Horizons and Null Infinity

Abhay Ashtekar
Physics Department, Penn State; & Perimeter Institute

Organization

1. Weakly Isolated Horizons (WIHs)
2. Horizons of Black Holes in Equilibrium
3. Null Infinity as a WIH
4. Opportunities for the LQG Community & References

(First 3 parts: Joint work with Simone Speziale: 2401.15618; 2402.17977)

Loops 24, Ft. Lauderdale, May 6-10

0. Preamble

- A change of gears vis a vis previous 3 talks on black holes.

Goal: To point directions for future work by the LQG community

to address the issue of entanglement and information recovery that has not received sufficient attention in LQG. (Emphasis will be on null surfaces.)

- Horizons, Δ, of BHs in equilibrium, and Null Infinity, $\mathscr{I}^+$, of asymptotically flat space-times are null 3-manifolds but have drastically different physical connotations: Typically Δ lies in the strong field region and there is no radiation flux across it, while $\mathscr{I}^+$ lies in the asymptotic, weak field regime with possibly large fluxes of radiation across it! Yet, surprisingly, they share a large number of geometric properties, making them both Weakly Isolated Horizons (WIHs) $\mathfrak{h}$. At first this is quite shocking.

- But the dramatic differences in their physics emerge from the same central equation governing the dynamics of the connection on WIHs $\mathfrak{h}$. This unified perspective helps relate properties of black hole horizons and null infinity in the classical theory. Since the gravitational connection plays a central role in this discussion, this framework well-tailored for LQG. Suitable extensions of well-established results on quantum geometry, and of the ongoing work on null surfaces, will provide sharp tools to correlate observables at the horizon and at null infinity and address issues related to entanglement during BH evaporation.

1. Geometrical WIHs

• A *Non Expanding Horizon (NEH)* in 4-d Space-time $(\bar{M}, \bar{g}_{ab})$ is a null submanifold $\mathfrak{h}$, topologically $\mathbb{S}^2 \times \mathbb{R}$, such that:

(i) Every null normal $\bar{k}^a$ to $\mathfrak{h}$ is expansion-free, $\theta_{\bar{k}} = 0$;   and,

(ii) On $\mathfrak{h}$ the Ricci tensor satisfies $\bar{R}_a{}^b \bar{k}^a = \alpha \bar{k}^b$ for some function α.

• Raychaudhuri Eq. implies that shear of $\bar{k}^a$ also vanishes $\Rightarrow$ the intrinsic (degenerate) metric $\bar{q}_{ab}$ satisfies $\mathcal{L}_{\bar{k}} \bar{q}_{ab} = 0$; it is 'time independent'. As a result, by pull-back, the space-time derivative operator $\bar{\nabla}$ induces a canonical intrinsic derivative $\bar{D}$ on $\mathfrak{h}$: $\underleftarrow{\bar{\nabla}} = \bar{D}$. It satisfies: $\bar{D}_a \bar{q}_{ab} = 0$, and, $\bar{D}_a \bar{k}^b = \bar{\omega}_a \, \bar{k}^b$ for some 1-form $\bar{\omega}_a$.

• We can always restrict ourselves to geodesic null normals $\bar{k}^a \bar{D}_a \bar{k}^b = 0$. Then NEH conditions $\Rightarrow$ $\mathcal{L}_{\bar{k}} \bar{\omega}_a = 0$; $\bar{\omega}_a$ is also 'time-independent'. Furthermore, we can now severely restrict the rescaling freedom in $\bar{k}^a$ by demanding that $\bar{\omega}_a$ be divergence-free, i.e, $\bar{q}^{ab} \bar{D}_a \bar{\omega}_b = 0$. Only remaining freedom: $\bar{k}^a \rightarrow c\bar{k}^a$ where c is a positive constant.

• Thus, every NEH can be naturally equipped with a (small) equivalence class of null normals $[\bar{k}^a]$ (where $\bar{k}'^a \approx \bar{k}^a$ iff $\bar{k}'^a = c\bar{k}^a$) such that $\mathcal{L}_{\bar{k}} \bar{\omega}_a = 0$. An NEH equipped with such an equivalence class $[\bar{k}^a]$ is called a *Weakly Isolated Horizon (WIH)*. The triplet $([\bar{k}^a], \bar{q}_{ab}, \bar{D})$ is said to constitute the geometry of the WIH $\mathfrak{h}$.

Time Dependence on Geometric WIHs

- On any geometrical WIH $(\mathfrak{h}, [\bar{k}^a], \bar{q}_{ab}, \bar{D})$, fields $\bar{q}_{ab}, \bar{\omega}_a$ are time independent :

$$\dot{\bar{q}}_{ab} := \mathcal{L}_{\bar{k}}\, \bar{q}_{ab} = 0 \quad \text{and} \quad \dot{\bar{\omega}}_a := \mathcal{L}_{\bar{k}}\, \bar{\omega}_a = 0 \;\Leftrightarrow\; \dot{\bar{D}}_a \bar{k}^b = 0.$$

Furthermore, one can show that, given any horizontal 1-form h_a (i.e. $h_a \bar{k}^a = 0$),

$$\dot{\bar{D}}_a h_b := (\mathcal{L}_{\bar{k}} \bar{D}_a - \bar{D}_a \mathcal{L}_{\bar{k}}) h_b = 0.$$

- Thus, time dependence of $\bar{D}$ is completely determined by $\dot{D}_a \bar{j}_b$ for any 1-form $\bar{j}_b$ satisfying $\bar{j}_a \bar{k}^a = -1$. It is given by:

$$\dot{\bar{D}}_a \bar{j}_b = \left(\bar{D}_a \bar{\omega}_b + \bar{\omega}_a \bar{\omega}_b\right) + \left(\bar{k}^c\, \bar{C}_{\underleftarrow{cab}}{}^d\, \bar{j}_d + \tfrac{1}{2} \bar{S}_{\underleftarrow{ab}} + \alpha\, \bar{q}_{ab}\right) \qquad (1).$$

($\bar{C}_{abc}{}^d$ and $\bar{S}_{ab}$ are the 4-d Weyl and 4-d Schouten tensors and $\bar{R}_a{}^b \bar{k}^a = \alpha\, \bar{k}^b$.) None of the terms on the right hand side vanishes on $\mathfrak{h}$! Thus part of the geometry of a WIH is dynamical. This dynamics is driven by the pull-back to $\mathfrak{h}$ of the 4-d curvature tensor $\bar{R}_{abc}{}^d$, since $\bar{\omega}_a$ is part of $\bar{D}$.

- So far no field equations have been imposed; we only have a geometric condition on $\bar{R}_a{}^b$. So Eq. (1) holds both on BH horizons Δ and null infinity $\mathscr{I}^+$. We will find that the diametrically opposite physics of Δ and $\mathscr{I}^+$ emerges from the fact that field equations imply that complementary terms on the right side of (1) vanish in the two cases.

2. BH WIHs Δ

- To discuss these horizons, let us now assume Einstein's vacuum field equations on them. Following literature, we will drop 'bars' over symbols $\bar{g}_{ab}$, $\bar{R}_{abc}{}^{d}$, $\bar{D}$ and use the notation: $\mathfrak{h} \to \Delta;$ $\bar{k}^a \to \ell^a;$ $\bar{j}_b \to n_a.$

- Now the Ricci tensor terms in equation (1) for $\dot{D}$ vanish. Furthermore, already on geometric WIHs $\mathfrak{h}$, the the Weyl term that enters the equation (1) for $\dot{D}$ is given by $\ell^c\, \bar{C}_{c\underleftarrow{ab}}{}^{d}\, n_d = -(\frac{1}{4}\mathcal{R}\, q_{ab} + D_{[a}\omega_{b]})$ (with $\mathcal{R}$ the 2-d scalar curvature). Therefore,

$$\dot{D}_a n_b = D_{(a}\omega_{b)} + \omega_a\omega_b - \tfrac{1}{4}\mathcal{R}\, q_{ab} \qquad (2)$$

- Thus, even on BH WIHs Δ, the derivative operator D is time-dependent! But this dependence is highly constrained, because the right side of (2), is time independent. $(\mathcal{L}_\ell q_{ab} = 0, \quad \mathcal{L}_\ell \omega = 0,\ (\text{and } \omega_a \ell^a = 0).)$ Therefore, (q_{ab}, D) on Δ are completely determined by its values on a 2-sphere cross-section; they are 'corner data', representing 'Coulombic fields'. There are no 3-d degrees of freedom that are hallmarks of radiation. The LQG quantum geometry provides novel insights for spectra of Coulombic observables by making their spectra discrete.

3. Asymptotic WIH $\mathscr{I}^+$

• A physical space-time $(M,\, g_{ab})$ is said to be asymptotically flat at future at future infinity if it admits a conformal completion $(\hat{M},\, \hat{g}_{ab})$, where $\hat{M} = M \cup \mathscr{I}^+$ is a manifold with a boundary $\mathscr{I}^+$, topologically $\mathbb{S}^2 \times \mathbb{R}$, and $\hat{g}_{ab} = \Omega^2 g_{ab}$ on M s.t.

(i) At $\mathscr{I}^+$, we have $\Omega \hat{=} 0$ and $\hat{\nabla}_a \Omega \hat{\neq} 0$; and,

(ii) g_{ab} satisfies Einstein's equations $G_{ab} = 8\pi G\, T_{ab}$, with $\Omega^{-2}\, T_{ab}$ admitting a smooth limit to $\mathscr{I}^+$.

These conditions imply: (a) $\mathscr{I}^+$ is null with null normal $\hat{n}^a := \hat{\nabla}^a \Omega$; and, (b) we can always choose Ω such that $\hat{\nabla}_a \hat{n}^a = 0$. As is standard, let us work with these divergence-free conformal frames. (If in addition the conformal factor is such that the (degenerate) metric on $\mathscr{I}$ is a unit 2-sphere metric, we are in a Bondi conformal frame: $g_{ab} \mathrm{d}x^a \mathrm{d}x^b \hat{=} 2\mathrm{d}u\mathrm{d}r + \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2$).

• Conformal Einstein's equations at $\mathscr{I}^+$ imply: (1) $\hat{R}_a{}^b \hat{n}^b \propto \hat{n}^a$ at $\mathscr{I}^+$, and, (2) $\hat{\nabla}_a \hat{n}^b = 0$ at $\mathscr{I}^+$. Thus $\mathscr{I}^+$ is a null 3-manifold, for which the expansion $\theta_{\hat{n}}$ of the null normal vanishes, and the Ricci tensor is such that it is an NEH. Also, condition (2) $\Rightarrow$ $\hat{\nabla}_a \hat{n}^b \equiv \hat{\omega}_a \hat{n}^b = 0$, whence $\hat{\omega}_a = 0$, and $(\mathscr{I}^+,\, \hat{n}^a)$ is a WIH.

Thus $(\mathscr{I}^+,\, \hat{n}^a,\, \hat{q}_{ab},\, \hat{D})$ is a WIH in the conformally completed space-time $(\hat{M}, \hat{g}_{ab})$.

Time dependence of the WIH geometry of $\mathscr{I}^+$

• Since $\mathscr{I}^+$ is a WIH, the metric $\hat{q}_{ab}$ is time independent. Recall that on any WIH, the time dependence of $\bar{D}$ is given by

$$\dot{\bar{D}}_a \bar{j}_b = \bar{D}_a \bar{\omega}_b + \bar{\omega}_a \bar{\omega}_b + \bar{k}^c\, \bar{C}_{\underleftarrow{cab}}{}^d\, \bar{j}_d + \tfrac{1}{2} \bar{S}_{\underleftarrow{ab}} + \alpha\, \bar{q}_{ab} \qquad (1).$$

In the notation used at $\mathscr{I}^+$: $\bar{k}^a \to \hat{n}^a,\ \bar{j}_b \to \hat{\ell}_b,\ \bar{D} \to \hat{D},\ \bar{\omega}_a \to \hat{\omega}_a.$

Interestingly, since $\hat{\omega}_a = 0,$ and $\hat{C}_{abc}{}^d = 0$ at $\mathscr{I}^+$, all the terms that contribute to $\dot{D}$ on BH horizons Δ now vanish at $\mathscr{I}^+$ and terms that vanish at Δ now survive! Thus, at $\mathscr{I}^+$ we have: $\dot{\hat{D}}_a \hat{\ell}_b = \frac{1}{2} \hat{S}_{\underleftarrow{ab}} + \alpha\, \hat{q}_{ab} \sim \frac{1}{2} \hat{N}_{ab}$

• Thus, the time dependence of the connection $\hat{D}$ at $\mathscr{I}^+$ is driven by the Ricci curvature of $\hat{g}_{ab}$ ($\sim$ Bondi news). Therefore, $\hat{D}$ has 2-degrees of freedom per point of $\mathscr{I}^+$. These are 3-d degrees of freedom, representing the radiative modes. Put differently, while the WIH geometry of Δ, carries only 'coulombic' information contained in the 'corner data', that at $\mathscr{I}^+$ carries 'radiative' information contained on all of $\mathscr{I}^+$. This diametrically opposite physics emerges from the same equation because Einstein's and conformal Einstein's equations set complementary terms in (1) to zero!

• Non-perturbative quantization radiative modes is also available: Asymptotic Quantization. Open Problem: LQG Description of the radiative modes on $\mathscr{I}^+$!

4. Opportunities for the LQG Community

- Physics at BH horizons Δ is drastically different from that at $\mathscr{I}^+$. Yet, both emerge from the same equation (1); in spite of dramatic diversity, there is striking underlying unity! This unity has already led the finding that there are strong correlations between dynamics at the horizon, and at null infinity in the classical theory, opening the field of gravitational wave tomography to read-off horizon dynamics (soon after a merger) from waveforms at null infinity (AA, Khera). In the quantum theory, the framework offers opportunities study correlations between observables at the horizon with those at null infinity, that lie at the heart of the 'information-loss issue'.

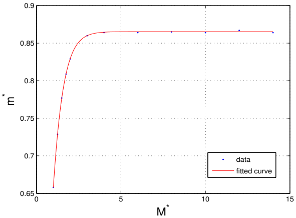
- Specifically, while there are many fascinating results on black hole evaporation in LQG, the focus tends to be on singularity resolution and the structure of the quantum-extended space-time, issues of entanglement and recovery of information, that originally sparked interest in black hole evaporation in the wider physics community, have not received as much attention in LQG. The WIH framework provides a natural springboard to rectify this situation. Because of its emphasis on gravitational connections D on Δ and $\hat{D}$ on $\mathscr{I}^+$, it leads to observables that are well suited to LQG. Investigations of correlations between them should lead to a sharper understanding of entanglement.

Opportunities for the LQG Community: Examples

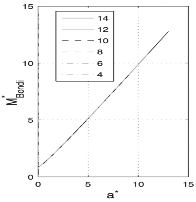
Quantum Geometry of LQG is especially well-suited for "Coulombic" observables:

(i) the Charge associated with the natural horizon time-translation symmetry ξ^a $(= \kappa v \ell^a)$ is $M_\Delta = \frac{\kappa}{8\pi G} A_\Delta$. Since area A_Δ is quantized in LQG, M_Δ would be naturally quantized. Multipoles: observables fully characterizing horizon geometry can also be promoted to operators in LQG.

CGHS Black Holes: Surprising universality in the Semi-classical phase of evaporation



The final Bondi mass vs ADM mass



Universal Correlation: Bondi mass vs horizon area

Macroscopic BHs (with Hawking evaporation time longer than Planck time) have a universal Bondi mass mass of $\mathrm{m}^\star = 0.86\ \mathrm{M_{Pl}}$ (per evaporation channel) at the end of the semi-classical phase. And the Bondi mass is universally correlated with the horizon area throughout the process. Suggests: (i) Universality of $\mathrm{m}^\star$ originates in the area gap! and, (ii) Horizon and $\mathscr{I}^+$ observables are correlated.

Opportunities for the LQG Community:Examples

(ii) Observable at $\mathscr{I}^+$ corresponding to M_Δ is $M_{\rm bms}$ associated with a BMS unit time translation $(\xi^a = \hat{n}^a)$. Even in presence of matter at $\mathscr{I}^+$, $M_{\rm bms}$ is expressible purely in terms of geometry, and therefore again well suited for LQG. Its usual expressions (involving Weyl curvature, shear and news) seem too complicated for the LQG Quantum Geometry. However, it can be recast as:

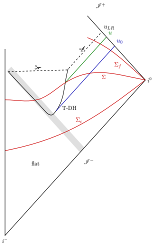
$$M_{\rm bms} = \frac{1}{16\pi G}\, \lim \oint_{r=r_\circ} (\theta_n + \tfrac{1}{r})\, d^2V \quad \text{as } r_o \to \infty \text{ along } u = u_\circ,$$

where θ_n is the rate of change of the area 2-form along inward-pointing null normal n^a. Therefore $M_{\rm bms}$ would be a well-defined operator in LQG, again with a discrete spectrum. During evaporation, it would be strongly correlated with M_Δ and A_Δ. (seen explicitly in the mean field approximation in the CGHS model).

- The evaporation process will appear as a correlated decrease in the discrete eigenvalues of M_Δ (encoded in the punctures on Δ) and discrete eigenvalues of $M_{\rm bms}$. Can we develop in some detail the LQG picture of the evaporation process as emission of area quanta/punctures from Δ to $\mathscr{I}^+$?

(iii) These considerations could be first developed in detail for the semi-classical phase in which, say, $1\,M_\odot$ initial black hole shrinks to Lunar mass $\sim 10^{-7}\,M_\odot$. This Process takes some 10^{67} years! During this adiabatic phase, it would be appropriate to let T-DH be a perturbed WIH and treat dynamics at $\mathscr{I}^+$ also perturbatively. Interestingly, symmetry groups of Δ and $\mathscr{I}^+$ are essentially the same and the expressions for fluxes associated with these symmetries are essentially identical in the classical theory (AA, Khera, Kolanowski, Lewandowski)

Credit: De Lorenzo



• Challenge and opportunity: Study the correlations between these observables in the adiabatically changing Unruh vacuum not only in the matter sector but also in the gravitational sector, which has not been investigated so far. These correlations will provide much deeper control on entanglement.

Source Material

- References most directly related to the talk:

1. AA & S. Speziale, Horizons and Null Infinity: A Fugue in 4 Voices, Phys. Rev. D (Letter) 109, L061501 (2024), arXiv; 2401:15618;

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3. AA, F. Pretorius & F. Ramazanaglu, Surprises in the Evaporation of 2-Dimensional Black Holes, Phys. Rev. Lett. 106, 161303 1-4 (2011), arXiv:1011.6442. Phys. Rev. arXiv: D83, 044040 1-18 (2011), arXiv:1012.0077.

- Further details on Weakly Isolated Horizons:

4. AA, C. Beetle, J. Lewandowski, Mechanics of Rotating Isolated Horizons, Physical Review D64 (2001) 044016; gr-qc/0103026

5. AA, N. Khera, M. Kolanowski and J. Lewandowski, Non-expanding horizons: Multipoles and symmetries, JHEP 01, 28 (2022), arXiv:2111.07873

6. AA, N. Khera, M. Kolanowski and J. Lewandowski, Charges and fluxes on (perturbed) non-expanding horizons, JHEP 02, 066 (2022) (38 pages), arXiv:2112.05608

- Further details on Null Infinity:

7. R. Geroch, Asymptotic Structure of Space-time, In: Volume edited by F. Esposito and L. Witten (Plenum Press, New York and London, 1976) pp. 1?105.

8. AA Geometry and physics of null infinity, prepared for Surveys in Differential Geometry, edited by L. Bieri and S. T.- Yau, pp99-122, (International press, Boston, 2015); arXiv: 1409:1800.