Loop Quantum Gravity and Cosmology

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a recent review:

Agulló, Wang, WE, *Loop quantum cosmology: relation between theory and observations*, in Handbook on Quantum Gravity (2023), Eds. Bambi, Modesto and Shapiro, Springer, `arXiv:2301.10215 [gr-qc]`.

Loops '24, Florida Atlantic University

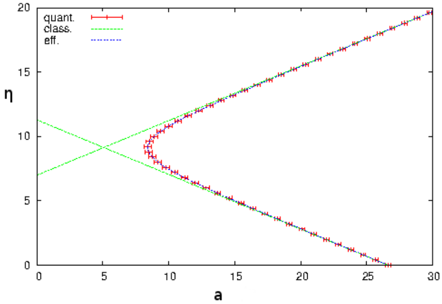
Questions from Cosmology

- Why is the universe nearly homogeneous at large scales?
- What are dark matter and dark energy?
- What sourced the CMB temperature anisotropies?
  - What generated the near scale-invariance?
  - Why are tensor modes and non-Gaussianities so small?
  - Are there other features/anomalies that can be understood?
- Can we describe the earliest moments of our universe?
  - Can quantum gravity resolve the big-bang singularity?
  - How are the dynamics of our universe modified?
  - How did the universe become classical?

The Loop Quantum Cosmology Approach

In loop quantum cosmology, the basic idea is to include effects due to holonomies and inverse triad operators in a symmetry-reduced setting [Bojowald 2001; Ashtekar, Bojowald, Lewandowski, 2003; ...].

Expressing the curvature in terms of holonomies around a loop of minimal area $\sim \ell_{\rm Pl}^2$ changes the dynamics in the deep Planck regime, replacing the big-bang singularity with a non-singular bounce [Ashtekar, Pawłowski, Singh, 2005; ...].



[Pawłowski, Pierini, WE, 2014]

Cosmology from Full Loop Quantum Gravity

LQC is not derived from LQG, but rather is motivated by it.

$\Rightarrow$ Is it possible to close the gap between LQG and LQC?

There has been considerable work in this direction:

- Canonical LQG [Alesci, Cianfrani, 2013; Bodendorfer, 2017; Dapor, Liegener, Pawłowski, 2019; ...],
  - Based on dynamical graphs [Han, Liu, 2021],
  - Connection to modified LQC [Yang, Ding, Ma, 2009; Dapor, Liegener, 2018; ...]?
- Spinfoams [Bianchi, Krajewski, Rovelli, Vidotto, 2011; Dittrich, Padua-Argüelles, 2023, Han, Liu, Qu, Vidotto, Zhang, 2024; ...],
- Group field theory (GFT).

Group Field Theory Cosmology

In the canonical form of GFT, creation and annihilation operators create spin-network nodes [Oriti, 2016; ...], for example

$$\hat{\varphi}_{\iota,\vec{j},\vec{m}}(\phi)^\dagger|0\rangle$$

creates a spin-network node with intertwiner ι, spins $\vec{j}$, magnetic numbers $\vec{m}$ and scalar field with the value ϕ. These can be connected through entanglement [Colafranceschi, Oriti, 2021; ...].

The homogeneity of cosmological spacetimes can be captured by considering condensate states, where all geometry quanta are in the same state [Gielen, Oriti, Sindoni, 2013]:

$$|\sigma\rangle = \exp\left(\sum_{\iota,\vec{j},\vec{m}} \sigma_{\iota,\vec{j},\vec{m}}(\phi)\varphi_{\iota,\vec{j},\vec{m}}(\phi)^\dagger\right)|0\rangle.$$

Cosmological Hydrodynamics

The cosmological dynamics are the hydrodynamics: calculate the collective observable $\langle\sigma|V_{tot}(\phi)|\sigma\rangle$ using ϕ as a relational clock.

In general the result is a non-singular bouncing cosmology with the correct classical limit (for appropriate choices of parameters in the GFT action) [Oriti, Sindoni, WE, 2016].

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For a single-spin condensate

$$\left(\frac{V'}{V}\right)^2 = 12\pi G\left(1 - \frac{\rho}{\rho_c}\right) + \frac{4EV_{j_0}}{V},$$

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Extensions: relational GFT [WE, 2019; Gielen, Polaczek, WE, 2019; Marchetti, Oriti, 2021; Marchetti, WE, wip], cyclic models [de Cesare, Pithis, Sakellariadou, 2016], anisotropies [de Cesare, Oriti, Pithis, Sakellariadou, 2018; Calcinari, Gielen, 2023; Oriti, Wang, 2023].

Answering Some Questions

- **Can quantum gravity resolve the big-bang singularity?**
- **How are the dynamics of our universe modified?**
  LQG predicts a non-singular bounce to occur in the Planck regime, with quantum gravity effects rapidly becoming negligible away from the bounce.

Answering Some Questions

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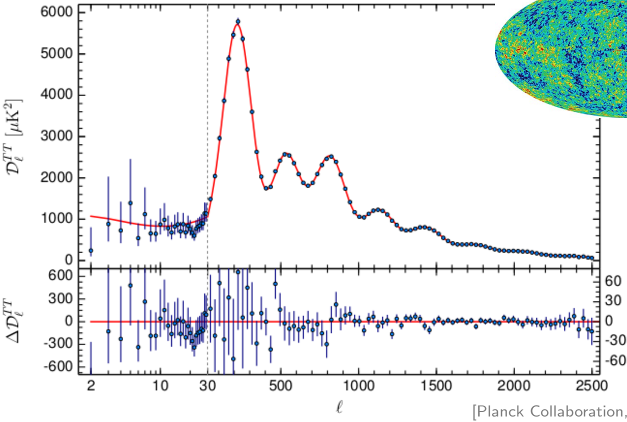
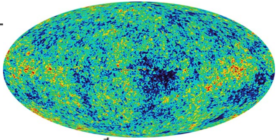
  LQG predicts a non-singular bounce to occur in the Planck regime, with quantum gravity effects rapidly becoming negligible away from the bounce.

- **How did the universe become classical?**
  This has often been asked about perturbations [Polarski, Starobinsky, 1996; Perez, Sahlmann, Sudarsky, 2006; Martin, Vennin, 2016; Ashtekar, Corichi, Kesavan, 2020; ...] but what about the homogeneous background?
  - $\Rightarrow$ In LQC, when the spatial volume is large it is possible for the relative quantum uncertainties to be very small [Rovelli, WE, 2014; ...],
  - $\Rightarrow$ In GFT cosmology, relative quantum uncertainties decrease as the universe expands [Marchetti, Oriti, 2021].

CMB Temperature Anisotropies

The temperature anisotropies observed in the cosmic microwave background give detailed information about the early universe.





[Planck Collaboration, 2015]

What can the CMB tell us about quantum gravity?

Different Approaches to Perturbation Theory

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- **Anomaly-free effective equations** [Bojowald, Hossain, Kagan, Shankaranarayanan, 2008; Cailleteau, Mielczarek, Barrau, Grain, 2012; ...]
- **Mixed quantization approaches: hybrid** [Fernandez-Mendez, Mena Marugán, Olmedo, 2012; ...] **and dressed metric** [Agulló, Ashtekar, Nelson, 2013; ...]
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  Consider modifications of the classical constraint algebra motivated by LQC, requiring a closed algebra.
- **Mixed quantization approaches: hybrid** [Fernandez-Mendez, Mena Marugán, Olmedo, 2012; . . .] **and dressed metric** [Agulló, Ashtekar, Nelson, 2013; . . .]
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  Perform a Fock quantization for the perturbations on an LQC FLRW background.
- **Separate universe approximation** [WE, 2012, 2016]

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- **Separate universe approximation** [WE, 2012, 2016]
  Perform a loop quantization of long-wavelength scalar perturbations.

Ambiguity in z''/z

In classical gravity, scalar perturbations v evolve as

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where $z = a\sqrt{\rho + P}/c_s H$.

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This can be calculated in the quantum separate universe and the anomaly-free effective approaches, with the same result: $z = a\sqrt{\rho + P}/c_s H$ [WE, 2012, 2016; Cailleteau, Mielczarek, Barrau, Grain, 2012].

Modifications to the Mukhanov-Sasaki Equation?

⇒ **Anomaly-free effective approach:**

The prefactor to k^2 changes, an effective 'signature change' near the bounce [Cailleteau, Mielczarek, Barrau, Grain, 2012; Bojowald, Paily, 2012; ...]. If vacuum initial conditions are set pre-bounce, short-wavelength modes will be amplified too much, ruling it out [Bolliet, Barrau, Grain, Schander, 2016]; but a reverse-engineered choice is viable [Li, Zhu, Wang, Kirsten, Cleaver, Sheng, 2019].

⇒ **Mixed quantization:**

⇒ **Separate universes loop quantization:**

Modifications to the Mukhanov-Sasaki Equation?

⇒ **Anomaly-free effective approach:**

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A Fock quantization by definition ignores LQG effects on the perturbations themselves and assumes the Mukhanov-Sasaki equation is unchanged.

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Modifications to the Mukhanov-Sasaki Equation?

⇒ **Anomaly-free effective approach:**

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⇒ **Separate universes loop quantization:**

It is possible to derive the LQC Mukhanov-Sasaki equation only for long-wavelength scalar modes; the resulting equation is unchanged [WE, 2012, 2016].

Inflation

In inflation, short-wavelength modes (initially in their vacuum state) exit the horizon at which point they feel the spacetime curvature and become nearly-scale invariant [Starobinsky, 1980; Mukhanov, Chibisov, 1981; . . .].

Quantum gravity signatures are possible:

1. for modes that were super-Hubble and felt the spacetime curvature at the bounce [Agulló, Ashtekar, Nelson, 2013; . . .],
2. due to modifications of the initial vacuum state for modes that were initially super-horizon or near-horizon [de Blas, Olmedo, 2016; Ashtekar, Gupt, 2017; Elizaga Navascués, Mena Marugán, Prado, 2020; Martín-Benito, Neves, Olmedo, 2021; . . .].

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$\Rightarrow$ LQC effects could be detected in the CMB—but if there are too many *e*-folds of inflation, then quantum gravity effects will be super-horizon today and unobservable.

Bouncing Alternatives to Inflation

Inflation is not the only mechanism to generate near scale-invariance: a matter-dominated cosmic bounce [Wands, 1999; Finelli, Brandenberger, 2002; ...] or an ekpyrotic universe [Khoury, Ovrut, Steinhardt, Turok, 2001; Finelli, 2002; ...] can also do this.

Given that they both rely on a bounce, they are nicely complementary to LQC and it is natural to consider them together [Cailleteau, Singh, Vandersloot, 2009; WE, 2013; Cai, WE, 2014; Li, Saini, Singh, 2021; ...].

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But in both scenarios, any background anisotropies generate a scale-invariant quadrupole moment in the perturbations [Agulló, Olmedo, WE, 2022].

$\Rightarrow$ This is highly constrained observationally, leading to a significant fine-tuning problem.

CMB Anomalies

Returning to inflation, where LQC effects occur at large scales that feel the spacetime curvature, it is intriguing that at large scales that there are some 'anomalies' in the CMB data [Planck, 2020]:

- Low power at large scales,
- Dipolar asymmetry,
- Bias for odd-parity correlations,
- Preference for lensing amplitude $A_L > 1$.

These anomalies have modest statistical significance; they could be due to cosmic variance.

Or perhaps they could be explained by quantum gravity?

Can LQC Explain the Anomalies?

- Choice of the initial vacuum state:
  - No oscillations in k [de Blas, Olmedo, 2016]
  - Quantum Penrose Weyl curvature conjecture [Ashtekar, Gupt, 2017]

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  Neglecting oscillations in k, non-Gaussian correlations generated by LQC between super-horizon and CMB modes can make all of the anomalous features more likely. Neglecting oscillations in k, template-matching strongly constrains this scenario [van Tent, Delgado, Durrer, 2023], but oscillations suppress this signature [Roshna, Sreenath, 2023]. Including oscillations, are the 'anomalies' still more likely?

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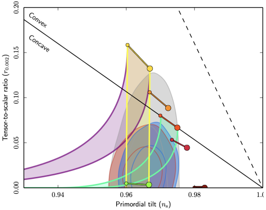
These proposals require ~ 70 inflationary e-folds.

CMB: Beyond Temperature Anisotropies

From CMB data, the amplitude of

- primordial gravitational waves,
- non-Gaussianities,

are constrained to be very small [Planck, 2018].



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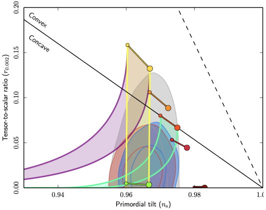

Some cosmological models are viable, but a large fraction of inflationary and alternative cosmological models are now strongly disfavoured by the data.

$\Rightarrow$ Strong observational constraints on many inflationary models and alternatives,

$\Rightarrow$ Fine-tuning problem for background anisotropies in ekpyrotic and matter bounce scenarios.

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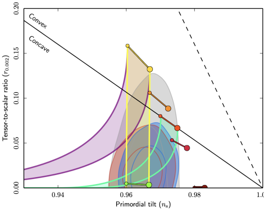

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What about quantum gravity? **Can quantum gravity generate near scale-invariance on its own?**

And from Full LQG

There has been progress on cosmological perturbation theory working in full LQG:

- Group field theory cosmology,
  - Relational framework [Gielen, Oriti, 2018; Marchetti, Oriti, 2022],
  - Separate universe approach [Gerhardt, Oriti, WE, 2018],
  - Entangled coherent states [Jercher, Marchetti, Pithis, 2024],
- Canonical LQG [Han, Li, Liu, 2020],
- Spinfoam models [Gozzini, Vidotto, 2021; Frisoni, Gozzini, Vidotto, 2023].

Next steps: combine with various cosmological scenarios, or perhaps obtain scale-invariance directly. . .

Trans-Planckian Modes

The expansion of the universe red-shifts perturbations:

$$\lambda \propto a(t).$$

As the universe has expanded considerably, modes that are today cosmologically relevant initially had a much shorter wavelength. In particular, modes with $\lambda(t_i) < \ell_{\rm Pl}$ will eventually reach cosmological scales.

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What is the physics of these trans-Planckian modes?

- Can trans-Planckian modes co-exist with discrete quantum geometry?
- Are new modes $\lambda \sim \ell_{\rm Pl}$ continuously 'created' as the universe expands?

Some Thoughts on the Dark Sector

- **Dark Matter:**
  Could dark matter be entirely or partially due to black holes?
  - Primordial black holes or black hole remnants from the pre-bounce era [Carr, Coley, 2011; Quintin, Brandenberger, 2016; Rovelli, Vidotto, 2018; ...],
  - LIGO has found a black hole in the 'mass gap' $2M_\odot$–$5M_\odot$ where none were expected [LIGO, 2024].

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- **Dark Energy:**
  The simplest explanation is that dark energy is just a cosmological constant [Bianchi, Rovelli, 2010].
  - But BAO observations prefer an increasing equation of state at 2.6σ from initially $\omega < -1$ to $\omega > -1$ today [DESI, 2024],
  - And the Hubble tension is alleviated by increasing $\Lambda(t)$ [Perez, Sudarsky, WE, 2021].

  So keep an eye on what future measurements of dark energy have to say...

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A perturbative analysis suggests this is not the case: perturbations grow in a contracting universe and their amplitude is preserved across the LQC bounce [WE, 2016; . . .].

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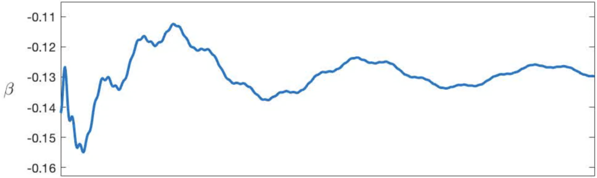
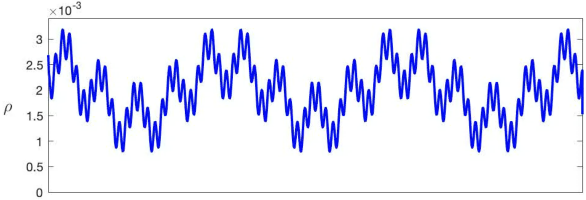
What about a non-perturbative analysis? This can be done, for example using the loop quantization of the Lemaître-Tolman-Bondi spacetime [Husain, Kelly, Santacruz, WE, 2022; ...], corresponding to spherically symmetric dust.

Non-Perturbative Homogenization

Preliminary results:





Inflation can blow a small patch up to cosmological scales.

Summary

- LQC is sufficiently mature to make contact between the theory and cosmological observations.
- Considerable progress has been made in connecting LQC and full LQG, in each of the GFT, canonical, and spinfoam approaches.
- Open questions remain:
  - Further understand the relation between LQC and LQG,
  - Can we get scale-invariant perturbations from QG alone?
  - Trans-Planckian modes,
  - . . .
- Applications: there are many open problems in cosmology—quantum gravity may turn out to be key in answering at least some of them.

Outlook: Questions from Cosmology

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