FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

Effective Spin foam models

An overview

Seth Kurankyi Asante, FSU Jena

[*Bianca Dittrich, Hal Haggard, Sebastian Steinhaus, José Diogo Simão, Alexander Jercher, José Padua-Argüelles, Taylor Brysiewicz**]

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Fort Lauderdale, Florida, USA

May 9, 2024

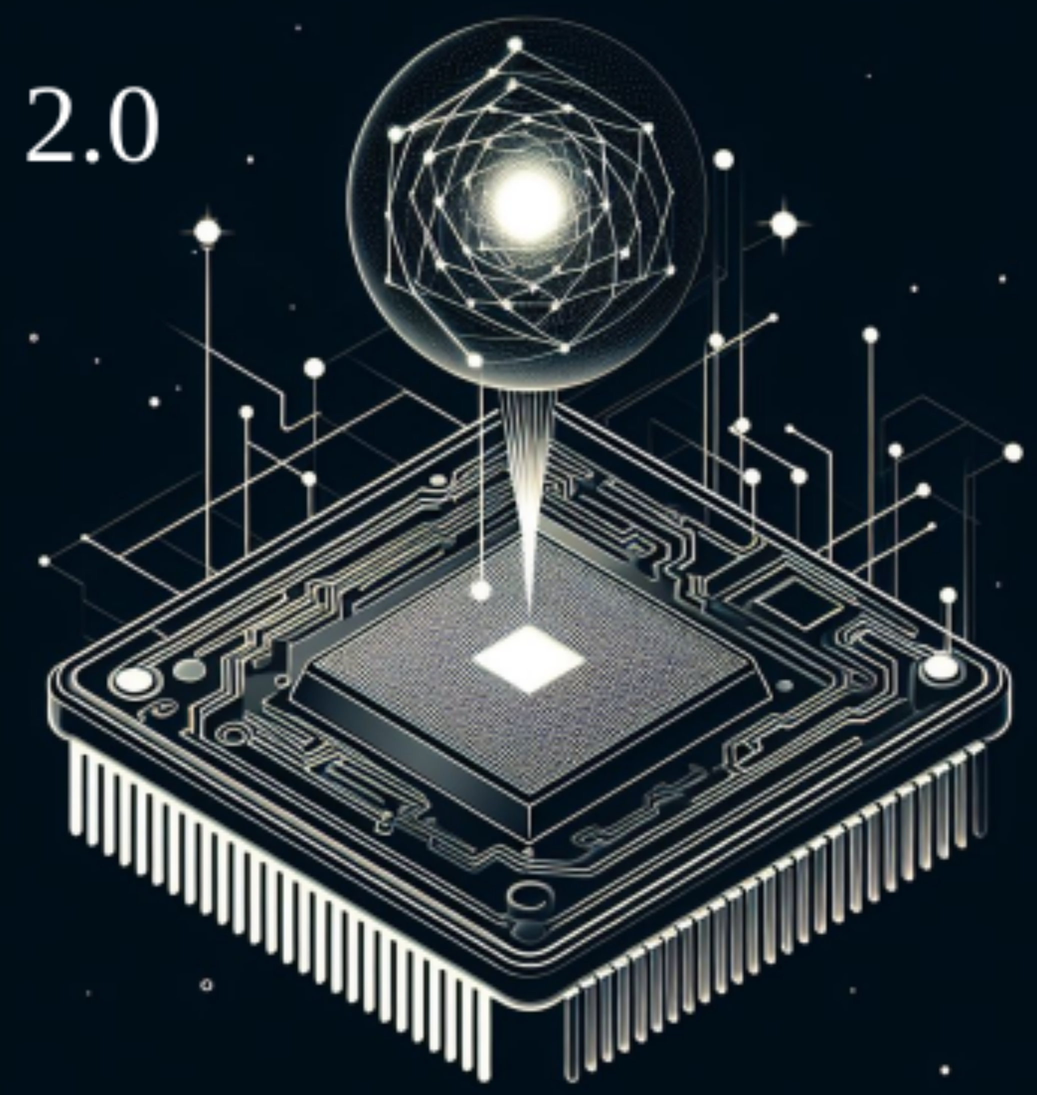
Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

Quantum Gravity on the Computer 2.0

September 9 - 13

FSU Jena



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What can we learn from (discrete) gravitational path integrals?

Quantum geometry

Quantum gravity

Semi-classical approximations

Topological transitions

Early universe Cosmology applications

Black hole dynamics

Renormalization/Continuum limit

Quantum field theory in curved space time

What are ‘effective spin foam models’?

They are defined as discrete geometric path integrals (sums) for quantum gravity

- maintain the ‘key dynamical principles’ of quantum geometry à la LQG

They are **effective** because:

- they provide spin foam models with efficient and computable dynamics
- they provide a general family of models by imposing constraints differently

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They can provide many insights into discrete path integral models

- easy construction of Lorentzian and Euclidean models
- applications to semi-classical geometries, cosmology, ...
- avenue to study **continuum limit** of discrete models

Outline

- **Constructing effective spin foams**
  - Discrete quantum geometries
- **Testing the model**
  - Semi-classical analysis
- **Cosmology applications**
  - Mini superspace models
- **Continuum limit**
  - First Steps: Perturbative

Effective Spin foams

Constructing the models

[SKA, Dittrich, Haggard, Padua-Argüelles, Brysiewicz]
arXiv: *2004.07013, 2011.14468, 2104.00485, 2402.17080*

The Construction

Features of quantum geometry (LQG)

[Ashtekar, Rovelli, Smolin, Thiemann, Lewandowski, Perez, Bianchi, Freidel, Corichi, Dittrich, Varadarajan, Livine, Bonzom…

many more ….

Construct a simple spin foam model where:

- area variables fundamental

  - enlarged space of (discrete) length geometries

support from: LQG, black hole entropy, thermodynamics, generalized geometry, geometric entanglement, strings

[Bekenstein, Ryu, Takayanagi, Cattaneo, Perez, Jacobson, Headrich, Zweibach, Schuller, Wohlfahrt,…]

[Rovelli, Smolin, Ashtekar, Lewandowski, Corichi, Wieland, Freidel, Geiller, Pranzetti…]

- spectra for area operators

Space-like areas

$$a_S = \gamma \ell_P^2 \sqrt{j(j+1)} \sim \gamma \ell_P^2 j, \quad j \in \mathbb{N}/2$$

Time-like areas

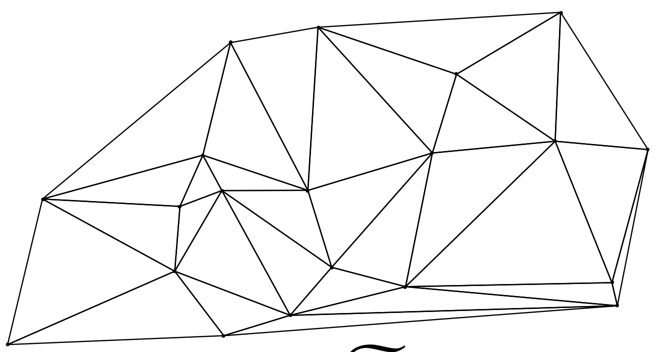
$$a_T = \ell_P^2 n, \quad n \in \mathbb{N}/2$$

The Construction

Area Regge Calculus

Simple action (4D) [Regge ’61; Rovelli ’93; Mäkelä ’94, Barrett, Roček, Williams ’97, SKA, Dittrich, Haggard ’18…]

Area Regge action: $S_{\text{ARC}} = -\sum_{t\in \text{bulk}} a_t\, \epsilon_t(a_{t'}) - \sum_{t\in \text{bdry}} a_t\, \psi_t(a_{t'})$



discrete GR action for area configurations

relevant for LQG and spin foam models [Rovelli ’93; Barrett et al…]

Classical dynamics: $\epsilon_t(a_{t'}) = 0$ does **not** reproduce discrete GR dynamics

vanishing curvature + non-shape matching

The Construction

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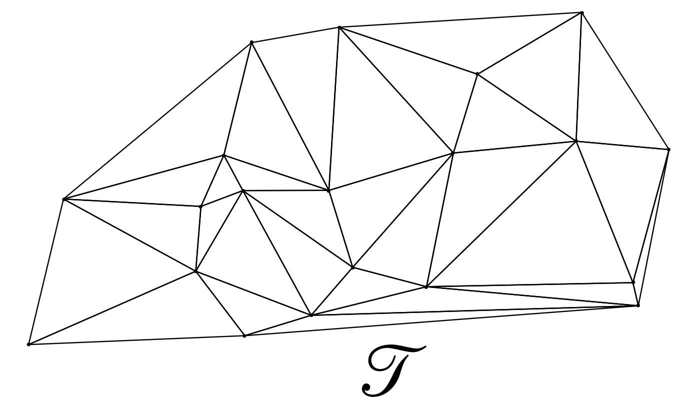

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Preliminary state-sum model: $Z = \sum_{\{a\}} \mu(a)\, \exp(\, i\, S_{\text{ARC}}(a))$

discrete areas

Add constraints to reproduce discrete GR [Dittrich, Speziale ’07]

The Construction

Constrain area variables

Area-length constraints set of polynomial equations

$$\left\{ a_t^2 = \frac{1}{16}(4l_1^2 l_2^2 - (l_1^2 + l_2^2 - l_3^2)^2), \ \forall t \right\}$$

cf. (part of) simplicity constraints

[*Heron of Alexandria, AD 70*]

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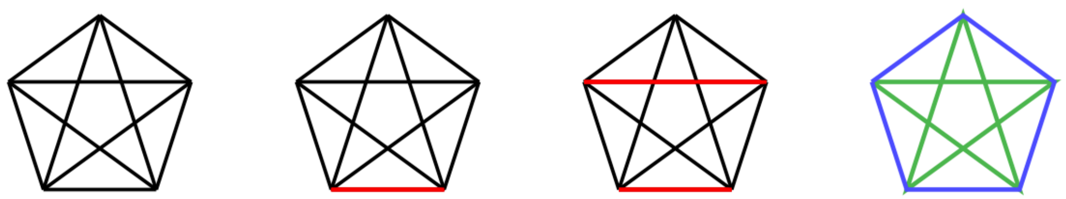
Homotopy continuation (tools from numerical algebraic geometry) [Brysiewicz, SKA '24]

A **4-simplex** with 10 areas has up to 64 length configurations



Galois group

$G = \mathbb{Z}^2 \wr \mathbb{S}^{32}$

$\Longrightarrow$ not solvable

Generically, area-length constraints has no/few solutions

Diophantine equations for discrete areas [Dittrich, Haggard, SKA '21]

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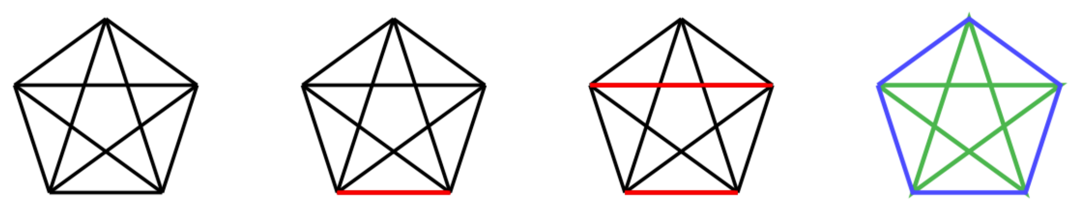

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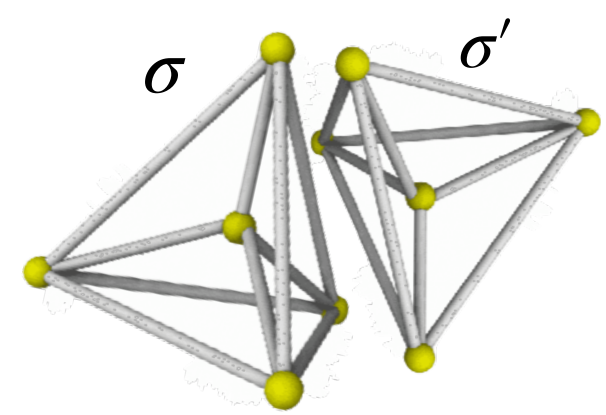
Constraints between neighbouring simplices

$$\Phi_{e_i}^{\tau,\sigma}(a_t) = \Phi_{e_i}^{\tau,\sigma'}(a_t), \quad i = 1,2$$

match pair of 3d dihedral angles



The Construction

Weak implementation of constraints

[courtesy of: *Hal Haggard*]

Forced between

Scylla: Reduce too much density of states

Charybdis: Impose dynamics that doesn't match GR

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Charybdis: Impose dynamics that doesn't match GR

(localized geometric constraints)

$$\mathscr{C}_i^\tau := \phi_{e_i}^\tau - \Phi_{e_i}^{\tau,\sigma}(a_t) = 0$$

(second-class constraints)

$$\{\mathscr{C}_i^\tau, \mathscr{C}_j^\tau\} = \gamma\,(9/2)\mathrm{Vol}_\tau$$

[*Dittrich, Speziale, Ryan, Haggard,…*]
[*Kapovich-Millson*]

γ - an anomaly parameter

Impose constraints '<u>weakly</u>': as strongly as allowed by uncertainty relation

[SF: *Engle-Perriera-Rovelli-Livine, Perez*]

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Use coherent states:

$$|\,K(\phi^\tau, \Phi_{e_i}^{\tau,\sigma})\,\rangle$$

'Integrate out' ϕ^τ variables →

coupling between σ and σ'

$$G_\tau = \langle K_{\Phi_{e_i}^{\tau,\sigma}} \,|\, K_{\Phi_{e_i}^{\tau,\sigma'}} \rangle$$

ansatz $\sim \mathscr{N}_k \exp\left(-\frac{\mathscr{C}^2}{4\Sigma^2(j)}\right)$

[*Livine, Speziale '17*]

[*Steinhaus, Simão, SKA '22*]

The Construction

Effective spin foam model

Combine simple amplitude and impose constraints 'weakly' [Dittrich, Haggard, Padua-Argüelles, SKA]

Effective spin foam models are discrete geometrical path integrals for quantum gravity

$$Z_{\mathrm{ESF}} = \sum_{\{a_t\}} \mu(a) \exp\left(\mathrm{i}\, S_{\mathrm{ARC}}(a)\right) \prod_{\tau} G_{\tau}^{\sigma,\sigma'}(a) \prod_{\sigma} \Theta_{\sigma}^{\mathrm{tr}}(a)$$

simplex inequalities

$\mu(a)$ - measure on space of discrete areas

Oscillatory behaviour controlled by area Regge action

Weak imposition of constraints by Gaussian terms localized on tetrahedra

Spin foam amplitudes may be cast into a similar form [Steinhaus, Simão, SKA '22]

(have to integrate normal vectors)

Features of the model

Regge calculus   Gluing terms

$$Z_{\mathrm{ESF}} = \sum_{\{a_t\}} \mu(a) \, \exp\left(\mathrm{i}\, S_{\mathrm{ARC}}(a)\right) \prod_{\tau} G_{\tau}^{\sigma,\sigma'}(a) \prod_{\sigma} \Theta_{\sigma}^{\mathrm{tr}}(a)$$

[Dittrich, Haggard, Padua-Argüelles, SKA]

- ‘Effective’ dynamics of quantum geometries

  keep dynamic principles of LQG and spin foam models

- Computationally very efficient

  fast numerical computations cf. to BF and EPRL models, restricted spin foam models

  [Speziale, Dona, Sarno, Gozzini, Frisono; Han, Liu, Qu, Huang; Bahr, Steinhaus, Simão, SKA...]

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- ✦ Related to spin foam models for higher gauge group

  $Z \propto \cos(S_{\mathrm{Regge}})$ [Baez, Girelli, Pfeiffer, Popescu; Baratin, Freidel; Miković, Vojinović; Dittrich, Girelli, Riello, Tsimiklis, SKA]

- ✦ Control: can test many interesting features

  [3D SFs: Simão '24, Jercher, Steinhaus, Simão]

  Simple construction of Lorentzian model: allows spacelike and timelike area configurations

  Cosmology applications*

Configurations

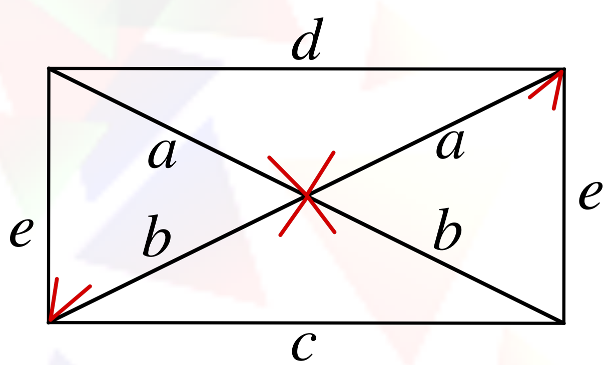
Configurations can be grouped into two sets: Regular and Irregular

according to light cone structure of faces

2D Examples (Hinge causality)

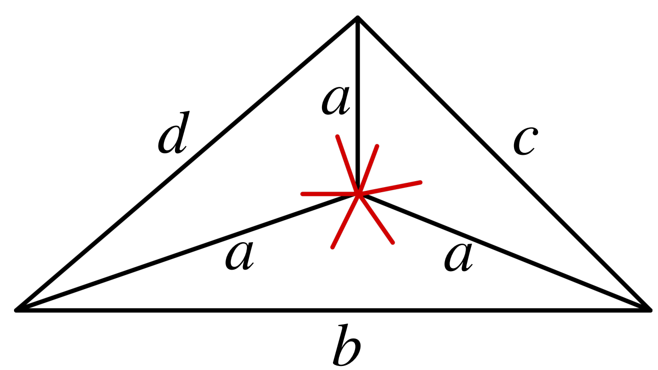
[Louko-Sorkin '95, Sorkin '19]

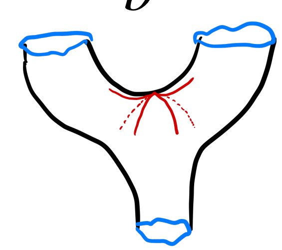
Regular configuration



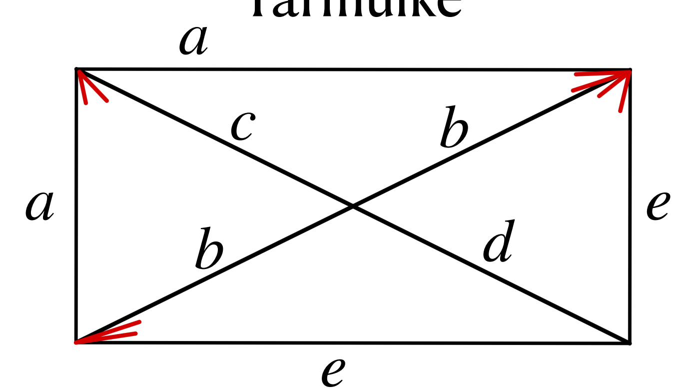
Irregular configurations

Trouser-like

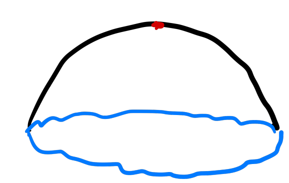




Yarmulke



related to:
Topology change



[Sorkin'19, Dittrich, Padua-Argüelles, SKA '21]

Lorentzian geometries

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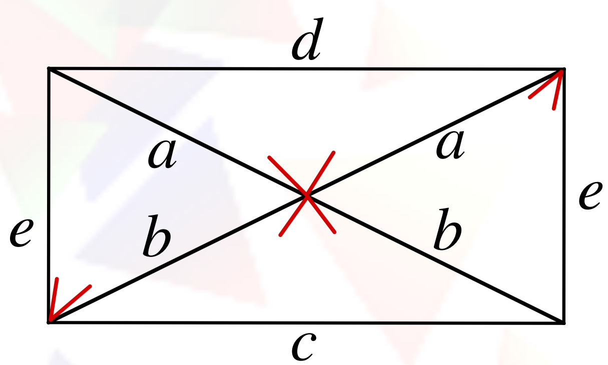

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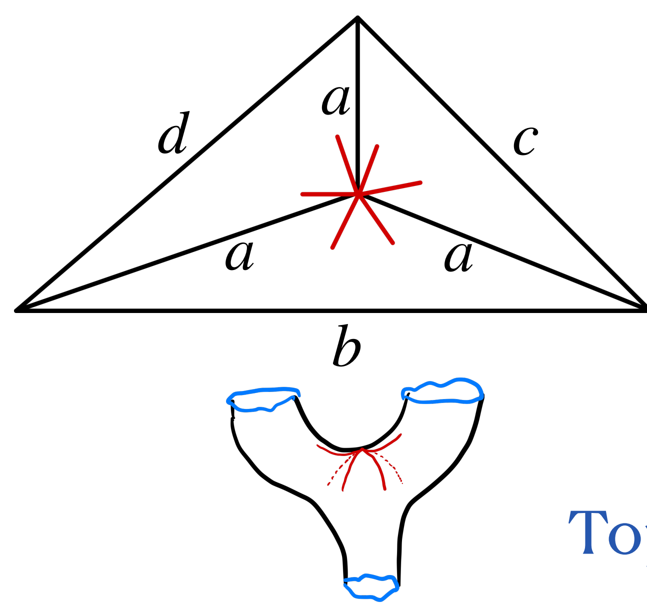

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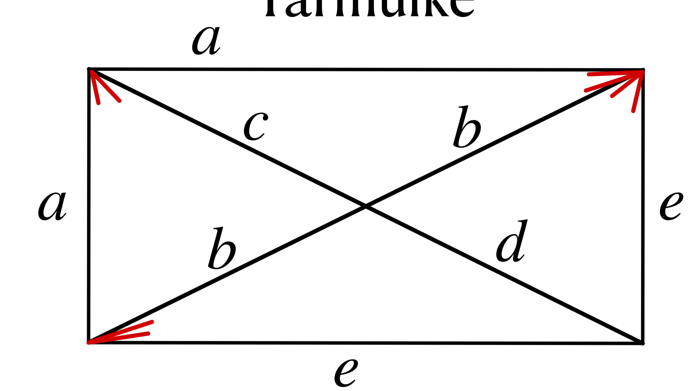

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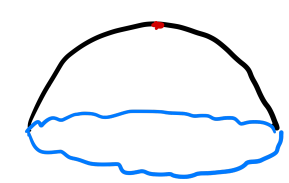


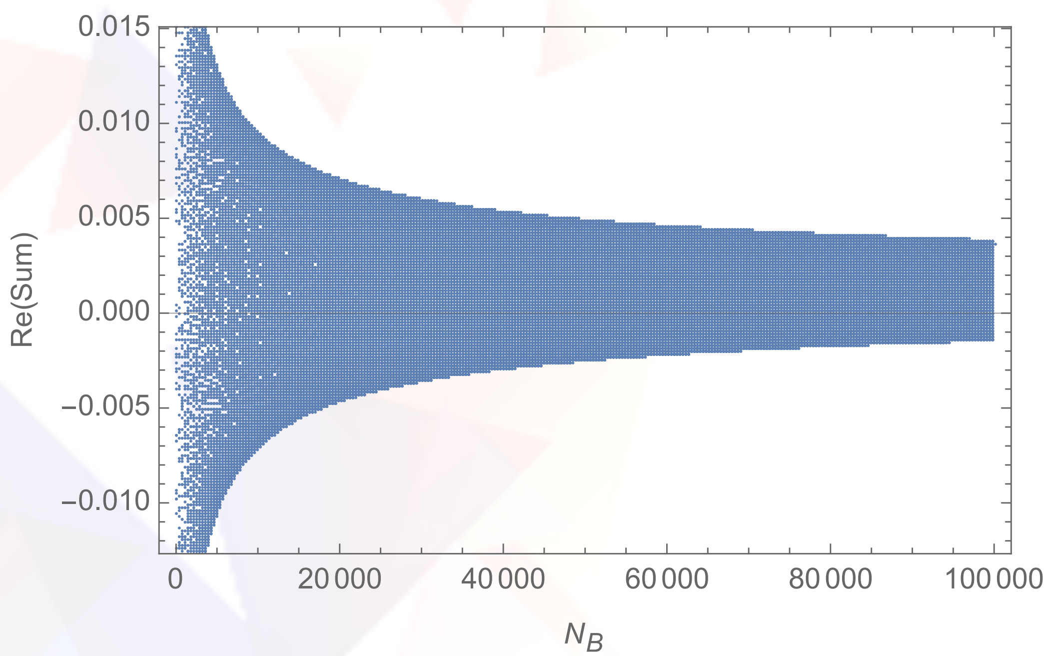
Higher Dimensions: Other causality conditions Edge causality, Vertex Causality [Jordan, Loll '13] [Borgolte, SKA wip]

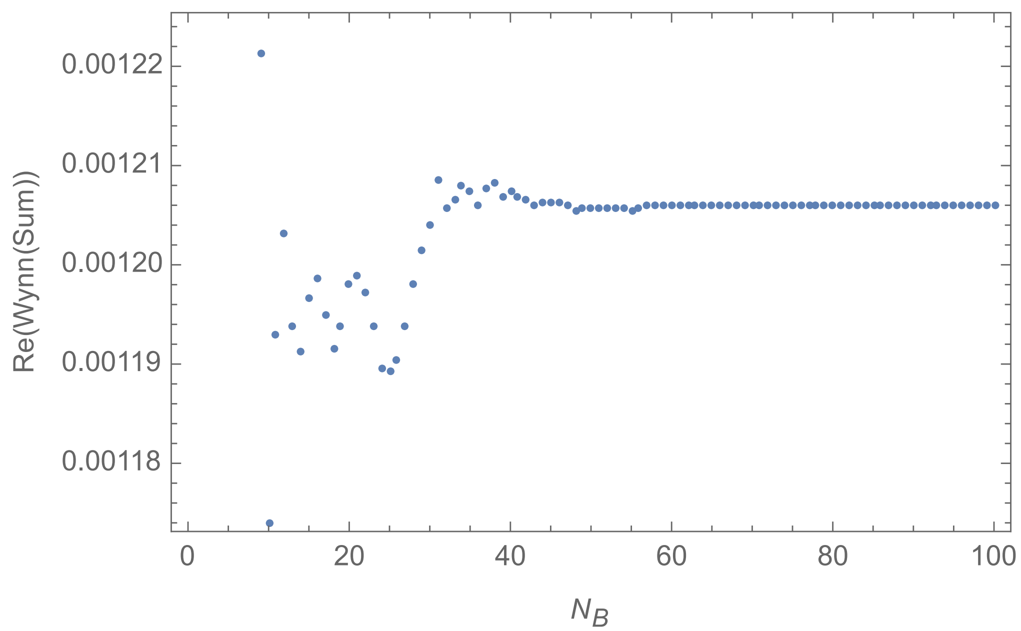
Irregular configurations leads to complex valued Regge action [Sorkin'19, Dittrich, Padua-Argüelles, SKA '21]

Convergence — Techniques to deal with complex amplitudes

Methods for treating complex amplitudes: Applications to Lorentzian path integrals

- Picard-Lefschetz, Holomorphic-gradient flow [*Han, Wan, Huang, Liu, Qu: Jia: Dittrich, Padua-Argüelles, SKA '22*]
- Tensor network techniques [*Bahr, Steinhaus, Dittrich, Cunningham, Mizera, Kaminski, Martin-Benito*]
- Quantum simulations, machine learning techniques
- Monte Carlo techniques [*Dona, Frisoni '23; Steinhaus '24*]
- Acceleration operators [EPRL: *Speziale, Dona, Sarno, Gozzini, Frisoni*] [ESF: *Dittrich, Padua-Argüelles '23*]
  - Shanks transform, Wynn epsilon algorithm, Aitken's delta process





[plots by: *Dittrich, Padua-Argüelles*]

Effective Spin foams

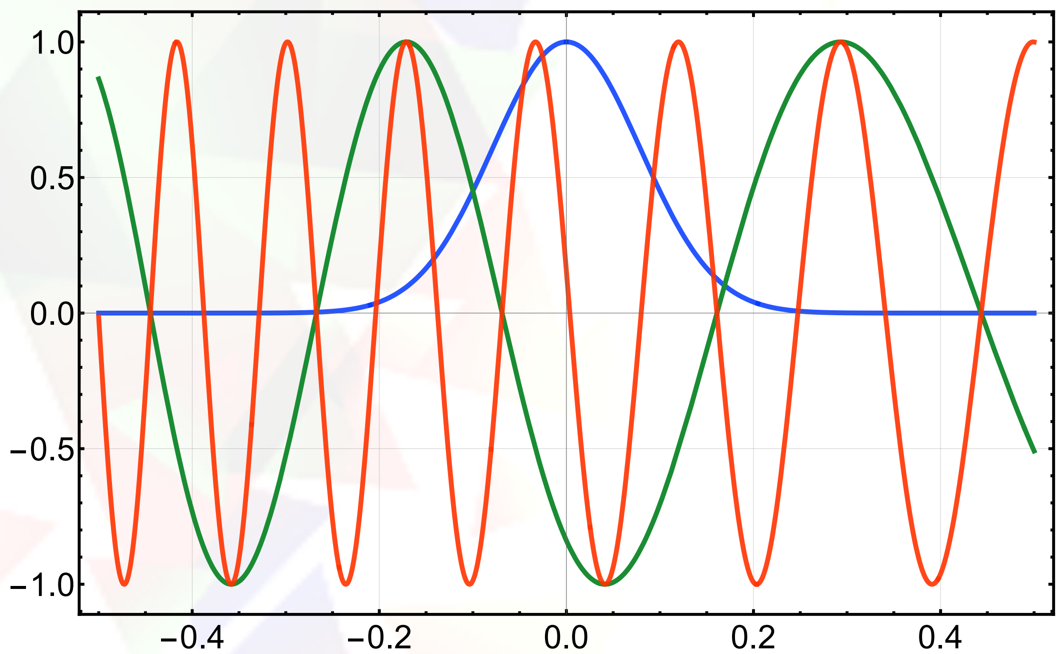
Testing the models

[SKA, Dittrich, Haggard, Padua-Argüelles]
arXiv: *2004.07013, 2011.14468, 2104.00485*

Discrete gravity dynamics

Semi-classical

Weakly imposed constraints



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Oscillations

Gaussians
peaked on constraints

Semi-classical limit:

Few oscillations over Gaussian needed

$$\gamma \sqrt{a_t}\, \mathrm{curv}_t \lesssim \mathcal{O}(1)$$

[*SKA, Dittrich, Haggard*]
[SF: *Han 13*]

[*Dittrich, Haggard, SKA '20*] [SF: *Han, Huang, Liu, Qu '21*]

Alternative idea: 'true' critical points are complex

Imaginary part of saddle point controlled by γ has to be small

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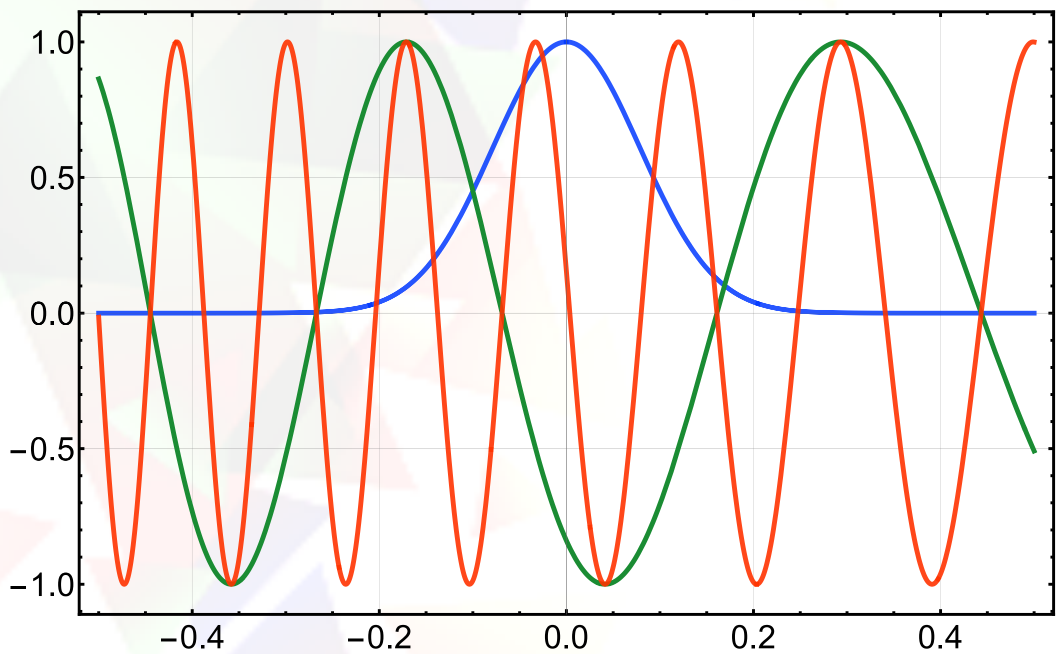

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[*Dittrich, Haggard, SKA '20*] [SF: *Han, Huang, Liu, Qu '21*]

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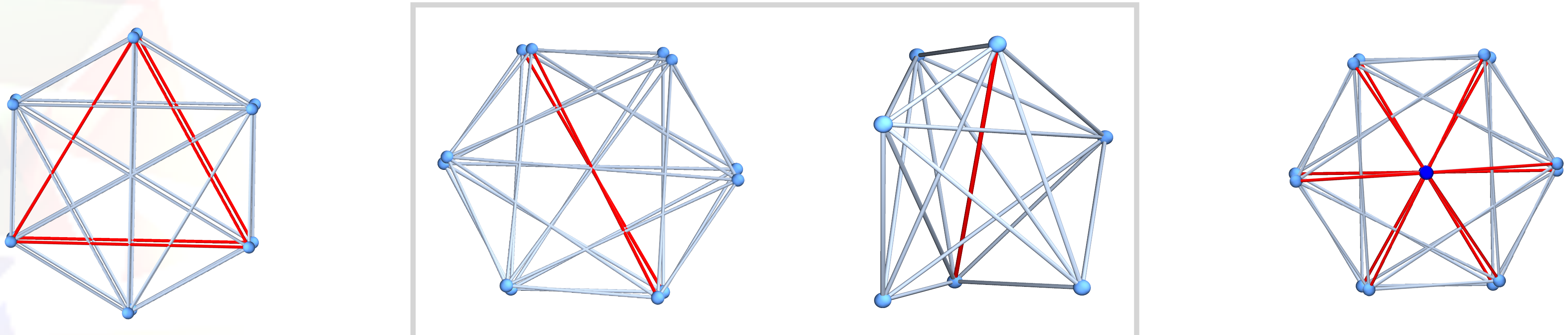
What are some effects of discrete area spectrum and 'weakly' imposed constraints ?

Testing ESF model

Numerical Tests

✦ Early results from explicit evaluations **Non-perturbative tests**

Several examples of discrete geometries with curvature



explicit sum over discrete areas, implement weakened constraints

Compute expectation values of bulk variables testing discrete EOMs

- Recover discrete gravity dynamics for certain range of parameters

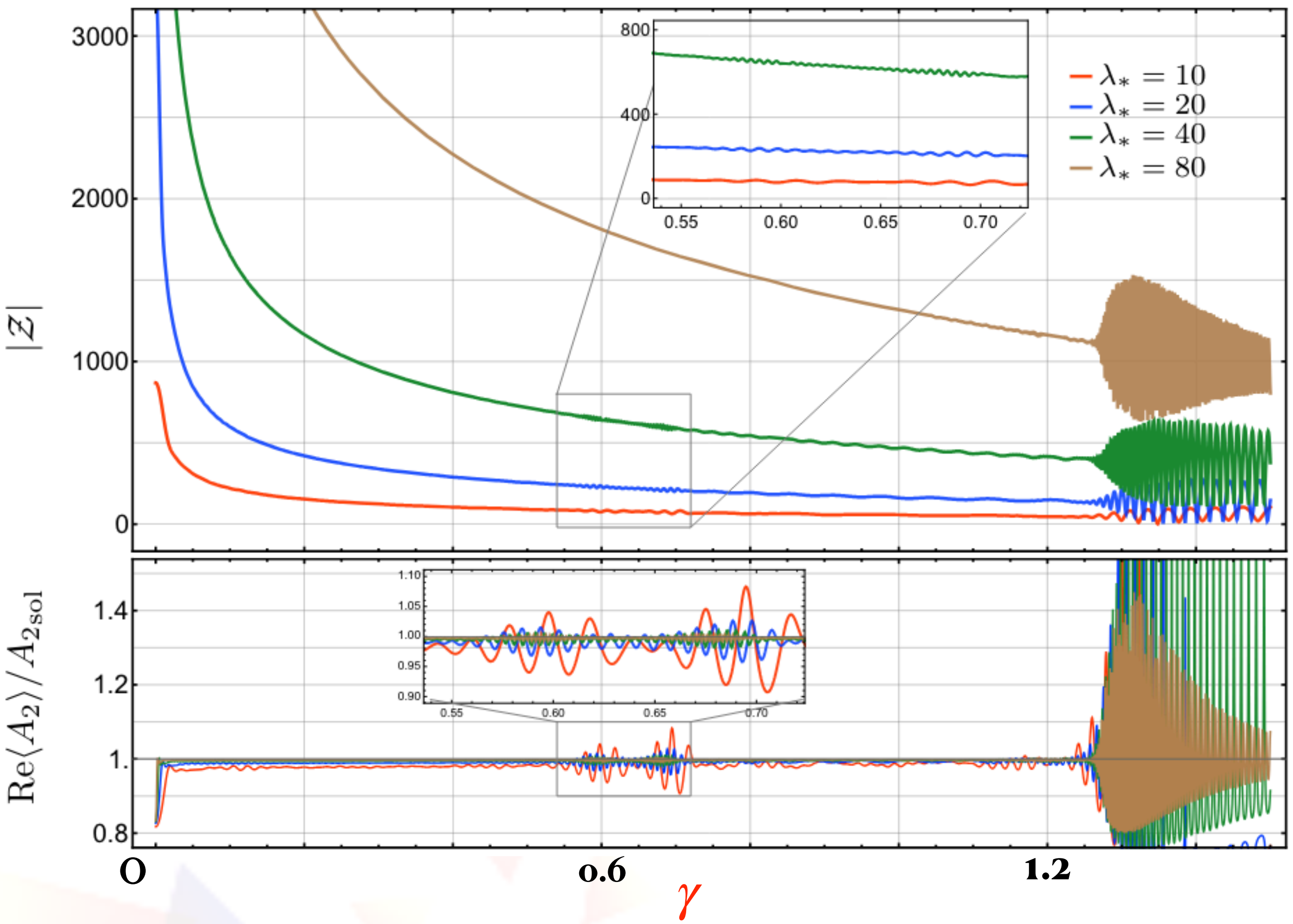
  interesting effects beyond saddle point evaluation

  larger acceptable range for γ than expected

Numerical results

Bulk-Edge

Small curvature



Oscillations due to interplay between discrete areas and constraint imposition

Nice surprises:

Large acceptable range for γ for the expectation values

Better matching of classical EOMs for large boundary scales

Numerical results

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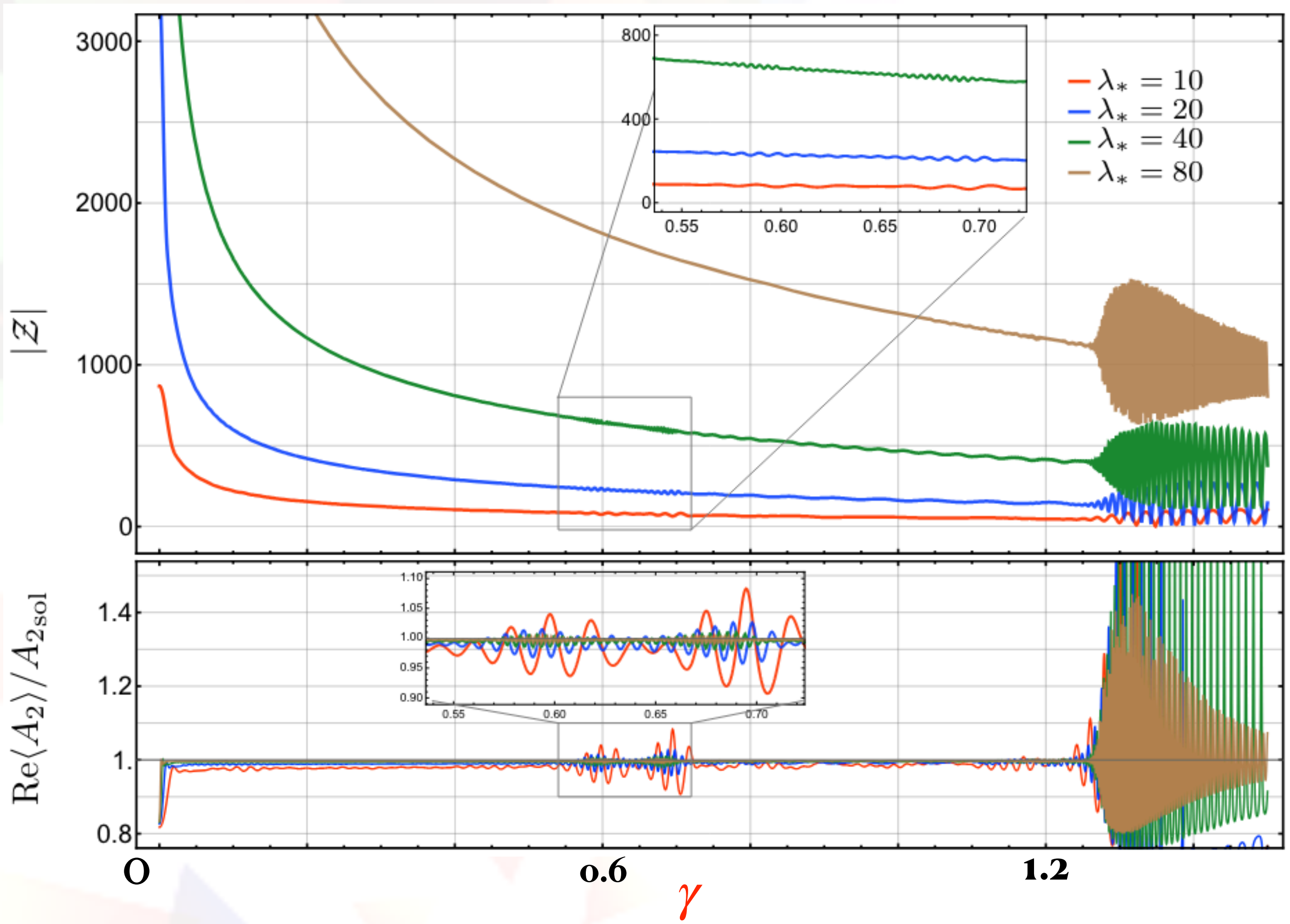

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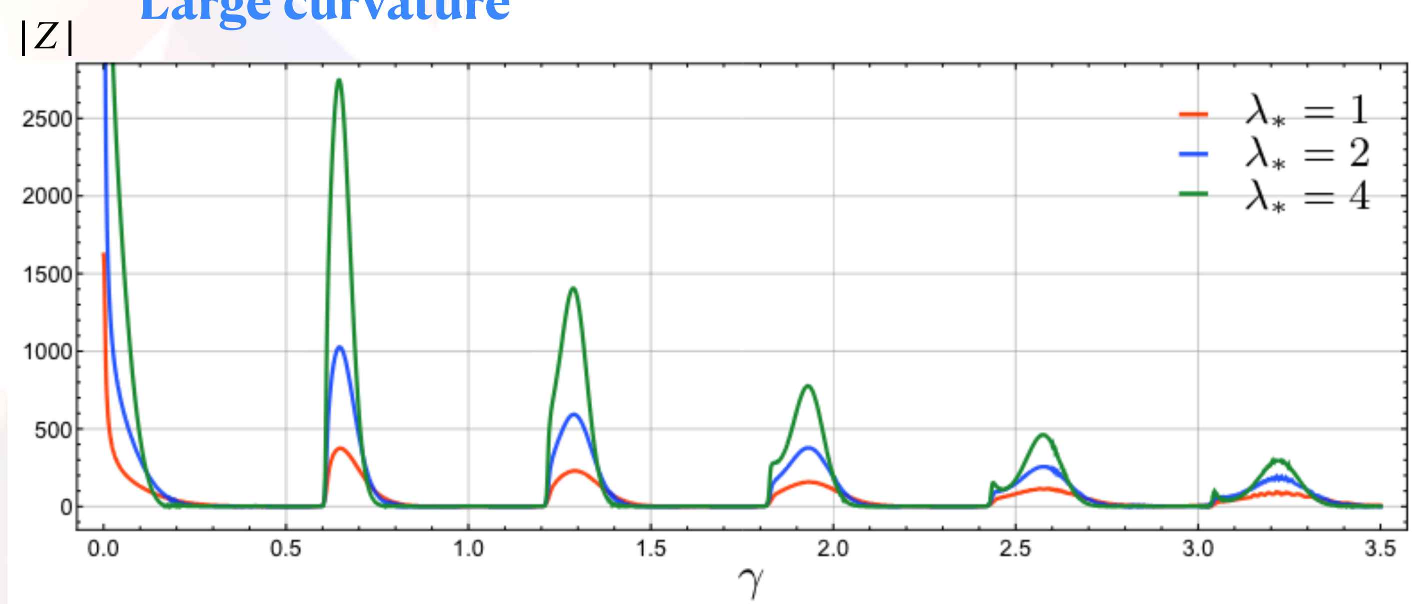
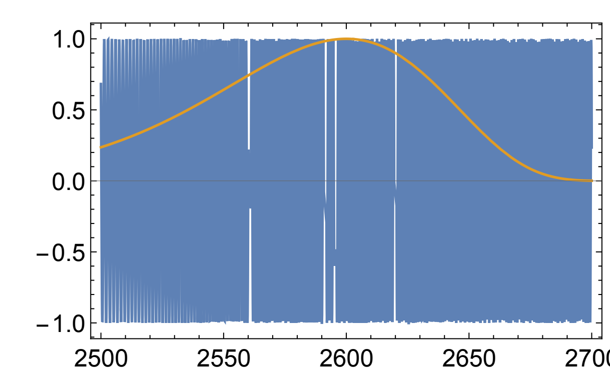
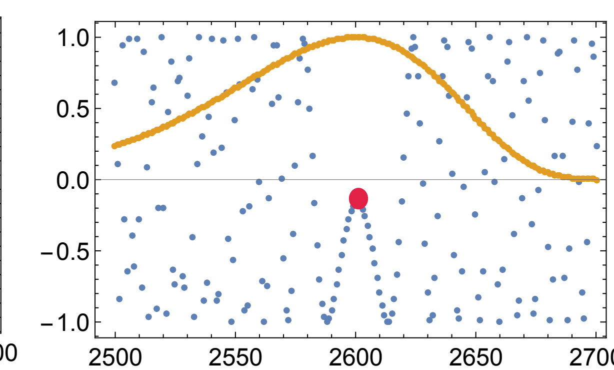
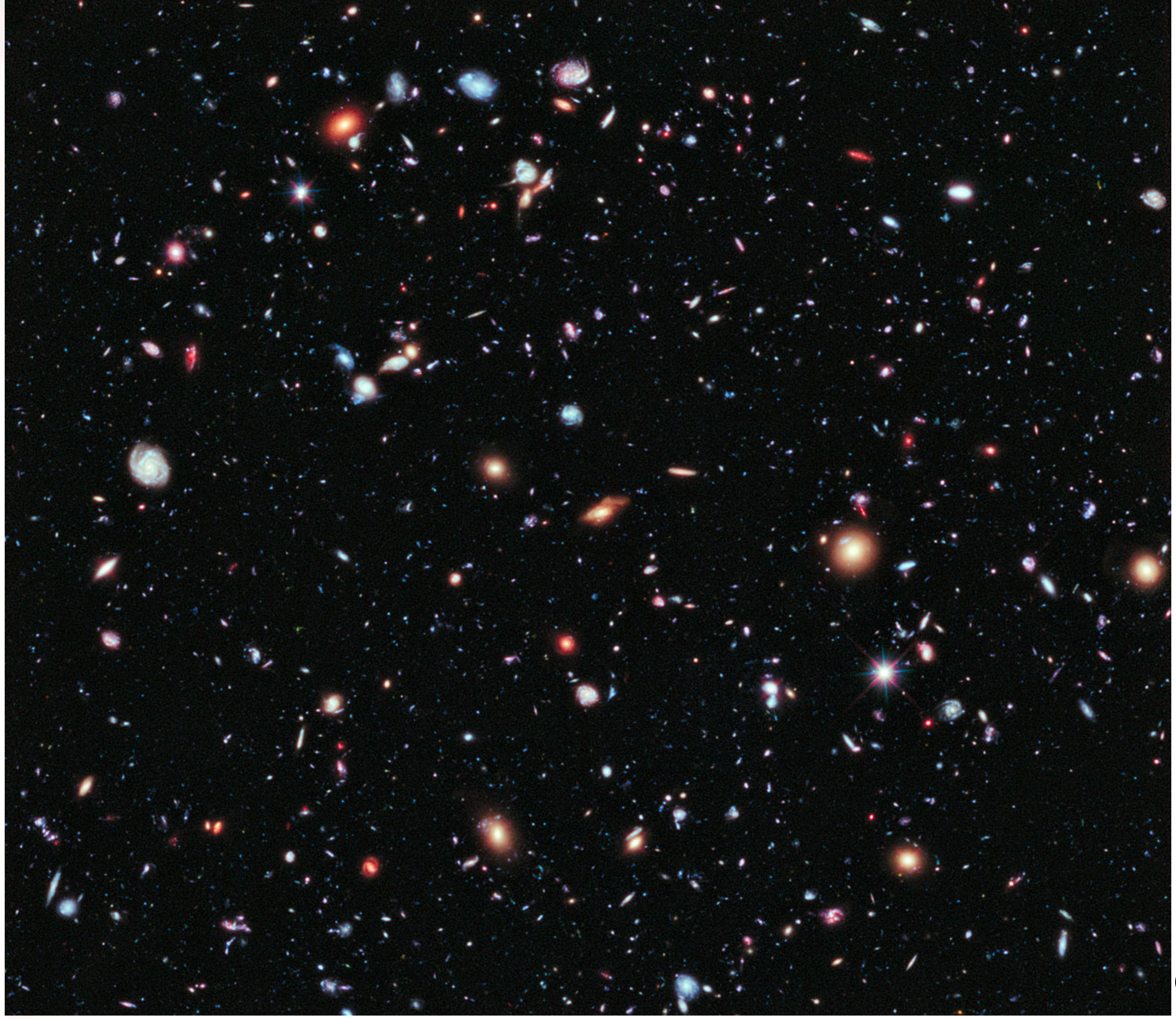


Peaks due to discretization effects: pseudo-saddle points







Credit: NASA, ESA, HUDF09 Team

Effective Spin foams

Cosmology applications

[*SKA, Dittrich, Padua-Argüelles, Gielen, Schander, Steinhaus, Jercher*]

arXiv: *2109.00875, 2112.15387, 2306.06012, 2312.11639*

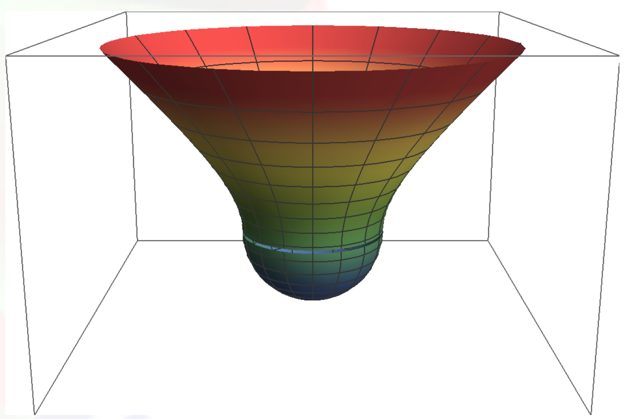
Cosmology Applications

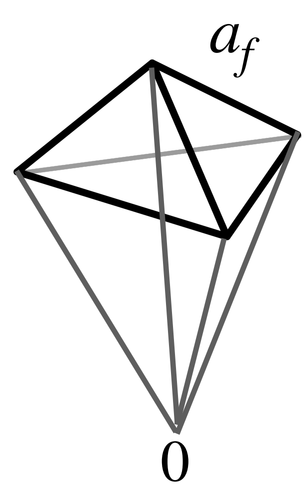
Mini-superspace models

[Hartle, Hawking, Feldbrugge, Lehners, Turok, Dorronsoro, Halliwell, Hertog, Jansen,….]

[Williams, Lui, Collins, Dittrich, Gielen, Schander, Vidotto, Gozzini, Frisoni,….]

No boundary proposal

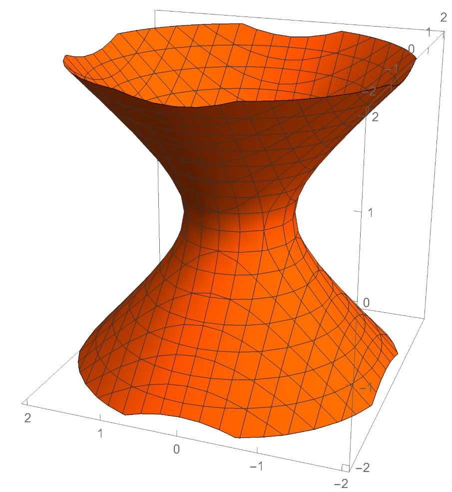


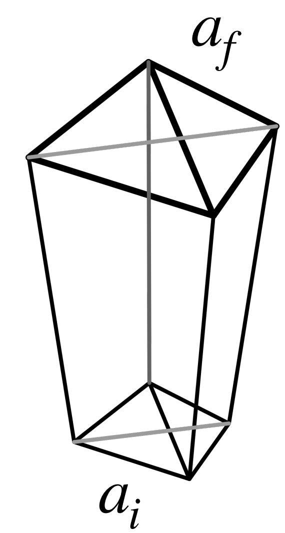


[Dittrich, Gielen, Schander '21]

partial discretization

deSitter cosmological spacetime

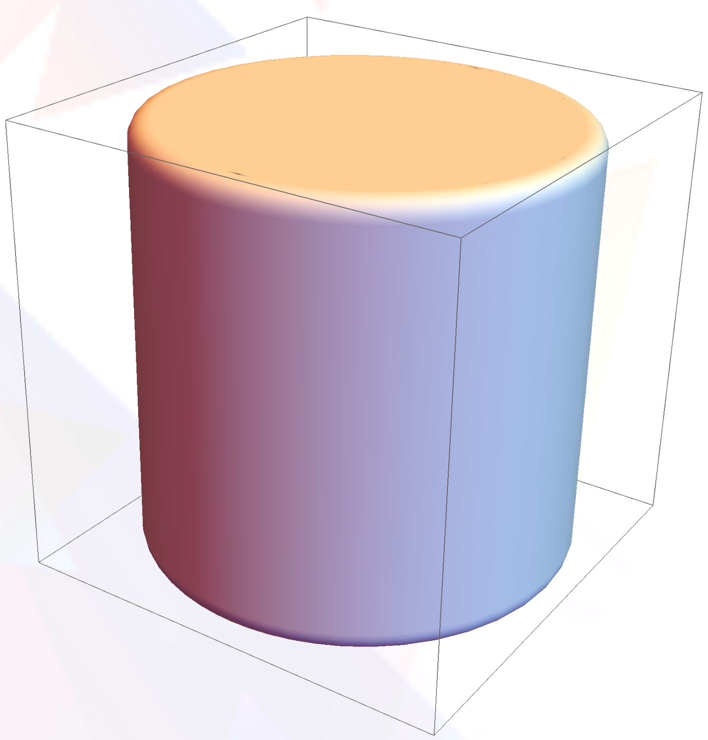


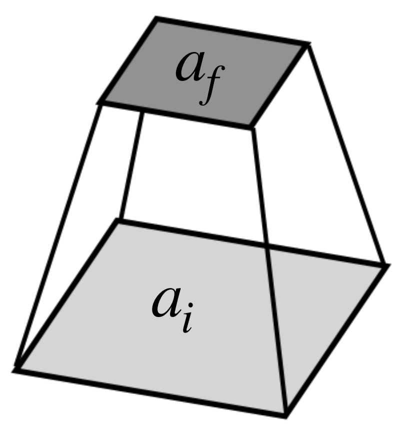


partial discretization

[Dittrich, Padua-Argüelles '23]

Spatially flat cosmology [Steinhaus , Jercher]





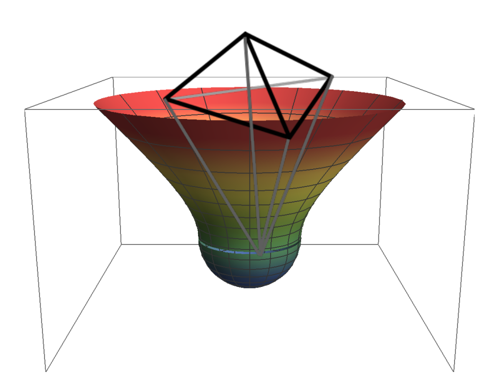
Alexander Jercher's talk!

Spin foam cosmology

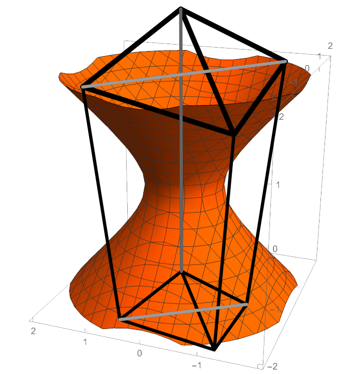
[Han, Liu, Qu, Vidotto, Zhang]

Cosmology Applications

[Dittrich, Padua-Argüelles '23]



Ball model



Shell model

Effective spin foam model

Results from deSitter cosmology

Deal with slowing converging and diverging sums

Unbounded sum over lapse variable (regular and irregular configurations)

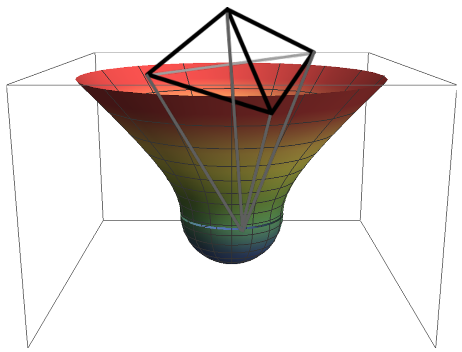
Acceleration techniques (Wynn's algorithm): Speeds up convergence of discrete sums

- works well for actions linear in summation variable (also for spin foams)

Cosmology Applications

[Dittrich, Padua-Argüelles '23]

Effective spin foam model



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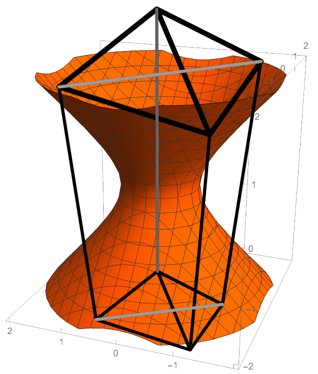

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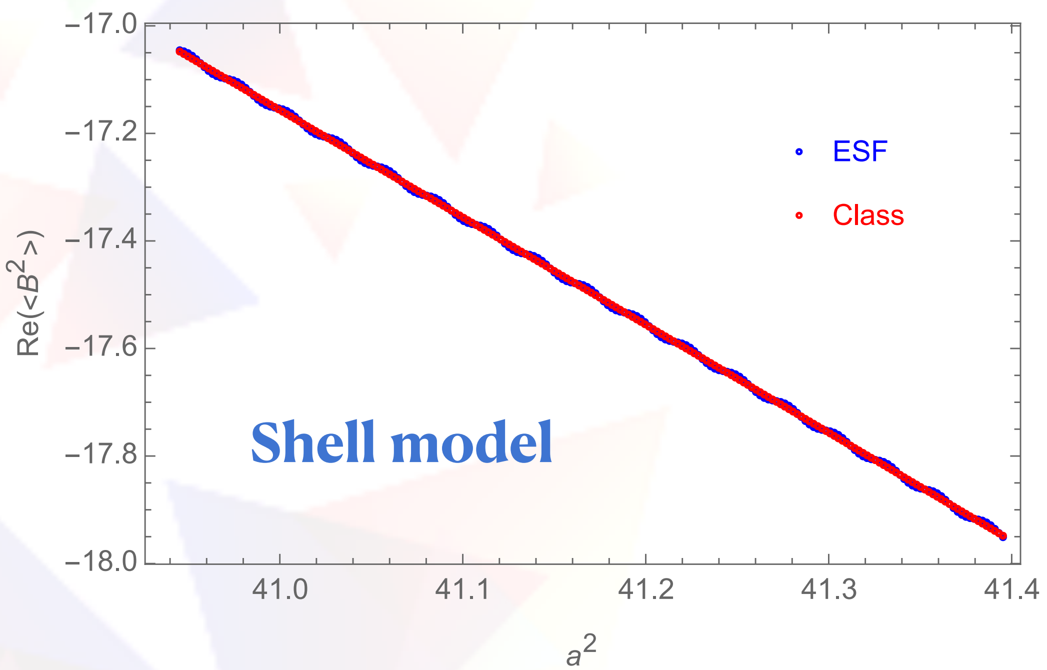
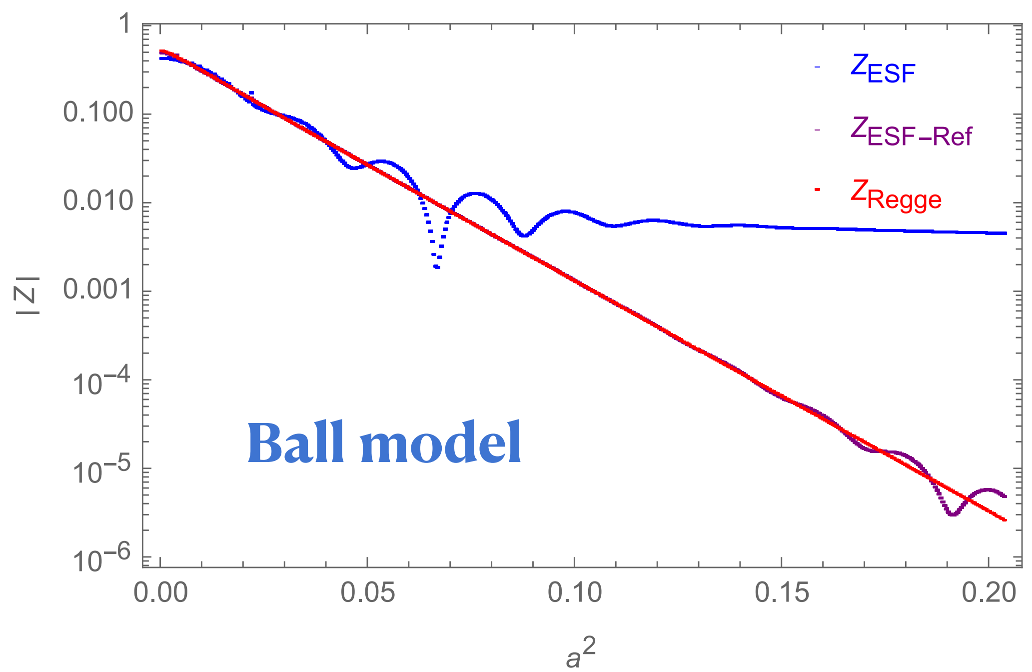
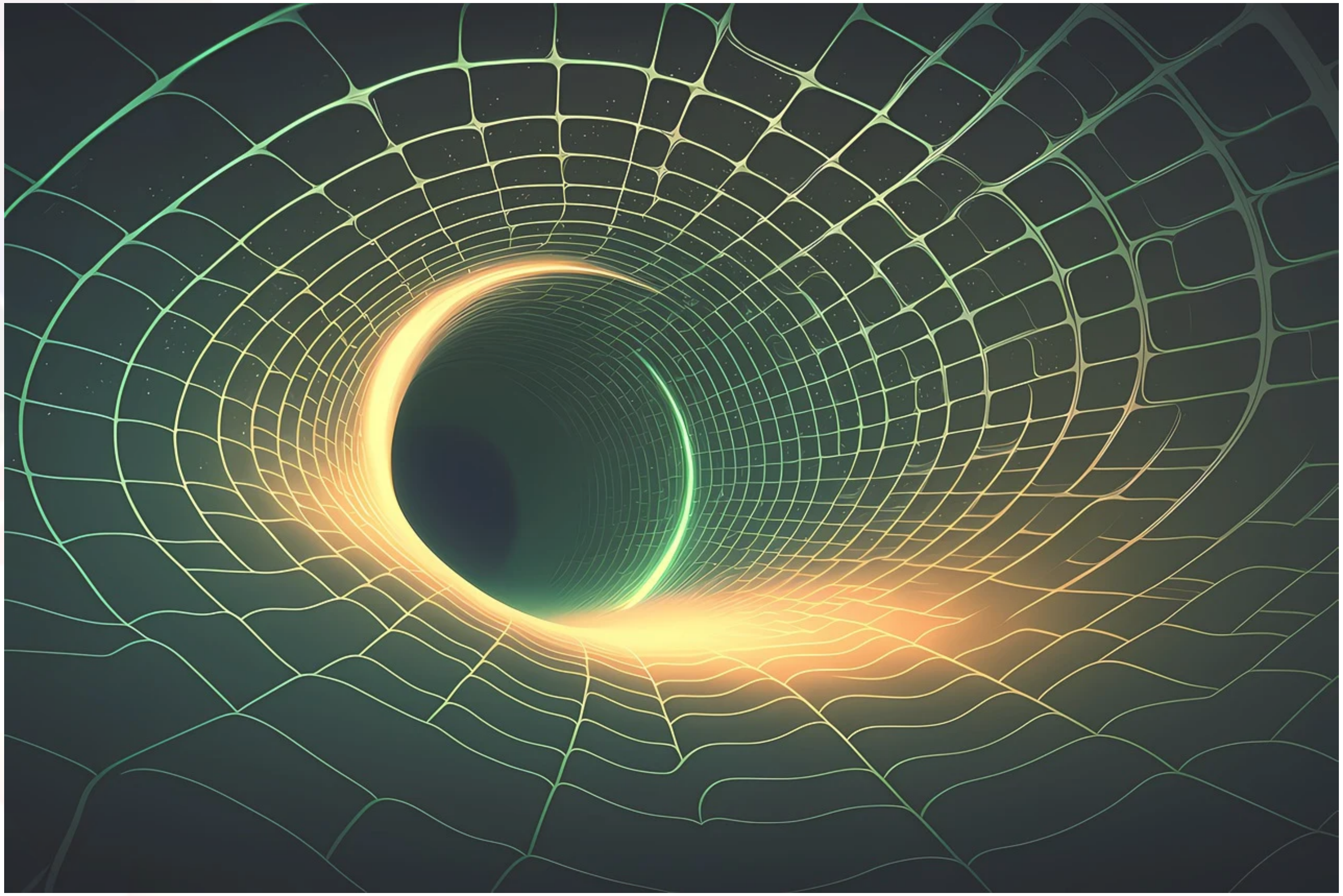




Partition function and expectation values sensitive to discrete area spectrum



Credit: Fzalai

Effective Spin foams

Continuum limit

[*Dittrich, Kogios, Borissova, Krasnov, Steinhaus, SKA*]

arXiv: *2105.10808, 2203.02409, 2207.03307, 2211.09578, 2312.13935*

Continuum limit

First Steps

[Dittrich '21, Dittrich-Kogios '22]

Action: $S = S_{\text{ARC}}(a) - i \sum_{\tau} \ln G_\tau(a)$

[Handbook: *Dittrich, Steinhaus, SKA*]

First attempts: Linearize area Regge calculus around a flat background on hyper cubic lattice(s)

Interesting Results

[Dittrich '21, Dittrich-Kogios '22]

Linearized continuum limit **Area Regge Calculus ~ GR + Weyl^2**

leading order in lattice derivatives ∂_k^2 (→ GR)

sub-leading ∂_k^4 (→ Weyl^2)

- Also holds starting from effective spin foams action

Continuum limit

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GR: leading order in lattice derivatives ∂_k^2

Weyl^2: sub-leading ∂_k^4

- Also holds starting from effective spin foams action

Related work (Area metric formulations)

✦ Modified Plebanski actions [*Borissova, Dittrich '22*]

✦ Continuum area-metric actions [*Borissova, Dittrich, Krasnov '23, Borissova, Ho '24*]

Continuum limit

Area Regge Calculus

Mechanism Compute Hessian around flat background

Quantum Regge Calculus [Williams, Roček '81]

Hessian $$S^{(2)} = \sum_t \frac{\partial a_t}{\partial l_e} \frac{\partial \epsilon_t}{\partial l_{e'}} \delta l_e \delta l_{e'}$$

Quantum: Area Regge Calculus

$$S^{(2)} = \frac{\partial \epsilon_t}{\partial a_{t'}} \delta a_t \delta a_{t'}$$ [Dittrich, Haggard, SKA '18]

Strategy: Organize variables according to order of lattice derivatives in Hessian

Leading order contributions results from variables forming an **area metric**

20 area metric variables per point $= h$ (10) $+ \chi$ (10)

trace part — trace-free part

Integrate Integrate out non-metric variables → Effective action for metric variables

$$S_{\text{eff}} = h \cdot (H_{hh} - H_{h\chi} H_{\chi\chi}^{-1} H_{\chi h}) \cdot h$$

Summary & outlook

What can we learn from (discrete) gravitational path integrals?

Using ‘effective spin foam models’

Quantum geometry

Quantum gravity

Semi-classical approximations

Topological transitions

Early universe Cosmology applications

Black hole dynamics

Quantum field theory in curved space time

Renormalization/Continuum limit

Look for predictions/observable signatures from area variables

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THANK YOU !