15j(l_f, k_e)$$

Linear combination of SU(2) invariants weighted by booster functions (encode all details of the model)

S. Speziale 1609.01632

sl2cfoam-next

Open source – bit.ly/sl2cfoam-next

Fast – C code

Modular & Scalable – mix & match

Optimized for HPC – parallelizable, GPU

User friendly – Julia interactive interface (step by step guide NEW) P.D., P. Frisoni 2202.04360

Example:

```
using SL2Cfoam
Immirzi = 1.2
data_folder = "path_data_folder"
configuration = SL2Cfoam.Config(VerbosityOff, VeryHighAccuracy, 100, 0)
SL2Cfoam.cinit(data_folder, Immirzi, configuration)
boundary_spins = ones(10)
Dl = 10
Av = vertex_compute(boundary_spins, Dl)
```

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P.D., P. Frisoni 2202.04360

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Unavoidable approximation – truncation

$$\sum_{l_f=j_f}^{\infty} \to \sum_{l_f=j_f}^{j_f+\Delta l}$$

Resource demanding – computational cost exponential in the number of faces

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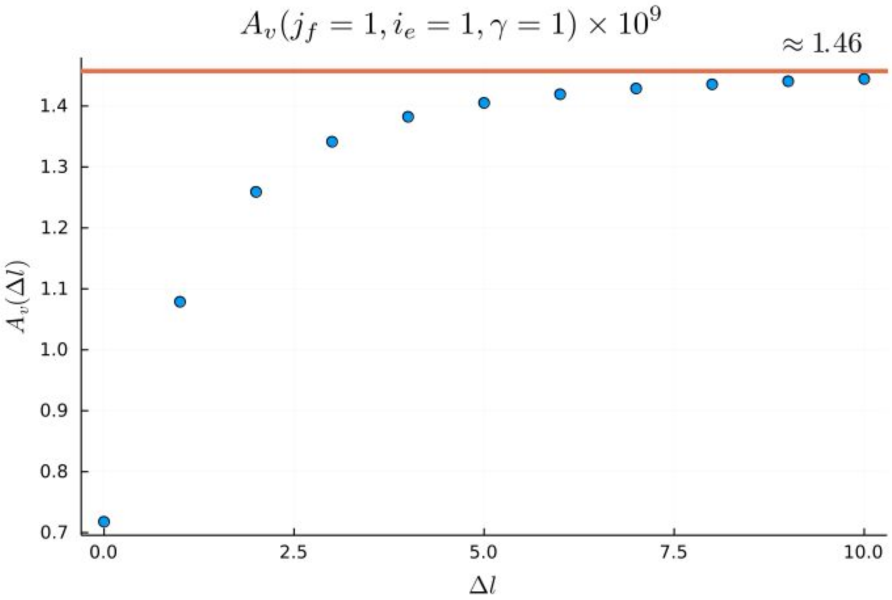
Unavoidable approximation – truncation

$$\sum_{l_f=j_f}^{\infty} \to \sum_{l_f=j_f}^{j_f+\Delta l}$$

Resource demanding – computational cost is exponential in the number of faces

NEW - We have solutions!

Removing the truncation



Convergent sequence - the amplitude is finite

Convergence acceleration techniques - extrapolate the limit

Aitken delta squared method

$$A^{(ex)}(\Delta l) = \frac{A(\Delta l)A(\Delta l - 2) - A(\Delta l - 1)^2}{A(\Delta l) - 2A(\Delta l - 1) + A(\Delta l - 2)} \approx \lim_{\Delta l \to \infty} A(\Delta l)$$

Saving resources with MC

Computational cost scales exponentially in the number of faces

Not vertices in general as the amplitudes generally repeat

Back of the envelope estimate. F faces and “typical” spins j

$(2j+1)^F$ assuming $F=10$ $j=10$ $T[A_v]\approx 1\mu s$ result $T[A_\Delta]\approx 6\,\mathrm{months}$

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Back of the envelope estimate. F faces and “typical” spins j

$(2j+1)^F$ assuming $F = 10 \quad j = 10 \quad T[A_v] \approx 1\mu s$ result $T[A_\Delta] \approx 6\,\text{months}$

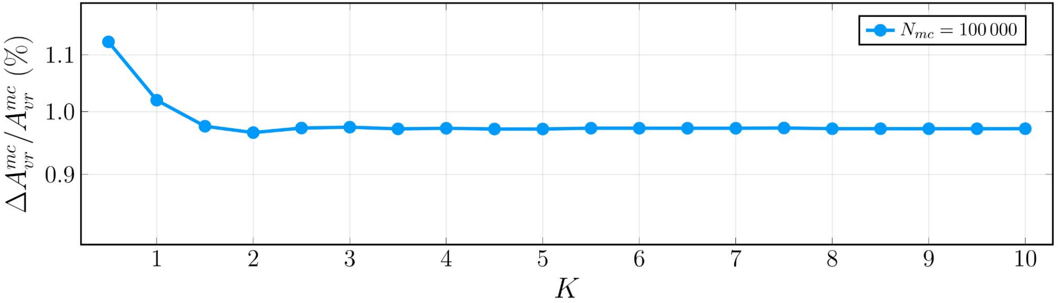
Uniform sampling on the bulk spins

Divide the bulk spins into layers

Each amplitude is equally important

Sampling over all bulk configurations

Surprisingly good results



Coherent Intertwiners?

Excellent progress by the Jena group S. Steinhaus, S. Asante, J. D. Simao 2403.04836

Applications

Large spins limit

Confirm analytical results - Emergence of the Regge action in the vertex amplitude asymptotics

Help developing new results - Revision of the semiclassical limit as a double scaling limit

Suggest new challenges - Can we characterize quantitatively the semiclassical limit?

IR Divergences

Confirm analytical results - Cutoff divergence of BF SU(2) topological theories

Help developing new results - Melonic divergence of the EPRL model

Suggest new challenges - Bubble convergence of the EPRL model?

Other Results

Correlations in primordial cosmology - Non trivial correlations emerges from spinfoam dynamics (1 vertex, 6 vertices, 8 vertices with complex connections)

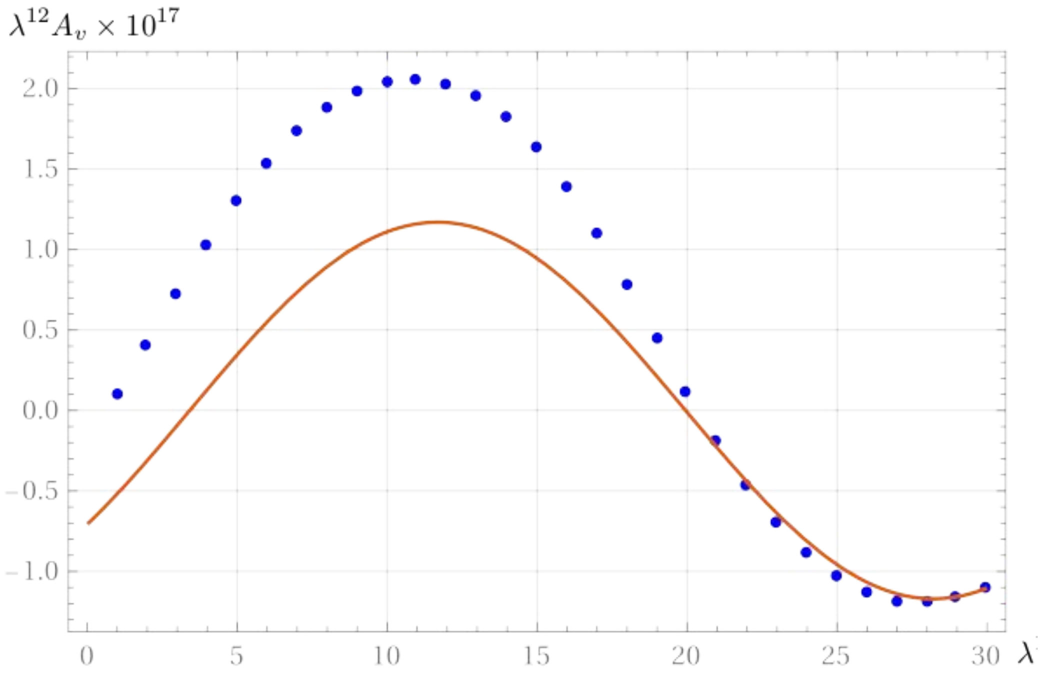
Francesca Vidotto - Today 2:15 PM
Thanos Kogios - Today 4:30 PM
2402.09038, 2312.02399, 2207.02881, 1906.02211

B2W transition amplitude - small spins regime

Pietropaolo Frisoni - 2304.02691

Large spins limit

P.D., G. Sarno, S. Speziale 1903.12624
F. Gozzini 2107.13952



Single vertex asymptotics

One EPRL coherent vertex amplitude

Uniform boundary spins rescaling

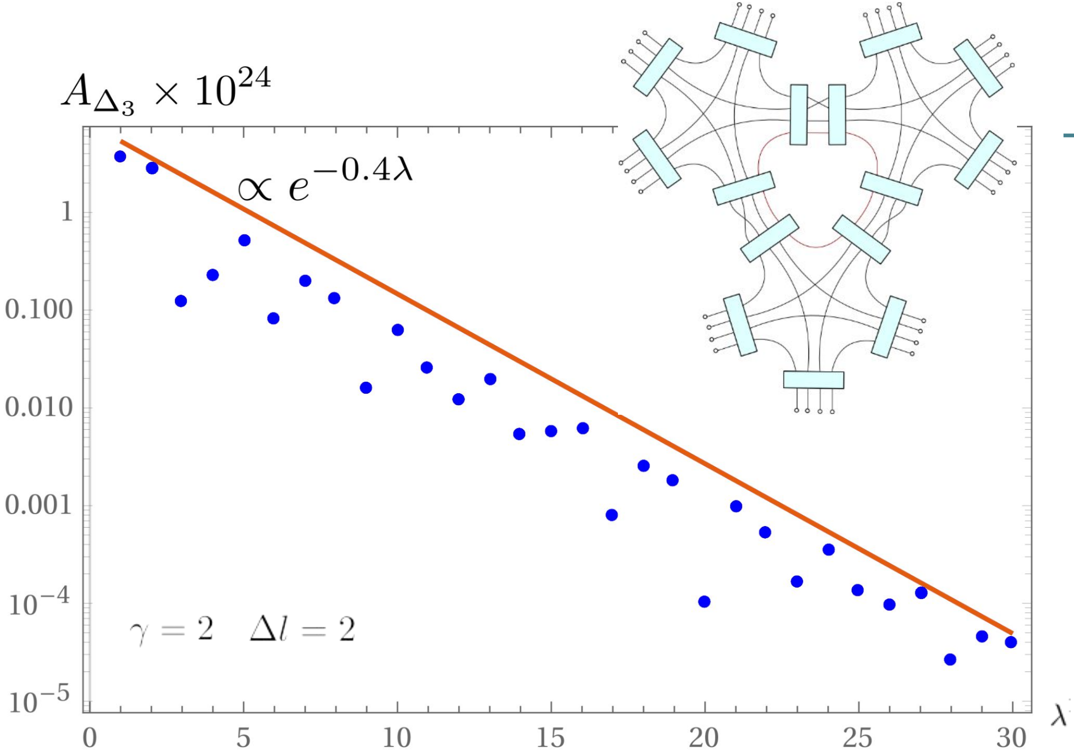
Prescribe boundary data corresponding to a Lorentzian 4-simplex (isosceles)

Power law suppression and oscillations (frequency equal to the Regge Action)

Confirm analytical results: emergence of the Regge Action

Large spins limit

P.D., G. Sarno, F. Gozzini 2004.12911
F. Gozzini 2107.13952



Three vertices asymptotics

Three EPRL coherent vertex amplitude

Sum over the spin of the bulk face

Scale the boundary spins

Exponential suppression if the *curvature* is too large

The semiclassical limit is a double scaling limit (large spins & locally small curvature) $\lambda \gamma \epsilon \ll 1$

Help developing new results: **redefine the semiclassical limit**

Double scaling limit

w.i.p.

Goal

- Study the “semiclassical” window as a function of:

  Number of vertices **V** . Continuum limit?

  Boundary scale λ. Large spins regime

  Small curvature ϵ . Smooth regime.

Setting

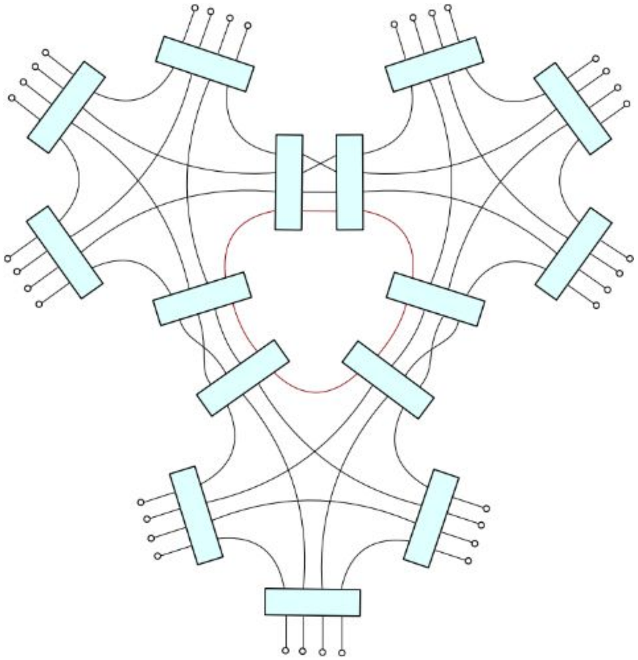
- Simplest possible

  Many vertices and one bulk face (D3 generalization)

  Symmetric boundary data

  Expectation values and fluctuations of the bulk spin

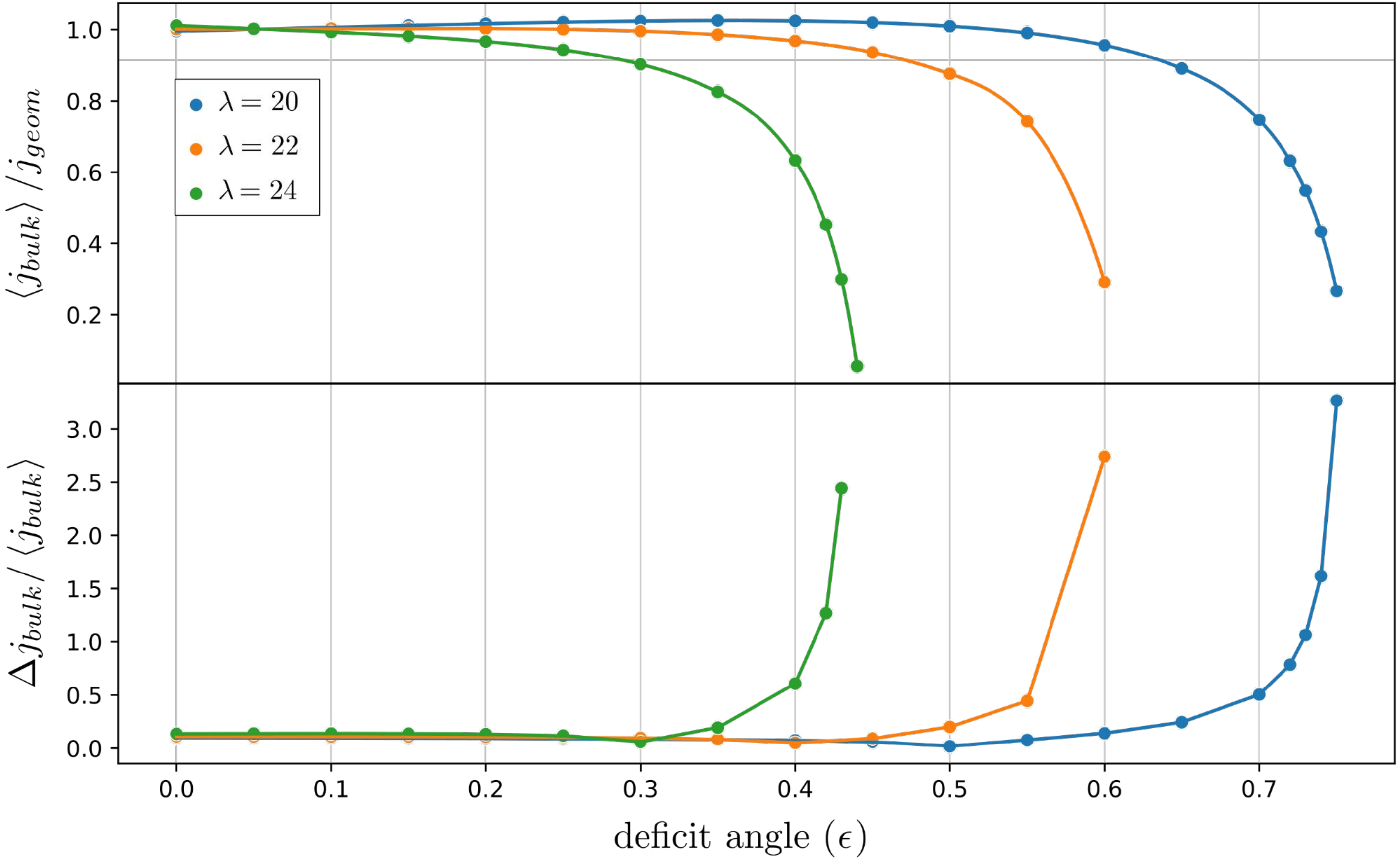
  Operational definition of classicality (geometric expectation values and small fluctuations)



Suggest new challenges: characterize the double scaling limit?

Double scaling limit

Very preliminary



$V = 3$

$BF\ SU(2)$

Sym bdry data

Critical ϵ ?

Repeat for V=5, 7, 9 and extract dependence on V

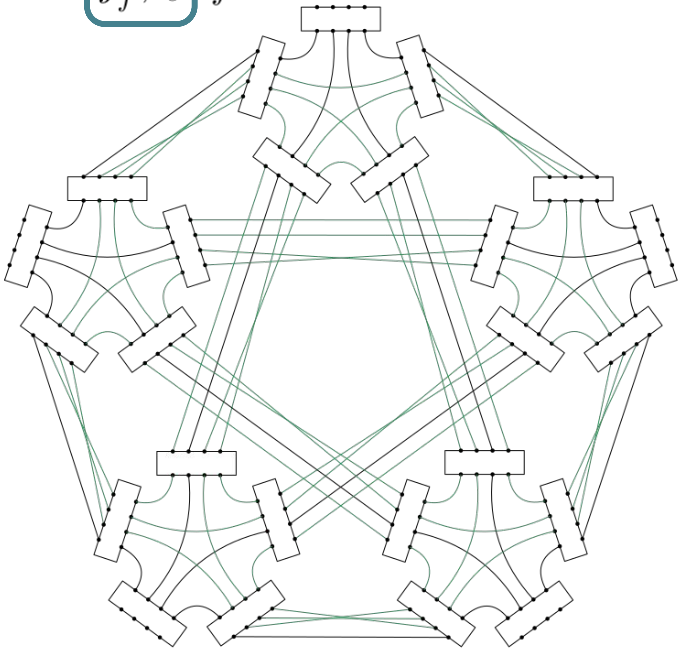
IR divergences

P.D. P. Frisoni 2302.00072

$$A_\Delta = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v(j_f, i_e)$$

Divergence for BF SU(2) models analytic, redundant delta functions, unfixed gauge symmetry

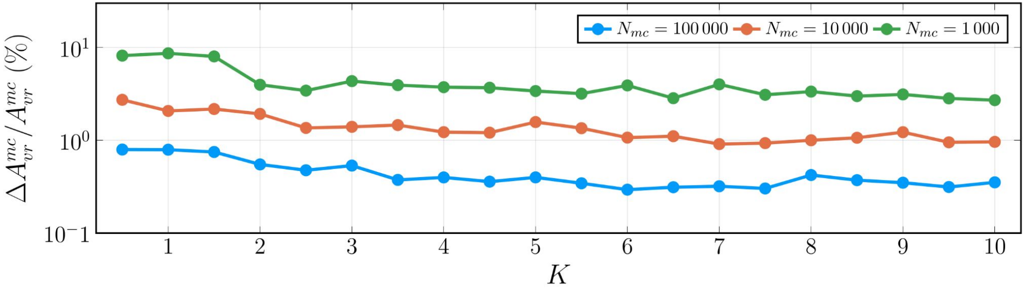
L. Freidel 0212001



$A_{ball} \propto K^{12}$

$F = 10$

MC uniform sampling on the bulk quantum numbers is crucial. There are $30\,788\,382\,715 \approx O(10^{10})$ terms



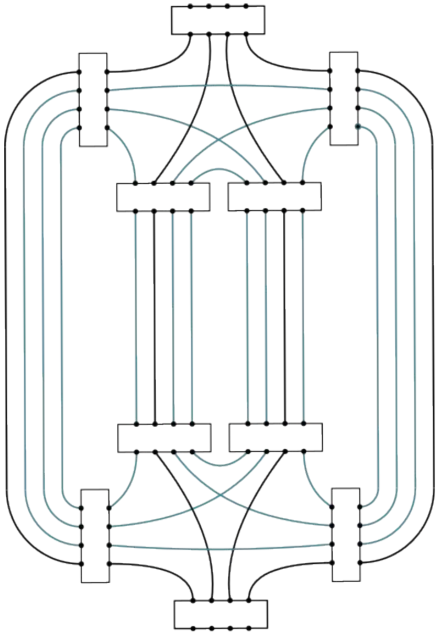
Confirm analytical results: reproduce BF SU(2) divergences

IR divergences

P.D. P. Frisoni 2302.00072

Divergences of the EPRL model are difficult to compute analytically (there are estimates)

A. Riello 1310.2174
P.D. 1803.00835

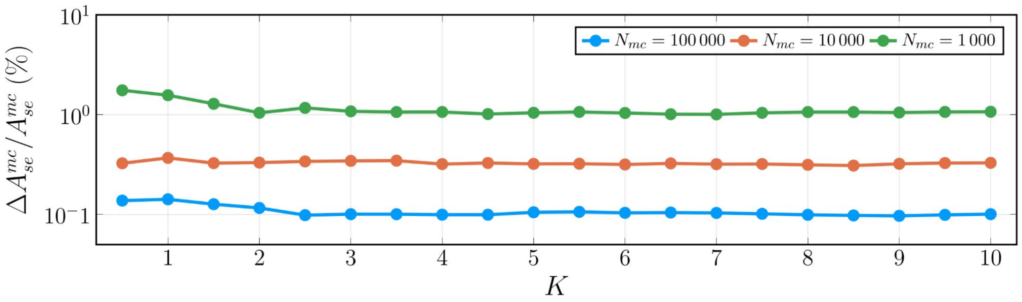


Previous numerical work (NO MC) estimated the melonic divergence

P. Frisoni, F. Gozzini, F. Vidotto 2112.14781

Huge numerical effort (months of CPU time) $A_{melon} \propto K$

Confirmed the result with a night of calculations (with MC)

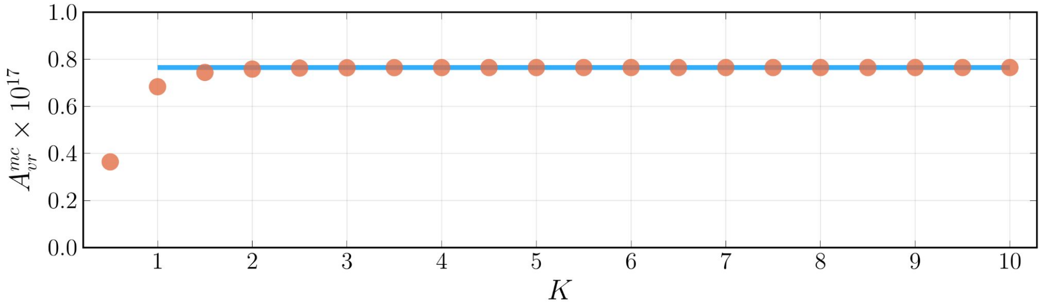


Help developing new results: numerical evidence of linear divergence

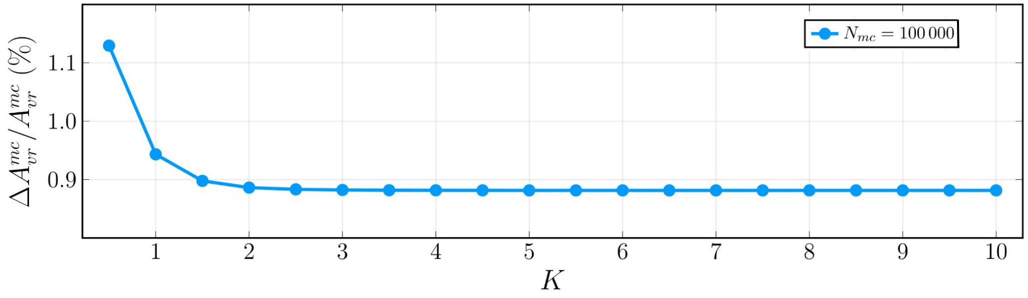
IR divergences

P.D. P. Frisoni 2302.00072

No prior estimate of the EPRL ball diagram divergence (vertex renormalization)



Surprising convergence



Good error control (sub 1%)

Suggest new challenges: explain convergence? General formula?

Conclusions

A mature framework

Direct calculation of EPRL spinfoam amplitudes is within everyone's reach

Another tool in our arsenal (complements existing ones)

Old technical obstacles are now under control (MC on spins and intertwiners)

Diverse applications

Semiclassical limit, IR divergences, correlations in spinfoam cosmology, B2W hole transition

Characterization of the double scaling limit? Explain the convergence of the Ball diagram?

Other numerical approaches

No competition, different tools work best for different problems (sl2cfoam-next $j < 50$)

Quantum Gravity on the Computer 2.0 Jena Sep 9-13 2024