Effective Gravitational Collapse

May 7, 2024

Outline

- Introduction: models of gravitational collapse in QG
- An explicit collapse-bounce spacetime
- Shock waves: some background
- An effective model: simulations
- Summary & open questions

Collaborators

Effective models: Francesco Fazzini, Jarod Kelly, Robert Santacruz, Edward Wilson-Ewing

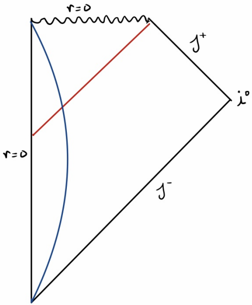
Phenomenological QG metric: Samantha Hergott, Saeed Rastgoo

Gravitational collapse in classical gravity is well-studied:

- Null dust: Vaidya and generalizations
- Particle models: $x(t), p(t)$ — Oppenheimer-Snyder, thin shell, and variations
- Field theory models: Lemaitre-Tolman-Bondi dust, GR + scalar field, GR + perfect fluid, $\cdots$; matter fields $\phi(r,t), p_\phi(r,t)$; metric:

$$ds^2 = -f(r,t)dt^2 + g(r,t)dr^2 + r^2 d\Omega^2$$

  and variations.



Represents all classical collapse models

Black holes evaporate. Hawking's conjectured "Information Loss" diagram:

Particle Creation by Black Holes 219

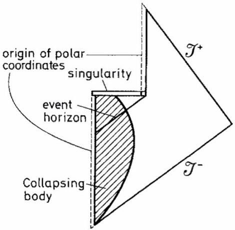


Fig. 5. The Penrose diagram for a gravitational collapse followed by the slow evaporation and eventual disappearance of the black hole, leaving empty space with no singularity at the origin

(from Hawking's paper)

There is no metric for which this is the conformal diagram: straw man.

Hawking Lifetime

black hole = black body

temperature $T \sim \dfrac{1}{M}$, area $A \sim M^2$

energy loss: Stefan-Boltzmann law

$$\frac{dM}{dt} \sim -AT^4 \sim -M^{-2}$$

$\longrightarrow$ blackhole lifetime $\sim M^3$

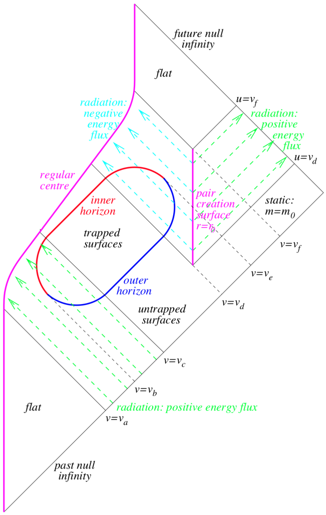
What should happen in quantum gravity?

- BH event horizon is a global classical object: discard in QG?
- Why quantize a classical black hole: we normally quantize classical theories, not special solutions of classical theories – too small a mini-superspace?
- Singularity resolution: matter reaches Planckian density and encounters resistance to further collapse. Bounce, or metastable "star" with collapse and bounce pressures balanced, repeated collapse and bounce, or BH $\longrightarrow$ WH, ...?
- What is the quantum theory of Choptuik collapse?

Penrose diagrams of conjectures

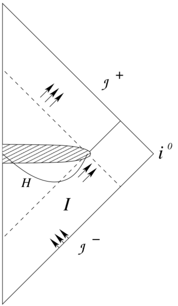
(Based on arguments, black hole quantization, metric gluing, ...)

I- Horizon bubbles



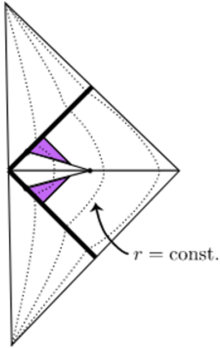
Hayward 2005

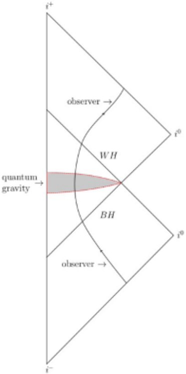
II- QG region replacing singularity

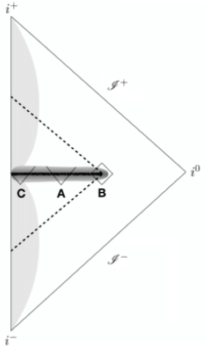


Ashtekar, Bojowald 2005

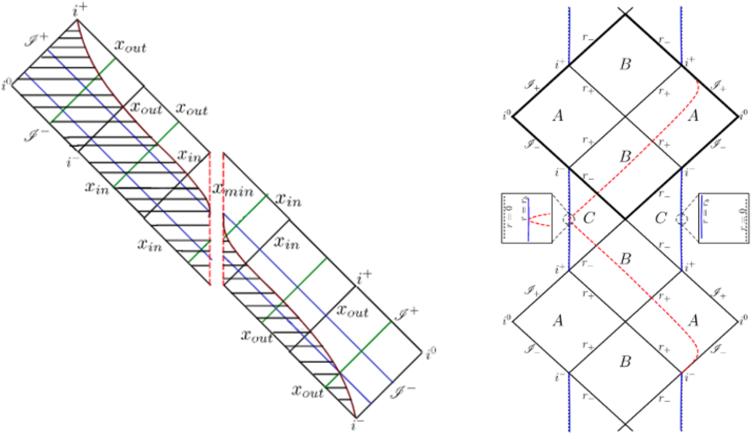
II- Black hole to White hole transitions







L: Haggard-Rovelli 2015; M: Olmedo et.al 2017; R: Bianchi et.al. 2018

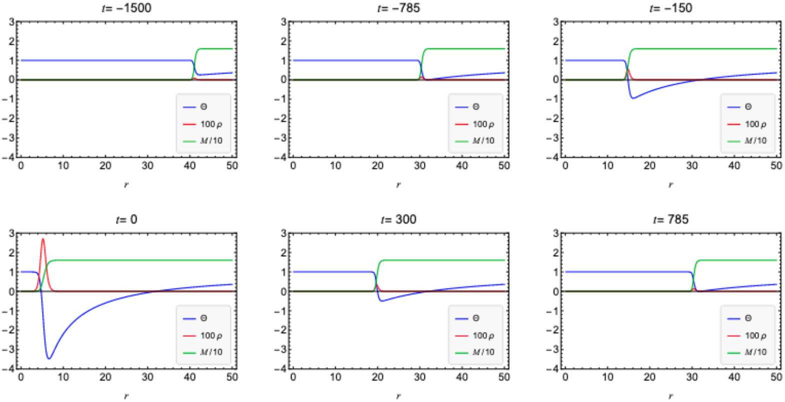


L: Munch 2021; R: Lewandowski et. al. 2023

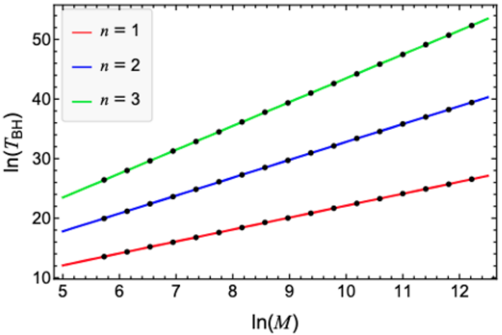
$$
\begin{aligned}
ds^2 &= -dt^2 + \left(dr + \sqrt{2Gm(r,t)/r}\right)^2 + r^2 d\Omega^2 \\
m(r,t) &= M_0 \left[1 + \tanh\left(\frac{r - r_0 - v(r,t)t}{\alpha\ l_0}\right)\right]^a \tanh\left(\frac{r^b}{l_0^b}\right) \\
v(r,t) &= \frac{A}{(1+r)^n} \tanh\left(\frac{t - t_0}{l_0}\right) \\
\rho(r,t) &= \frac{m'(r,t)}{4\pi r^2}
\end{aligned}
$$

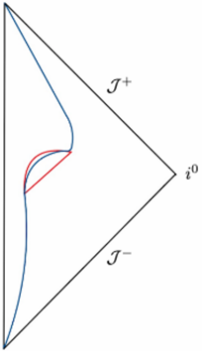
Describes smooth mass inflow $\rightarrow$ outflow, asymptotically flat, dynamical horizons, n determines lifetime of trapped region.

(S. Hergott, VH, S. Rastgoo, PRD106, 046012 (2022))



t= −1500
t= −785
t= −150
t= 0
t= 300
t= 785
Θ
100 ρ
M/10
r





$$T_{BH} \approx 2^{n+2} M^{n+1} \qquad M = \lim_{r \to \infty} m(r,t)$$

conformal diagram: blue line is peak of matter density $\rho = m'/4\pi r^2$; red line is dynamical horizon

Interlude: 2d Burgers' equation – the "harmonic oscillator" of shock waves

$$u_t + uu_x = 0$$

Solution using method of characteristics: let $u = u(x(s), t(s))$. Then

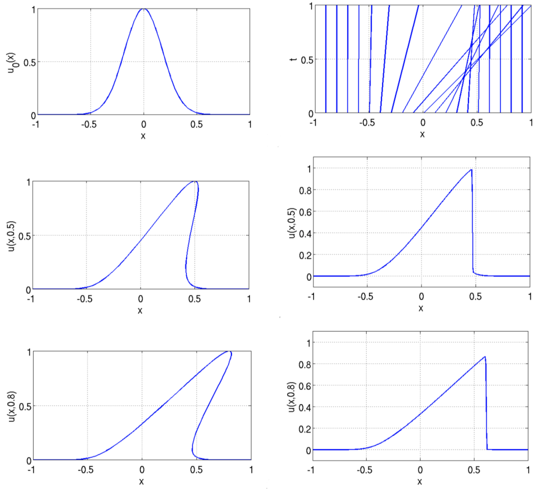
$$\frac{du}{ds} = u_t \frac{dt}{ds} + u_x \frac{dx}{ds} = 0$$

provided the characteristic equations hold:

$$\frac{dt}{ds} = 1, \quad \frac{dx}{ds} = u.$$

$\longrightarrow$ the implicit solution for initial data $u(x, 0) = f(x)$ is

$$u(x, t) = f(x - u(x, t)t)$$



L column: solution by method of characteristics (wave breaking)
Top right: crossing of characteristics
R column: shock wave solution (numerically generated weak solution)

A similar effect arises in effective dust collapse

—work with J. Kelly, R. Santacruz, E. Wilson-Ewing dust time gauge; areal gauge

canonical gauge fixed action:

$$S_{GF} = \int dt \int dx \, \left(g \; \dot{b} E^b - \mathcal{H}_{\rm phys} \right)$$

$$\mathcal{H}_{\rm phys} = -\frac{g}{2}\left[\frac{E^b}{\gamma x}\partial_x(xb^2) + \frac{\gamma E^b}{x} + \frac{\gamma x}{E^b}\right].$$

$(g = 1/G\gamma, \; E_3^{\phi} = E^b, \; E_2^{\theta} = E^b \sin\theta)$

dust density:

$$\rho = - \, \frac{\mathcal{H}_{\rm phys}}{4\pi x \, E^b}$$

metric:

$$ds^2 = -dt^2 + \left(\frac{E^b}{x}\right)^2 \left(dx - \frac{b}{\gamma} \; dt\right)^2 + x^2 d\Omega^2$$

$$\mathcal{H}^{\rm eff}_{\rm phys} = -\frac{g}{2}\left[\frac{E^b}{\gamma \ell_{\rm Pl}^2 x}\partial_x\left(x^3 \sin^2 \frac{\ell_{\rm Pl} b}{x}\right) + \frac{\gamma x}{E^b} + \frac{\gamma E^b}{x}\right]$$

$$\begin{aligned} \dot{b} &= \frac{x}{2E_b^2} - \frac{1}{2x} - x \sin\frac{b}{x}\left[\frac{3}{2}\sin\frac{b}{x} + x\,\partial_x \sin\frac{b}{x}\right] \\ \dot{E}_b &= -\frac{x^2}{2}\,\partial_x\left(\frac{E_b}{x}\right)\sin\frac{b}{x}\ \cos\frac{b}{x} \end{aligned}$$

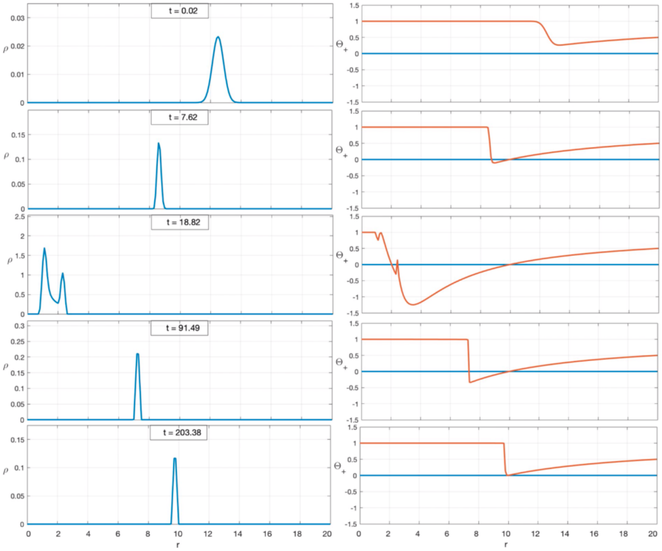
Special class of solutions: $E^b = x$:

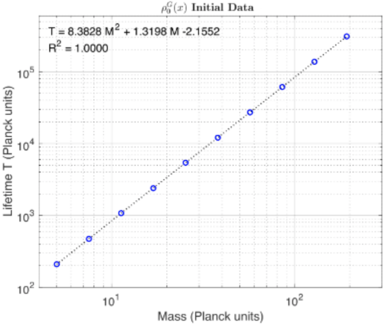
$$\dot{b} + \frac{1}{2x}\partial_x\left(x^3 \sin^2\frac{b}{x}\right) = 0$$

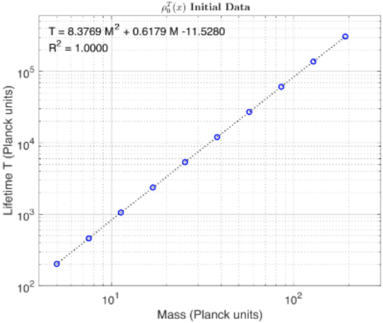
With $B = xb$ this becomes a conservation eqn. like Burgers:

$$\dot{B} + \frac{1}{2}\partial_x\left(x^2 \sin^2\left(\frac{B}{x^2}\right)\right) = 0.$$

Results







$$T_{BH} \approx M^2$$

(Code available online)

Comment on non-areal gauge

$$ds^2 = -dt^2 + \frac{(\partial_R r(R,t))^2}{1+\mathcal{E}(R,t)} dR^2 + r(R,t)^2 d\Omega^2.$$

Effective LTB equations: (Giesel, Liu, Singh, and Weigl)

$$\frac{\partial m(R,t)}{\partial t} = 0, \qquad \frac{\partial \mathcal{E}(R,t)}{\partial t} = 0,$$

$$\left(\frac{\dot{r}}{r}\right)^2 = \left(\frac{2Gm}{r^3} + \frac{\mathcal{E}}{r^2}\right)\left[1 - \Delta\left(\frac{2Gm}{r^3} + \frac{\mathcal{E}}{r^2}\right)\right] \equiv \frac{f^2(r,m,\mathcal{E})}{r^2},$$

Observation: (VH, Fazzini, Wilson-Ewing)

- These are the characteristic eqns of the pdes

$$\partial_t m + f(r,m,\mathcal{E})\,\partial_r m = 0,$$
$$\partial_t \mathcal{E} + f(r,m,\mathcal{E})\,\partial_r \mathcal{E} = 0$$

- Like Burgers' eqn, characteristic crossings occur.

Summary

- BH ends in shock wave (with our gauge choices)
- BH lifetime proportional to M^2
- Any pde of the type $u_t + [f(u)]_x = 0$ can give shock waves via weak solutions

Future

Other matter types, gauges, covariant models, $\cdots$

All nonlinear wave phenomena exhibit shock waves: why should gravitational waves be an exception?