Quant. Oppenheimer-Snyder & Swiss Cheese Model

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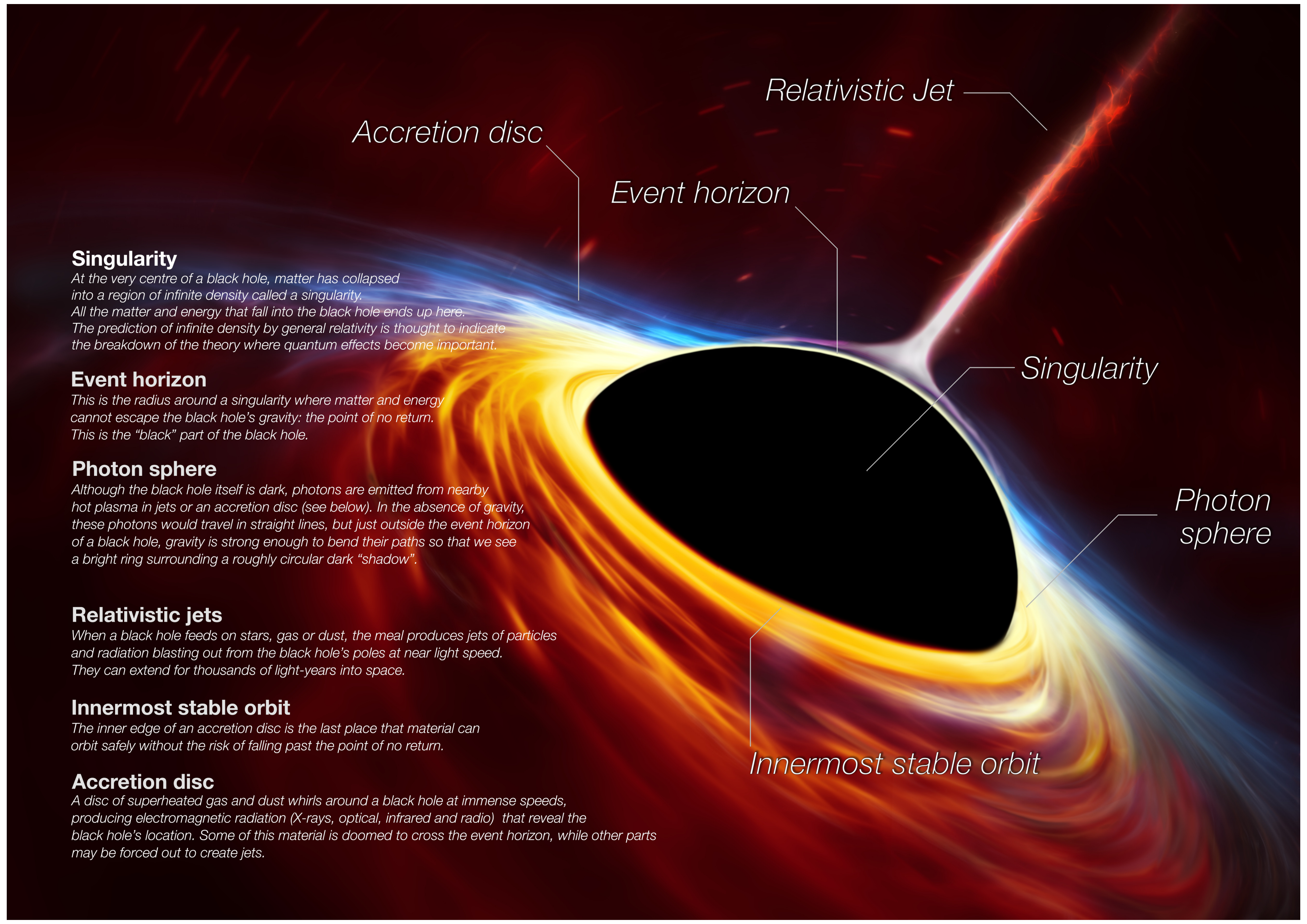
In collaboration with Jerzy Lewandowski, Yongge Ma, Jinsong Yang, Muxin Han and Dongxue Qu

Based on: PRL 130, 101501; PRD 109, 064012; PRD 108, 104004; EPJC 83, 619; and ArXiv: 2404.02796;

Motivation and background

The black holes of nature are the most perfect macroscopic objects there are in the Universe

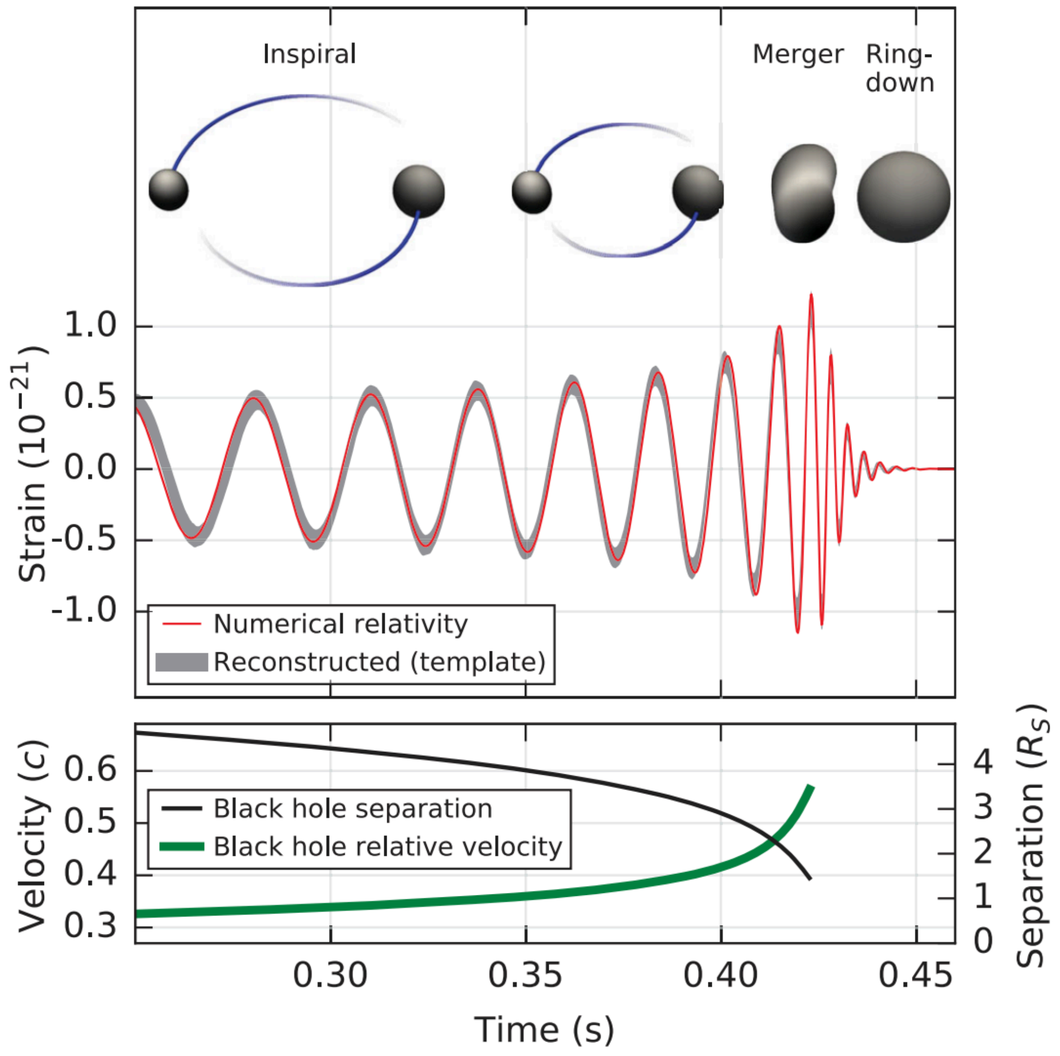
—Subrahmanyan Chandrasekhar



https://en.wikipedia.org/wiki/Black_hole

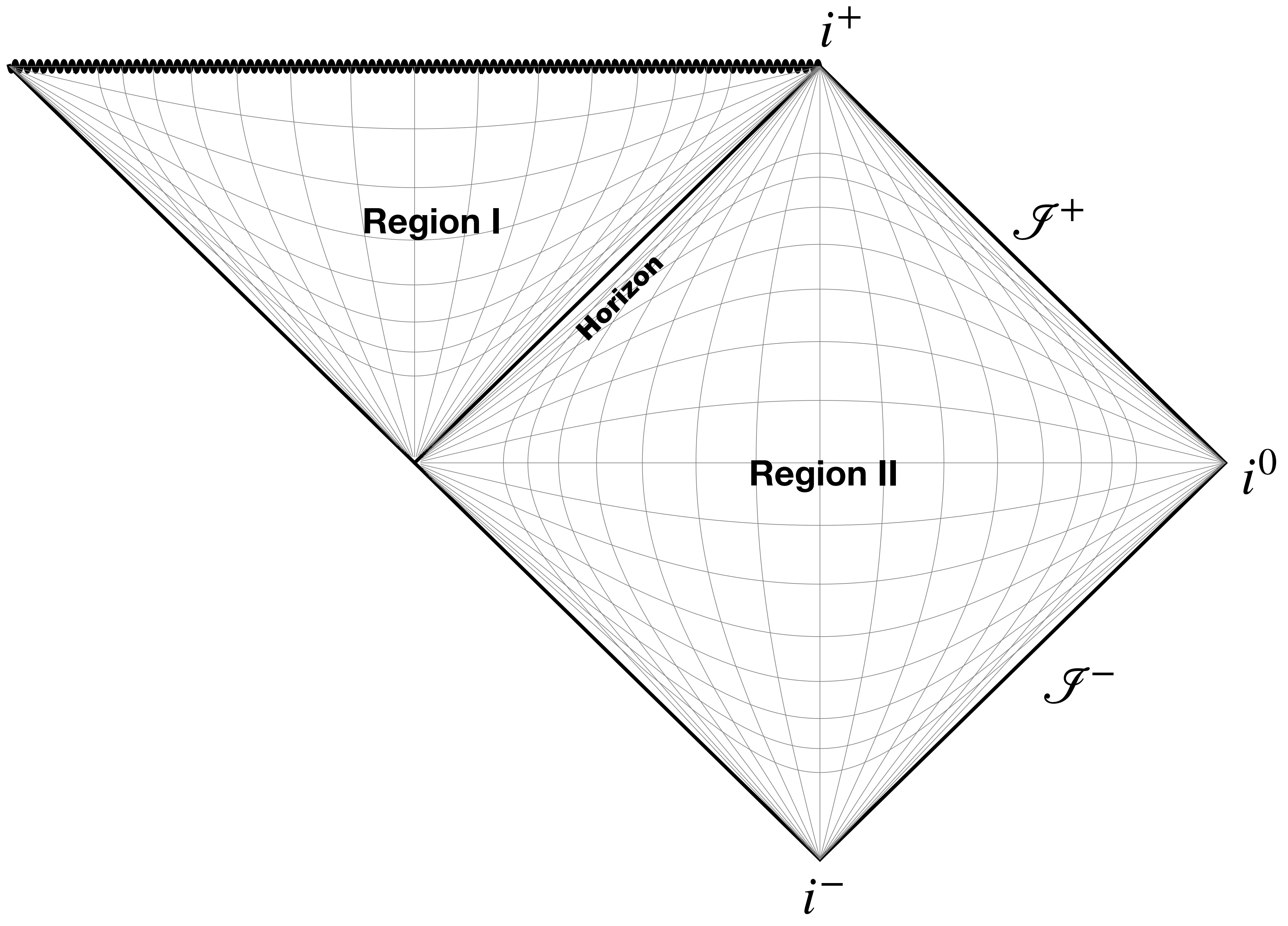
BHs have been observed due to the development of observational technique

Image of a BH at the core of M87
[https://eventhorizontelescope.org/]



Estimated gravitational-wave strain amplitude from GW150914 [PRL 116, 061102 (2016)]

Motivation and background

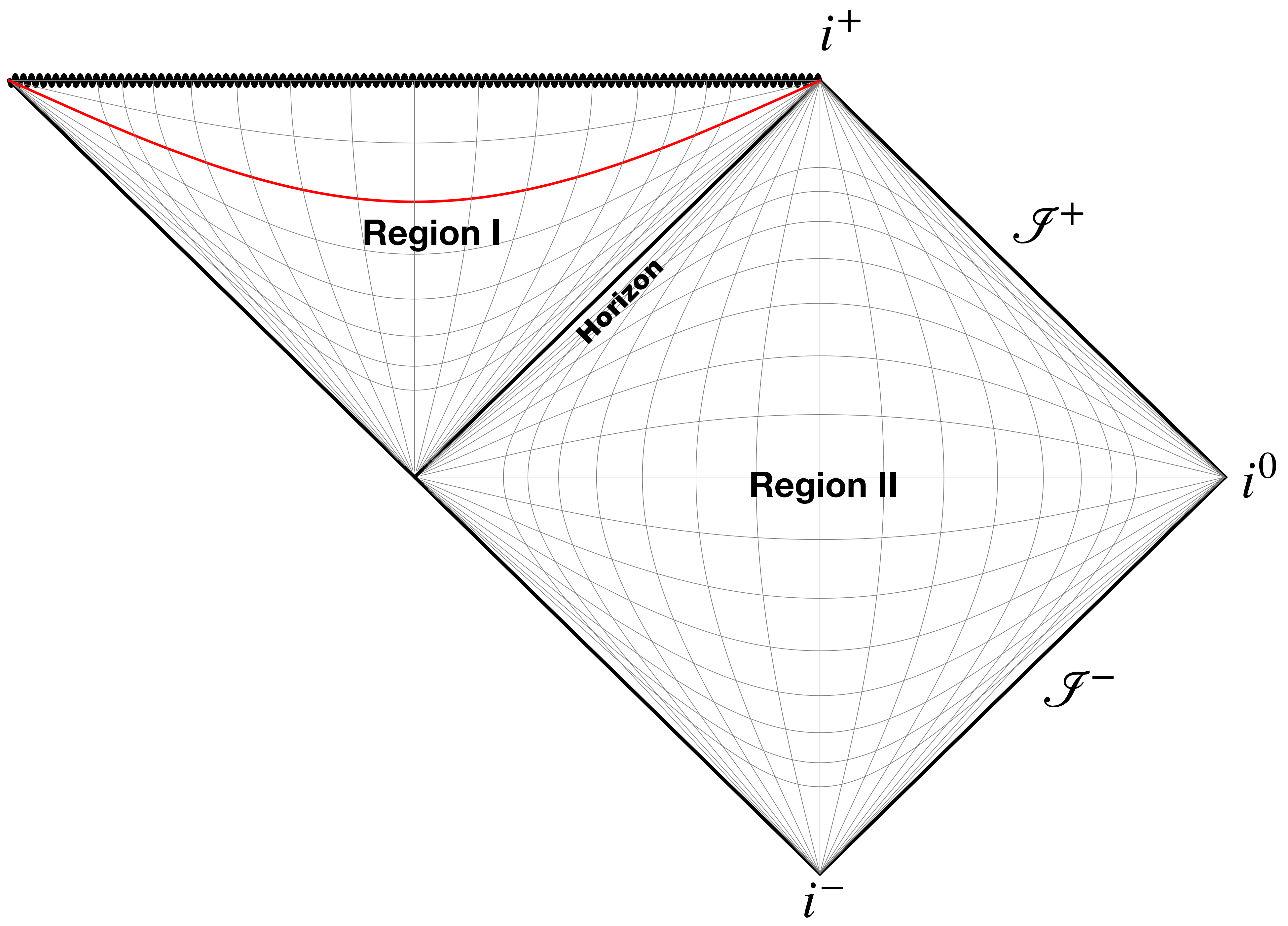


Penrose Diagram of a Schwarzschild BH

- The existence of singularities in BHs motivate us to introduce QG in BH physics;
- In loop quantum gravity, our answer on quantum BH haven't form a unique picture; There are, e.g., Ashtekar-Bojowald paradigm **[Ashtekar & Bojowald 05']**, the SF qBH model **[Rovelli, Harggard, Christodoulou, Speziale, Vilensky etc. 15', 16', Han, Qu, CZ 24' and so on]** and different loop quantum symmetry-reduced models **[Ashtekar, Bojowald, Bodendorfer, Boehmer, Chiou, Giesel, Gambini, Han, Husian, Li, Liu, Lewandowski, Modesto, Ma, Mehdi, ,Mena Marugan, Olmedo, Pullin, Singh, Vandersloot, Wang, Wilson-Ewing, Yang, Zhang and so on]**

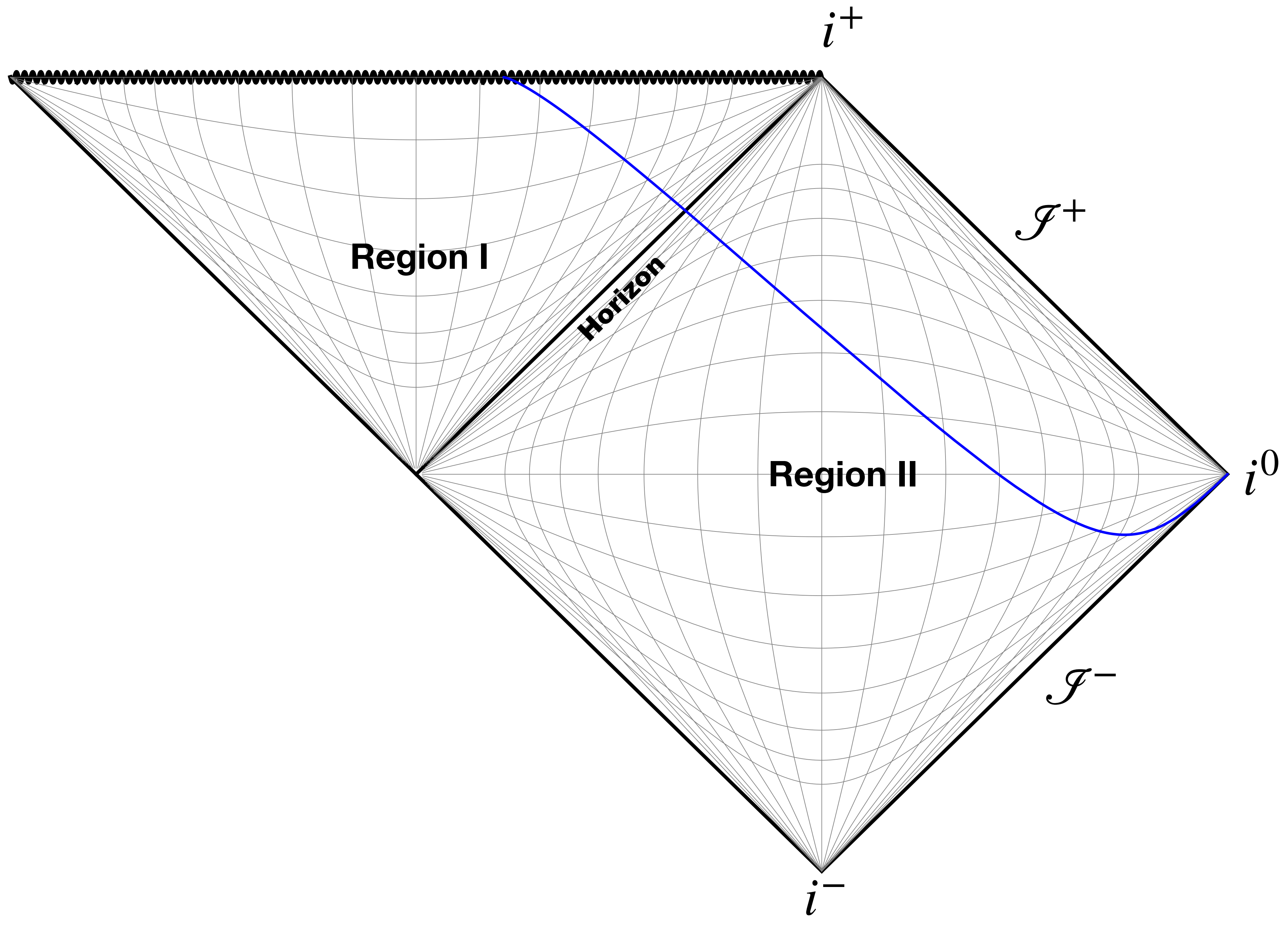
Motivation and background

Homogeneous Model



- $\mathbb{R} \times \mathrm{S}^2$ **with symmetry** $\mathbb{T} \times \mathrm{SO}(3)$ **:** $\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \frac{p_b^2}{|p_c| L_0^2}\mathrm{d}x^2 + |p_c|\,\mathrm{d}\Omega^2$**;**
- **Quantization: promote** p_b **and** p_c **to operators acting on a Hilbert space.**

Spherically symmetric Model



- $\mathbb{R} \times \mathrm{S}^2$ **with symmetry** $\mathrm{SO}(3)$ **:** $\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \frac{(E^b)^2}{E^c}(\mathrm{d}x + N^x\mathrm{d}t)^2 + E^c\mathrm{d}\Omega^2$**;**
- **Quantization: promote** $E^b(x)$ **and** $E^c(x)$ **to operators acting on a Hilbert space.**

Loops quantization: choose the polymer Hilbert space as the home of the operators.

Motivation and background

- **Exciting results based on effective dynamics: singularity resolution with quantum extension of the Schwarzschild spacetime** [e.g. Ashtekar, Olmedo & Singh 18', Achour, Lamy, Liu & Noui 18', Gambini, Olmedo & Pullin 20', Han & Liu 22', Husain, Kelly, Santa Cruz & Wilson-Ewing 22', and so on]

- **Quantum dynamics are studied** [e.g., Cortez, Navascués, Mena Marugán, Velhinho 23', CZ, Ma, Song & Zhang 20' & 22', CZ 21']
  - **The mass spectrum of BH is discrete and has non-vanishing minimal value** [based on the homogeneous model, see CZ, Ma, Song & Zhang 20' & 22'; different results by Cortez, Navascués, Mena Marugán, Velhinho 23']
    ⇒ stable remnant at the end of the BH evaporation.
  - **The usual effective dynamics is only valid for BH with large mass** [based on the spherically symmetric model, see CZ 21']
    ⇒ phase transition during the BH evaporation when BH mass decreases from a large one to a smaller one.

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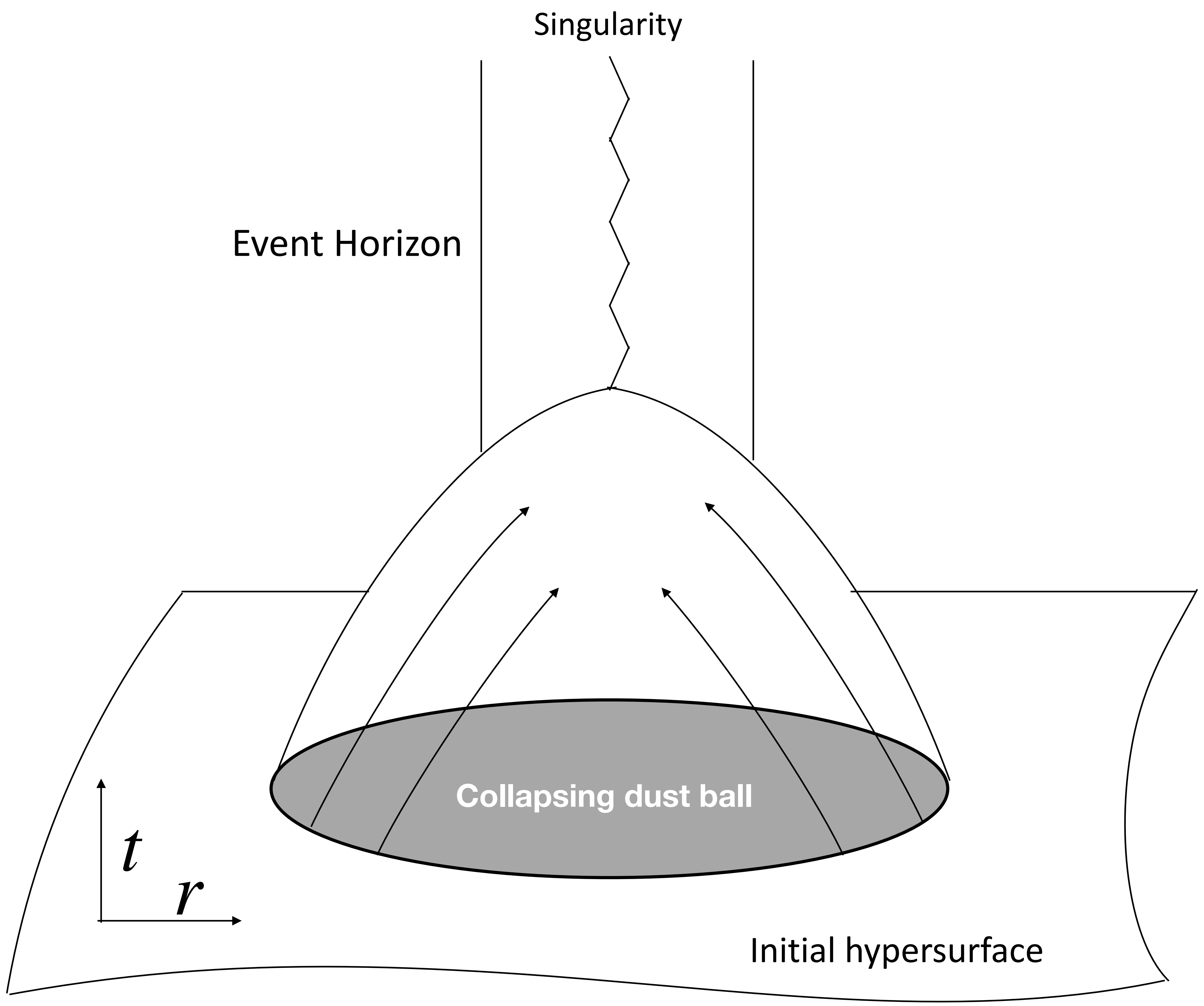
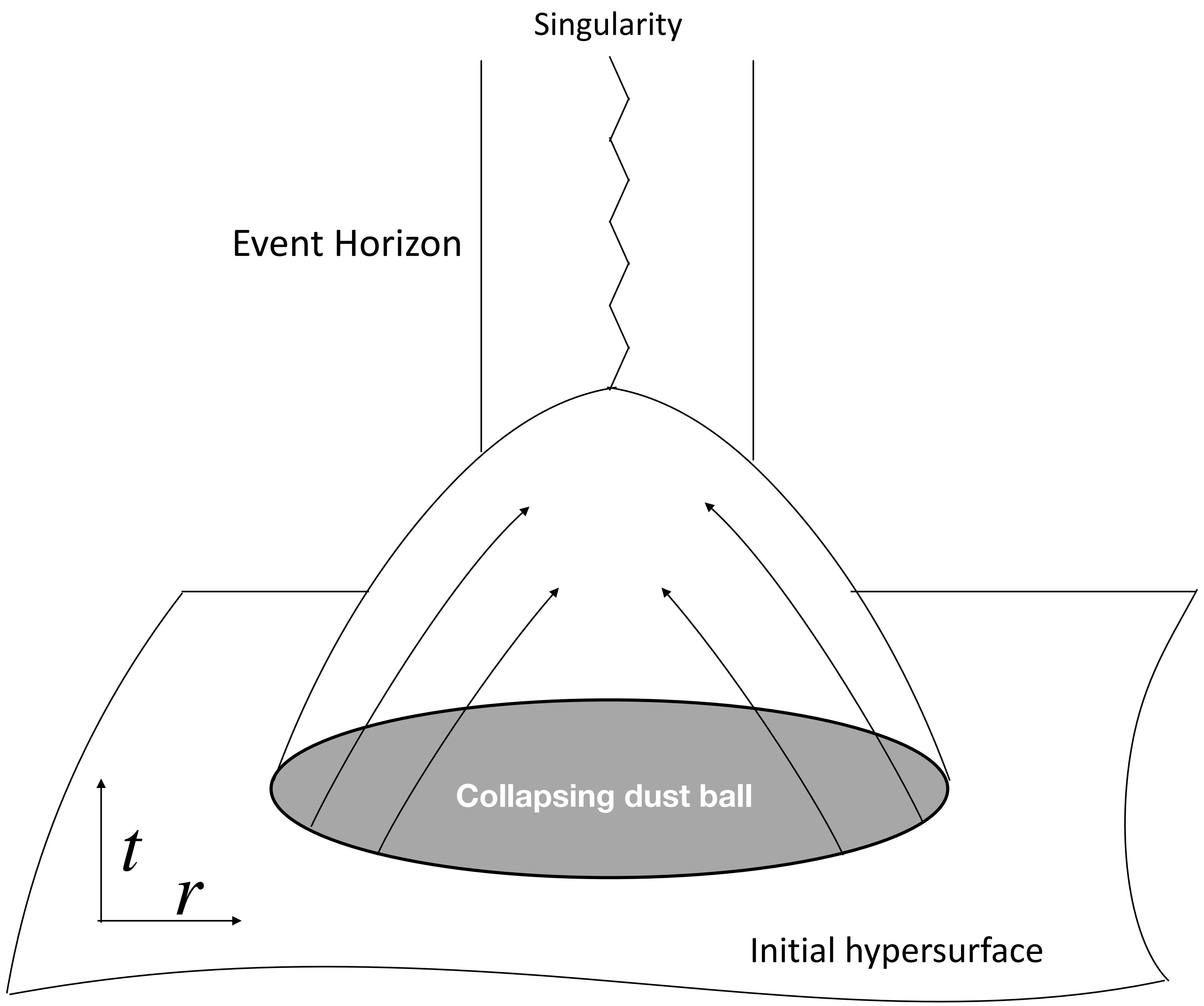
Most of the models don't consider the formation of a BH, motivating us to consider a BH formed by collapsing dust, so we are interested in:
What is a BH spacetime containing a collapsing matter ball like?

Oppenheimer-Snyder model



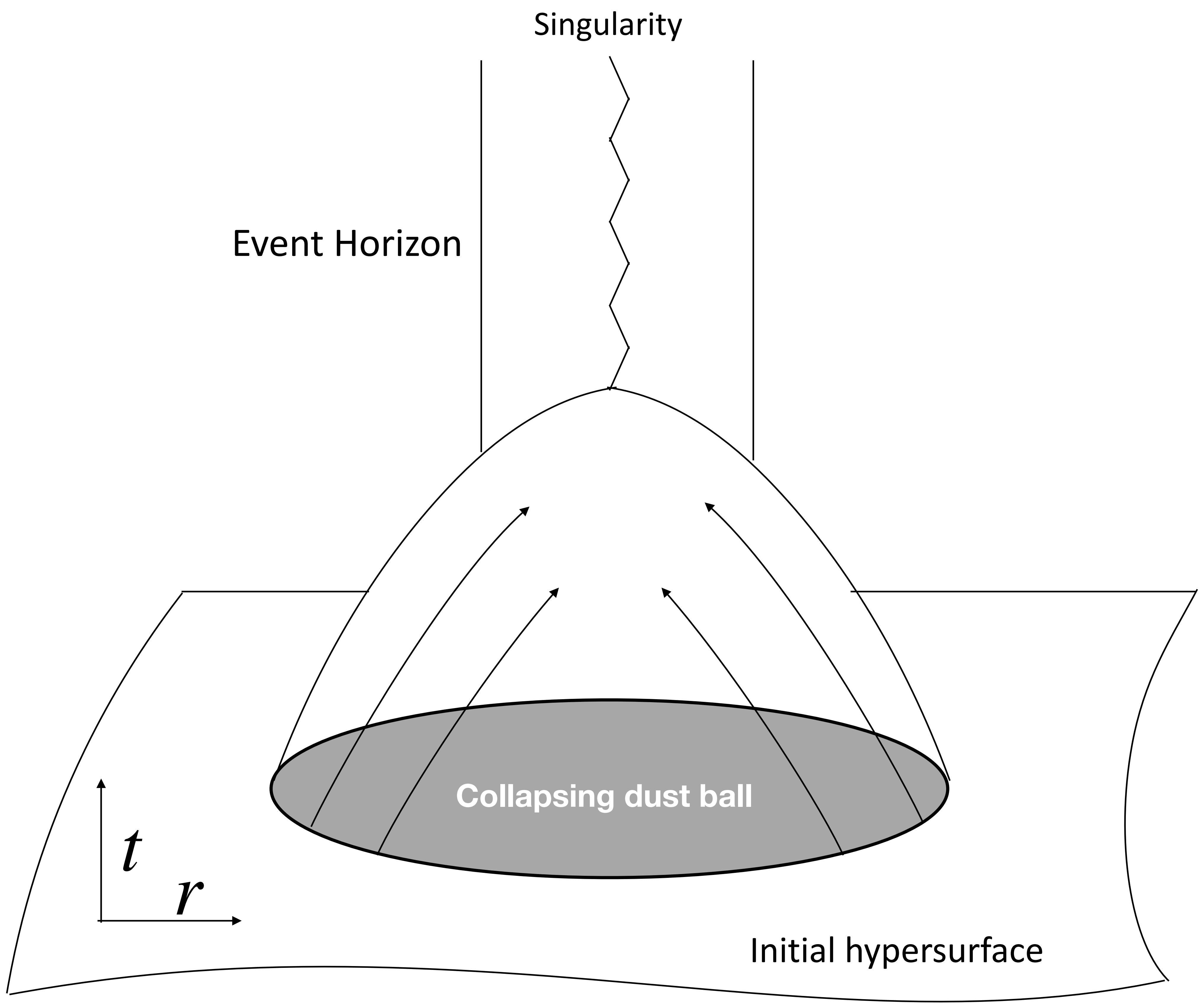
Oppenheimer-Snyder model



Some facts:

- **The dust ball takes the metric** $ds^2 = -d\tau^2 + a(\tau)^2 ds_E^2$ **;**
- $a(\tau)$ **is governed by:** $\mathbb{H}^2 = \frac{8\pi G}{3}\rho$ **and** $\partial_\tau(\rho a^3) = 0$ **;**
- **The Schwarzschild outside is the unique spherically symmetric and stationary metric that can be glued to the dust ball metric by the junction condition. This is the result without necessary to consider the EOM.**

Oppenheimer-Snyder model



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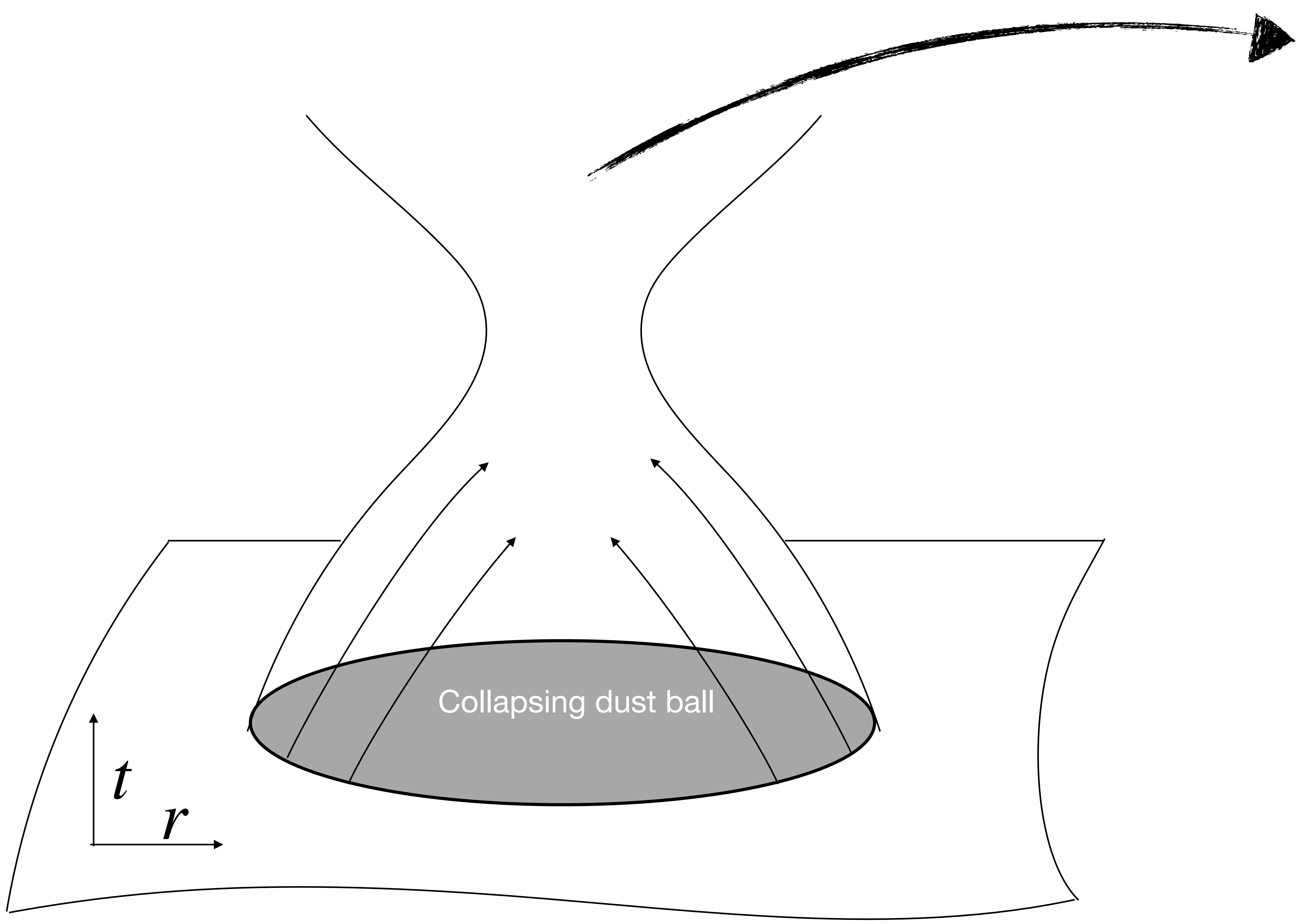
What will happen if the dust ball is a LQC one?

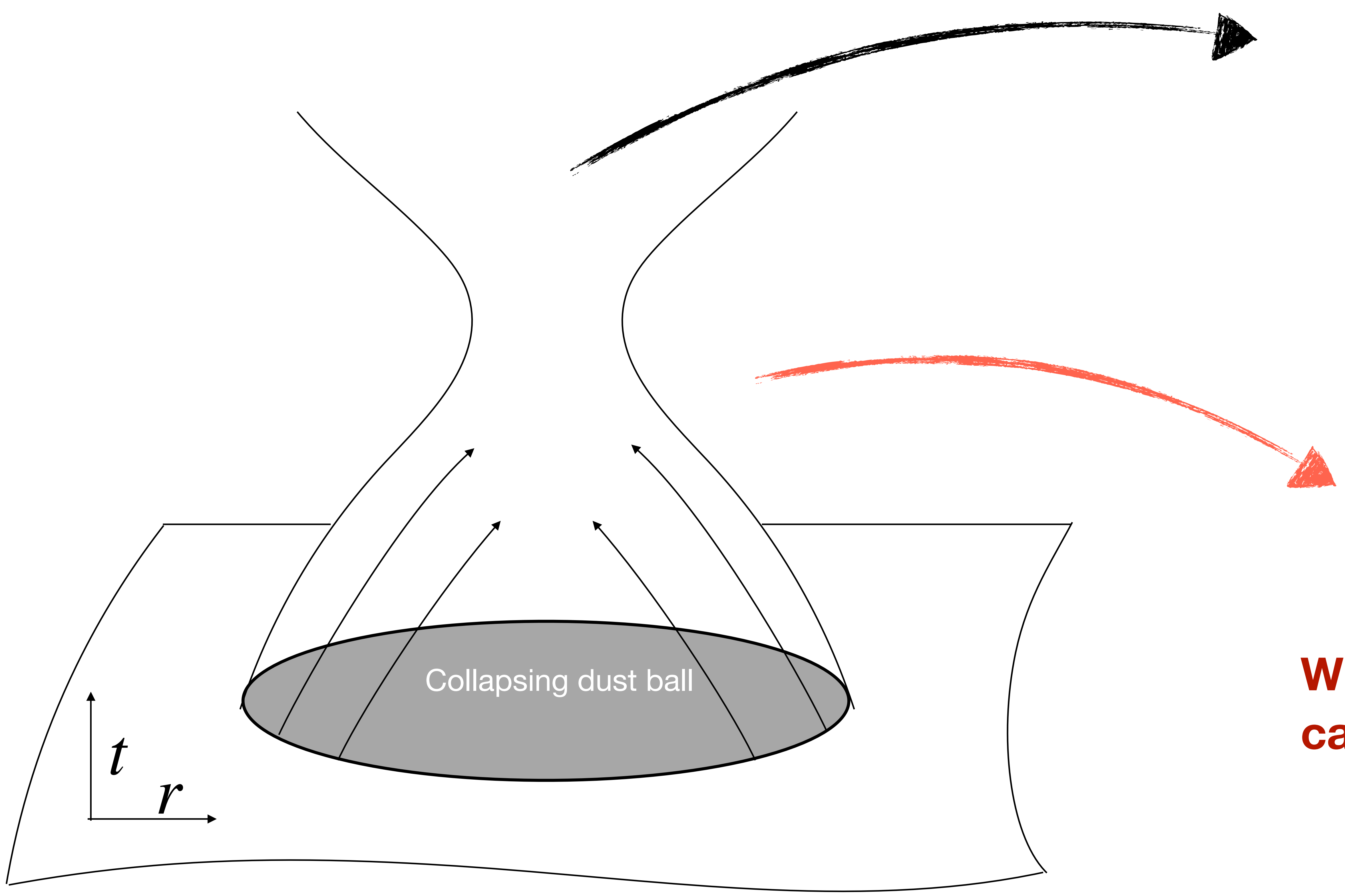
Quantum Oppenheimer-Snyder model



$$ds^2 = -d\tau^2 + a(\tau)^2 ds_E^2$$

$$\mathbb{H}^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$ **and** $$\partial_\tau(\rho a^3) = 0$$

Quantum Oppenheimer-Snyder model



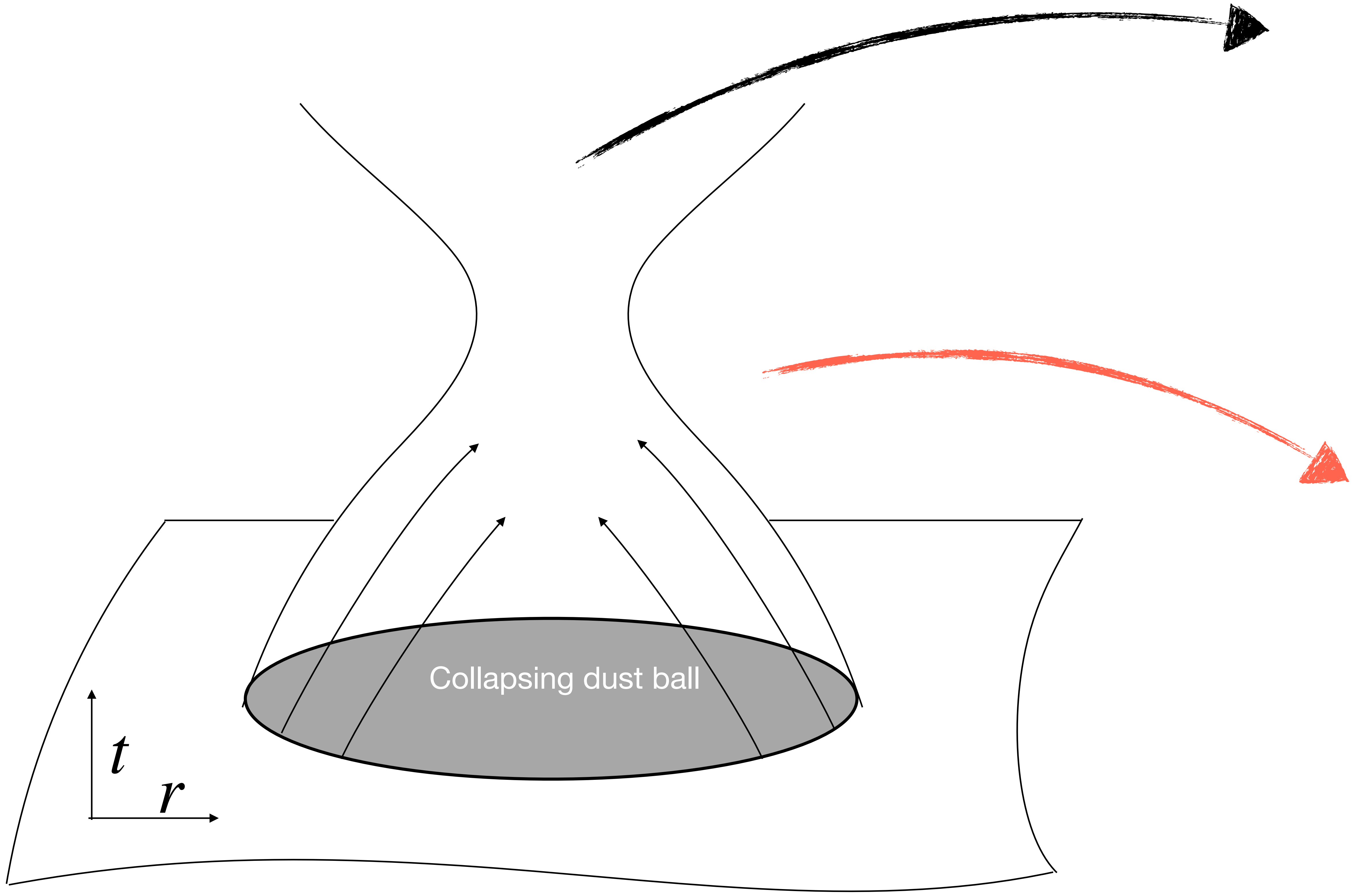
$$ds^2 = -d\tau^2 + a(\tau)^2 ds_E^2$$

$$\mathbb{H}^2 = \frac{8\pi G}{3}\rho(1 - \frac{\rho}{\rho_c}) \text{ and } \partial_\tau(\rho a^3) = 0$$

$$ds^2 = -f(r)dt^2 + g(r)^{-1}dr^2 + r^2 d\Omega^2$$

What is the expression for $f(r)$ and $g(r)$ so that the outside can be glued with the inside by the junction condition?

Quantum Oppenheimer-Snyder model



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$\mathbb{H}^2 = \frac{8\pi G}{3}\rho(1 - \frac{\rho}{\rho_c})$ **and** $\partial_\tau(\rho a^3) = 0$

$$ds^2 = -f(r)dt^2 + g(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$f(r) = g(r) = 1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}$$

$$\alpha = 16\sqrt{3}\pi\gamma^3\ell_p^2$$

[Lewandowski, Ma, Yang, CZ 23']

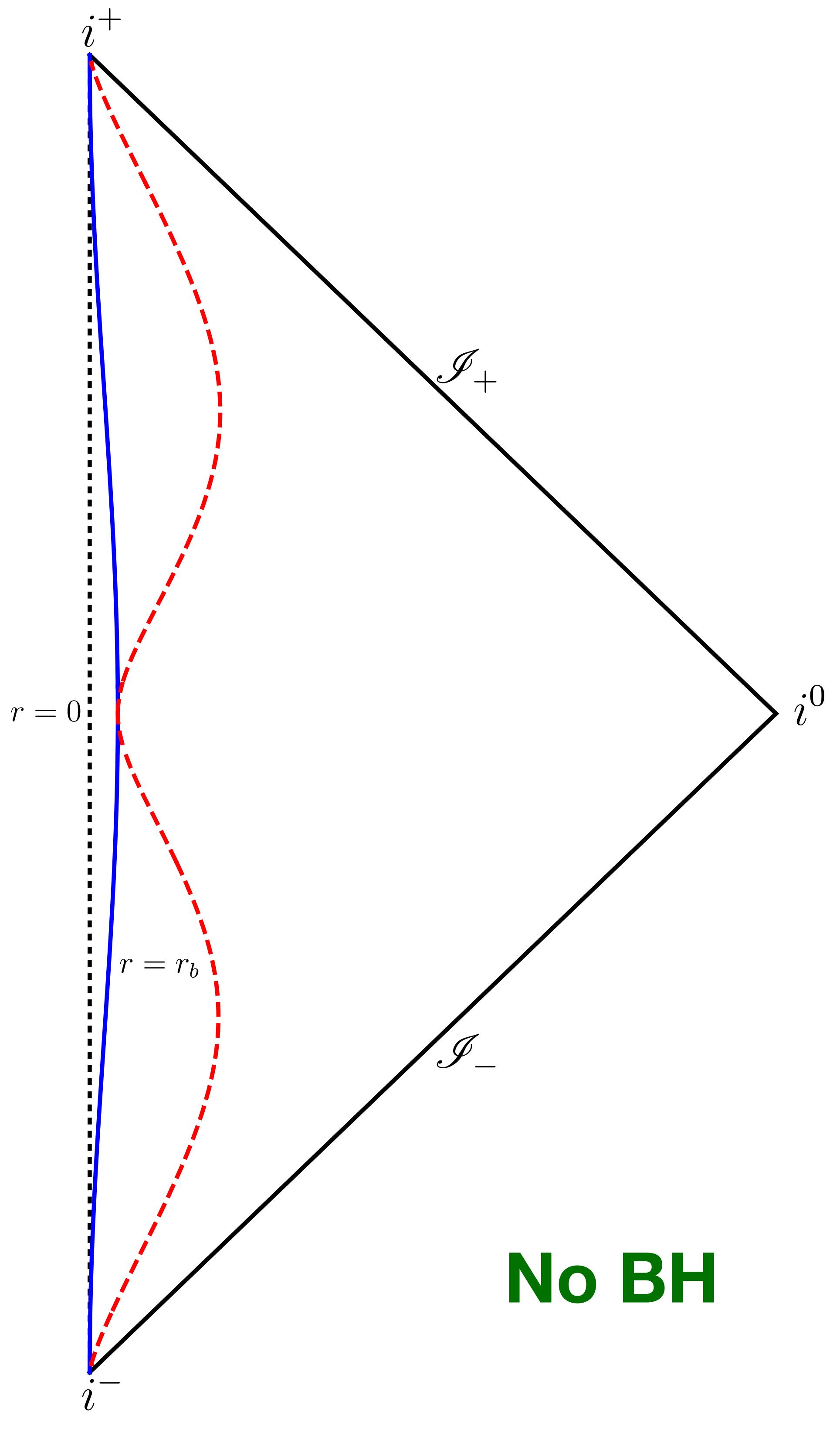
Quantum Oppenheimer-Snyder model

- **The outside metric is uniquely determined by the modified Friedemann equation,**

  for $\mathbb{H}^2 = \dfrac{8\pi G}{3}\rho X(\rho)$**, we get** $f(r) = g(r) = 1 - 2GMr^{-1}X(3M/(4\pi r^3))$

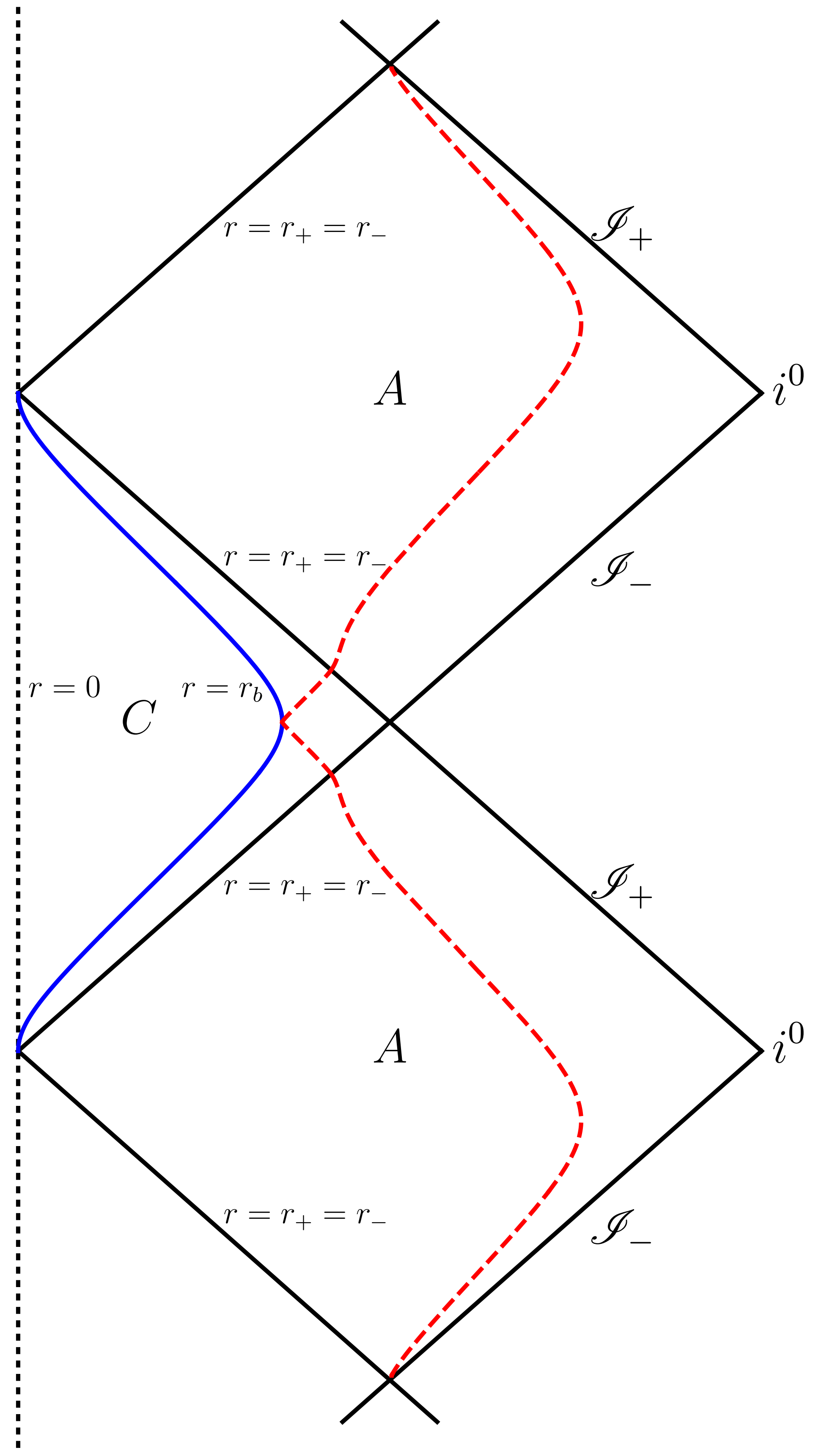
- **The same metric is obtained by other people from various approaches** [e.g., Marto, Tavakoli & Moniz 15', Kelly, Santacruz & Wilson-Ewing 20', Bobula & Powłowski 23', and Giesel, Liu, Rullit, Singh & Weigl 23', see also Viqar's and Hongguang's talks]
- **The Penrose diagram of the maximally extended spacetime is studied as follows:**

Quantum Oppenheimer-Snyder model



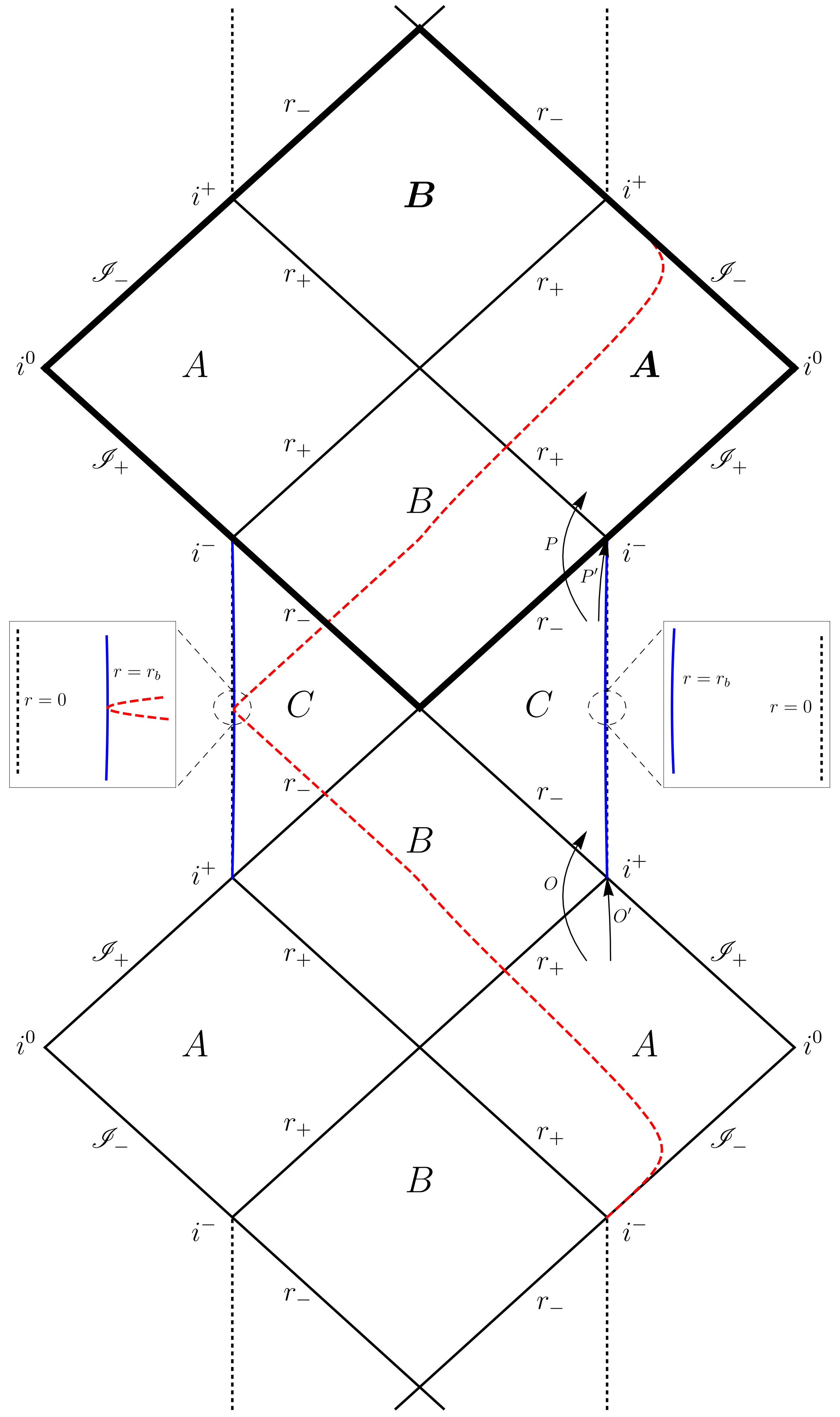
No BH

$M < M_{\text{min}}$



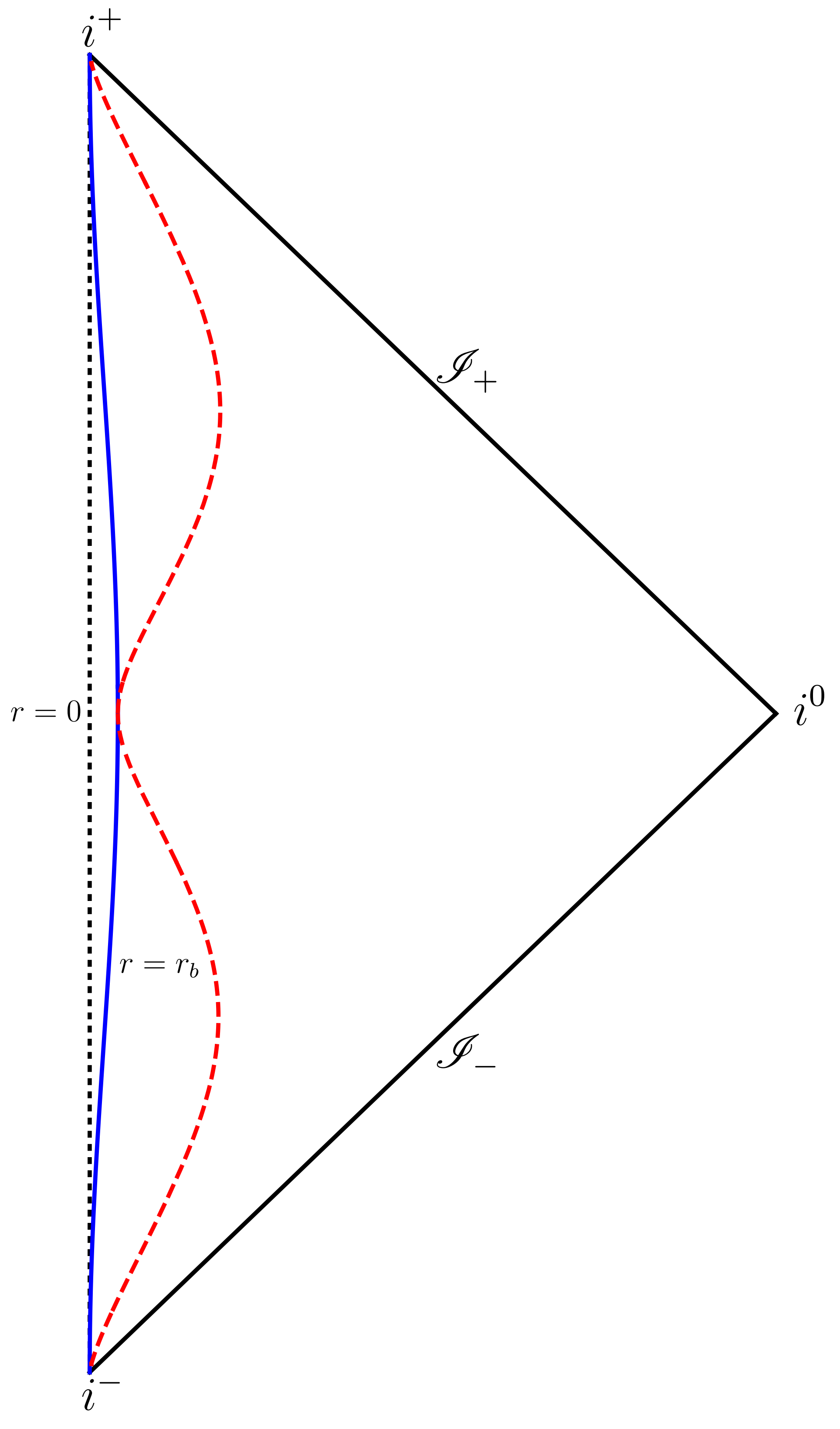
$M = M_{\text{min}}$

BHs exist

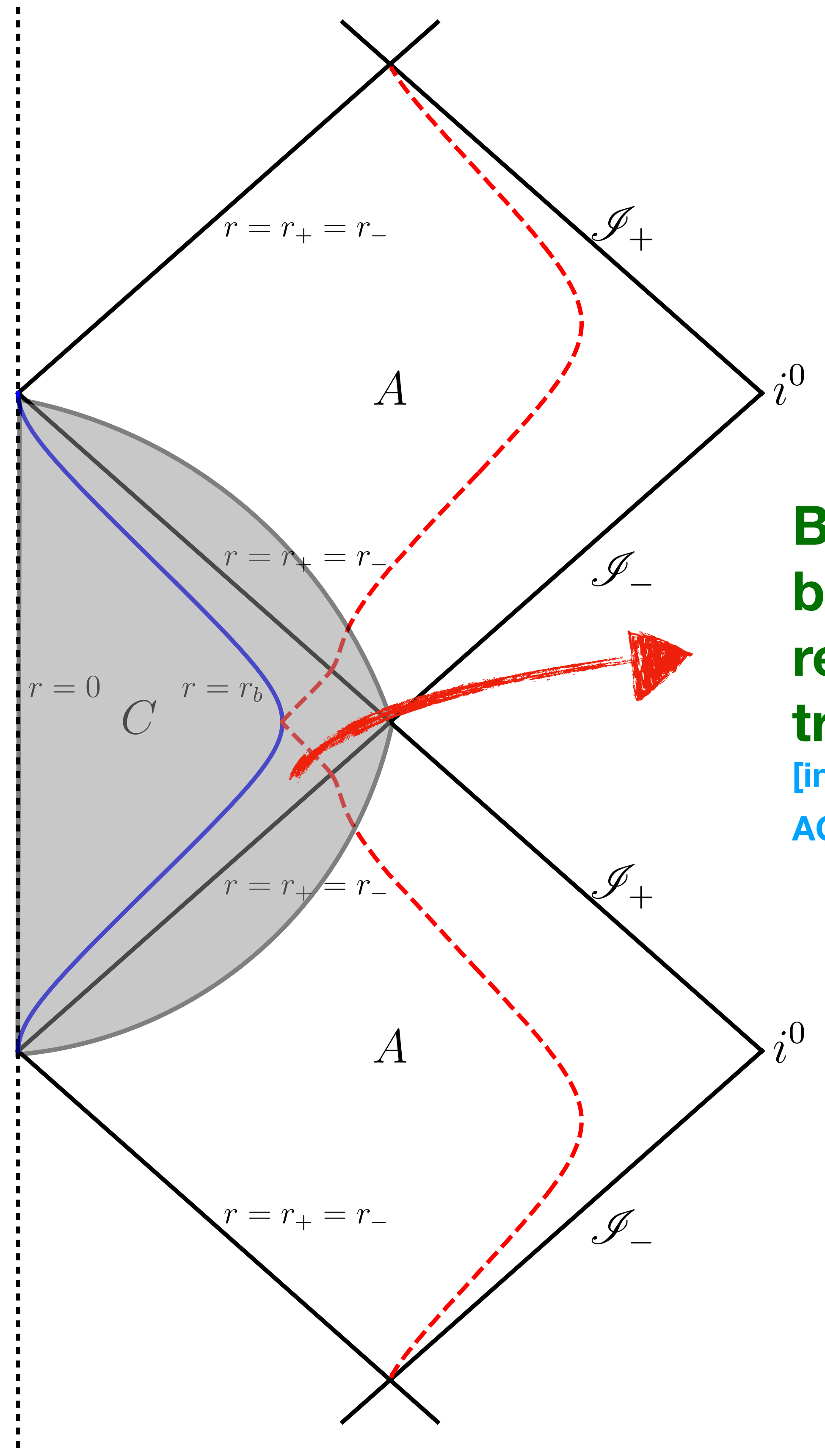


$M > M_{\text{min}}$

Quantum Oppenheimer-Snyder model



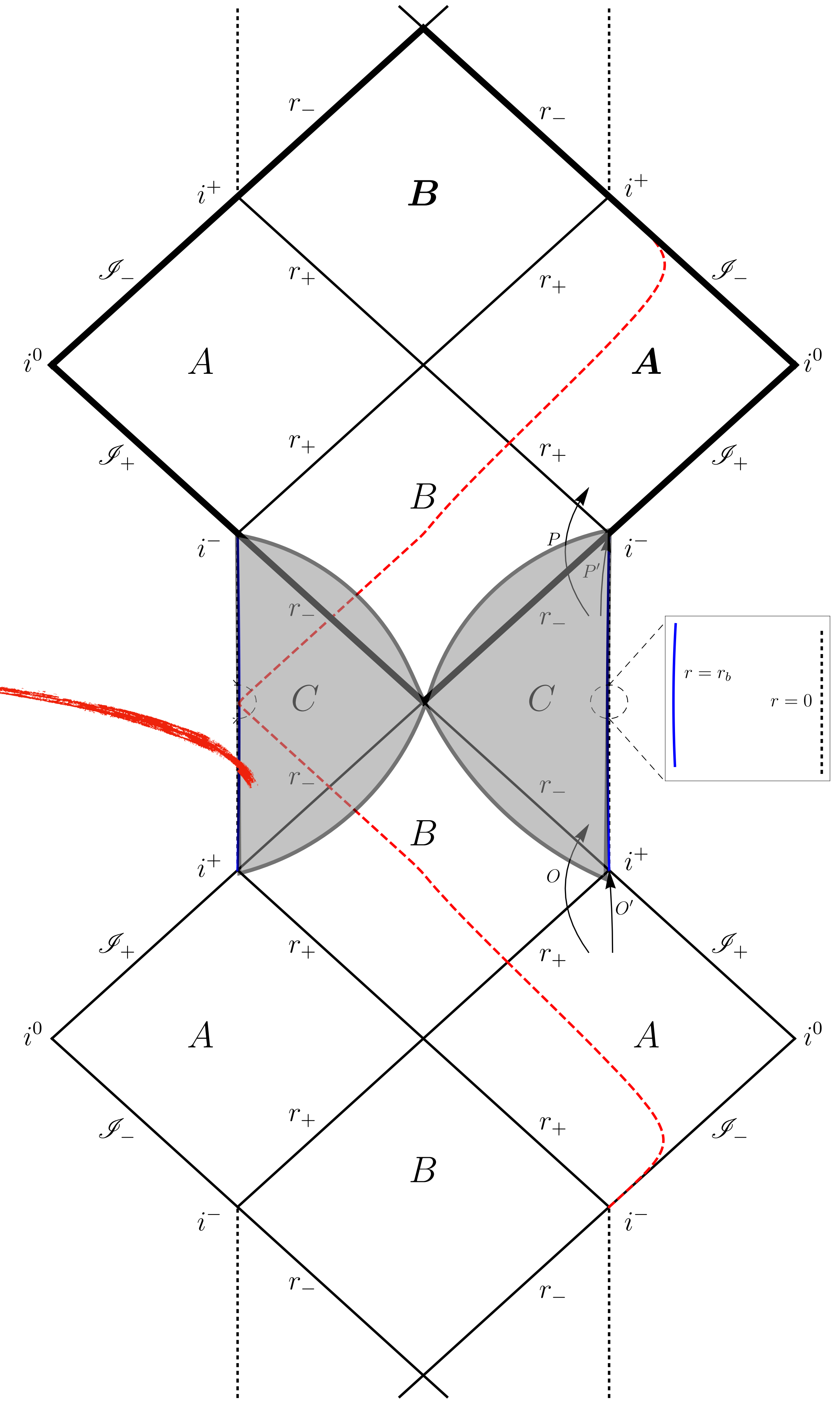
$M < M_{\rm min}$



$M = M_{\rm min}$

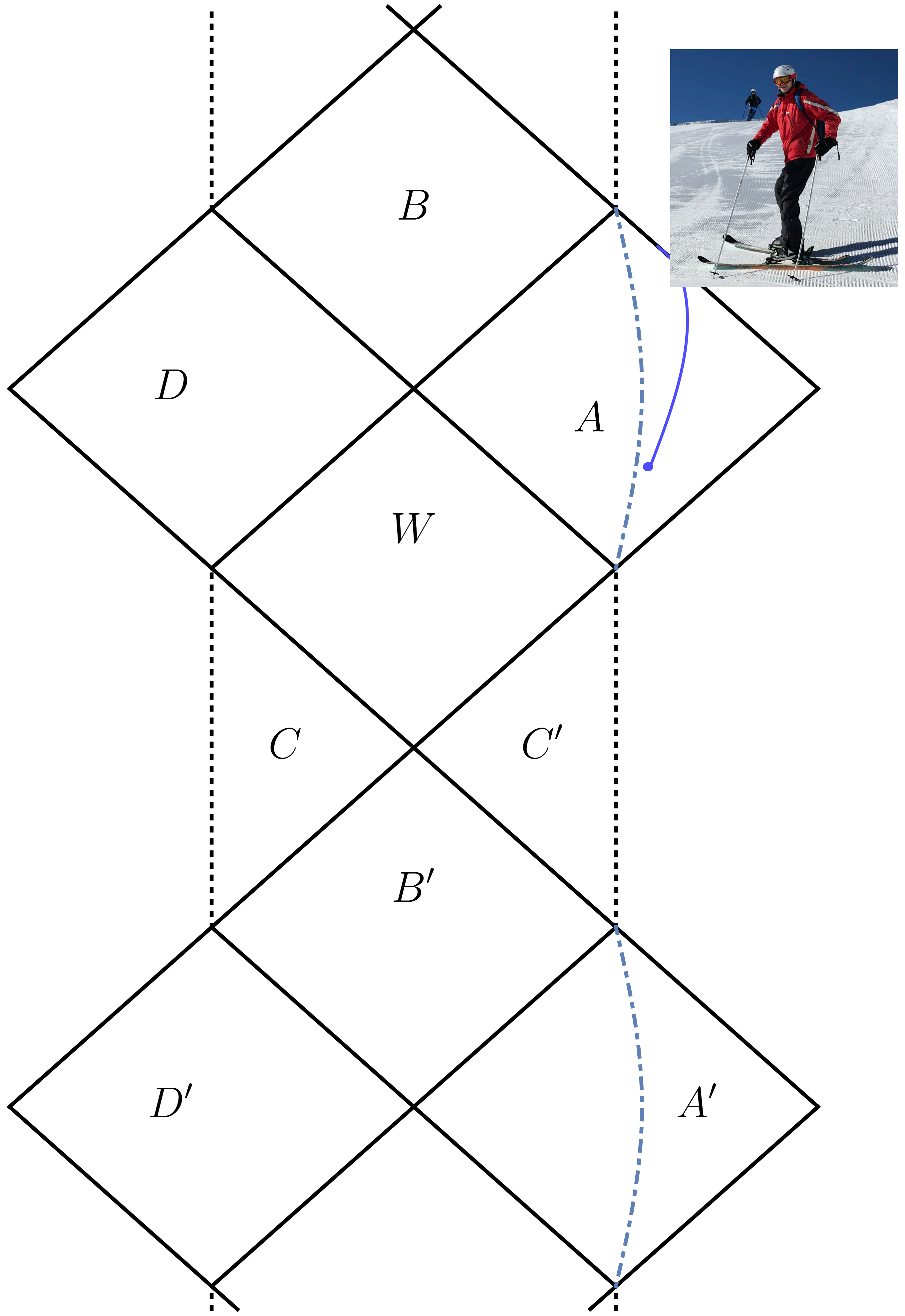
BH-WH transition, but singularity is replaced by a transition region

[in comparison with, e.g., AOS 18']



$M > M_{\rm min}$

Observational effects of quantum correction



[Yang, CZ, Ma 23']

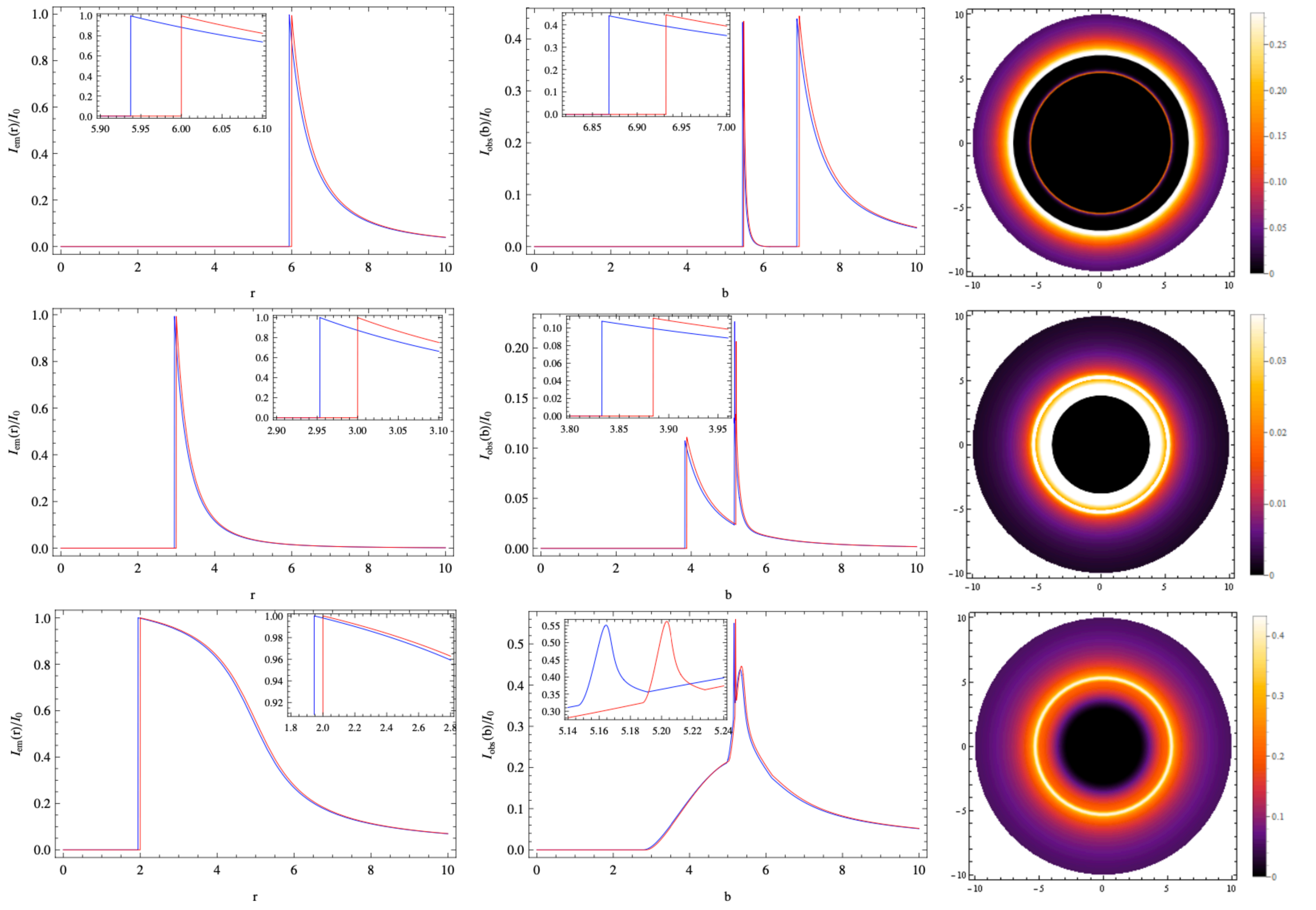
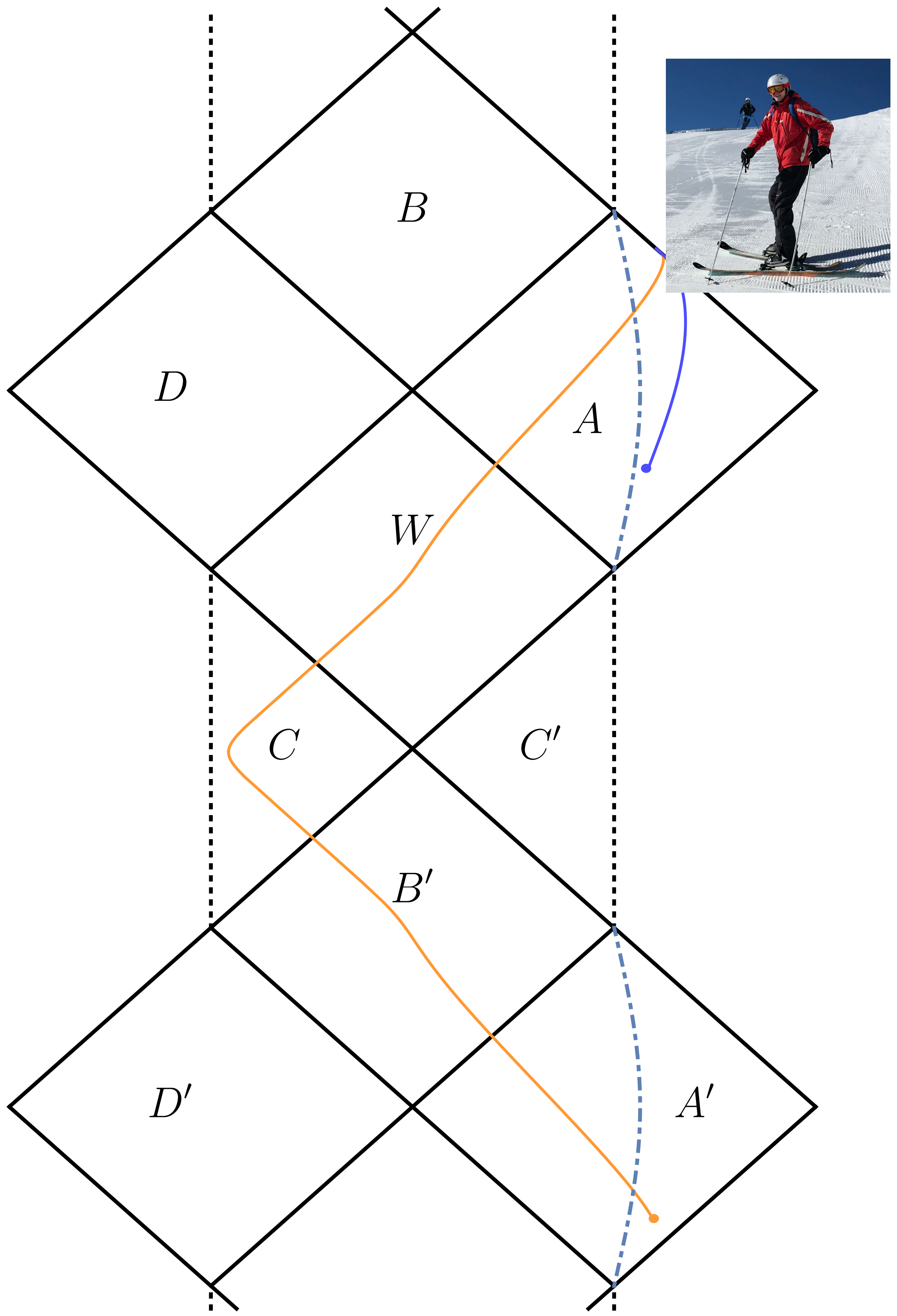
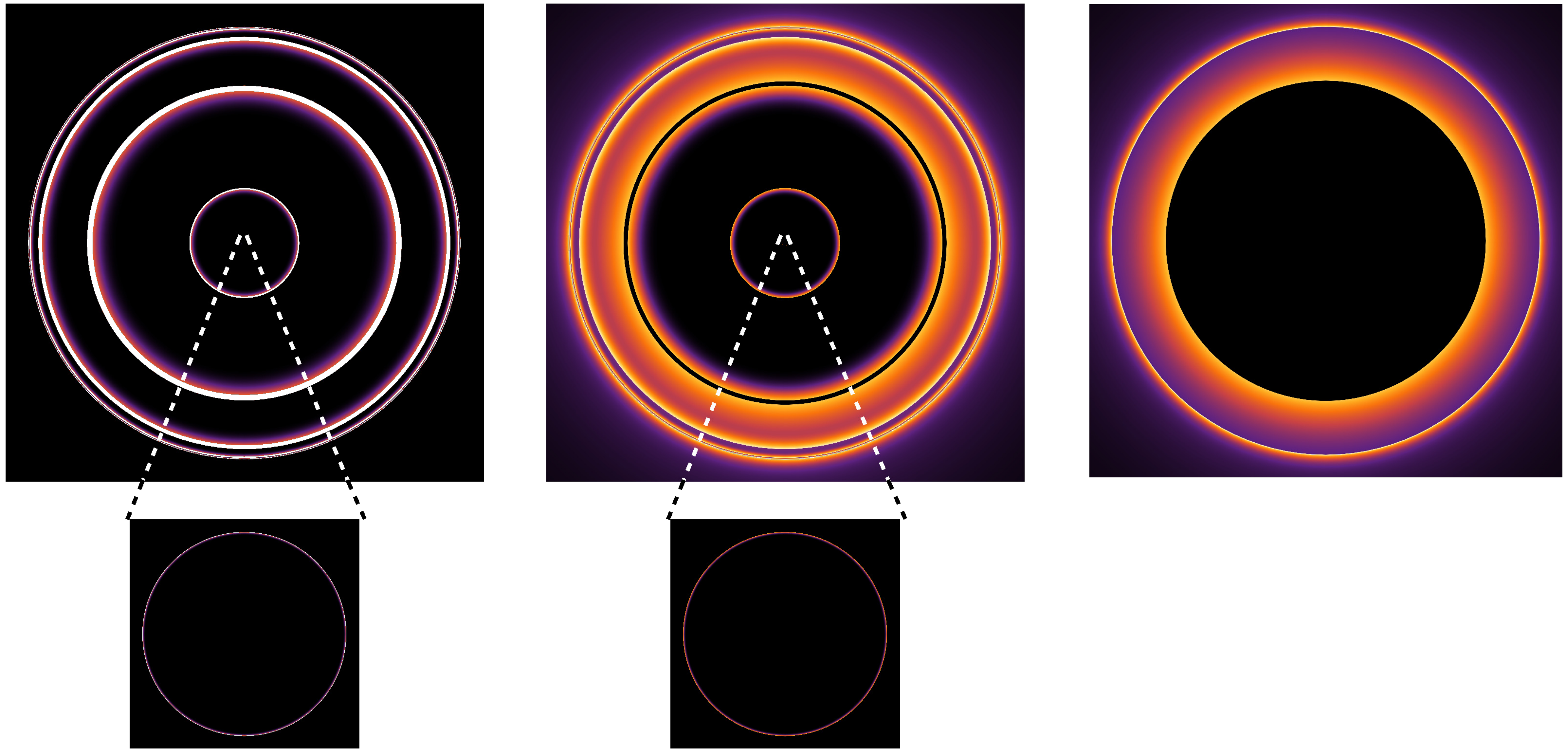


FIG. 6. The observational appearances of the thin disk near the BHs with the three different profiles. In each row, the first two panels show the emission intensity $I_{\rm em}/I_0$ and observational intensity $I_{\rm obs}/I_0$, normalized to the maximum value I_0, of a thin disk near the quantum-corrected BH (blue) compared to those of the Schwarzschild BH (red), and the third panel depicts the density plot of $I_{\rm obs}/I_0$ of a thin disk near the quantum-corrected BH. The parameters are $R_s = 2$, $\gamma = 1$ and $\Delta = 0.1$.

Observational effects of quantum correction



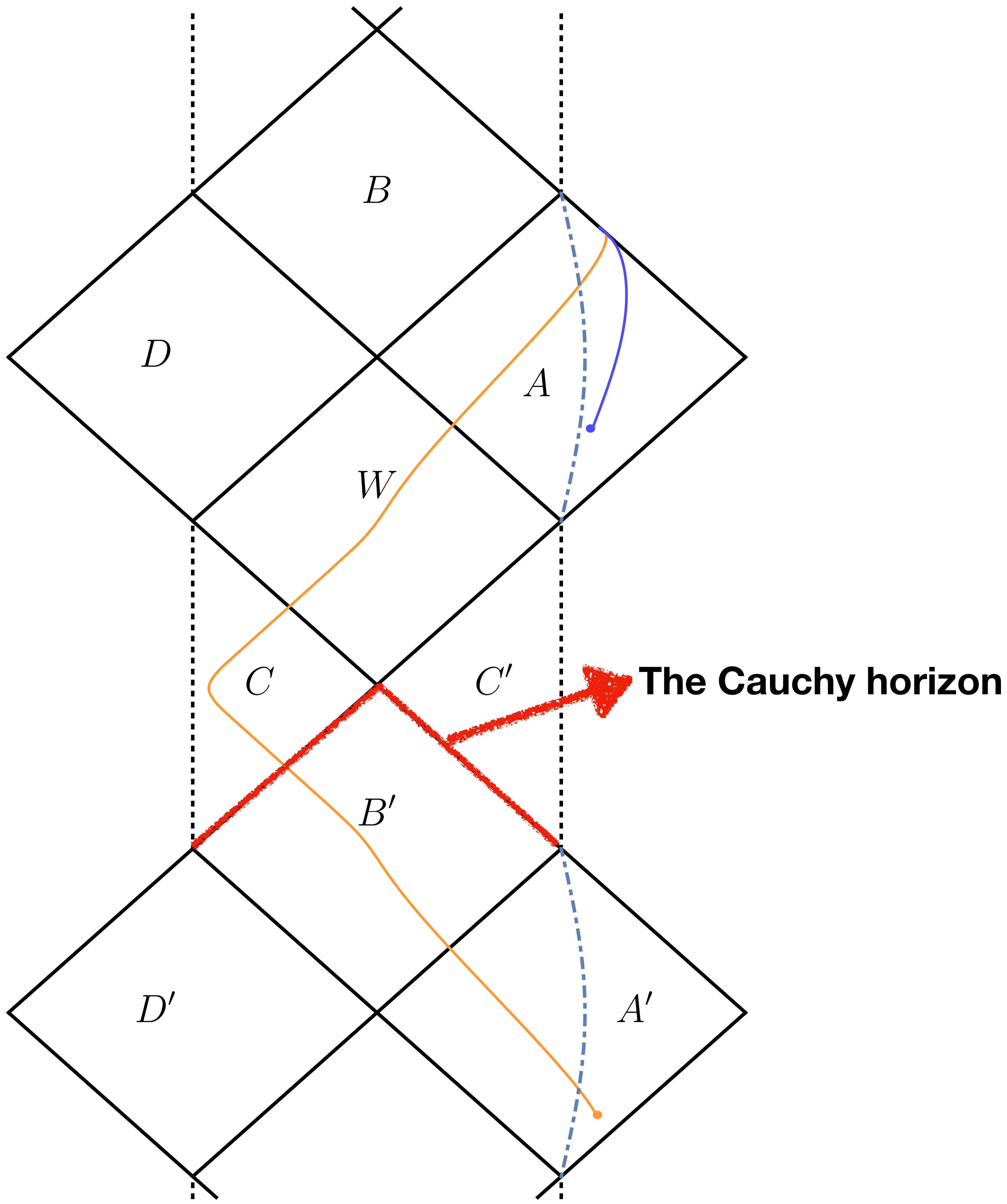
[CZ, Ma, Yang 23’]



By measuring the position and width of the light rings, we could get the details of the quantum correction.

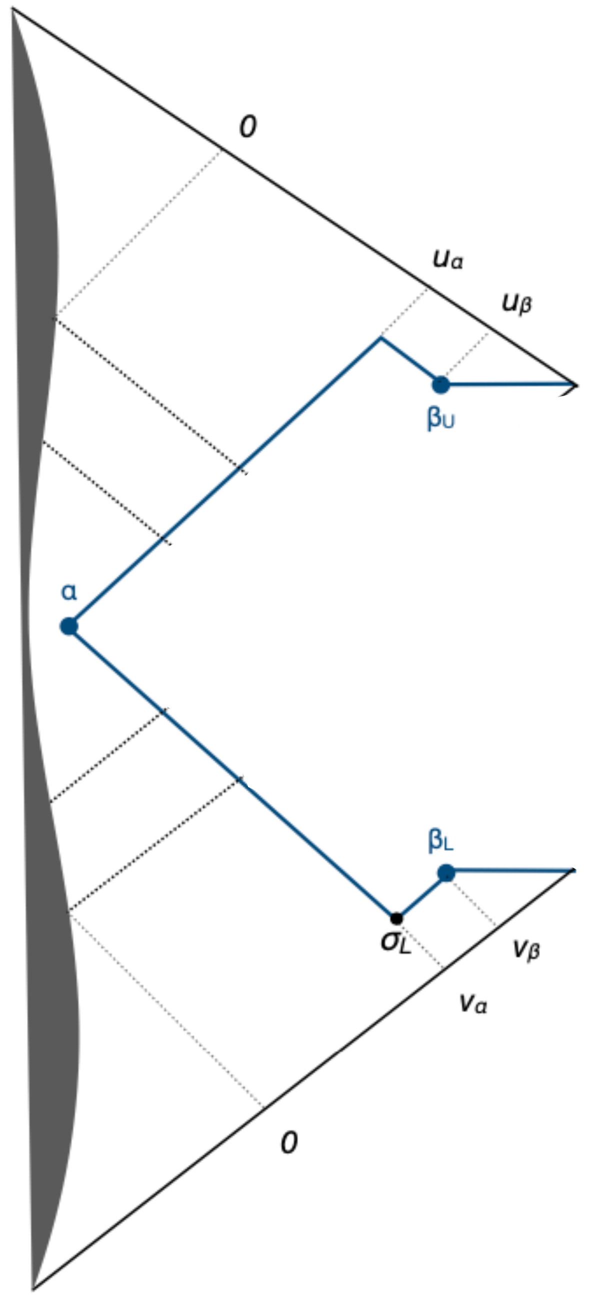
[see Cao, Li, Liu, Zhou 24’ for similar work in regular BH]

BH model with spinfoam



While the spacetime offers distinct advantages, it is not without debates:
The existence of Cauchy horizon implies that the spacetime could be unstable under perturbation [Cao, Li et.al. 23' and 24', Shao, CZ, et.al. (2023)].

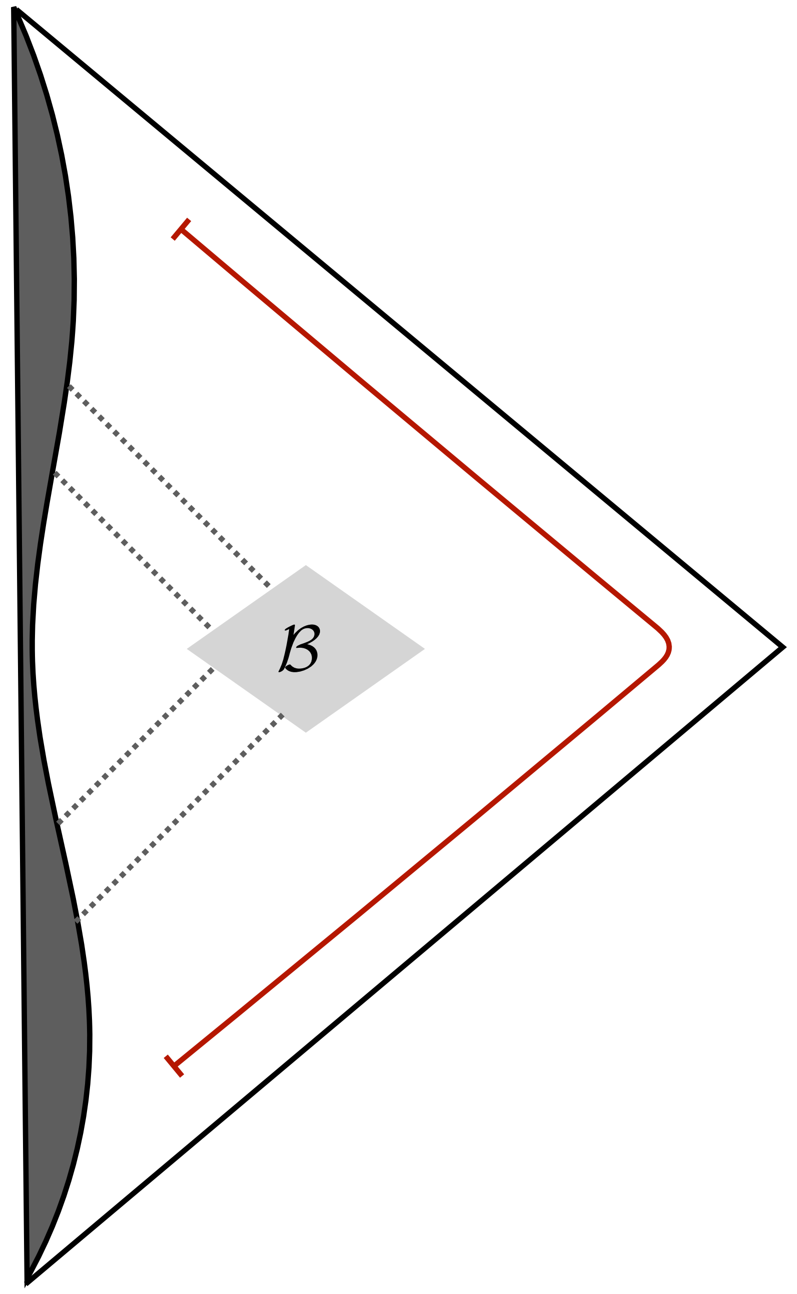
BH model with spinfoam



Consider the QG effect at
the end of evaporation

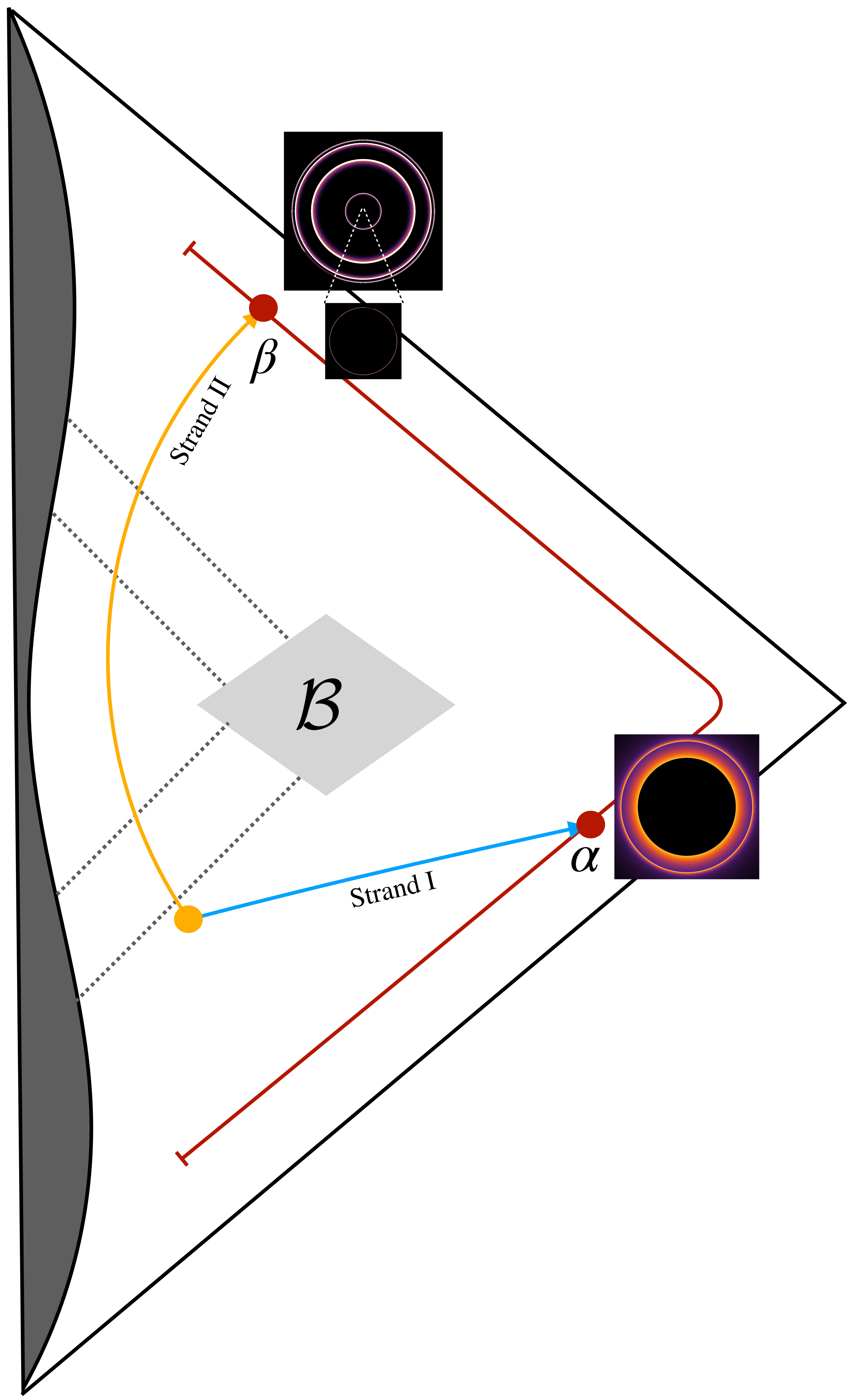
[Han, Rovelli & Soltani 23']

[Carlo's lecture and Farshid's talk]



- The metric is locally the same as ours except for the B-Region in the new spacetime;
- No Cauchy horizon.

BH model with spinfoam



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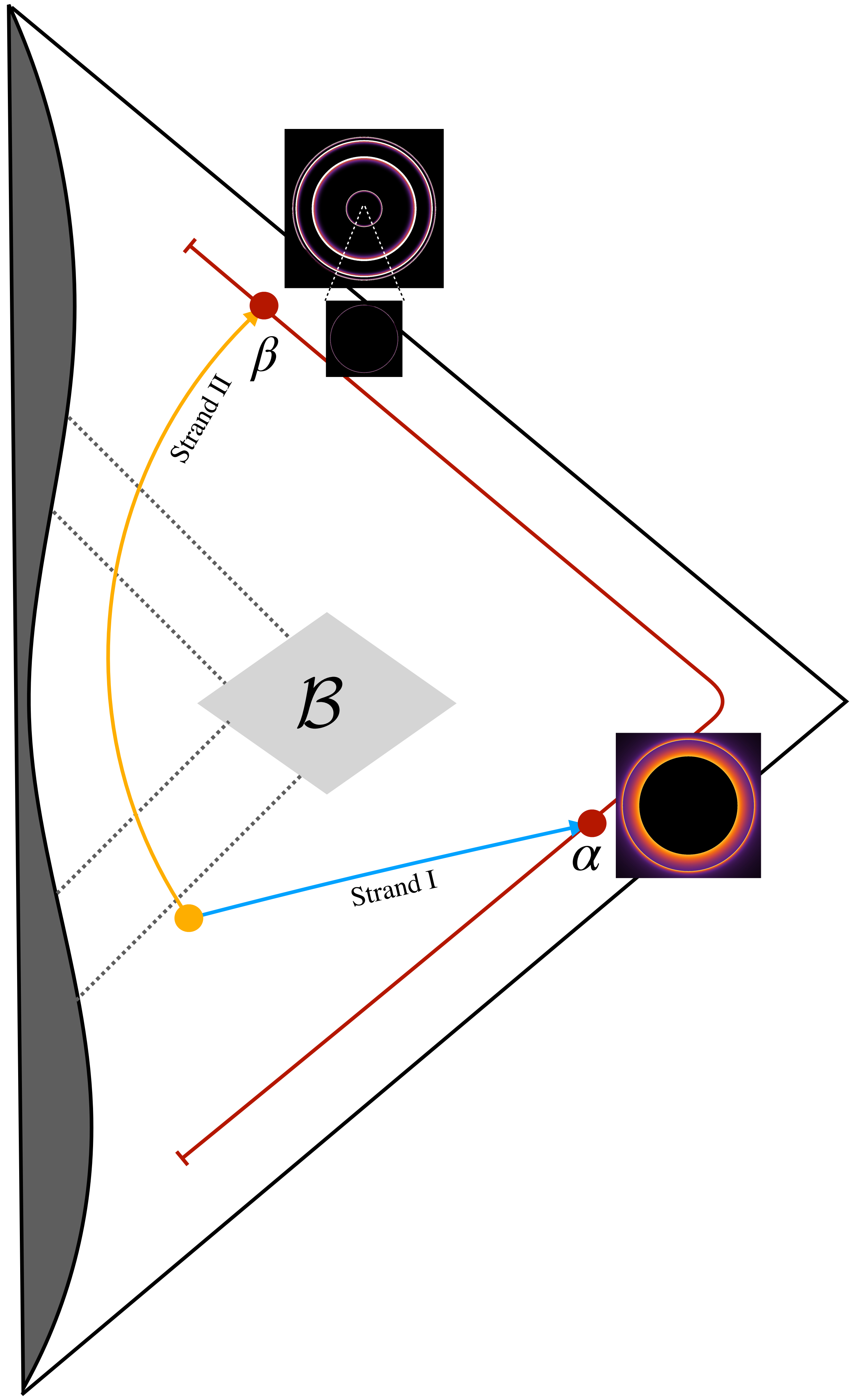
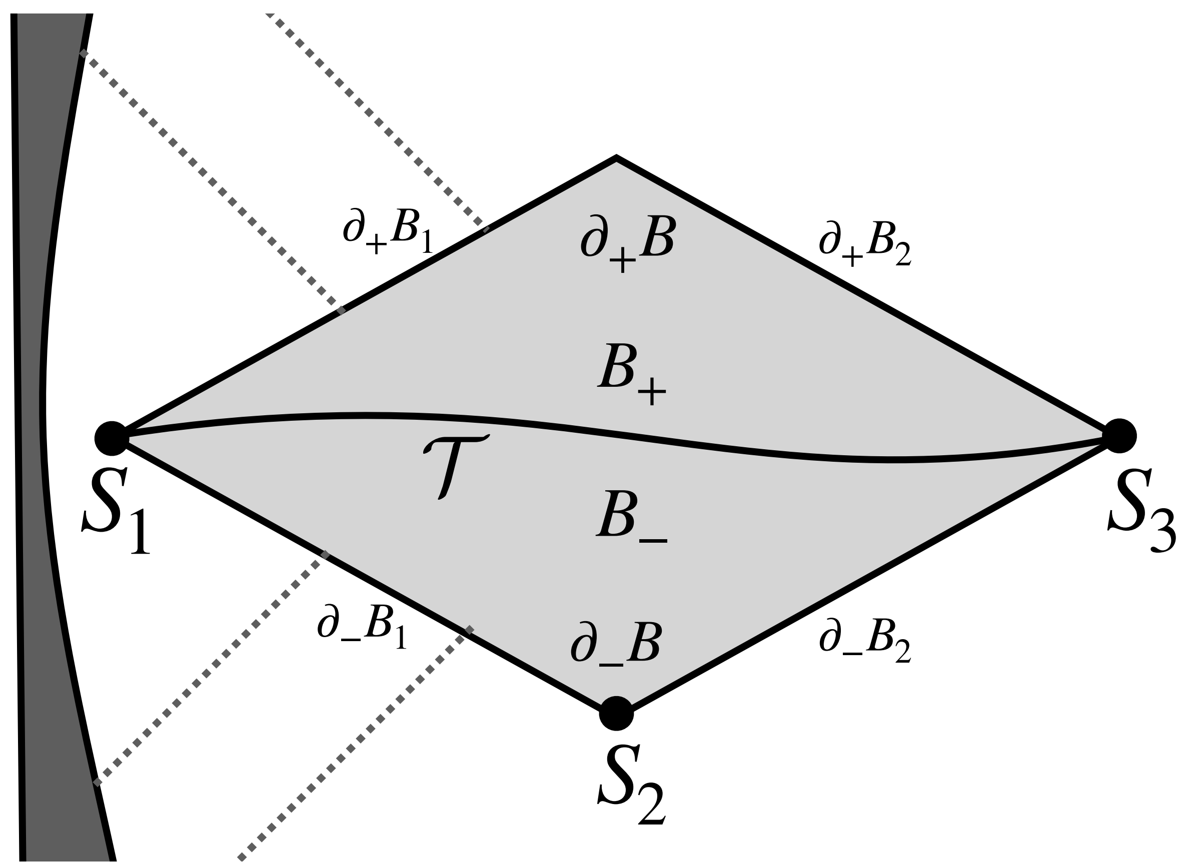


What is the dynamics in the B region?

BH model with spinfoam

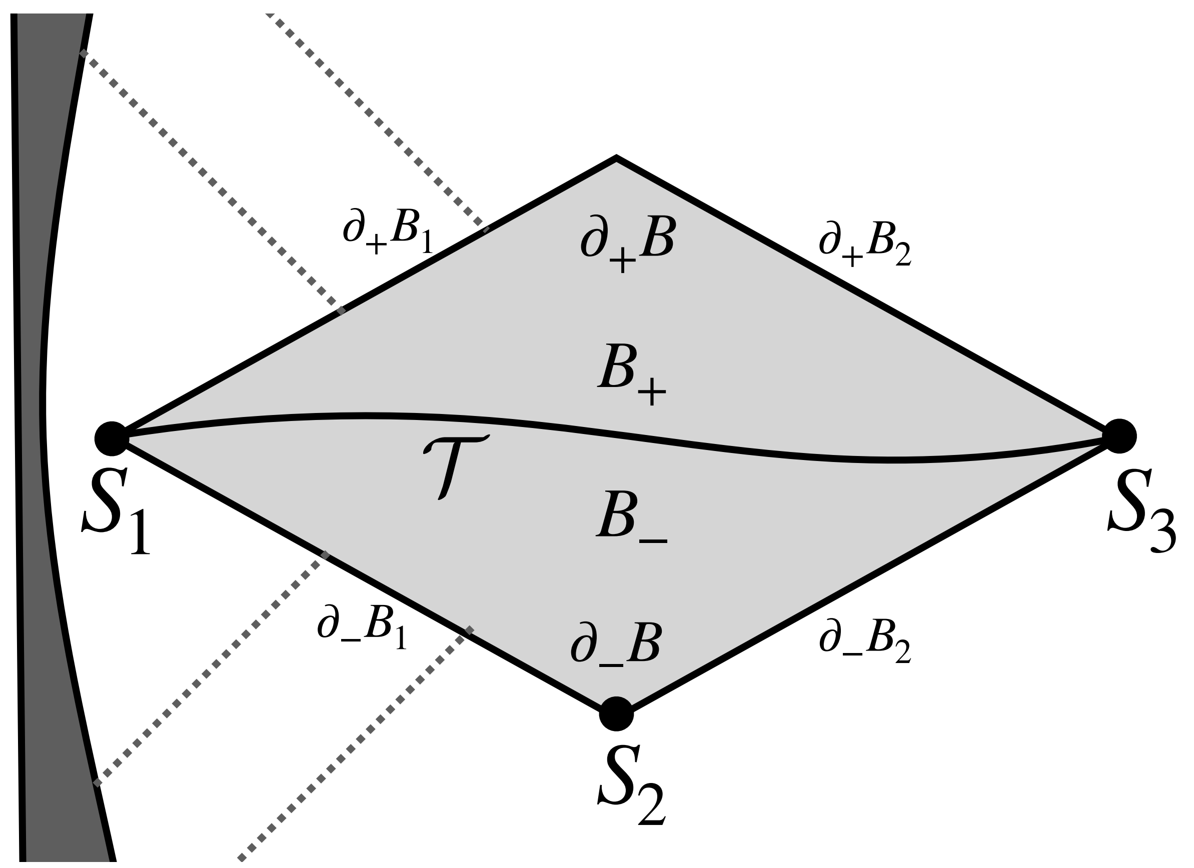


The dynamics of B region is governed by the spinfoam model [Carlo's lecture]

The SF amplitude can be numerical calculated with various algorithm [Hongguang's lecture]**:**
Small spin regime: e.g. Soltani, Rovelli & Martin-Dussaud 21', Donà & Frisoni 23', Pietro's talk;
Large spin regime: Han, Liu & Qu 23', Dongxue's talk;

BH model with spinfoam

[Han, Qu & CZ 24']



- ∂B is located in the semiclassical region, so that the boundary state can be chosen as the coherent state "labelled" by (e_a^i, K_a^i) with spread t.
- We consider the amplitude as $t \to 0$, equivalent as $j \to \infty$;
- In LQG, $\pm e_a^i$ are regarded as different states due to the $\mathrm{SU}(2)$ gauge;
- $\pm e_a^i$ give the same 3-D metric q_{ab};
- The boundary state is proposed as the superposition

$$\left(\psi_{(e_-,K_-)} + \psi_{(-e_-,K_-)}\right) \otimes \left(\psi_{(e_+,K_+)} + \psi_{(-e_+,K_+)}\right)$$

$$A = A\left(\psi_{(K_+,e_+)} \otimes \psi_{(K_-,e_-)}\right) + A\left(\psi_{(K_+,-e_+)} \otimes \psi_{(K_-,e_-)}\right) + A\left(\psi_{(K_+,e_+)} \otimes \psi_{(K_-,-e_-)}\right) + A\left(\psi_{(K_+,-e_+)} \otimes \psi_{(K_-,-e_-)}\right)$$

- We consider a non-degenerate 2-complex containing 56 vertices in our work;
- The first two terms dominate the amplitude;
- The first two terms imply the transition $\pm e_- \to \pm e_+$ with $\det(e_+) = -\det(e_-)$;
- Tunneling between opposite orientations accompanying the BH-WH transition;
- The value of the effective action in the amplitude is computed with the results:

$$S^{(++)} = -0.0458193513442056, S^{(--)} = -0.0458193513442275,$$

where the parameter is chosen as $t = 1/246.34$, and $GM = 2 \times 10^5 \sqrt{\beta\kappa\hbar}, \quad \beta = \dfrac{1}{10}$.

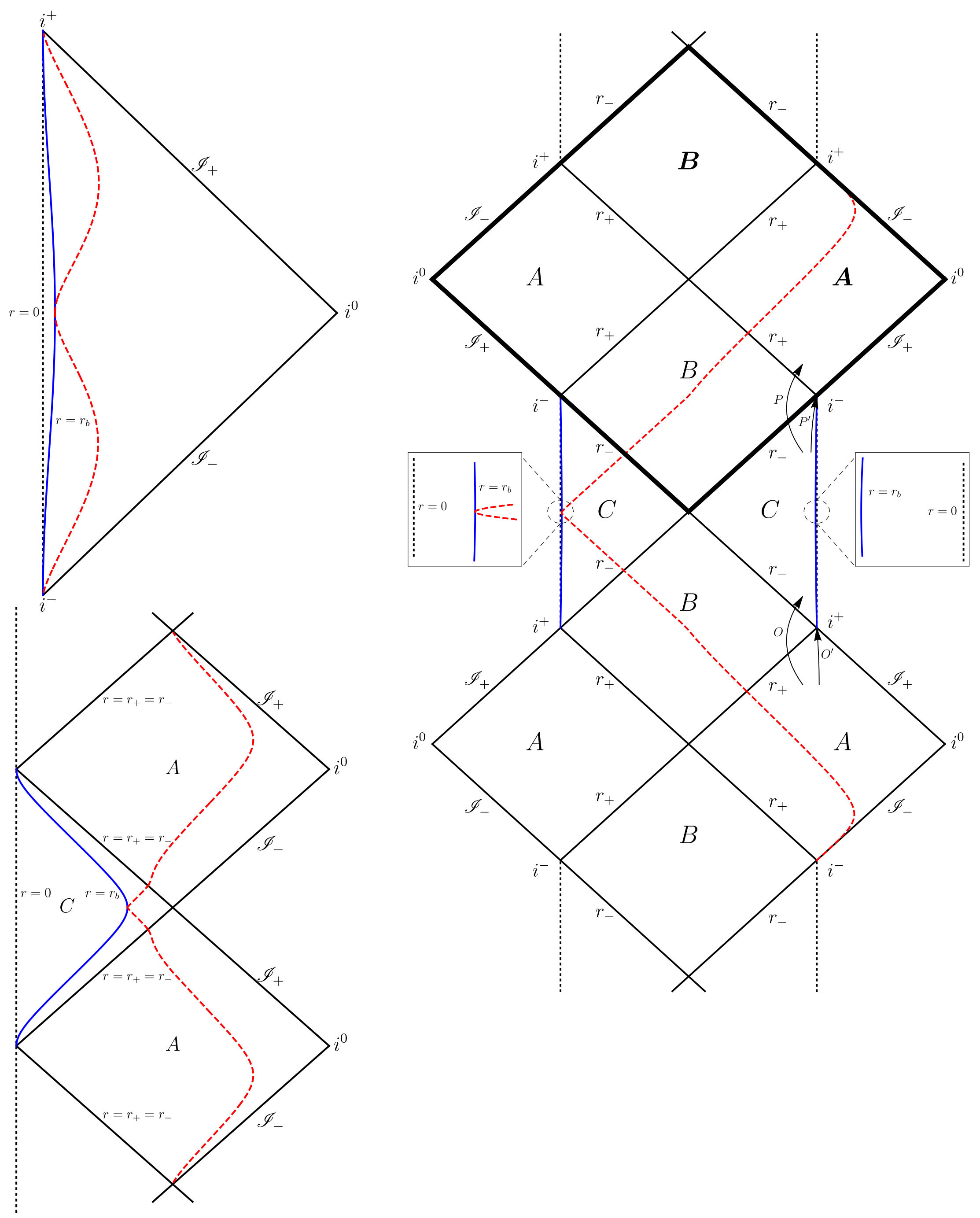
[https://github.com/czhangUW/BH2WHTranstionInSF]

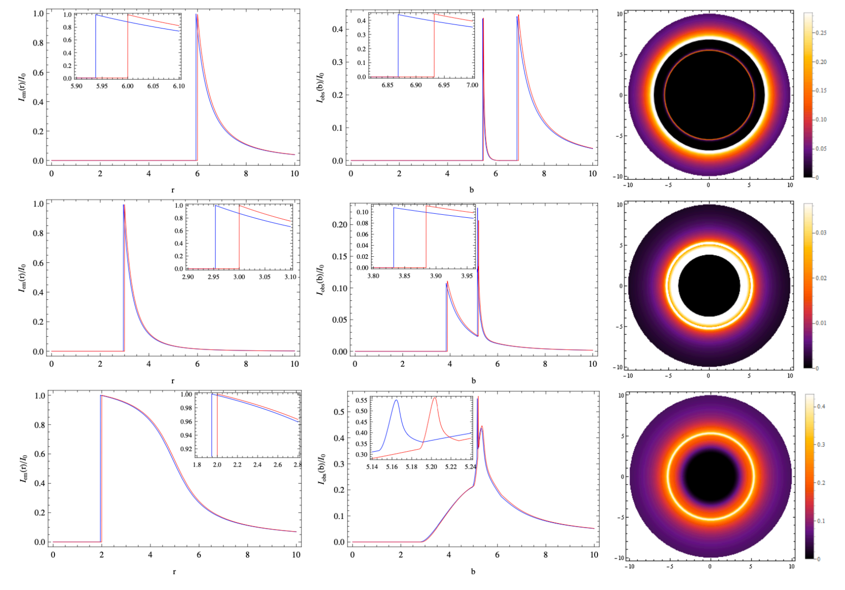
Summary

We introduced our works related to the quantum OS model with the results:

$$ds^2 = -f(r)dt^2 + g(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$f(r) = g(r) = 1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}$$





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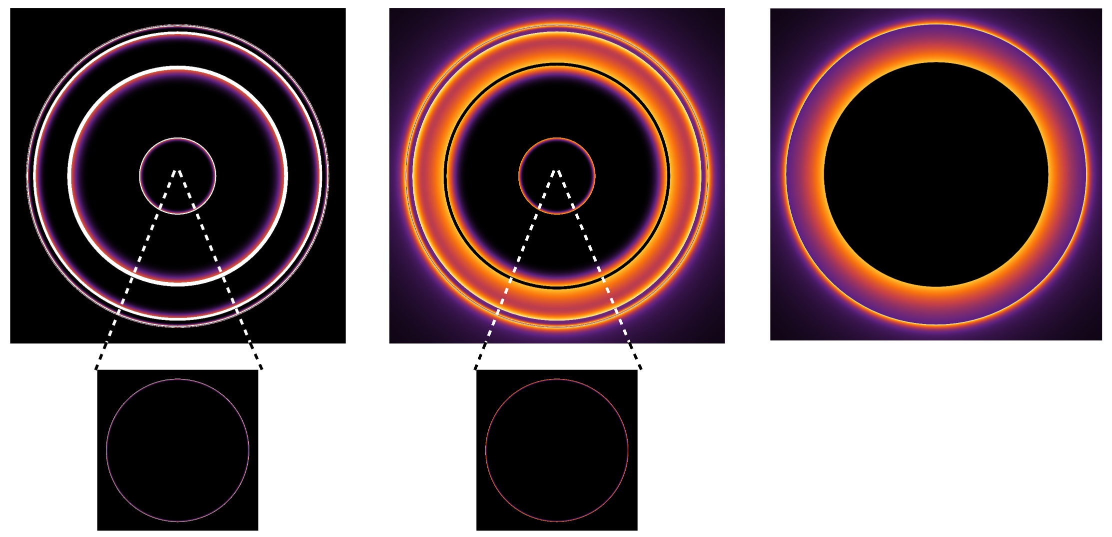

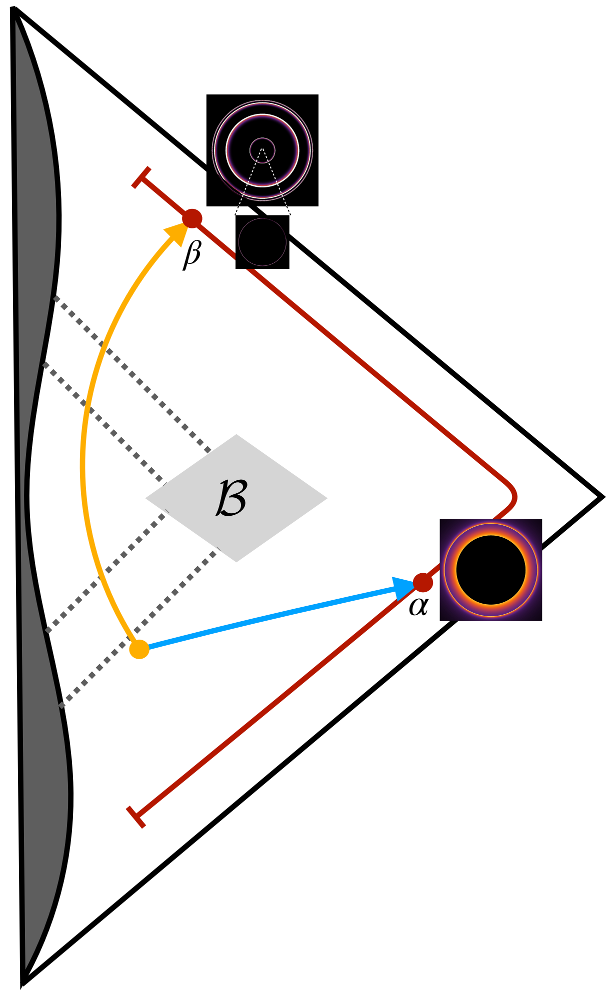

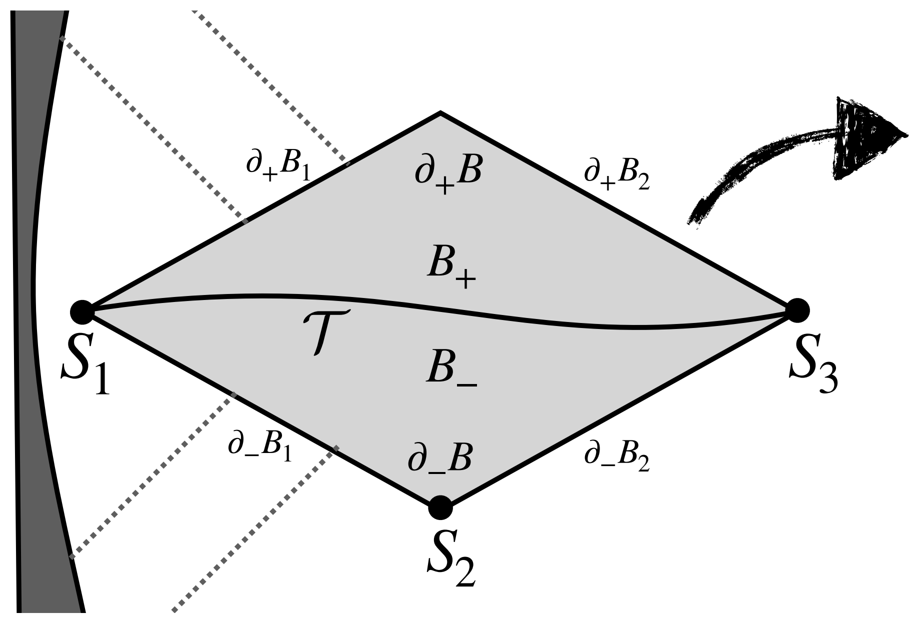

The SF dynamics with the complex critical point method

Thank you for your attention !