Physics of Loop Quantum Cosmology: Some New Answers to Some Old Questions

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Based on works with Bao-Fei Li, Meysam Motaharfar, Eklavya Thareja, and Sahil Saini

Motivation

Singularities such as the big bang are the limits of validity of Einstein's beautiful theory of GR. Incorporating quantum gravity necessary. How does quantum nature of spacetime affect the classical singularities and the physics of very early universe?

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Goal of this talk: Explore non-trivial consequences of quantization choices on physical implications. (Some surprises).

Outline

- Overview
- Implications of quantization ambiguities
  - Unfinished business: a forgotten loop area assignment ambiguity (some useful lessons)
  - Regularization ambiguities: (some surprises)
    - Distinct signatures
    - Can the universe be cyclic?
- Conclusions

Loop quantum cosmology

Symmetry reduce connection and triads at classical level, and then loop quantize geometry. Various kinematical features of LQG exported to LQC **(Bojowald; Ashtekar, Bojowald, Lewandowski (2001-03))**.

Rigorous quantization of the spacetime results in a singularity resolution due to non-perturbative effects **(Ashtekar, Pawlowski, PS (06))**. Results generalized to various spacetimes.

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- Mathematical aspects: **(Ashtekar, Brunnemann, Campiglia, Henderson, Lewandowski, Kaminski, Noui, Pawlowski, Perez, Thiemann, Szulc, Varadarajan ...)**
- Quantization of iso, aniso, pol. Gowdy $\mathbb{T}^3$ models: **(Ashtekar, Corichi, Martin-Benito, Mena Marugán, Olmedo, Pawlowski, PS, Vandersloot, Wilson-Ewing, ...)**
- Connection with LQG: **(Beetle, Bruno, Engle, Hogan, Fleishchack, Mendonca, Vilensky)**
- (Towards) Cosmological sector from LQG: **(Assaniousi, Dapor, Han, Li, Liu, Liegner, Kaminski, Ma, Elizega Navascués, Pawlowski, PS, Thiemann, Wang, ...)**
- Effective dynamics: **Copeland, Garriga, Maartens, Tsujikawa, Vilenkin, & many authors**
- Numerical techniques: **(Cartin, Diener, Gupt, Khanna, Joe, Megevand, PS, ...)**
- Perturbations: **(Agullo, Ashtekar, Barrau, Bojowald, Bolliet, Hossain, Grain, Gomar, Li, Nelson, Maartens, Martin-Benito, Mena Marugán, Olmedo, PS, Tsujikawa, Wang, Wilson-Ewing, ...)**

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Evolution determined by a quantum difference equation. Arises from expressing field strength in terms of holonomies computed over loops in quantum geometry.

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For spatially-flat isotropic FLRW model **(Ashtekar, Pawlowski, PS (2006))**

$$\mathcal{C}^+(v)\Psi(v+4,\phi) + \mathcal{C}^0(v)\Psi(v,\phi) + \mathcal{C}^-(v)\Psi(v-4,\phi) = \hat{\mathcal{H}}_\phi \Psi(v,\phi)$$

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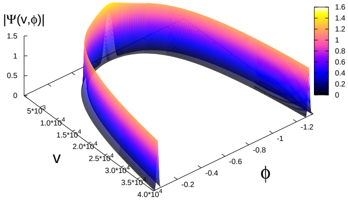

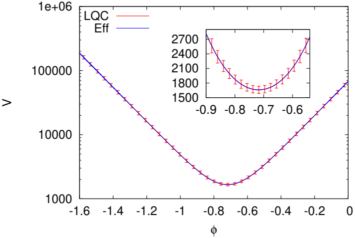

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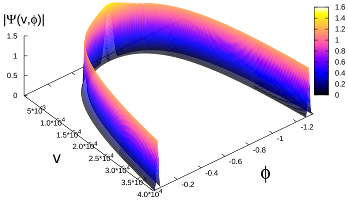

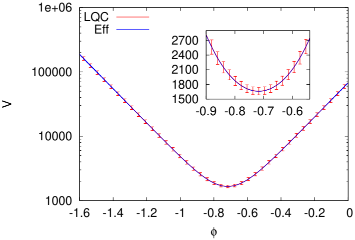

Big bang replaced by bounce at $\rho_{\max}=3/8\pi G\lambda^2\approx 0.41\rho_{\mathrm{Planck}}$. This is supremum in phys. Hilbert space **(Ashtekar, Corichi, PS(08))**; Consistent quantum probability of bounce is unity **(Craig, PS(13))**; Rigorously tested effective dynamics using HPC **(Diener, Gupt, Joe, Megevand, PS(14,18))**.

Physics from LQC: some main results

- Genericness of singularity resolution: All strong curvature singularities are resolved for isotropic, Bianchi-I, II, IX and Kantwoski-Sachs spacetimes **(PS (09,11); PS, Vidotto (10); Saini, PS (16-19))**

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- Observable effects in CMB: Excellent agreement with observations for different inflationary models **(Agullo, Ashtekar, Gupt, Li, Martin-Benito, Mena Marugán, Olmedo, PS, Wang ...)** Potential explanation of anomalies with different initial states **(Ashtekar, Gupt, Jeong, Sreenath (16-20); Martin-Benito, Neves, Olmedo (21))** and with inclusion of non-gaussianities **(Agullo, Bolliet, Kranas, Sreenath (18-20))**

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- Insights on resolution of black hole singularities: Has inspired rigorous quantizations of Schwarzschild spacetime **(Ashtekar, Bojowald, Corichi, García-Quismondo, Giesel, Li, Mena-Marugán, Modesto, Olmedo, Rastgoo, Saini, PS, Wang, ...)**, spherically symmetric models **(Alonso-Bardaji, Bojowald, Brizuela, Gambini, Giesel, Han, Husain, Lewandowski, Li, Liu, Ma, Pullin, PS, Wilson-Ewing, Zhang ...)**

  (Talks by V. Husain, H. Liu, C. Zhang)

Some old questions:

- What about quantization ambiguities? Can they be ruled out phenomenologically?
- There have been earlier paradigms of bouncing and cyclic models but with singularity problems. Does quantum geometry help in resolution of singularities of the cyclic models such as Ekpyrotic model (Steinhardt, Turok, ...)?

Some old and new questions:

- What about quantization ambiguities? Can they be ruled out phenomenologically? Do they result in distinct signatures?
  - revisiting a loop area assignment ambiguity
  - Regularization ambiguities
- There have been earlier paradigms of bouncing and cyclic models but with singularity problems. Does quantum geometry help in resolution of singularities of the cyclic models such as Ekpyrotic model (Steinhardt, Turok, ...)? Since such models are restricted observationally, does LQC make them viable? Or any other constraints?

Quantization ambiguities: loop area assignment

In isotropic models ambiguities related to assigning minimum loop area severely restricted using independence from fiducial structures and classicality at late times **(Corichi, PS (08); Engle, Vilensky (19))**. Unique quantization in standard LQC – $\bar{\mu}$ scheme **(Ashtekar, Pawlowski, PS(06))**

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In anisotropic models, single choice if spatial manifold $\mathbb{R}^3$ **(Ashtekar, Wilson-Ewing (09); Corichi, PS (09); Engle, Vilensky (18))**

- $\bar{\mu}_i$ scheme: $\bar{\mu}_i \propto (|p_j p_k / p_i|)^{1/2}$

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Singularity resolution and recovery of large volume universe after the bounce for both. Energy conditions not violated. Classicality for both (shear conserved) **(Chiou, Vandersloot (08))**. $\bar{\mu}_i$ scheme results in universal bounds on shear **(Corichi, PS (09))** and is topology free (preferred choice), but $\bar{\mu}_i'$ gives a much simpler quantization.

Implications of quantization ambiguity in Bianchi-I model

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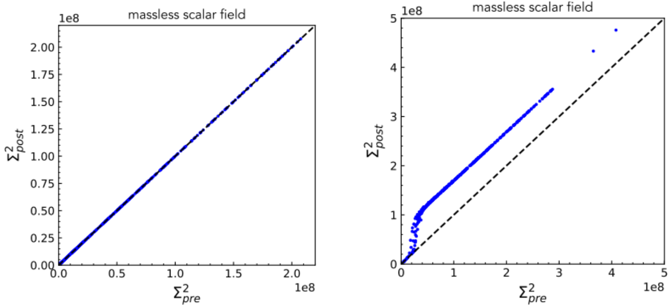
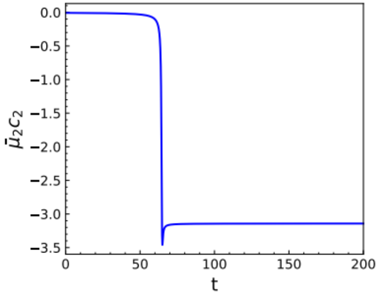


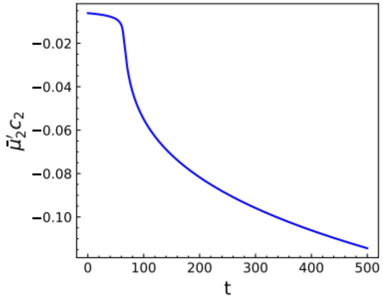
Shear scalar is not preserved across the bounce for a large class of initial conditions for the $\bar{\mu}'_i$ scheme **(Motaharfar, Thareja, PS (23))**

Effect difficult to observe for small anisotropic shear, therefore missed in earlier works.

Resolution of quantization ambiguity: Bianchi-I model

Shear problem in $\bar{\mu}_i'$ scheme arises because one of the connections never becomes classical after the bounce!





Even though the universe becomes macroscopic, it retains quantum character after bounce in one direction even after a very long time.

Gives cyclic evolution even for dust when it is spatially flat! Even though non-singular and resulting in a macroscopic universe on both sides of the bounce, $\bar{\mu}_i'$ scheme is not classically viable.

Quantization ambiguities: different regularizations

(Yang, Ding, Ma (09); Assanioussi, Dapor, Liegener, Pawlowski (18); Li, PS, Wang (18); Han, Liu (20))

In standard LQC, Euclidean and Lorentzian terms of Hamiltonian constraint combined before quantization (Ashtekar, Bojowald, Lewandowski (03))

$$\mathcal{C}_{\rm grav} = \mathcal{C}_{\rm grav}^{(E)} - (1+\gamma^2)\mathcal{C}_{\rm grav}^{(L)}$$

where

$$\mathcal{C}_{\rm grav}^{(E)} = \frac{1}{2}\int \mathrm{d}^3x\, \epsilon_{ijk} F^i_{ab} \frac{E^{aj}E^{bk}}{\sqrt{\det(q)}}$$

and

$$\mathcal{C}_{\rm grav}^{(L)} = \int \mathrm{d}^3x\, K^j_{[a} K^k_{b]} \frac{E^{aj}E^{bk}}{\sqrt{\det(q)}}$$

Thiemann's regularization of the Hamiltonian constraint: treat these terms independently (Thiemann (98); Giesel, Thiemann (07))

- Quantize using identities on classical phase space and express in terms of holonomies (closer to LQG): (mLQC-I)
- Use $K^i_a = \gamma^{-1} A^i_a$ in spatially flat, then quantize: (mLQC-II)

Quantization ambiguities: different regularizations

In Thiemann's regularization of Hamiltonian constraint one finds an emergent deSitter pre-bounce phase in mLQC-I, and a very similar bounce as in LQC in mLQC-II.

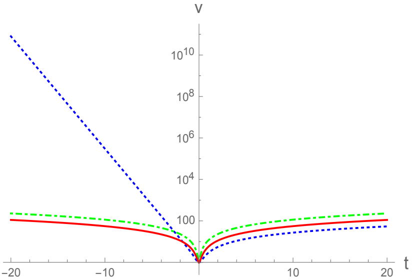


Figure: LQC (red), mLQC-I (blue), mLQC-II (green)

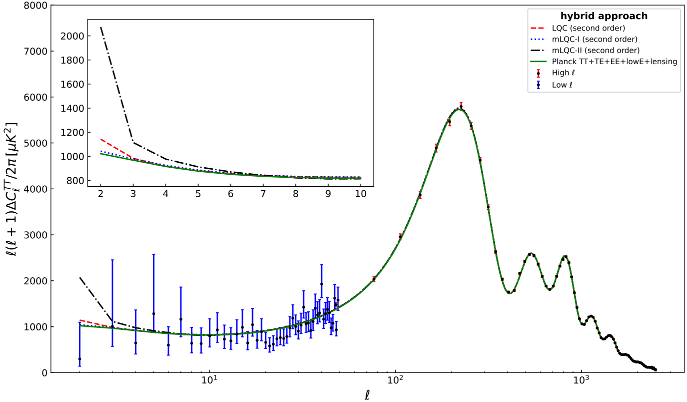
Bounce at nearly same density, but non-trivial modified Friedmann dynamics (Li, PS, Wang (18)). All strong curvature singularities resolved (Saini, PS (18)). Inflationary attractors (Li, PS, Wang (20)). In pre-inflation, post-bounce evolution very quickly agrees with LQC.

Effects in primordial power spectrum explored: (Agullo (19); Li, PS, Wang (20), Li, Olmedo, PS, Wang (20)). (B-F Li's talk)

Even though post-bounce physics almost identical within few Planck seconds, mLQC-I/II potentially yield distinct signatures.

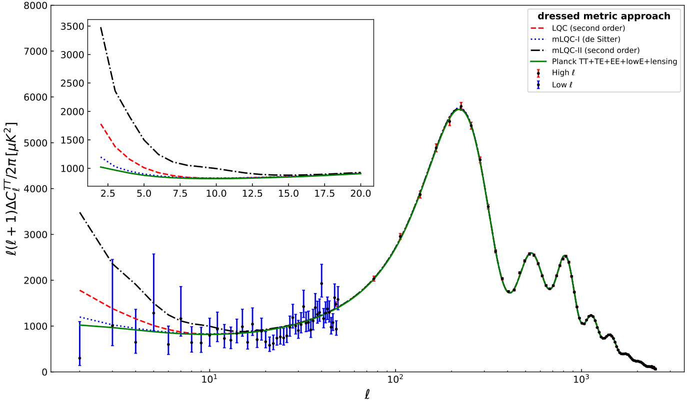
mLQC-II leads to significant power amplification at large scales. Competition between LQC and mLQC-I for a better fit.

(Li, Motaharfar, PS (to appear))



(M. Motaharfar's talk)

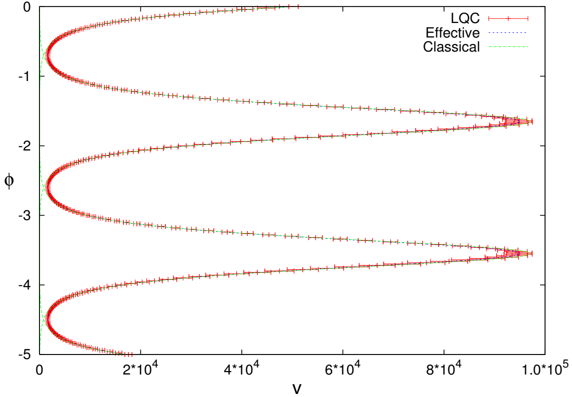
Results do not change if one uses dressed or hybrid metric approach.



mLQC-II can be ruled out!

Cyclic models?

Conventional wisdom: cyclic models are straightforward to construct with negative potential or positive spatial curvature if big bang/crunch singularity is resolved.



The closed FLRW model has big bang/crunch singularities which are successfully resolved by LQC without violating any energy conditions. **(Ashtekar, Pawlowski, PS, Vandersloot; Szulc, Kaminski, Lewandowski (2007))**

Cyclic models?

In a spatially flat universe add a negative cosmological constant or a negative potential to the matter density with $w > -1/3$:

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}(\rho - \rho_\Lambda), \qquad \rho_\Lambda = \frac{|\Lambda|}{8\pi G}$$

Hubble rate vanishes in a finite future evolution causing a recollapse which is followed by a big crunch in GR.

Replaced by bounce in standard LQC, leads to a continuous cycle **(Bentivegna, Pawlowski (07))**.

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Ekpyrotic/cyclic models can be made non-singular in standard LQC **(PS, Vandersloot, Vereshchagin (06); Wilson-Ewing (13); Li, Saini, PS (20))**, even in presence of anisotropies **(Cailleatau, PS, Vandersloot (09), Brown, PS (to appear))**

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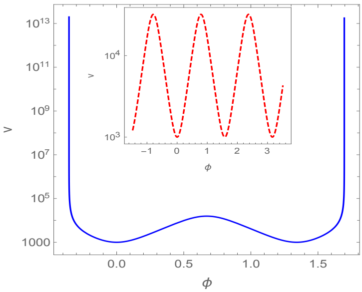
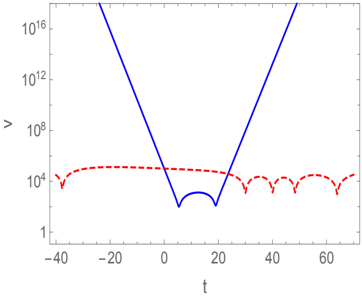
But LQC can not help with viability. Spectrum still not scale invariant after quantum geometric corrections in simple realizations **(Cai, Wilson-Ewing (14); Li, Saini, PS (20))**

Cyclic models?

Unlike standard LQC where cyclic models are dynamically possible, Thiemann regularized LQC (mLQC-I) severely restricts cyclic models. Emergent quantum geometric deSitter phase not only in pre-bounce but appears also post-bounce after recollapse. **(Li, PS (22))**



Negative Lambda



Ekpyrotic potential

Cycles impossible unless negative potential is Planckian!

Some new answers:

- Can quantization ambiguities be ruled out phenomenologically? Yes
  Do they result in distinct signatures? Yes
  - a forgotten loop area assignment ambiguity. Not classically viable as earlier believed. Ruled out but gave useful lessons.
  - Regularization ambiguities. mLQC-II can be ruled out! mLQC-I seems promising.
- Does quantum geometry help in resolution of singularities of the cyclic models such as Ekpyrotic model (Steinhardt, Turok, ...)? Yes. Since such models are restricted observationally, does LQC make them viable? Not so far. Or any other constraints? Yes! If Thiemann regularization is correct, no cyclic models in spatially flat universes. Thiemann regularized LQC (mLQC-I) favors inflationary paradigm.

Summary

- LQC offers a platform to extract rigorous physics at the Planck scale. Non-singular dynamics. Extensive phenomenology studied. Many robustness checks.
- There are well justified quantization ambiguities which at first sight may seem like standard LQC but can result in very different physical implications.
- Some lessons:
  - Recovery of macroscopic universe not enough for classicality even when energy conditions satisfied. Precise classicality requirements important.
  - Explore wide range of parameters.
  - Consider different types of matter.
  - Quantizations beyond standard LQC are complex/rich. Many opportunities.