Regular black holes and their relationship to polymerized models and mimetic gravity

Hongguang Liu FAU Erlangen-Nürnberg

Joint work with K. Giesel, P. Singh and S. Weigl, arXiv:2405.03554

Also based on arXiv: 1712.03876, 2212.04605, 2308.10949, 2308.10953

Loops' 24 Fort Lauderdale, Florida

Regular BH models

I : Classical regular BHs
e.g. Bardeen, Hayward

II: quantum BHs
e.g. LQC insipred models

Viqar Husain's talk
Cong Zhang's talk
Panel: Quantum BHs
Parallel sessions: BHs

Can we have a unified framework
that be able to discribe and investigate both kinds of regular BH models?

Yes if we **extend** the polymerization to more generic class of functions

The formalism developed in [2308.10949] (embedding LTB into polymerized models) can be used

seminal works see [Bojowald et al, 08' 09'

I: Classical regular BHs

Can we build an effective (high-derivative) Lagrangian inspired from QG?

Regular BHs: resolve the central singularity that is present in black hole solutions in GR

Phenomenological models with GR + (usually quite exotic) matter

Most famous ones: Bardeen (1968) and Hayward (2005)

$$\mathcal{G}(r)^2 = \frac{r_s r^2}{(r^2+l^2)^{\frac{3}{2}}} \qquad \mathcal{G}(r) = \frac{r_s r^2}{r^3+\alpha^2 r_s}$$

Schwarzschild solution $\mathcal{G}(r)^2 = \frac{r_s}{r}$

In Schwarzschild-like coord $$\mathrm{d}s^2 = -(1-\mathcal{G}(r)^2)\mathrm{d}\tau^2 + \frac{1}{(1-\mathcal{G}(r)^2)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 .$$

and many others can be understood as GR + Non-linear electrodynamics [Bronnikov, 23 for a review

- Are they related to QG/Quantum geometries?
- What are more physical situations, e.g. inhomogeneous gravitational collapse beyond static/stationary?
- GR has Birkhoff theorem, which states that vacuum solution is uniquely given by stationary Schwarzschild BH. Can they be explained as vacuum solutions for some effective theory of QG?
- Perturbations? still in classical GR

II: quantum BHs

Effective regular BHs from backgroud independent QG:

To extract observational effects:

- QG observables are defined **relationally** e.g. matter field/observer defines the notion of time/gauge fixing [Brown and Kuchar 95', Giesel Thiemann 07, Husain, Pawlowski 11, Witten, 2022
- We need a **emergent spacetime in continuum** gravitational physics continuum limit for discrete approach

really hard to derive dynamics from a full fundenmental QG → Related to renormalization, Really hard for full QG theory

One way: **symmetry reduced models** to derive an effective dynamics for the continuum geometry: e.g. loop inspired cosmology and (static) spherically symmetric spacetime [Ashtekar, Brizuela, Boehmer, Bojowald, Bodendorfer, Campiglia, Chiou, Corichi, Elizaga Navascué, Gambini, Giesel, Han, Husain, Li, Lewandowski, Ma, Mena Marugán, Pullin, Vandersloot, Olmedo, Oriti, Perez, Rovelli, Singh, Wang, Wilson-Ewing, Yang, Zhang.....

However, it is hard to go beyond symmetry reduced spacetime:

- Can not encode **QG effects for perturbations**,
- **4d covariant** Lagrangian is missing : problem of **general covariance**, and **coordinate transformations**
  (even if the algebra is closed, the coordinate transformations are generally not the classical ones, unless one reproduces exactly the diffeo algebra)
- Consistently embedding of **dust collapse** models (e.g. junction conditions are generally modified in the effective dynamics, Oppenheimer-Snyder may not be hold in general.) Concrete example: [Giesel, HL, Singh, Rullit and Weigl, 2308.10953

Solution: effective 4d (high-derivative) covariant Lagrangian with LTB embedding

Regular black holes, Polymerized models and Modified gravity

Modified gravity as emergent effective theory of underlying QG

However, usually it is hard to investigate (exact) black hole solutions in modified gravity, especially with (inifite) higher derivatives which captures the UV compeletion of underlying QG

What we propose:

Extended Mimetic theory ⟷ **Polymerized Hamiltonian**

- ✓ (Inifite) **higher derivatives** encoded in the mimetic potential (QG and UV complete/asymptotic safe)
- ✓ Mimetic field plays the role of **observer** (relational framework for background independent QG)
- ✓ Allow decoupled dynamics s.t. we have **decoupled EoMs** with dust comoving frame (Lemaître–Tolman-Bondi (LTB) coordinates), gluing along dust geodesics is allowed (Oppenheimer-Snyder)
- ✓ One can prove a **Birkhoff-like theorem** for the polymerized vacuum solution (uniqueness/stationary)
- ✓ Allow to **reconstruct** the theory consistently (Hamiltonian and 4d mimetic lagrangian) from static solutions
- ✓ A **limiting curvature mechanism** to have **regular** BHs/cosmology
- ✓ Encode the **inhomogeneous dust collapse solutions** and their reduction to cosmological dynamics (1+1d emdeding)
- ✓ **“Exact” (inhomogeneous dust collapse) solutions** as an inverse function of an integration (modified Friedmann Eq)
- ✓ Effective dynamics where we assume it captures the main quantum effects and holds for all radius (e.g. at r=0 for BHs with a regular center)

Extended Mimetic Gravity

Higher order derivative scalar tensor modified gravity theory which only propagates 2 (gravity) +1 (scalar) d.o.f. (subclass of DHOST)

$$S[g_{\mu\nu},\phi,\lambda]=\frac{1}{8\pi G}\int_{\mathcal{M}_4}\mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R}^{(4)}+L_\phi(\phi,\chi_1,\cdots,\chi_n)+\frac{1}{2}\lambda(\phi_\mu\phi^\mu+1)\right]$$

higher derivative coupling

Non rotational dust

[Achour, Lamy, HL, Noui, 17
[Langlois, Mancarella, Noui, Vernizzi, 18
[Han, HL, 22

contraction of higher order derivatives

$$\chi_n\equiv\sum_{\mu_1,\cdots,\mu_n}\phi^{\mu_2}_{\mu_1}\phi^{\mu_3}_{\mu_2}\cdots\phi^{\mu_n}_{\mu_{n-1}}\phi^{\mu_1}_{\mu_n},\quad \phi_\mu=\nabla_\mu\phi,\quad \phi_{\mu\nu}=\nabla_\mu\nabla_\nu\phi.$$

Only 2 independent terms in spherically symmetric spacetime

$g_{\mu\nu}$: gravity, ϕ : mimetic scalar, λ : lagrangian multiplier (dust density)

dust density

Non-rotational dust: dust collapse

Modified Einstein Eq: $G^{(\alpha)}_{\mu\nu}:=G_{\mu\nu}-T^{\phi}_{\mu\nu}=-\lambda\partial_\mu\phi\partial_\nu\phi$ Non rotational dust energy-momentum

Effective Einstein tensor

higher derivative coupling
QG effect

Polymerized vacuum: $\lambda=0$

Equations of motion for ϕ is not independent: gauge fixing for ϕ commute with variation, e.g. $\delta S|_{\phi=t,N=1}=\delta(S|_{\phi=t,N=1})$

Two folds role of ϕ:

- a nature observer (clock field), $\phi\to t$ as gauge fixing (unitary gauge, $N\equiv g_{tt}=1$) under this gauge, $\phi^\mu_\nu=K^\mu_\nu$ (extrinsic curvatures)
- introduce the QG effects in higher derivatives.

Extended Mimetic Gravity

Higher order derivative scalar tensor modified gravity theory which only propagates 2 (gravity) +1 (scalar) d.o.f. (subclass of DHOST)

$$S[g_{\mu\nu},\phi,\lambda]=\frac{1}{8\pi G}\int_{\mathcal{M}_4}\mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R}^{(4)}+L_\phi(\phi,\chi_1,\cdots,\chi_n)+\frac{1}{2}\lambda(\phi_\mu\phi^\mu+1)\right]$$

[Achour, Lamy, HL, Noui, 17
[Langlois, Mancarella, Noui, Vernizzi, 18
[Han, HL, 22

higher derivative coupling — Non rotational dust

contraction of higher order derivatives

$$\chi_n\equiv\sum_{\mu_1,\cdots,\mu_n}\phi^{\mu_2}_{\mu_1}\phi^{\mu_3}_{\mu_2}\cdots\phi^{\mu_n}_{\mu_{n-1}}\phi^{\mu_1}_{\mu_n},\quad \phi_\mu=\nabla_\mu\phi,\quad \phi_{\mu\nu}=\nabla_\mu\nabla_\nu\phi.$$

Only 2 independent terms in spherically symmetric spacetime

$g_{\mu\nu}$: gravity, ϕ : mimetic scalar, λ : lagrangian multiplier (dust density)

dust density — Non-rotational dust: dust collapse

Modified Einstein Eq: $G^{(\alpha)}_{\mu\nu}:=G_{\mu\nu}-T^\phi_{\mu\nu}=-\lambda\partial_\mu\phi\partial_\nu\phi$ Non rotational dust energy-momentum

Effective Einstein tensor

higher derivative coupling
QG effect

Polymerized vacuum: $\lambda=0$

Equations of motion for ϕ is not independent: gauge fixing for ϕ commute with variation, e.g. $\delta S|_{\phi=t,N=1}=\delta(S|_{\phi=t,N=1})$

Two folds role of ϕ:

- a nature observer (clock field), $\phi\to t$ as gauge fixing (unitary gauge, $N\equiv g_{tt}=1$) under this gauge, $\phi^\mu_\nu=K^\mu_\nu$ (extrinsic curvatures)
- introduce the QG effects in higher derivatives.

Hamiltonian under the gauge $\phi\to t$: $H^\alpha=\int dx\, C^\alpha+N^xC_x$ in spherically symmetric case

$$C^\alpha\equiv\frac{1}{2G}E^\varphi\sqrt{E^x}\left(\left(1+f^\alpha\left(\frac{K_\varphi}{\sqrt{E^x}},\frac{\sqrt{E^x}K_x}{E^\varphi}\right)\right)\left(\frac{K_\varphi K_x}{E^\varphi}+\left(\frac{K_\varphi}{2\sqrt{E^x}}\right)^2\right)-\frac{1}{2}R^{(3)}\right)$$

Diffeo generates coords. trans along x as a field theory

$$C_x\equiv\frac{1}{G}\left(E^\varphi\partial_xK_\varphi-K_x\partial_xE^x\right)$$

$C^\alpha=\lambda E^\varphi\sqrt{E^x}$, λ dust density

$\bar\mu$ polymerization function f^α, α can be identified as $\alpha_\Delta=\gamma\sqrt{\Delta}$ in loop inspired models

$$ds^2=-\mathrm{d}t^2+\frac{E^\varphi(t,x)^2}{E^x(t,x)}\left[\mathrm{d}x+N^x(t,x)\mathrm{d}t\right]^2+E^x(t,x)\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right)$$

LTB spacetime

Dust driven collapse: non-interacting, pressure-less field (for simplicity, we assume marginally bound case here)

A special coordinates system:

dust as free falling observers form a decoupled system

Lemaître–Tolman-Bondi (LTB) coordinate

$(E^x)' - 2\sqrt{1+\mathcal{E}(x)}E^\phi$

$$-dt^2 + \frac{(E^\phi)^2}{r^2}(dr + N^x dt)^2 + r^2 d\Omega^2$$

$$-dt^2 + \frac{(R')^2}{1+\mathcal{E}(x)}dx^2 + R^2 d\Omega^2$$

Cosmology $R = xa(t)$

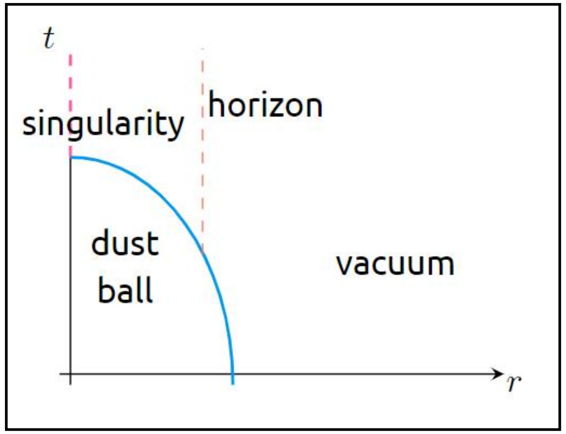
Dynamics is given by

$$C = \partial_x \tilde{H},$$

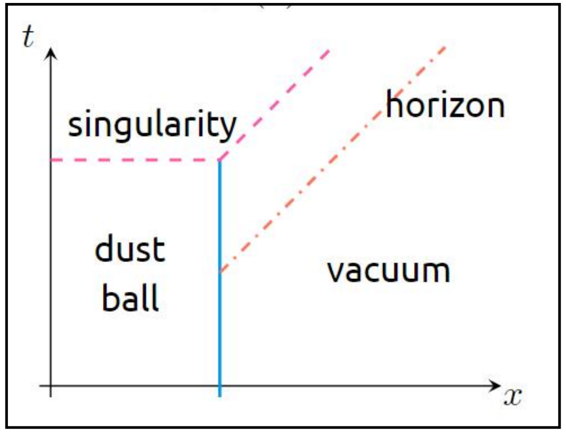
$v = R^3$

cosmological Hamiltonian

$$\tilde{H} = \frac{1}{2G} v\, b^2$$



PDE



ODE

- ✓ Decoupled ODE instead of PDE
- ✓ Friedmann equation for each x
- ✓ Conserved quantity $\tilde{H}(x) = M(x)$ relates to the gravitational mass inside the sphere at radius x (can do inhomogeneous collapse)
- ✓ General solution $R = \sqrt{E^x} = \left[\frac{9}{4}\sqrt{2GM(x)}(s(x) - t)^2\right]^{\frac{1}{3}}$
- ✓ Vacuum: Birkhoff theorem (vacuum solution $M(x) = m$ is unique & asymptotically static (Killing vec. field along s(x)+t)

LTB ⟷ Existence of non-flat vacuum solution and Birkhoff theorem

Can we do this for effective models? Yes, but only for a subclass of polymerization

[Giesel, HL, Singh, Rullit and Weigl, 2308.10949 for details

Extended Mimetic theory with LTB

Requiring the existence of LTB further constrains the model [Stefan Weigl's talk yesterday

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + (\partial_x R(t,x))^2 \mathrm{d}x^2 + R(t,x)^2 \mathrm{d}\Omega^2$$

A subclass of the model: no K_x polymerization

$$C^\alpha = \frac{1}{2G} E^\varphi \sqrt{E^x}\left(\left(\frac{\sqrt{E^x}K_x}{E^\varphi}\tilde{f}^{(2)}(b) + \left(\tilde{f}^{(1)}(b)\right)^2\right) - \frac{1}{2}R^{(3)}\right)$$

$$\tilde{f}^{(1)} + b\,\tilde{F}' - 3\tilde{F} = 0, \tilde{F}'(b) = 2\tilde{f}^{(2)}(b)$$

$b = \frac{K_\varphi}{2\sqrt{E_x}}$

$C^\alpha = \lambda\, E^\varphi \sqrt{E^x},\ \lambda$ dust density

$\lambda = 0$ polymerized vacuum

with mimetic lagrangians expressed in 1+1d (mimetic-dilaton gravity, $\psi = \frac{1}{2}\log E^x$)

$$S_2 = \frac{1}{4G}\int_{\mathscr{M}_2} \mathrm{d}^2x\, \det(e)\{e^{2\psi}\left(R_h + 2h^{ij}\partial_i\psi\partial_j\psi\right) + 2 + e^{2\psi}\left[L_\phi(X,Y) + \frac{1}{2}\lambda\left(\partial_j\phi\partial^j\phi + 1\right)\right]\}$$

impose constraint on Y $L_\phi(X,Y) = 2XY - Y^2 + \tilde{f}^{(1)}\left[(\tilde{f}^{(2)})^{-1}(-Y)\right] + 2X(\tilde{f}^{(2)})^{-1}(-Y).$

X, Y functions of χ_1, χ_2

Decoupled system

effective QG dynamics at cosmology, e.g. LQC

$\tilde{H}$ defines a Hamilton on each x given by polymerization functions $\tilde{F}(b)$

$$C^{(\alpha)}(x)\Big|_{LTB} = \partial_x \tilde{H}^{(\alpha)}(x) \qquad -M(x) = \tilde{H}^{(\alpha)}(x) = -\frac{1}{2G}\left[v\tilde{F}(b)\right](x), \quad v := (E^x)^{\frac{3}{2}} \equiv R^3$$

Conserved quantity $M(x)$

The model is compeletely detemrmined by $\tilde{F}(b)$

Oppenheimer-Snyder and inhomogeneous dust collapse is naturally encoded in the model

see our paper for decoupling in non-marginally bound case

underlying dynamics to the model [Lewandowski, Ma, Yang, Zhang 22, (Cong Zhang's talk)

Decoupled dynamics

Decoupled system

$$C^{(\alpha)}(x)\Big|_{LTB} = \partial_x \widetilde{H}^{(\alpha)}(x)$$

effective QG dynamics at cosmology, e.g. LQC

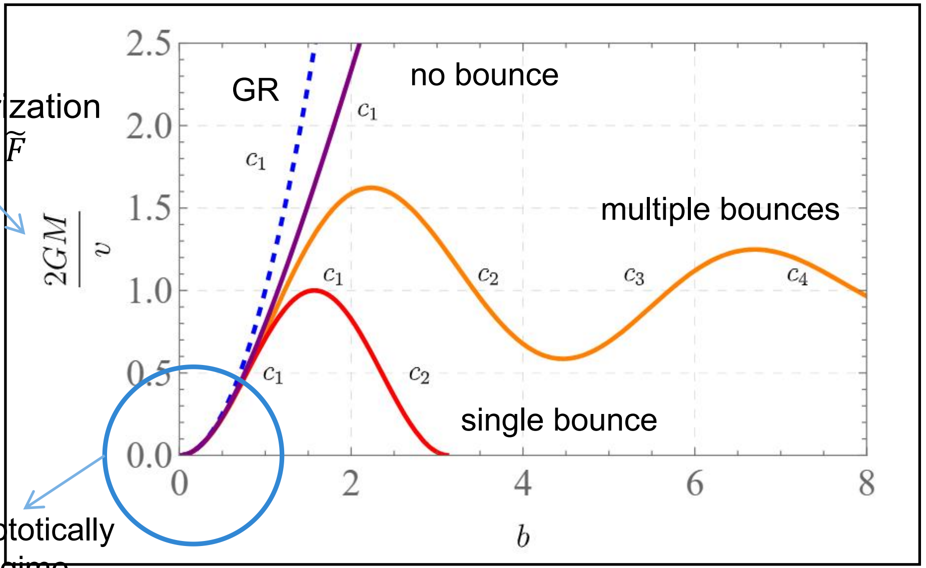
$$-M(x) = \widetilde{H}^{(\alpha)}(x) = -\frac{1}{2G}\left[v\tilde{F}(b)\right](x), \quad v := (E^x)^{\frac{3}{2}} \equiv R^3$$

The model is compeletely detemrmined by $\widetilde{F}(b)$

Conserved quantity $M(x)$

In general non-linear dynamics   possible phase space trajectories

Polymerization function $\widetilde{F}$



The asymptotically classical regime

no bounce: $\widetilde{F}(b)$ monotonic and unbounded

Bouncing: $\widetilde{F}(b)$ not monotonic (usually bounded)

for each c^i: $\quad b(t,x) = \widetilde{F}^{-1}_{(i)}\left(\frac{2GM(x)}{v}\right)$

Modified Friedmann equation for each c^i

$$\frac{\dot{R}}{R} = -\frac{1}{2}\tilde{F}'\left[\tilde{F}^{-1}_{(i)}\left[\frac{2GM(x)-\left(\Xi(x)^2-1\right)R}{R^3}\right]\right]$$

General solution in marginally bound case for each c^i

$$R(t,x) = \mathcal{F}^{-1}_{(i)}\left(s(x)-t\right), \quad \mathcal{F}_{(i)} = \int_{R_0}^{R} \frac{2\mathrm{d}r}{r\tilde{F}'\left[\tilde{F}^{-1}\left[\frac{2GM(x)}{r^3}\right]\right]}$$

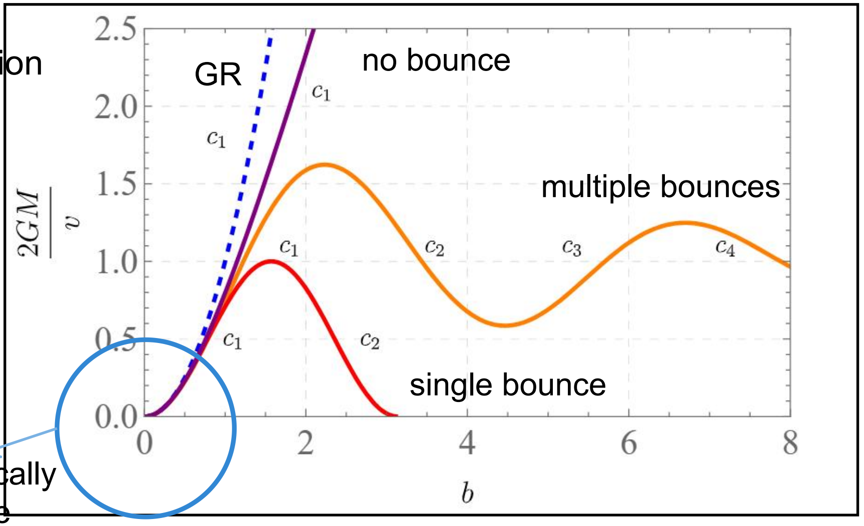
R is a continous function expressed in piecewise segments for each c^i

unique asymptotically stationary (static) solution ⟹ Birkhoff-like theorem

Asymptotic flatness for c^1 is ensured by requiring the theory has correct asymptotic limit at $b \to 0, \widetilde{F} \to b^2$

Polymerized vacuum in Schwarzschild-like coordinates

Polymerization function $\widetilde{F}$



The asymptotically classical regime

Coordinate transformation:

$(t, x) \rightarrow (\tau, r = \sqrt{E_x})$

$$ds^2 = -dt^2 + (\partial_x R(t,x))^2 dx^2 + R(t,x)^2 d\Omega^2$$

$$ds^2 = -(1 - \mathcal{G}_{(i)}(r)^2)d\tau^2 + \frac{1}{(1 - \mathcal{G}_{(i)}(r)^2)}dr^2 + r^2 d\Omega^2$$

$\mathcal{G}_{(i)}(r)^2 = \dot{R}^2$ ← Modified Friedmann equation

Only well defined for each monotonic segment c^i

For each monotonic segment c^i we have a $\mathcal{G}_{(i)}$, they may be in different forms: different Schwarzschild-like solution

Lemma 1 (Birkhoff-like theorem 1)

The model admits a unique asymptotically flat vacuum LTB solution labeled by mass *m* which has an extra Killing field (asymptotically stationary and static).

For symmetric bounce and unbouncing solutions, the Schwarzschild-like solution is unique

Corollary 1 (Birkhoff-like theorem 2)

The vacuum solution in Lemma 1 expressed in Schwarzschild-like coordinates may not be unique, but countably (possibly infinitely) many. Each Schwarzschild-like metric corresponds to a piecewise monotonic segment in t of the unique vacuum solution in LTB coordinates in Lemma 1.

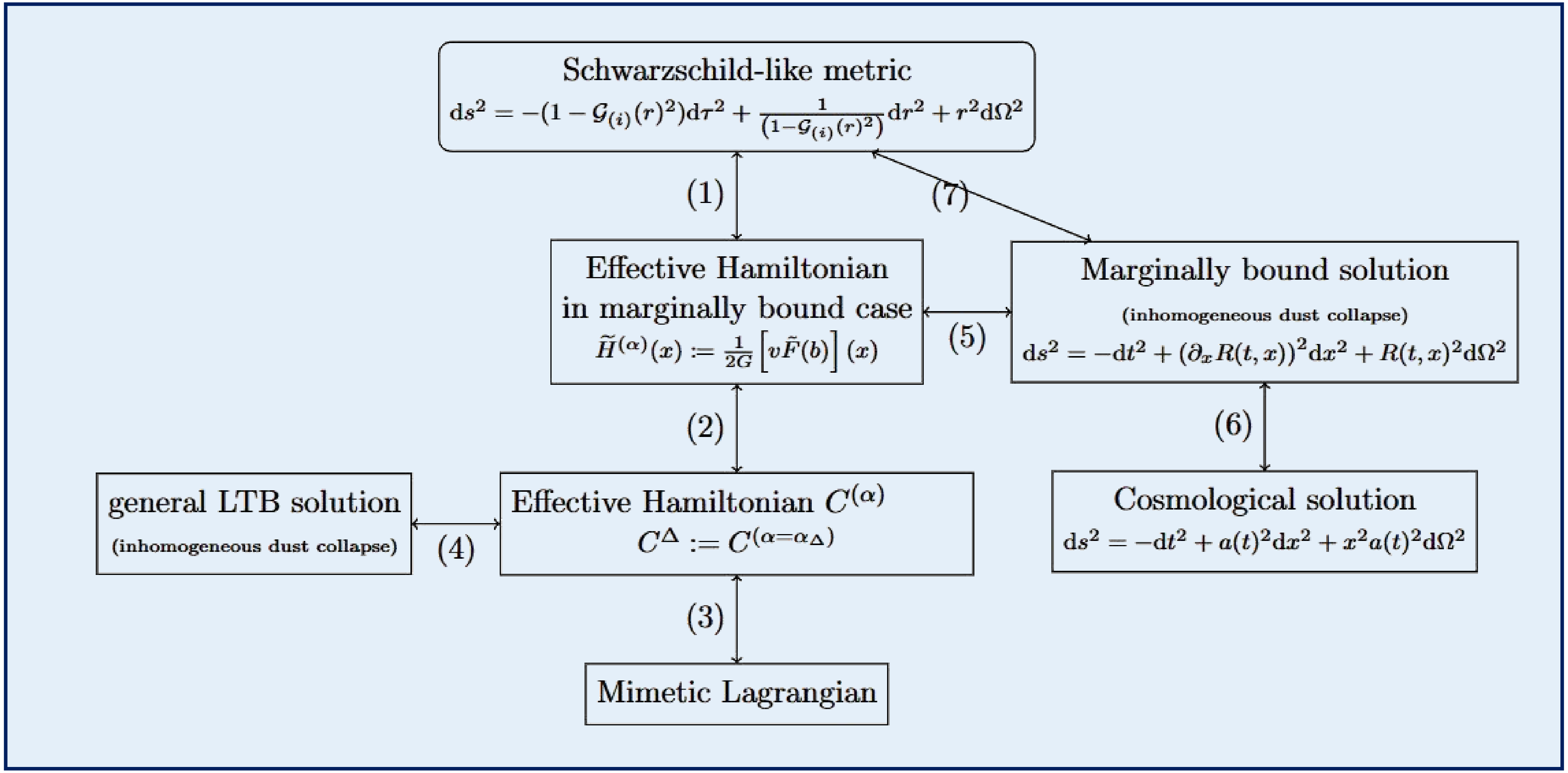
Only one Schwarzschild-like solution $\mathcal{G}_{(1)}$ connected with asymptotics

Reconstruction algorithm

See our paper for a more general reconstruction with

$$\mathrm{d}s^2 = -(1-\mathcal{G}_{(i)}(r)^2)\mathrm{d}\tau^2 + \frac{1}{g_{(\alpha)}(r)^2\left(1-\mathcal{G}_{(i)}(r)^2\right)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

However, in such case we do not have corresponding mimetic gravity



See our paper for a more general reconstruction with

Limiting curvature mechanism for regular BHs

Mimetic again: $S_2=\frac{1}{4G}\int_{\mathcal{M}_2}\mathrm{d}^2x\,\det(e)\{e^{2\psi}\left(R_h+2h^{ij}\partial_i\psi\partial_j\psi\right)+2+e^{2\psi}[L_\phi(X,Y)+\frac{1}{2}\lambda\left(\partial_j\phi\partial^j\phi+1\right)]\}$

In marginally bound case $Y=\frac{\dot{R}}{R},\quad X=2Y+\frac{RY'}{R'},$

$$\mathrm{d}s^2=-\mathrm{d}t^2+(\partial_x R(t,x))^2\mathrm{d}x^2+R(t,x)^2\mathrm{d}\Omega^2$$

Curvature scalars become

$$\mathcal{R}=\frac{2\dot{Y}'(X-2Y)}{Y'}+8XY+6\dot{Y}-4Y^2,$$

$$\mathcal{K}=\frac{4\left(Y'\left(2XY+\dot{Y}-3Y^2\right)+\dot{Y}'(X-2Y)\right)^2}{Y'^2}+8\left(Y^2\left(-2XY+X^2+2Y^2\right)+2\dot{Y}Y^2+\dot{Y}^2\right)+4Y^4.$$

Polymerization may not remove singularities
[Idrus Belfaqih's talk yesterday

Curvature scalars will be bounded if Y and X (or R') (and their derivatives) are bounded

$\tilde{F}'(b)=2\tilde{f}^{(2)}(b)$,
polymerization function $\tilde{F}$ can be unbounded

We only impose constraint on Y in the subclass

$$L_\phi(X,Y)=2XY-Y^2+\tilde{f}^{(1)}\left[(\tilde{f}^{(2)})^{-1}(-Y)\right]+2X(\tilde{f}^{(2)})^{-1}(-Y).$$

⟹ A bounded $\tilde{f}^{(2)}$

As a result, R' can be 0 thus X can diverge. If X diverges, this is a shell-crossing singularity where $R'=0$ [Viqar Husain's talk

Weak singularity where spacetime can be extended beyond it using weak solutions/shock waves

- For bouncing solutions shell-crossing can happen in the polymerized vacuum, center singularity ⟹ shell-crossing singularity (⇑ shock wave)
- For no bouncing solutions, shell-crossing only happens with no trivial matter profiles (general dust collapse) and can be avioded by choosing a good matter profile (similar to GR) — center singularity is removed

To remove all singularities, we also need to put bound on X, this is related to polymerize K_x, see [Han, HL, 2212.04605,
However, we do not have LTB reduction in that case

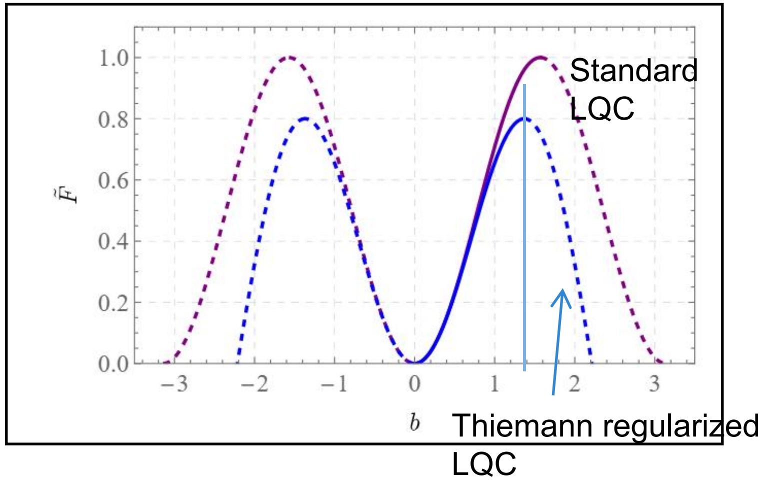
Examples: Bouncing solution

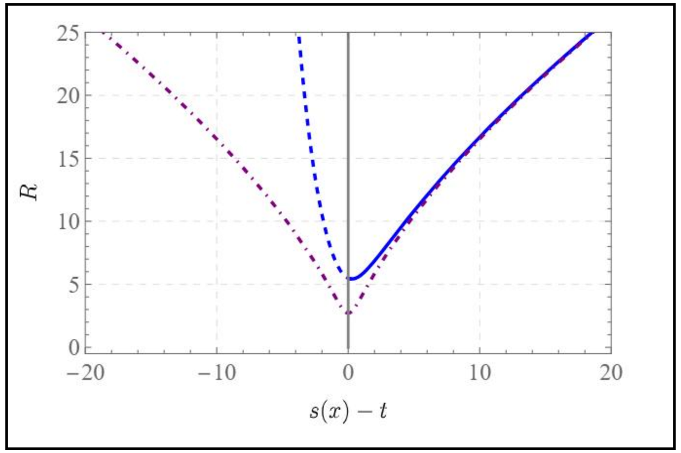
(Yang, Ding, Ma; Dapor, Liegener; Li, Singh, Wang...)

Two different Schwarzschild-like solution before and after bounce

	standard LQC	Thiemann regularized LQC	
metric	$\mathcal{G}(r)^2 = \frac{r_s}{r} - \frac{\alpha_\Delta^2 r_s^2}{r^4}$	$\mathcal{G}_\pm(r) = \frac{r\sqrt{\frac{1}{2}-\gamma^2 x_0}\sqrt{x_0 \pm \sqrt{1-2\gamma^2 x_0}+1}}{\alpha_\Delta(\gamma^2+1)}$ $\quad x_0 := 2\alpha_\Delta^2(\gamma^2+1)\left(\frac{r_s}{r^3}\right)$	r_{min} from $\mathcal{G}^2 \geq 0$
Polymerization function	$\tilde{F}(b) = \frac{\sin^2(\alpha_\Delta b)}{\alpha_\Delta^2}$	$\tilde{F}(b) = \frac{\sin^2(\alpha_\Delta b\gamma)\left(1-(\gamma^2+1)\sin^2(\alpha_\Delta b\gamma)\right)}{(\alpha_\Delta\gamma)^2}$	r_{min} from $\tilde{F} = \frac{r_s}{r^3}$
Marginally bound solution	$R(t,x) = (2GM(x))^{\frac{1}{3}}\left(\alpha_\Delta^2 + \frac{9}{4}z^2\right)^{\frac{1}{3}}$ $s(x) - t = z$ [Fazzini, Rovelli, Soltani 23’ [Giesel, HL, Singh and Weigl 2308.10953	$R(t,x) = \sqrt[3]{\frac{2GM(x)\left(4\alpha_\Delta^2\gamma^2+9\eta^2\right)^2}{18\eta^2 - 8\alpha_\Delta^2\gamma^4}}$ $s(x) - t = \eta - \frac{2}{3}\alpha_\Delta(\gamma^2+1)\tanh^{-1}\left(\frac{2\alpha_\Delta\gamma^2}{3\eta}\right)$	r_{min} from $z = 0$ $\eta = 0$
Minimal radius	$r_{\min} = (r_s\alpha^2)^{\frac{1}{3}}.$	$r_{\min} = 2^{2/3}\sqrt[3]{\alpha_\Delta^2\gamma^2(\gamma^2+1)r_s}$	

LTB solution obtianed from either segments by analytic continuation





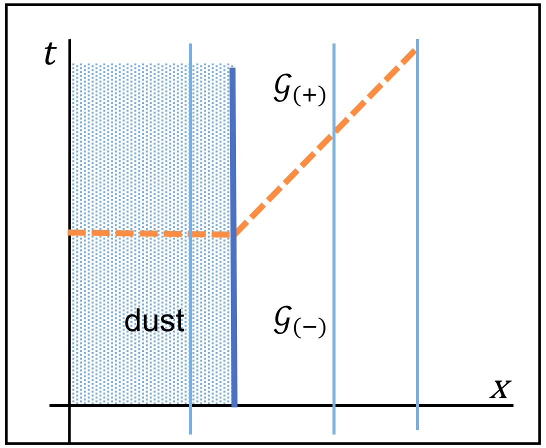
Examples: Bouncing solution

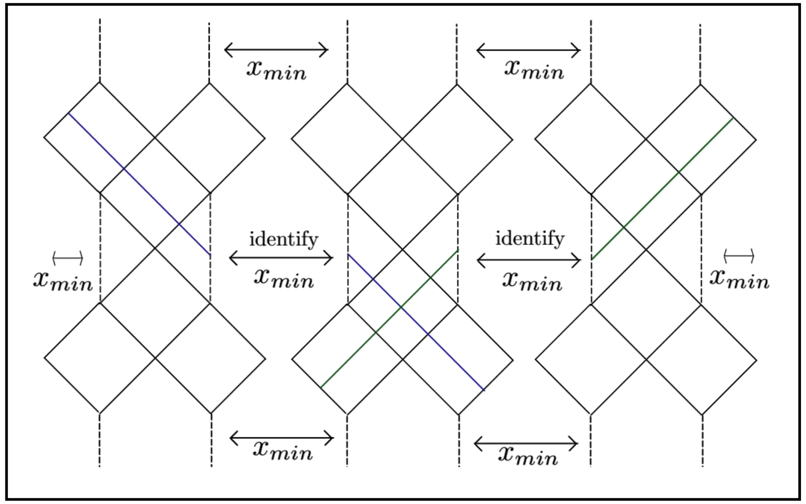
$$\mathrm{d}s^2 = -\mathrm{d}t^2 + (\partial_x R(t,x))^2 \mathrm{d}x^2 + R(t,x)^2 \mathrm{d}\Omega^2$$

shell-crossing

At $\partial_x R = 0$, LTB is not a good coordinates! For vacuum solution since $\partial_t R = \partial_x R$, this happens exactly at the bounce!

Geodesics do not stop at bounce and do not intersect after the bounce





Polymerized vacuum [Johannes Münch, 21

We can check the curvature invariants for any dust matter profile (marginally bound case) with the help of LTB solution

standard LQC

$$\mathcal{K} = \frac{432}{(9z^2+4\alpha^2)^4 \mathcal{S}^2}(\dots) \qquad \mathcal{S} = M'(x)\,(9z^2+4\alpha^2) + 18M(x)\beta'(x)z$$

In geneal, shell crossing singularity always appear as $z \in (-\infty, +\infty)$ (e.g. after the bounce)

Special case: vacuum in LQC, there is no singularity!

Lewandowski, Ma, Yang, Zhang, 22'
Fazzini, Rovelli, Soltani 23'
Giesel, HL, Singh and Weigl 2308.10953

Thiemann regularized LQC similar to standard LQC S linear in z, thus always have poles

Vacuum also **has** shell-crossing singularity, happens at bounce!

$$\mathcal{K} \sim \frac{81\gamma^2}{16\alpha^2(\gamma^2+1)^2(2\gamma^2+1)(\eta-\eta_0)^2}.$$

Schwarzschild singluarity is been replaced by shell crossing singularity, non-aviodable shell-crossing in general: shock solutions..

[Fazzini, Husain, Wilson-Ewing 23, (Viqar Husain's talk)

shell crossing prevents the appearance of other Schwarzschild-like solutions after the bounce!

Examples: solutions with a regular center

	Bardeen	Hayward	LQC
metric	$\mathcal{G}(r)^2 = \frac{r_s r^2}{(r^2 + \alpha^{\frac{4}{3}} r_s^{\frac{2}{3}})^{\frac{3}{2}}}$	$\mathcal{G}(r)^2 = \frac{r_s r^2}{(r^3 + \alpha^2 r_s)}$	$\mathcal{G}(r)^2 = \frac{r_s}{r} - \frac{\alpha_\Delta^2 r_s^2}{r^4}$ r_{min} from $\mathcal{G}^2 \geq 0$
Polymerization function	$\tilde{F}^{-1} = \frac{\alpha r_s}{2r^3} {}_2F_1\left(-\frac{3}{2}, -\frac{3}{4}, -\frac{1}{2}, -\left(\alpha^2 \frac{r_s}{r^3}\right)^{-\frac{2}{3}}\right) + \frac{\sqrt{\pi}\Gamma(\frac{3}{4})}{\alpha\Gamma(-\frac{3}{4})}$	$\tilde{F}^{-1} = \frac{2\eta\alpha + \sinh(2\alpha\eta)}{4\alpha}$, $\alpha\eta = \sinh^{-1}\sqrt{\frac{\alpha^2 r_s}{r^3}}$	$\tilde{F}(b) = \frac{\sin^2(\alpha_\Delta b)}{\alpha_\Delta^2}$ r_{min} from $\tilde{F} = \frac{r_s}{r^3}$
Marginally bound solution	$R(t,x) = (2GM(x))^{\frac{1}{3}}\sqrt{\eta^{\frac{4}{3}} - \alpha^{\frac{4}{3}}}, \quad \eta \geq \alpha$ $s(x) - t = \frac{2}{3}\eta + \alpha \tan^{-1}\left(\eta^{\frac{1}{3}}\alpha^{-\frac{1}{3}}\right) - \alpha \,\mathrm{Re} \tanh^{-1}\left(\eta^{\frac{1}{3}}\alpha^{-\frac{1}{3}}\right)$	$R(t,x) = \left(\frac{2GM(x)\alpha^2}{\sinh^2(\alpha\eta)}\right)^{\frac{1}{3}}, \quad \eta \geq 0$ $s(x) - t = \frac{2}{3}\alpha\left(\coth(\alpha\eta) - \alpha\eta\right)$	$R(t,x) = (2GM(x))^{\frac{1}{3}}\left(\alpha_\Delta^2 + \frac{9}{4}z^2\right)^{\frac{1}{3}}$ $s(x) - t = z$ r_{min} from $z = 0$
Curvature scalars	$\mathcal{R} = \frac{\mathcal{A}}{\eta^{14/3}\mathcal{S}}, \quad \mathcal{K} = \frac{\mathcal{B}}{\eta^{28/3}\mathcal{S}^2}$ $\mathcal{S} = M'(x)\eta + 3M(x)s'(x)$	$\mathcal{R} = \frac{\mathcal{A}}{4\alpha^3\mathcal{S}}, \quad \mathcal{K} = \frac{\mathcal{B}}{16\alpha^6\mathcal{S}^2}$ $\mathcal{S} = M'(x) + 3M(x)s'(x)\frac{\tanh(\alpha\eta)}{\alpha}$	$\mathcal{R} = \frac{\mathcal{A}}{\mathcal{S}}, \mathcal{K} = \frac{\mathcal{B}}{\mathcal{S}^2},$ $\mathcal{S} = M'(x)\left(9z^2 + 4\alpha_\Delta^2\right) + 18M(x)s'(x)z$

shell crossing similar to classical GR for η has a lower bound

shell crossing unaviodable as $z \in \mathbb{R}$ (except polymerized vacuum)

Mimetic potential

LQC $$L_\phi(X,Y) = 2XY - Y^2 + \frac{3\sin^2(\alpha_\Delta b) - \alpha_\Delta b \sin(2\alpha_\Delta b)}{\alpha_\Delta^2} + 2Xb, \quad b \equiv \sin_m^{-1}(-Y)$$

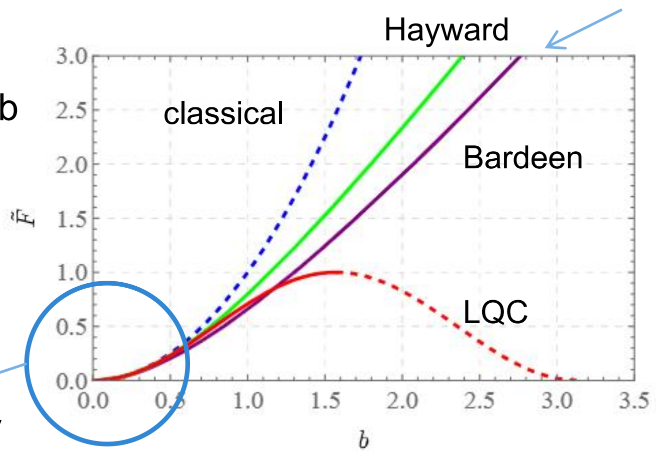
Hayward $$L_\phi(X,Y) = 2XY - Y^2 + \tilde{f}^{(1)}(b) + 2Xb, \qquad b = \frac{Y}{2\alpha^2 Y^2 - 2} - \frac{\tanh^{-1}(\alpha Y)}{2\alpha} \qquad \tilde{f}^{(1)}(b) = \frac{(3\sinh(\mathrm{Lsinh}_2(4\alpha b)) - 4\alpha b)\tanh\left(\frac{1}{2}\mathrm{Lsinh}_2(4\alpha b)\right)}{2\alpha^2}$$

Limiting curvature for regular BHs: e.g. Hayward, a tanh function present

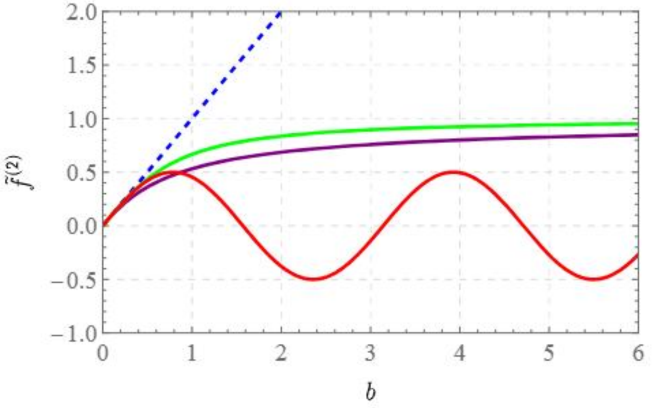
Examples: regular solution

linear for large b

classical case is quadratic in b

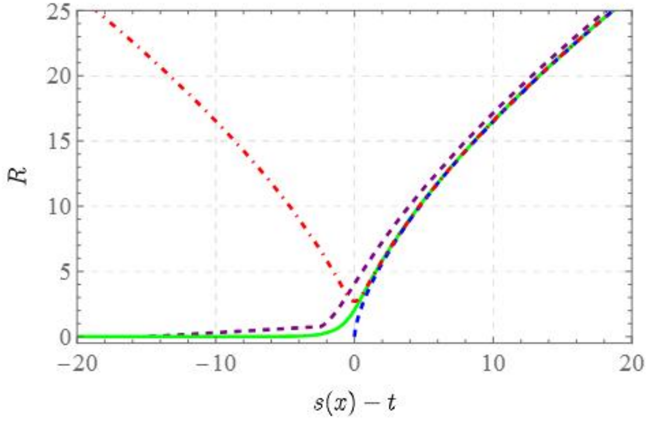


The asymptotically classical regime



Limiting curvature: bounded $\tilde{f}^{(2)}$

$$\tilde{F}'(b) = 2\tilde{f}^{(2)}(b)$$



To have a regular solution, unbounded polymerization functions need to be at most linear for large b

Summary

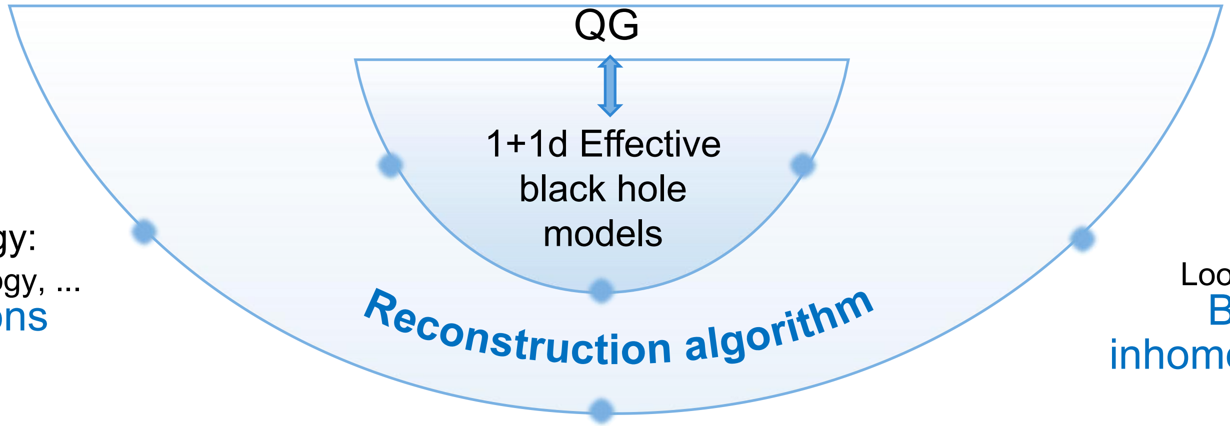
I : Classical regular BHs
e.g. Bardeen, Hayward

Unbounded polymerization function

II: quantum BHs
e.g. LQC insipred models

Bounded polymerization function

A general framework and classification of regular spherically symmetric BH theories



Regular cosmology:
LQC, bouncing cosmology, ...
“exact” LTB solutions

Regular black holes:
Loops BH, Bardeen, Hayward....
Birkhoff-like theorem
inhomogeneous dust collapse

Emergent modified gravity
4d Covariant Extended Mimetic Lagrangian
Limiting curvature for regular BHs

Summary

I : Classical regular BHs
e.g. Bardeen, Hayward

Unbounded polymerization function

II: quantum BHs
e.g. LQC insipred models

Bounded polymerization function

A general framework and classification of regular spherically symmetric BH theories

QG

1+1d Effective black hole models

Reconstruction algorithm

Regular cosmology:
LQC, bouncing cosmology, ...
“exact” LTB solutions

Regular black holes:
Loops BH, Bardeen, Hayward....
Birkhoff-like theorem
inhomogeneous dust collapse

Emergent modified gravity
4d Covariant Extended Mimetic Lagrangian
Limiting curvature for regular BHs

Perturbation theory beyond mixed approach	Gravitational wave/QNMS	Axially symmetric spacetime	Light-cone gauge collapse and evaporation	Extension to more broader class of models

many new possibilities!

THANKS!

Bouncing and no-bouncing?

Hayward as a geometric series

$$\mathcal{G}_{Hay}(r)^2 = \frac{r_s}{r}\frac{1}{1+\frac{\alpha^2 r_s}{r^3}} = \sum_{k=0}^{\infty}(-1)^k\left(\frac{\alpha^2 r_s}{r^3}\right)^k$$

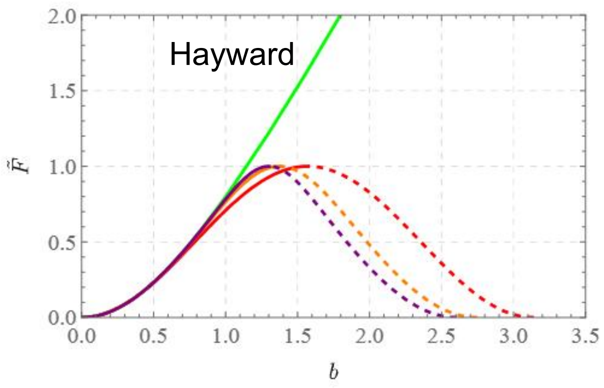
Converging for $r^3 > r^2_{\rm min} \equiv \alpha^2 r_s.$

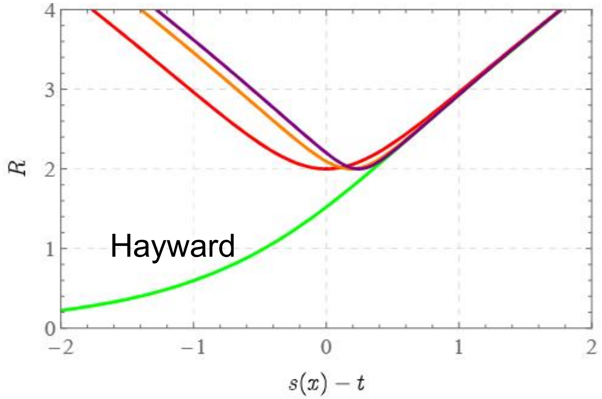
Truncated geometric series

$$\mathcal{G}^{(n)}_{Hay}(r)^2 := \frac{r_s}{r}\sum_{k=0}^{n}\left(-\frac{\alpha^2 r_s}{r^3}\right)^k = \frac{r_s}{r}\frac{1-\left(-\frac{\alpha^2 r_s}{r^3}\right)^{n+1}}{1-\left(-\frac{\alpha^2 r_s}{r^3}\right)}, \qquad n \in \mathbb{N}_+.$$

$$\mathcal{G}^{(1)}_{Hay}(r)^2 = \frac{r_s}{r}\left(1-\frac{\alpha^2 r_s}{r^3}\right)$$ ← LQC

All odd n truncations give bouncing solution (from the - sign)





We can not distinguish a no-bouncing solution with a regular center from a bouncing solution for $r > r_{min}$ (before the bounce), which is inside the horizon generally....

similar facts for Bardeen and general BHs with regular center, with binomial series..

For all truncated bouncing solutions, **polymerization functions** are still nonpolynomial and there is no truncation on it, but identifying coeffecients

expansion of sin^2 function

LQC →

$$\tilde{F}^{(1)}(b) = b^2 - \frac{1}{3}\alpha^2 b^4 + \frac{2}{45}\alpha^4 b^6 - \frac{1}{315}\alpha^6 b^8 + \frac{2}{14175}\alpha^8 b^{10} + O\left(\alpha^9\right)$$

$$\tilde{F}^{(3)}(b) = b^2 - \frac{1}{3}\alpha^2 b^4 + \frac{11}{45}\alpha^4 b^6 - \frac{73}{315}\alpha^6 b^8 + \frac{1973}{14175}\alpha^8 b^{10} + O\left(\alpha^9\right)$$

$\cdots$

$$\tilde{F}_{Hay}(b) = b^2 - \frac{1}{3}\alpha^2 b^4 + \frac{11}{45}\alpha^4 b^6 - \frac{73}{315}\alpha^6 b^8 + \frac{3548}{14175}\alpha^8 b^{10} + O\left(\alpha^9\right)$$

classical

Canonical analysis

$$\tilde{S}_2 = \frac{1}{2G}\int \mathrm{d}t\mathrm{d}x\, E^\varphi \sqrt{E^x}\left\{-\left[2XY - Y^2\right] + \tilde{L}(X,Y) + \frac{1}{2}R^{(3)}\right\} \quad X = \frac{\partial_t E^\phi + \partial_x\left(N^x E^\phi\right)}{E^\phi} \quad Y = \frac{\partial_t E^x + N^x \partial_x E^x}{2E^x}$$

momenta

$$\pi_\varphi = 2G\frac{\delta S_2'}{\delta \dot{E}^\varphi} = \sqrt{E^x}\left(\partial_X \tilde{L} - 2Y\right),$$

$$\pi_x = 2G\frac{\delta S_2'}{\delta \dot{E}^x} = \frac{E^\varphi}{2\sqrt{E^x}}\left(\partial_Y \tilde{L} - 2X + 2Y\right)$$

$$\{E^x(x), \pi_x(x')\} = \{E^\varphi(x), \pi_\varphi(x')\} = 2G\delta(x, x')$$

Diffeo $\pi_{N^x} = \frac{\delta S_2'}{\delta \dot{N}^x} = 0 \;\Longrightarrow\; C_x(x) = \frac{1}{G}\left(E^\phi K_\phi{}' - K_x E^{x\prime}\right)(x)$

Suppose we can invert $X = f_X\left(\frac{\pi_\varphi}{\sqrt{E^x}}, \frac{\sqrt{E^x}\pi_x}{E^\varphi}\right)$, $\quad Y = f_Y\left(\frac{\pi_\varphi}{\sqrt{E^x}}, \frac{\sqrt{E^x}\pi_x}{E^\varphi}\right)$, Legendre transformation gives a true hamiltonian in the form $H^\alpha = \int dx\, C^\alpha + N^x C_x$

$C^\alpha = \lambda\, E^\varphi\sqrt{E^x}$, λ dust density

$\lambda = 0$ polymerized vacuum

$$C^\alpha = \frac{1}{2G}E^\varphi\sqrt{E^x}\left(f_X\frac{\pi_\varphi}{\sqrt{E^x}} + 2f_Y\frac{\sqrt{E^x}\pi_x}{E^\varphi} + 2f_X f_Y - f_Y{}^2 - \tilde{L}\left(\frac{\pi_\varphi}{\sqrt{E^x}}, \frac{\sqrt{E^x}\pi_x}{E^\varphi}\right) - \frac{1}{2}R^{(3)}\right)$$

$$\equiv \frac{1}{2G}E^\varphi\sqrt{E^x}\left((1+f^\alpha)\left(\frac{\pi_\varphi\pi_x}{E^\varphi} + \left(\frac{\pi_\varphi}{2\sqrt{E^x}}\right)^2\right) - \frac{1}{2}R^{(3)}\right)$$

Polymerized hamiltonian

$f_X, f_Y, \tilde{L}$ combines gives the polymerization function $f^\alpha\left(\frac{\pi_\varphi}{\sqrt{E^x}}, \frac{\sqrt{E^x}\pi_x}{E^\varphi}\right)$

α (QG) parameter, s.t. $\lim_{\alpha\to 0} f^\alpha = 0$

Only combinations in the form $\frac{\pi_\varphi}{\sqrt{E^x}}, \frac{\sqrt{E^x}\pi_x}{E^\varphi}$ will appear in the Hamiltonian ---- (This is called $\bar{\mu}$ scheme for loop inspired models)