Group Field Theory and Spin Foam Renormalization

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in collaboration with

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2211.12768 (PRL 130, 141501, 2023), **2209.04297** (JHEP 2023, 74 (2023)),
2206.15442 (PRD 106, 066019, 2022), **2112.00091** (*JCAP* 01 (2022) 01, 050),
2110.15336 (JHEP 2021, 201 (2021)), **1904.00598** (PRD 98, 126006 (2018))
&
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DFG Deutsche Forschungsgemeinschaft

Outline

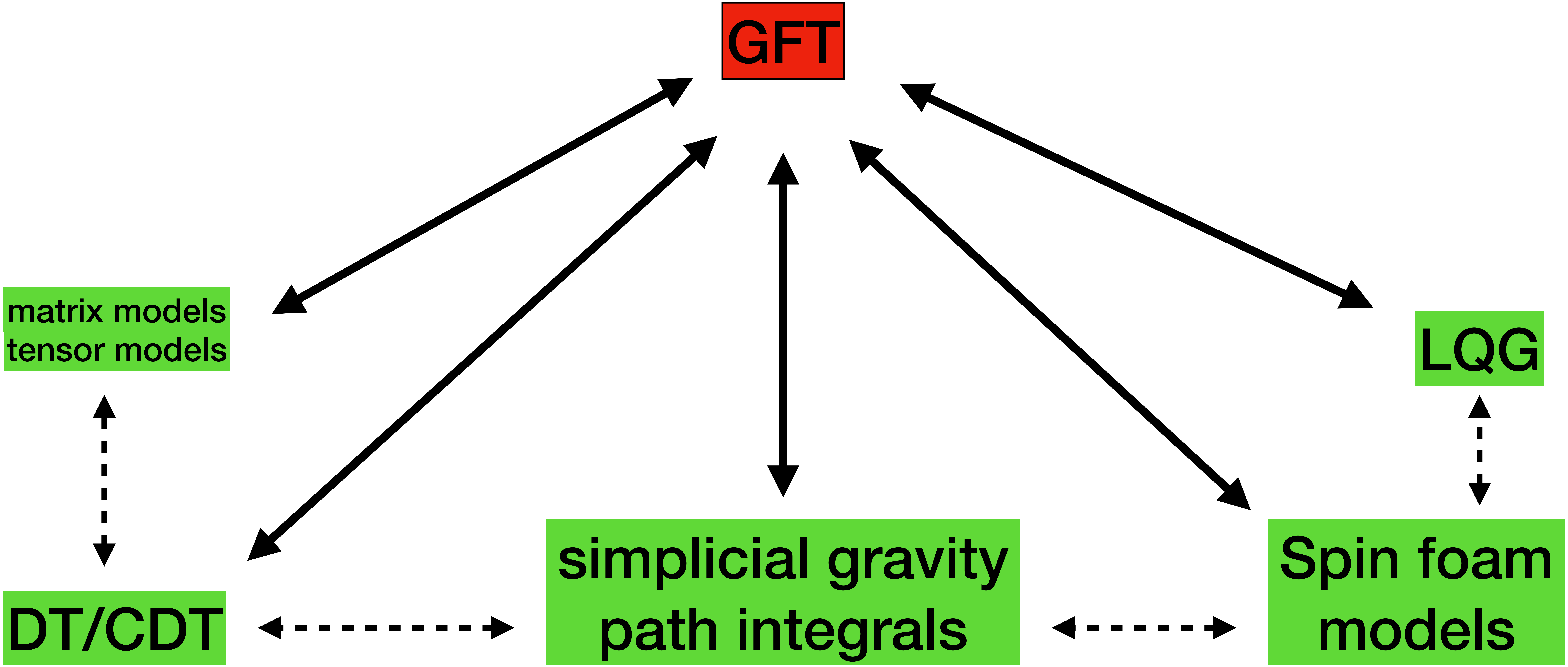
- **What is GFT?**
  - relation to spin foams and tensor models
- **GFT Renormalization**
  - power-counting, perturbative & non-perturbative renormalization and Landau-Ginzburg mean-field theory
- **Conclusions and outlook**

What is Group Field Theory?

Group Field Theory

(Freidel 2005; Oriti 2006; Krajewski 2011; Carrozza 2012; Carrozza 2016; Carrozza 2024)

at the confluence of various discrete quantum gravity approaches



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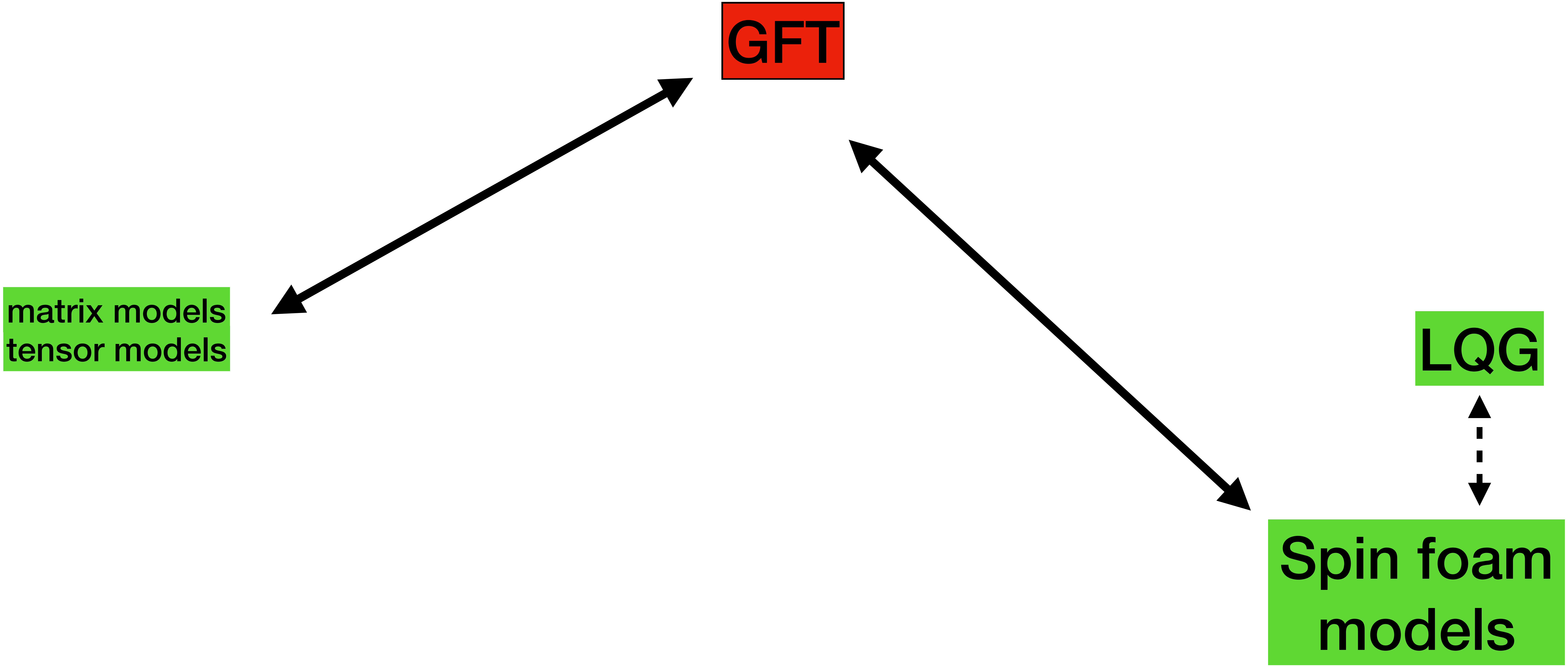

What are spin foams?

Group Field Theory via Spin Foams

- GFTs are **a** tentative completion of **spin foam models**:

  What are spin foam models?

  - provide lattice regularized quantum gravitational path integral
    - *sum over histories definition of quantum dynamics of spacetime*

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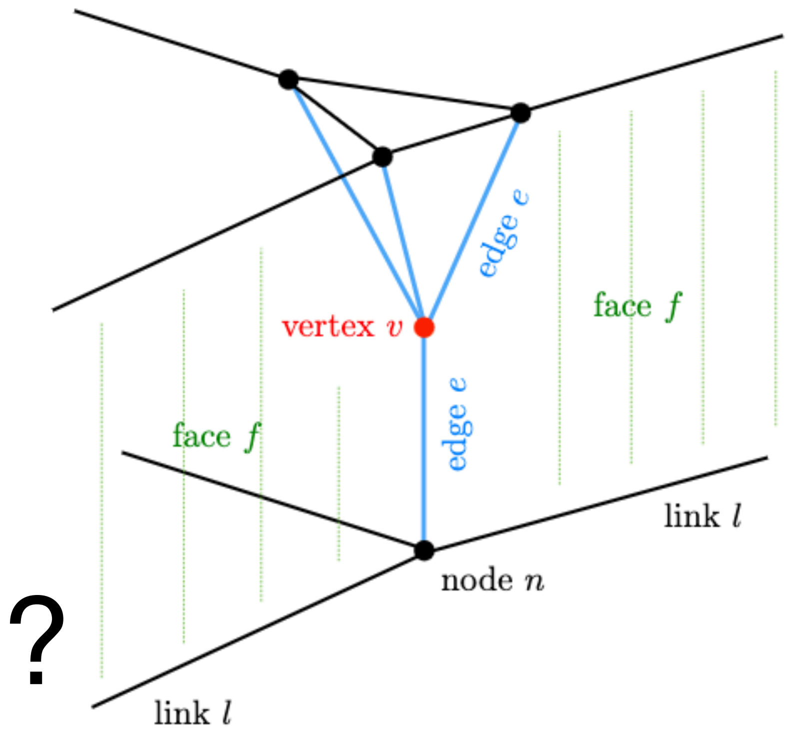
  - one quantum history = **spin foam** (specify complex dressed with algebraic data)

- **basic component** of spin foam model:
  - *quantum amplitude for given complex*

$$Z(\Gamma) = \sum_{\{J\},\{I\}|j,j',i,i'} \prod_f A_f(J,I) \prod_e A_e(J,I) \prod_v A_v(J,I) \equiv \mathcal{A}_\Gamma$$



Γ

- single + simplicial complex?, triangulation dependence?
- finite number of degrees of freedom

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    - 2) **organization principle** for amplitudes

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- GFT models provide a **QFT** generating function for spin foam amplitudes

→ **perturbative expansion of GFT = spin foam model with sum over complexes:**

$$Z_{GFT} = \sum_{\Gamma} \overset{\text{weights}}{w(\Gamma)} \mathcal{A}_{\Gamma} = \sum_{\Gamma} \frac{\overset{\text{GFT coupling}}{\lambda}^{\overset{\text{number of vertices}}{n(\Gamma)}}}{\mathrm{sym}(\Gamma)} \overset{\text{amplitude}}{\mathcal{A}_{\Gamma}}$$

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→ **perturbative expansion of GFT = spin foam model with sum over complexes:**

weights GFT coupling number of vertices amplitude

$$Z_{GFT} = \sum_{\Gamma} w(\Gamma)\mathcal{A}_{\Gamma} = \sum_{\Gamma} \frac{\lambda^{n(\Gamma)}}{\mathrm{sym}(\Gamma)} \mathcal{A}_{\Gamma}$$

- consistency: renormalization of amplitudes
- evaluate Z = control continuum limit

(details follow)

What are tensor models?

Group Field Theory via Tensor Models

- GFTs are algebraically enriched **tensor models**:

  (Ambjorn, Durhuus, Jonsson 1991; Sasakura 1991; Gross 1992)

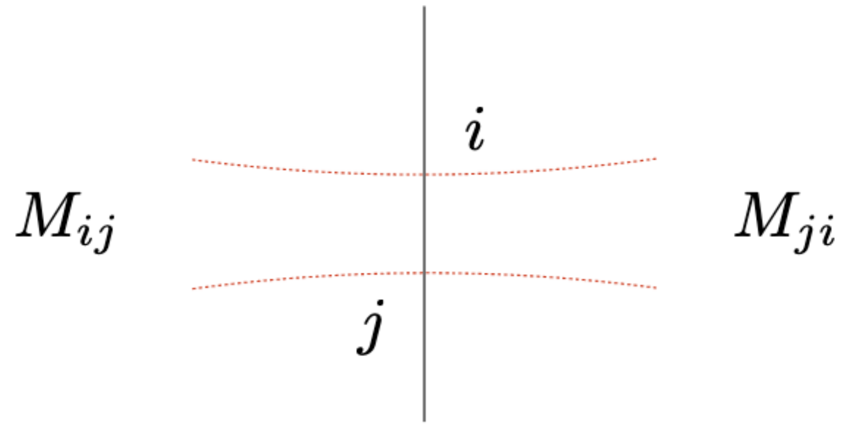
  - tensor models: higher dimensional generalization of matrix models

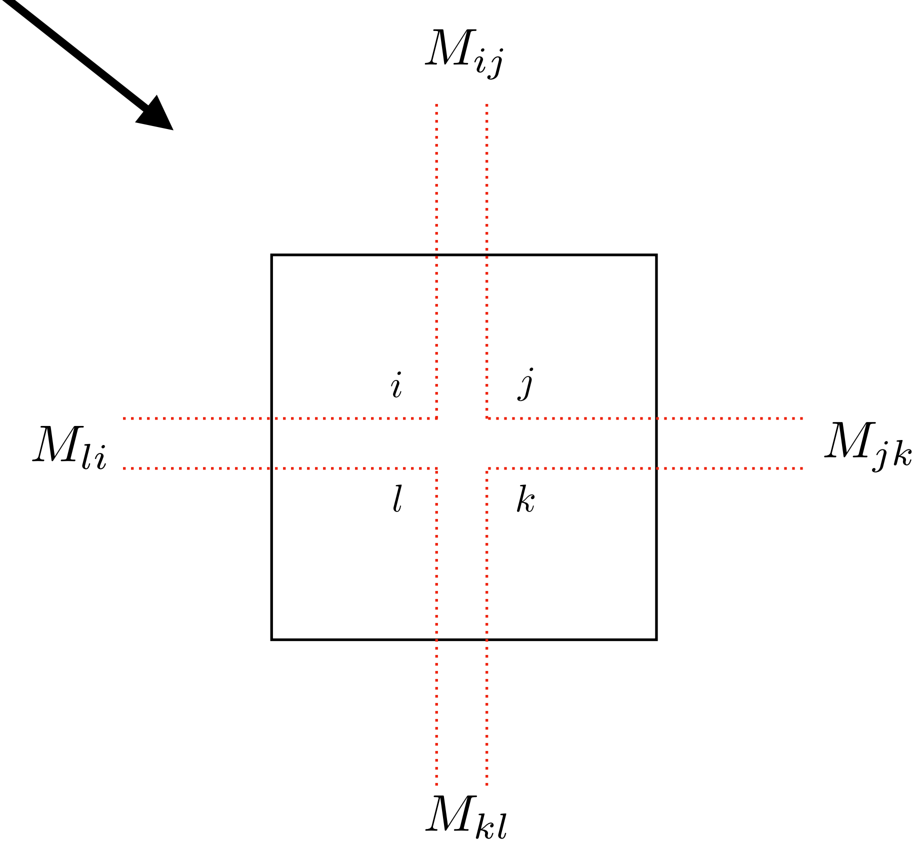
    ('t Hooft 1974; Brezin, Itzykson, Parisi, Zuber 1978; David 1985; Ambjorn, Durhus, Fröhlich 1985; Di Francesco, Ginsparg, Zinn-Justin 1995;….)

  - **matrix models:** $M: \; N \times N$ Hermitian matrix

action: $$S(M) = \frac{1}{2}\mathrm{tr}M^2 - \frac{\lambda}{4}N\mathrm{tr}M^4$$

partition function: $$Z = \int \mathrm{d}M \mathrm{e}^{-S(M)}$$





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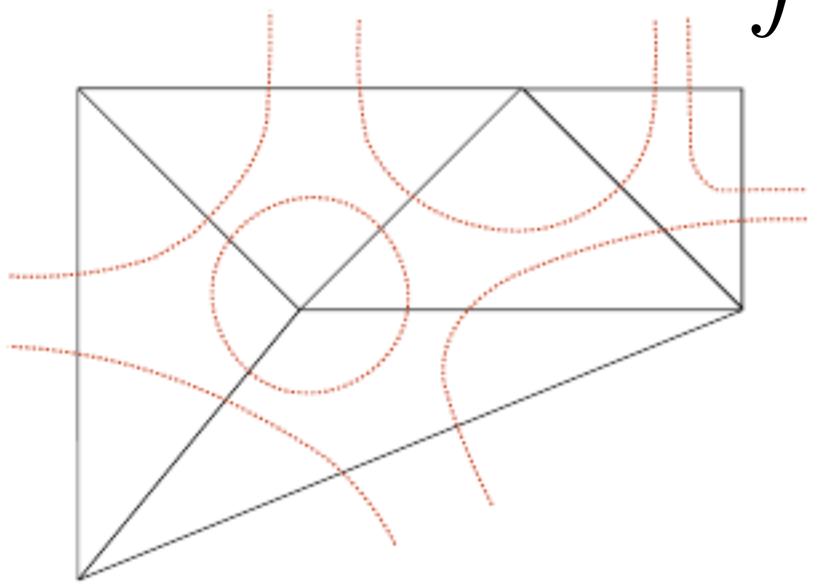
    - Feynman graphs are stranded diagrams
    - dual to tesselations of closed surfaces
    - due to rigid interplay between combinatorics + global structure of Feynman diagrams



snapshot of a Feynman graph

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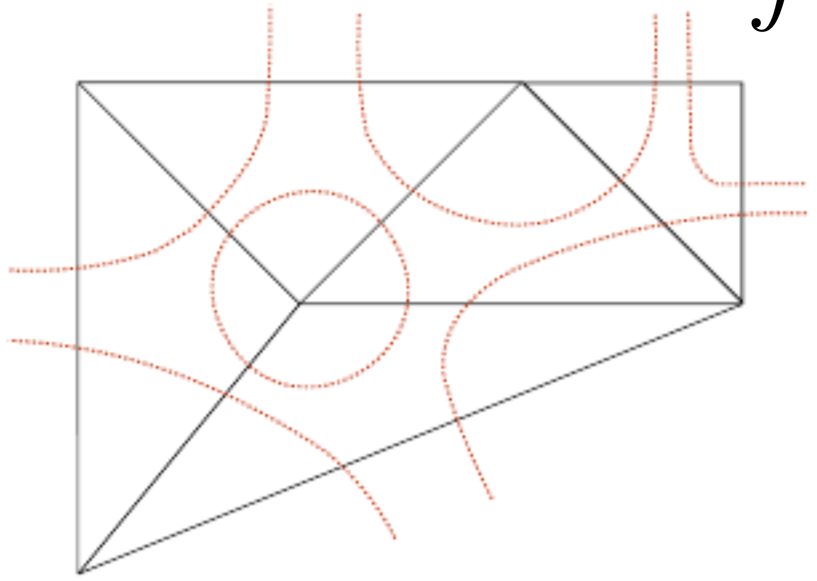

snapshot of a Feynman graph

    - Feynman graphs are stranded diagrams
    - dual to tesselations of closed surfaces
    - due to rigid interplay between combinatorics + global structure of Feynman diagrams
    - large-N expansion is topological genus expansion $$\log Z = \sum_{\Gamma_c} \frac{\lambda^{\overset{\text{number of vertices}}{\downarrow}\, n(\Gamma_c)}}{\mathrm{sym}(\Gamma_c)} \overset{\text{amplitude}}{\overset{\downarrow}{\mathcal{A}_{\Gamma_c}}} \stackrel{!}{=} \sum_{g \in \mathbb{N}_0} N^{2-\overset{\text{genus}}{\overset{\downarrow}{2g}}} \mathcal{F}_g(\lambda)$$
    - **critical regime/continuum limit:** 2d Euclidean (Liouville) quantum gravity

(Miller, Sheffield 2015; Gwynne 2020)

Group Field Theory via Tensor Models

- **colored** tensor models: $T^c: \; N^{\times r}, \; \text{colored tensor}, \; c = 0, ..., r$ (Gurau 2011)
    - colorization to ensure nice global properties like manifold structure

$$S(T^c, \bar{T}^c) = \sum_{c=0}^{r} \sum_{a_0,...,a_r} T^c_{a_1...a_r} \bar{T}^c_{a_1...a_r} - \frac{\lambda}{N^{r(r-1)/4}} \sum_{\{a_{cc'}, c<c'\}} \prod_{c=0}^{r} T^c_{\mathbf{a}_c} + \text{c.c.} \qquad \mathbf{a}_c = (a_{cc-1}, ..., a_{c0}, a_{cr+1}, ..., a_{cc+1}), \; a_{cc'} \equiv a_{c'c}$$

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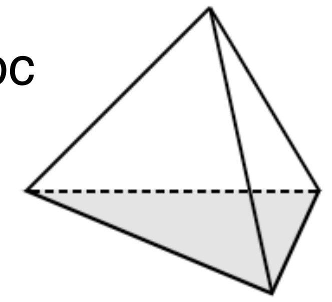
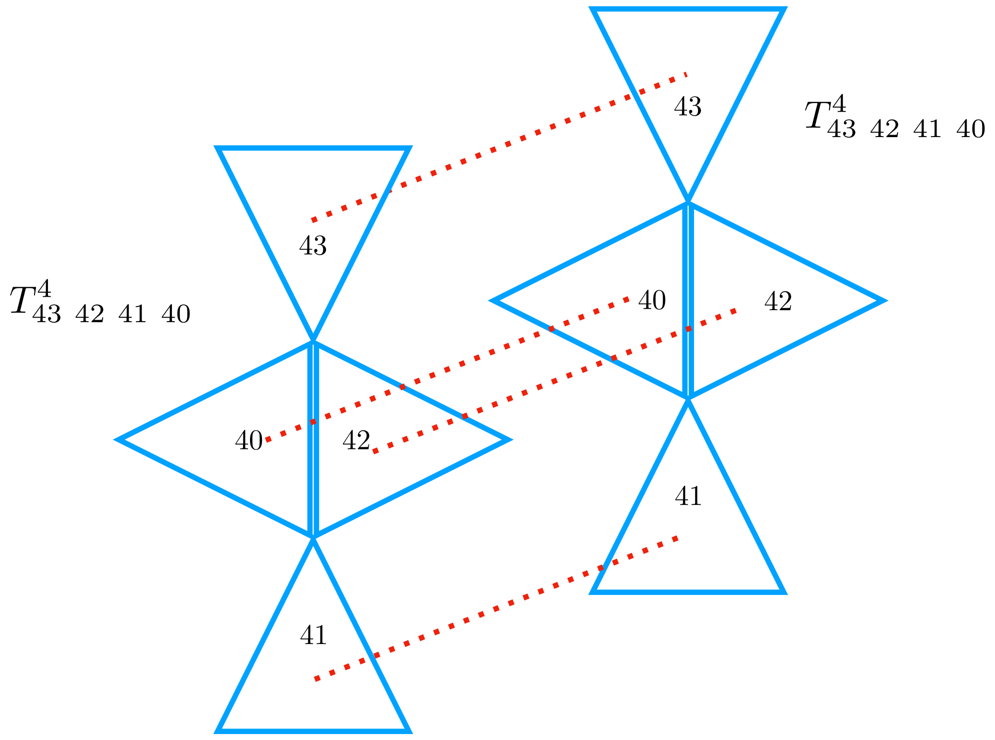
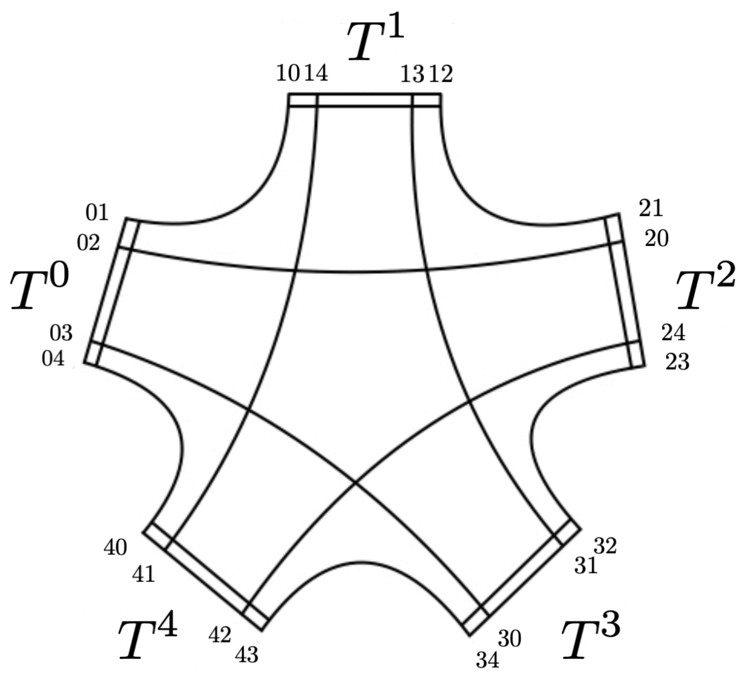
$r = 4$

$$S(T^c, \bar{T}^c) = \sum_{c=0}^{4} \sum_{a_0,...,a_4} T^c_{a_1a_2a_3a_4} \bar{T}^c_{a_1a_2a_3a_4} - \frac{\lambda}{N^3} \sum_{\text{all indices}} T^0_{04030201} T^1_{10141312} T^2_{21202424} T^3_{32313034} T^4_{43424140} + \text{c.c.}$$

$T \equiv$ 

tetrahedron





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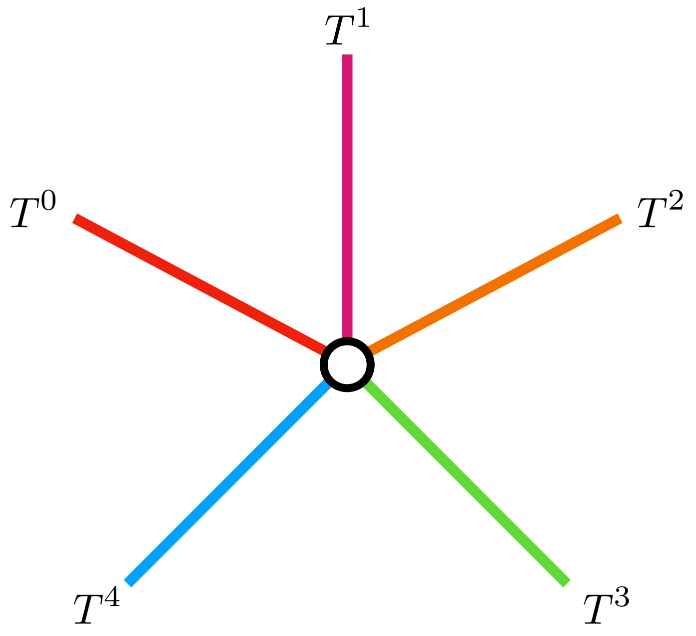

combinatorics of a 4-simplex

Group Field Theory via Tensor Models

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  - Feyman diagrams are edge-colored graphs with more rigid combinatorics

    (Gurau 2010; Bonzom, Gurau, Riello, Rivasseau 2011; Gurau, Ryan 2012)
    - they are dual to discrete orientable pseudomanifolds

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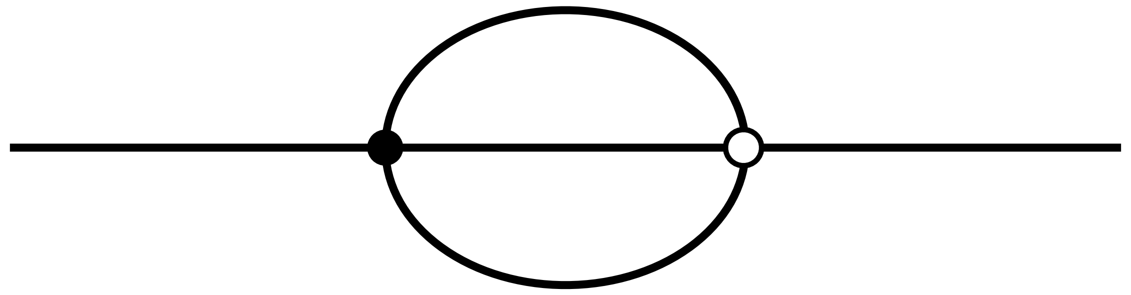
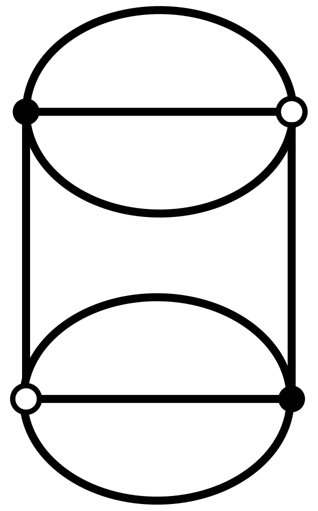
    - large-N expansion indexed by higher-dim. analogue of genus (Gurau 2011; Gurau, Rivasseau 2011; Gurau 2012)

$$Z = \int \prod_c \left(\mathrm{d}T^c \mathrm{d}\bar{T}^c\right) \mathrm{e}^{-S(T^c, \bar{T}^c)} \longrightarrow \log Z = \sum_{\omega \in \mathbb{N}_0} N^{r - \frac{2}{(r-1)!}\overset{\text{Gurau degree}}{\omega}} \mathcal{F}_\omega(\lambda\bar{\lambda})$$

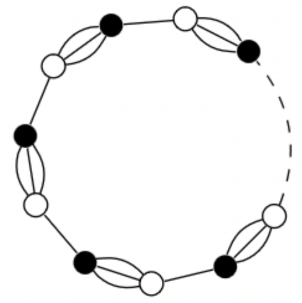
    - dominated by melonic diagrams (=triangulation of r-spheres) (Bonzom, Gurau, Riello, Rivasseau 2011)



open melon



closed 2-melon melon



closed chain of melons

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  - **continuum limit:** (mostly) branched polymer phase for such simple models (Gurau, Ryan 2014; Bonzom, Gurau, Ryan, Tanasa 2014; Bonzom, Delpouve, Rivasseau 2015; Lionni, Thürigen 2019)
    - open problem: find non-trivial metric space with dimension > 2

What is Group Field Theory?

Group Field Theory

- **Tensor models:**
  - Feynman expansion generates random discrete geometries
- **GFTs as enriched tensor models:**
  - 1) QFTs of tensor fields living on Lie group G
  - 2) closure condition
  - 3) specific combinatorial non-local action

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→ Feynman expansion defines lattice gauge theory on a random lattice

- GFTs as tentative completions of **spin foam models**

BONUS
- use standard QFT concepts and methods (e.g. renormalization and mean-field analysis) to unravel their phase structure and to extract physics (e.g. cosmology + black holes)

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rank r

group field $\quad \varphi(g_1, ..., g_r) : G^r \to \mathbb{R}, \mathbb{C}, \quad \varphi \in L^2(G^r)$

Lie group G

parallel transport $\quad g_I = \mathcal{P}\mathrm{e}^{\int_{e_I} A} \quad$ for $\quad I = 1, ..., r, \quad$ link e_I, connection A

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phase space: $T^*G^r \cong G^r \times \mathfrak{g}^r$

bi-vector/fluxes

dual formulation: $\quad \tilde{\varphi}(B_1,...,B_r) = \int (\mathrm{d}g)^r \varphi(g_1,...,g_r) \prod_{I=1}^{r} e_{g_I}(B_I)$

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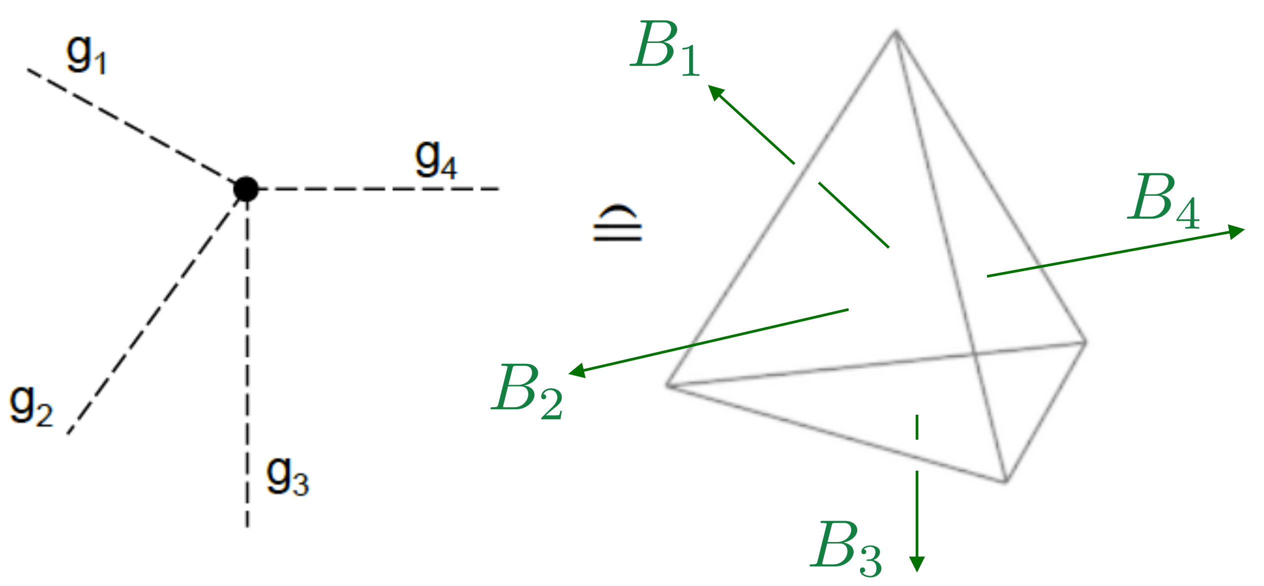
  - 2) closure condition

$$\varphi(g_1, ..., g_r) = \varphi(g_1 h^{-1}, ..., g_r h^{-1}), \quad \forall h \in G \longrightarrow \sum_{I=1}^{r} B_I = 0 \qquad r = 4$$

tetrahedron



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$$S_{\mathrm{GFT}} = \int (\mathrm{d}g)^r \bar{\varphi}(g_I)\mathcal{K}\varphi(g_I) + \mathcal{V}[\bar{\varphi}(g_I), \varphi(g_I)]$$

$\mathcal{K}:$ kinetic operator, $\mathcal{V}:$ non-linear and non-local interaction term (examples on next slide)

→ model specified by: G, rank r, $\mathcal{K}$, $\mathcal{V}$ and symmetries of φ

→ $\mathcal{K},\ \mathcal{V} \leftrightarrow \{A_f, A_e, A_v\}$ (amplitudes of respective spin foam model)

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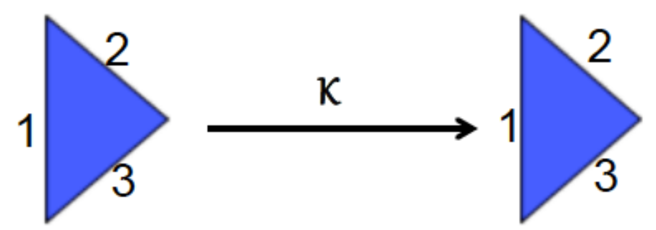
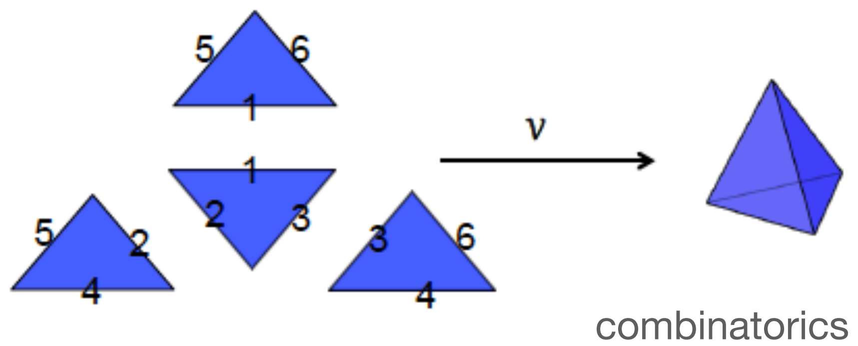
$\boxed{\text{example in 3d:}}$

(Boulatov 1992)

$$S = \int (\mathrm{d}g)^3 |\varphi_{123}|^2 + \frac{\lambda}{4!} \int (\mathrm{d}g)^6 \varphi_{123}\varphi_{245}\varphi_{156}\varphi_{364} + \text{c.c.} \quad , \; \varphi_{123} \equiv \varphi(g_1, g_2, g_3)$$





combinatorics of a 3-simplex

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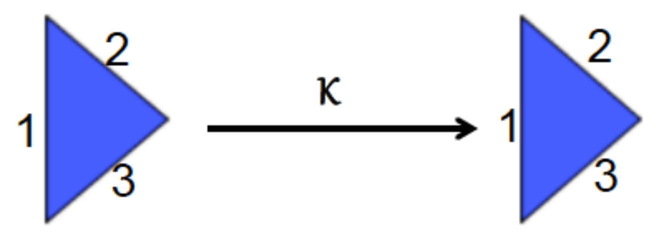
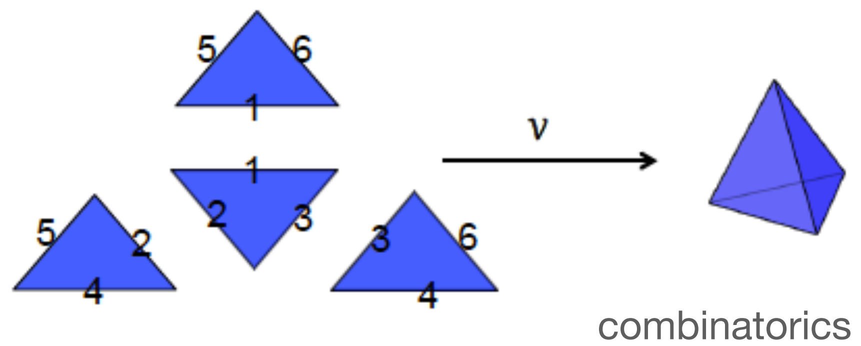


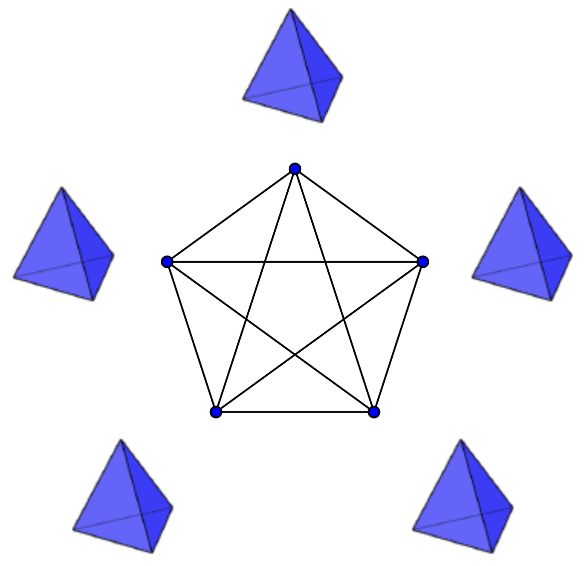


combinatorics of a 3-simplex

example in 4d:

(Ooguri 1992)

$$S = \int (\mathrm{d}g)^4 |\varphi_{1234}|^2 + \frac{\lambda}{5!}\int (\mathrm{d}g)^{10} \varphi_{1234}\varphi_{4567}\varphi_{7389}\varphi_{962(10)}\varphi_{(10)851} + \mathrm{c.c.}$$



combinatorics of a 4-simplex

also colored versions

(Gurau 2010; Gurau 2011; Gurau, Ryan 2012)

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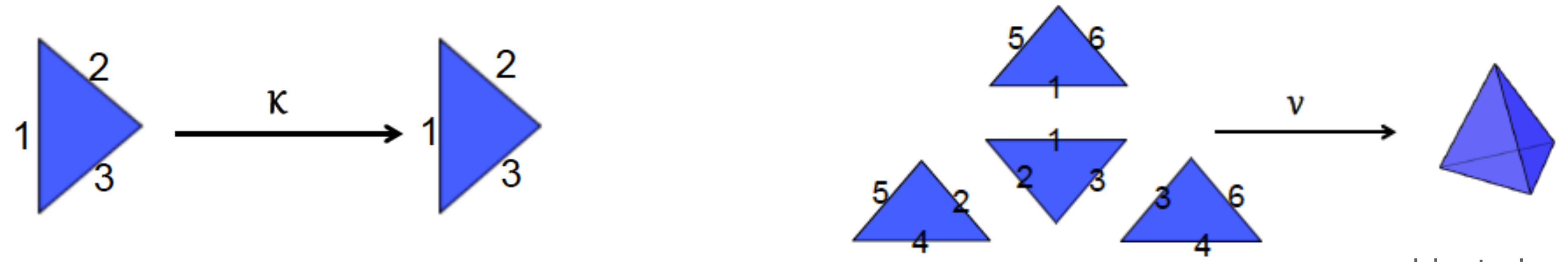
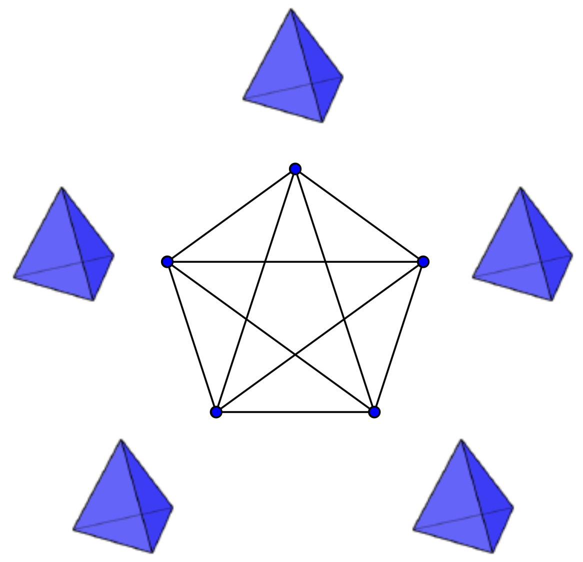


combinatorics of a 3-simplex

example in 4d:

(Ooguri 1992)

$$S = \int (\mathrm{d}g)^4 |\varphi_{1234}|^2 + \frac{\lambda}{5!} \int (\mathrm{d}g)^{10} \varphi_{1234}\varphi_{4567}\varphi_{7389}\varphi_{962(10)}\varphi_{(10)851} + \mathrm{c.c.}$$



combinatorics of a 4-simplex

also colored versions

(Gurau 2010; Gurau 2011; Gurau, Ryan 2012)

  - uncolored models with tensor-invariant interactions

(Gurau 2011; Carrozza 2014;...)

Group Field Theory

- **Tensor models:**
  - Feynman expansion generates random discrete geometries
- **GFTs as enriched tensor models:**
  - 1) QFTs of tensor fields living on Lie group G
  - 2) closure condition
  - 3) specific combinatorial non-local action
- Feynman expansion defines a lattice gauge theory on a random lattice
  - expansion generates random discrete geometries
  - weighted by amplitudes of specific lattice gauge theory with structure group G

number of vertices

$$Z = \int \mathcal{D}\varphi\mathcal{D}\bar{\varphi}\mathrm{e}^{-S[\varphi,\bar{\varphi}]} = \sum_{\Gamma} \frac{(\lambda\bar{\lambda})^{n(\Gamma)/2}}{\mathrm{sym}(\Gamma)} \mathcal{A}_{\Gamma}$$

GFT Feynman amplitude $\mathcal{A}_{\Gamma}$ = spin foam amplitude $Z(\Gamma)$

graph dual to cellular complexes dual to discrete orientable pseudomanifolds

sum over graphs/cellular complexes

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    - quantum BF+$r = 4, G = \mathrm{SL}(2, \mathbb{C})$+simplicity constraints: quantum gravity amplitudes
      - **GFT as tentative completions of BC, EPRL and other spin foam models**

(De Pietri, Freidel, Krasnov, Rovelli 2000; Reisenberger, Rovelli 2001; Baratin, Oriti 2011; Baratin, Oriti 2012; Ben Geloun, Gurau, Rivasseau 2010; Baratin, Oriti 2010; Baratin, Girelli, Oriti 2011; Oriti 2016; Finocchiaro, Oriti 2018; Jercher, Oriti, Pithis 2022)

What do we know about GFT renormalization?

Motivation for GFT renormalization

- GFTs suffer from divergences similar to local (perturbative) QFT:
  - divergences arise from short scale structure of configuration space
  - more difficult due to non-local interactions of GFT
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  - **step by step**, towards renormalizable models for 4d quantum gravity

Strategies for GFT renormalization

- **two** paths:
  1) use tensor model tools to study to **large-N** behavior
     - **Boulatov-Ooguri** type models: dominance of **melonic diagrams**
     - topologically singular spin foam structures convergent
     - beyond leading order: critical properties/continuum limit still not well understood

(Gurau 2011; Gurau, Rivasseau 2011; Gurau 2012; Ben Geloun, Krajewski, Magnen, Rivasseau 2010; Ben Geloun, Gurau, Rivasseau 2010; Freidel, Gurau, Oriti 2009; Magnen, Noui, Rivasseau, Smerlak 2009; Krajewski, Magnen, Rivasseau, Tanasa, Vitale 2010; Bonzom, Smerlak 2010, Bonzom, Smerlak 2012; Bonzom, Smerlak 2012; Carrozza, Oriti 2012; Carrozza, Oriti 2012; Baratin, Carrozza, Oriti, Ryan, Smerlak 2014)

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  2) use **full machinery of renormalization**

    - as in ordinary QFTs: definition of **scale** and set up of mode integration
      - via non-trivial propagator in action: spectrum of **Laplacian** on G

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    - **Landau-Ginzburg mean-field analysis**

NEXT SLIDES

GFT and power-counting

compared to local scalar field theory more involved !

degree of divergence

- understand how Feynman diagrams diverge in the UV
  - superficial degree of divergence captures their UV behavior $|\mathcal{A}_\Gamma^\Lambda| \propto |\lambda| \Lambda^\omega$
  - for power-counting renormalizability: degree should be bounded from above

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- derive from this understanding criteria of renormalizability
    - full classification of renormalizable melonic GFTs **with closure**:
        - focus on these since melonic diagrams most divergent
        - a general Abelian G power-counting theorem (Ben Geloun, Krajewski, Magnen, Rivasseau 2010; Carrozza, Oriti, Rivasseau 2014; Samary, Vignes-Tourneret 2014)
        - non-Abelian G (Bonzom, Smerlak 2010; Bonzom, Smerlak 2012; Carrozza, Oriti, Rivasseau 2014; Carrozza 2014)

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(taken from Carrozza 2016)

	Type	r	$\dim(G)$	$v_{\max}$	ω	Explicit examples	
just-renormalizable	A	3	3	6	$3 - N/2 - 2n_2 - n_4 + 3\rho$	$G = \mathrm{SU}(2)$	(Carrozza, Oriti, Rivasseau 2014; Carrozza 2015)
	B	3	4	4	$4 - N - 2n_2 + 4\rho$	$G = \mathrm{SU}(2) \times \mathrm{U}(1)$	(Carrozza 2015)
	C	4	2	4	$4 - N - 2n_2 + 2\rho$	(not yet exhibited)	
	D	5	1	6	$3 - N/2 - 2n_2 - n_4 + \rho$	$G = \mathrm{U}(1)$	(Samary 2013; Samary, Vignes-Tourneret 2014)
	E	6	1	4	$4 - N - 2n_2 + \rho$	$G = \mathrm{U}(1)$	(Samary 2013; Samary, Vignes-Tourneret 2014; Benedetti, Lahoche 2016)
super-renormalizable	F	3	2	arbitrary	$2 - 2V$	(not yet exhibited)	
	G	4	1	arbitrary	$2 - 2V$	$G = \mathrm{U}(1)$	(Carrozza, Oriti, Rivasseau 2014; Lahoche 2015)
finite	H	3	1	arbitrary	$1 - L - V < 0$	$G = \mathrm{U}(1)$	(Lahoche 2015)

r : rank

ρ : combinatorial quantity

N : number of propagator lines

V : number of interaction vertices

L : number of external lines

n_{2k} : number of interactions of valency k

$v_{\max}$: max. valency of renormalizable interaction

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- UV divergences from subgraphs; absorbable in tensor-invariant effective interactions
  - corresponds to coarse graining of the lattice

(Carrozza, Oriti, Rivasseau 2014; Samary, Vignes-Tourneret 2014; Carrozza, Oritit, Rivasseau 2014; Carrozza 2014; Lahoche, Oriti, Rivasseau 2015; Carrozza 2015; Carrozza 2016)

GFT and perturbative/non-perturbative renormalization

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  ELSE: explore theory space away from perturbative fixed point

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  - A.2) check **consistency** of theory **in deep UV** in non-perturbative scheme
    - If convergence to non-zero: theory is non-Gaussian = **asymptotic safety**
      - corresponds to non-trivial fixed point as UV completion of the theory

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  - **B)** study behavior **in deep IR:** fixed points, different phases, phase transitions
    - IR fixed points relate to continuum limits

GFT and Functional Renormalization

- address **these goals** for instance using the FRG methodology

(Wetterich 1993; Morris 1994)

scale-dependent partition function/generating functional

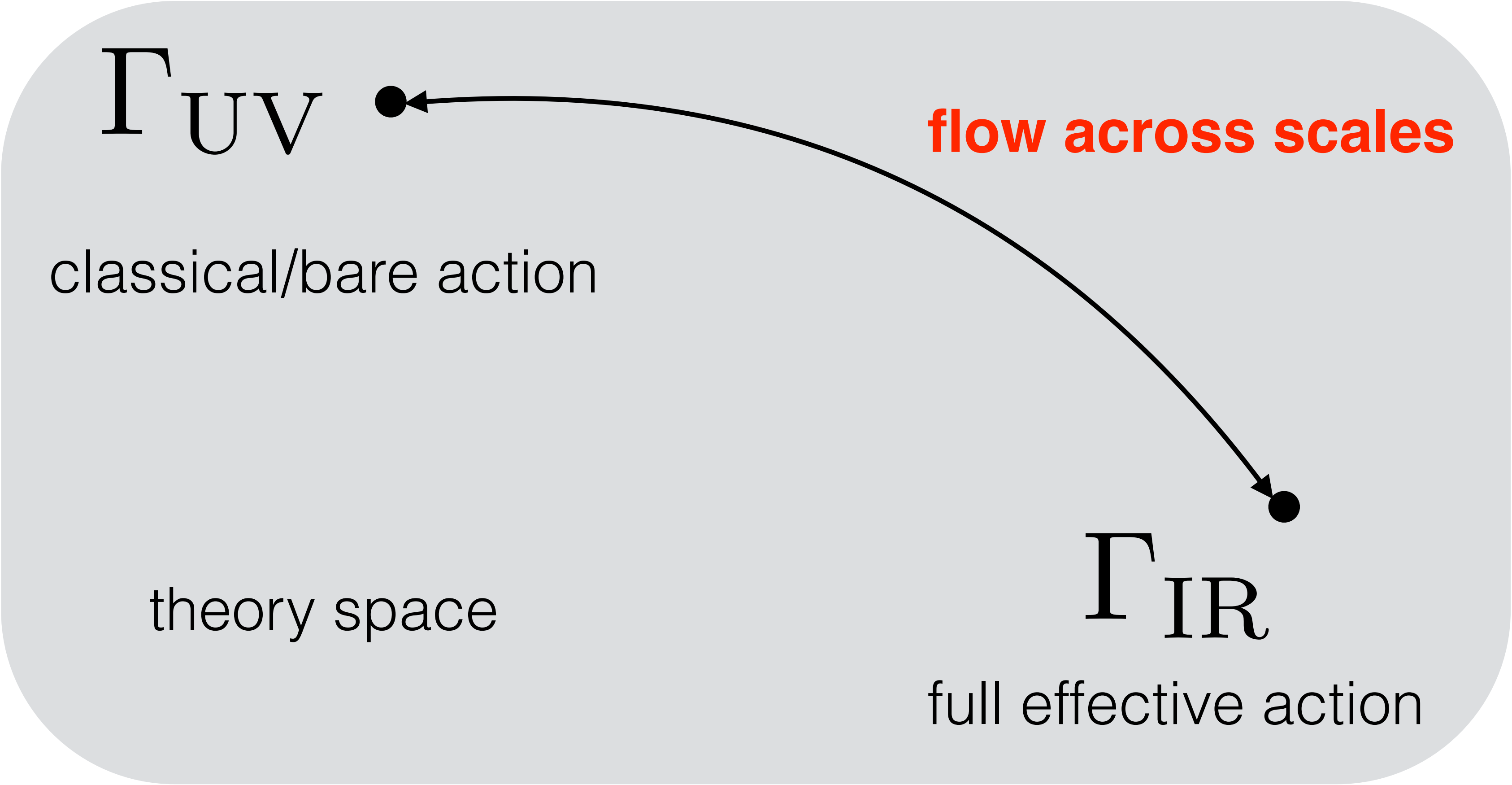
$$Z_k[J] = \mathrm{e}^{W_k[J]} = \int \mathcal{D}\phi \mathrm{e}^{-S[\phi]-(\phi,R_k\phi)+(J,\phi)}$$

RG scale
not an energy scale in GFT/spin foams

regulator function
eliminates UV (IR) modes

$$\Gamma_k[\phi] = \sup_J \left((J,\phi) - W_k[J]\right) - (\phi, R_k\phi)$$

effective average/flowing action



Idea: While moving along scales, the form of the action changes. Fields and coupling constants are re-written (= renormalized) with respect to the current scale.

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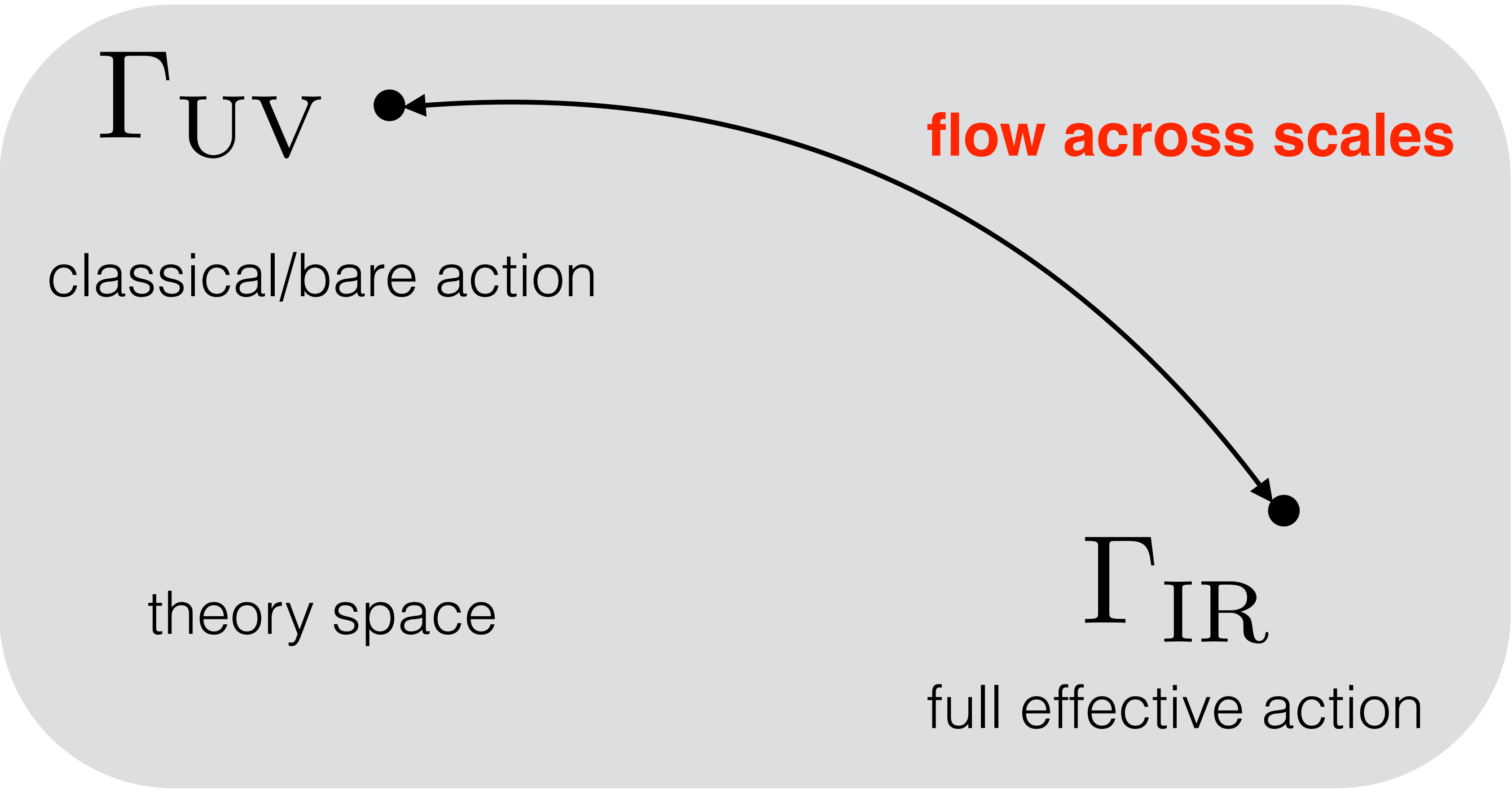
- implement flow via **Wetterich-Morris equation:**



$$k\partial_k\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi^2} + R_k\right)^{-1} k\partial_k R_k\right]$$

successfully transferred to GFTs, matrix and tensor models

(Koslowski, Sfondrini 2010; Eichhorn, Koslowski 2013; Benedetti, Ben Geloun, Oriti 2014)

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not an energy scale in GFT/spin foams

regulator function (R_k)
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effective average/flowing action (Γ_k)

- from WM equation one obtains $\beta-$functions for the running couplings
  - consistent set of equations only for dimensionless couplings
    - re-scale dimensionful couplings with powers of RG scale
    - precise re-scaling inferred from superficial degree of divergence

degree of divergence

$$|\mathcal{A}^k_\Gamma| \propto |\lambda| k^{\omega} \longrightarrow \bar{\lambda}(k) := \frac{\lambda(k)}{k^\omega}$$

  - set of functions **vanishes at UV/IR fixed points**

GFT and Functional Renormalization

- **challenges for application to GFTs:**
  - **combinatorics**:
    - re-scaling (degree of divergence) depends on precise combinatorics
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- **results** (depending on rank, group, group dimension, interaction) **so far**:

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- **in deep UV and IR**: $\beta-$functions become autonomous
- **in deep UV**: models with **asymptotic freedom** and **asymptotic safety**

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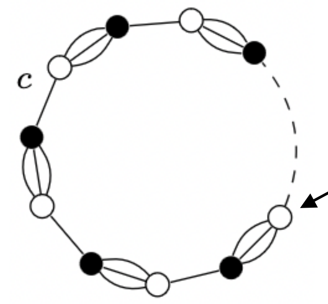
  - for rank 4 model on $G = \mathrm{SL}(2,\mathbb{C})$ + BC simplicity constraints (Jercher, Pithis, Thürigen wip)

Snapshot of FRG applied to Lorentzian cyclic melons + BC simplicity

(Jercher, Pithis, Thürigen wip)

$$\Gamma_k[\varphi,\bar\varphi] = \langle\varphi|\mathcal{K}_k|\varphi\rangle + \text{“}$$  ”

regulator on $G = \mathrm{SL}(2,\mathbb{C})$

$$k\partial_k\Gamma_k[\varphi,\bar\varphi] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)}[\varphi,\bar\varphi] + \mathcal{R}_k\mathbb{I}_2\right)^{-1}(k\partial_k)\,\mathcal{R}_k\mathbb{I}_2\right]$$

projection $|\varphi_0|^2 = v$

$$U_k(v) := \mu_k v + \sum_{c=1}^{4} V_k^c(v)$$

complex field

$$k\partial_k U_k(v) = Z_k k^2\left(\prod_{c=1}^{4}\int \mathrm{d}\rho_c \sum_{j_c,m_c}\right)|D^{(\rho_c,0)}_{j_c m_c 00}(e)|^2 \sum_{\alpha=0,1}\left[\frac{\left(1-\frac{\eta_k}{2}\left(1-\frac{\sum_c \mathrm{Cas}_{1,\rho_c}}{(ak)^2}\right)\right)\theta\left((ak)^2-\sum_c \mathrm{Cas}_{1,\rho_c}\right)}{Z_k k^2 + \underbrace{\mu_k + \sum_c \mathcal{O}^c_{(\boldsymbol{\rho},\boldsymbol{j},\boldsymbol{m})}V_k^{c\prime}(v) + \alpha\left(\prod_c \frac{1}{\rho_c^2}\delta(\rho_c - i)\delta_{0j_c}\delta_{0m_c}\right)2v\sum_c V_k^{c\prime\prime}(v)}_{\text{non-local operator due to GFT interaction}}}\right]$$

in the deep IR:

$$k\partial_k U_k \sim \left(1-\frac{1}{(ak)^2}\right)^{\frac{1}{2}}\theta\left(1-\frac{1}{(ak)^2}\right) \to 0$$

$\eta_k \to 0$

anomalous dimension

due to regulator on $G = \mathrm{SL}(2,\mathbb{C})$

results so far suggest:

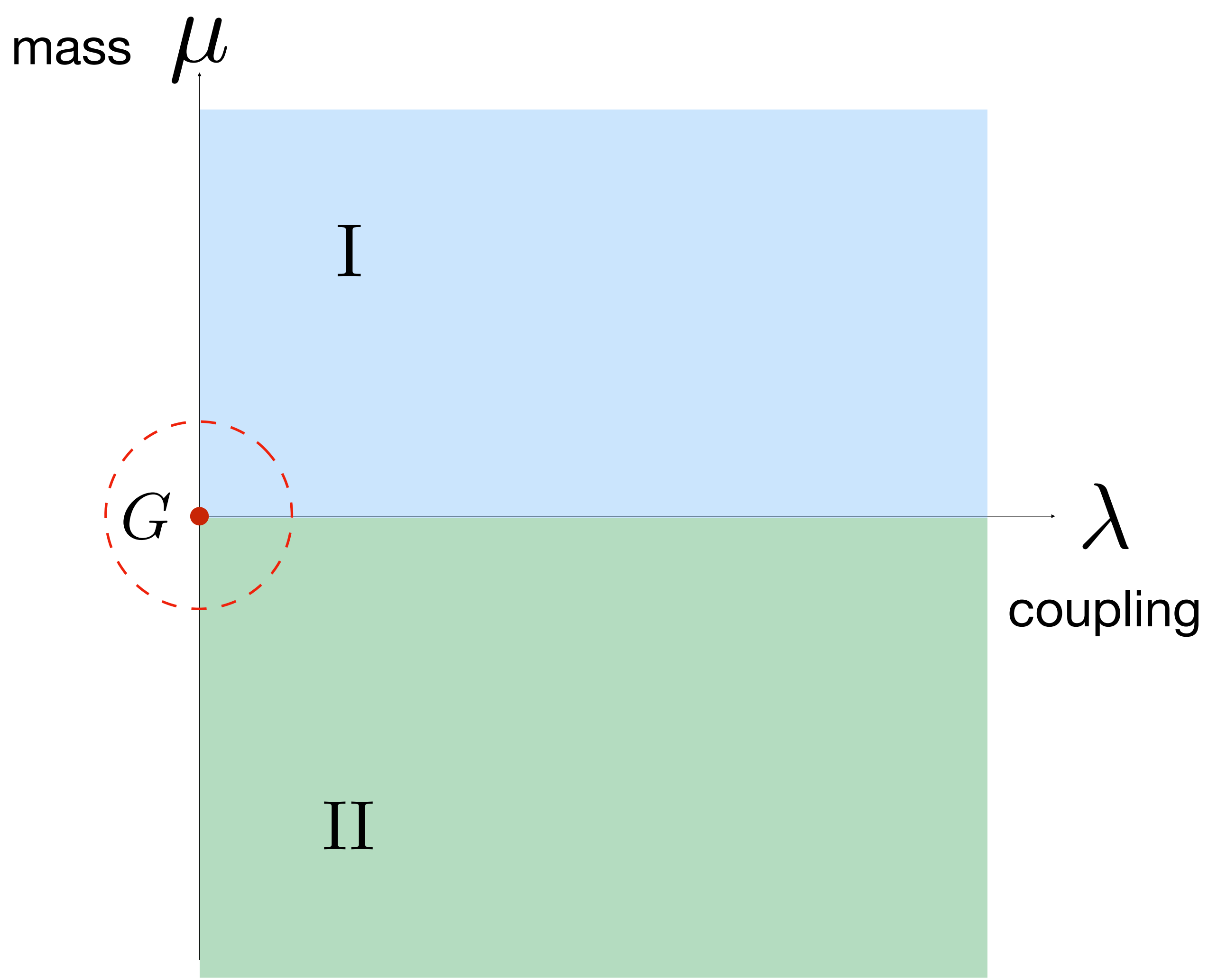
- **in IR only Gaussian FP: asymptotic freedom**
- **would also apply when other type of interaction used**
- **expect same for model with EPRL simplicity constraints**
- agreement with LG analysis results (see next slides)

GFT and Landau-Ginzburg mean-field theory

(Pithis, Thürigen 2018; Marchetti, Oriti, Pithis Thürigen 2021; Marchetti, Oriti, Pithis Thürigen 2023; Marchetti, Oriti, Pithis Thürigen 2023; Dekhil, Jercher, Oriti, Pithis 2024)

- full RG treatment is complex: approximate evaluation of $\boxed{Z = \int \mathcal{D}\varphi \mathrm{e}^{-S[\varphi]}}$
    - done via saddle-point evaluation
        - investigation of quadratic perturbations around saddle-point (=mean-field)
- yields coarse account of phase structure around Gaussian fixed point

phase diagram



allows to check for phase transition

$$\langle \varphi_0 \rangle = 0 \longleftrightarrow \langle \varphi_0 \rangle \neq 0$$

Interesting for GFT: ~ number of quanta large/infinite

~ continuum limit (for GFT tentatively)

GFT and Landau-Ginzburg mean-field theory

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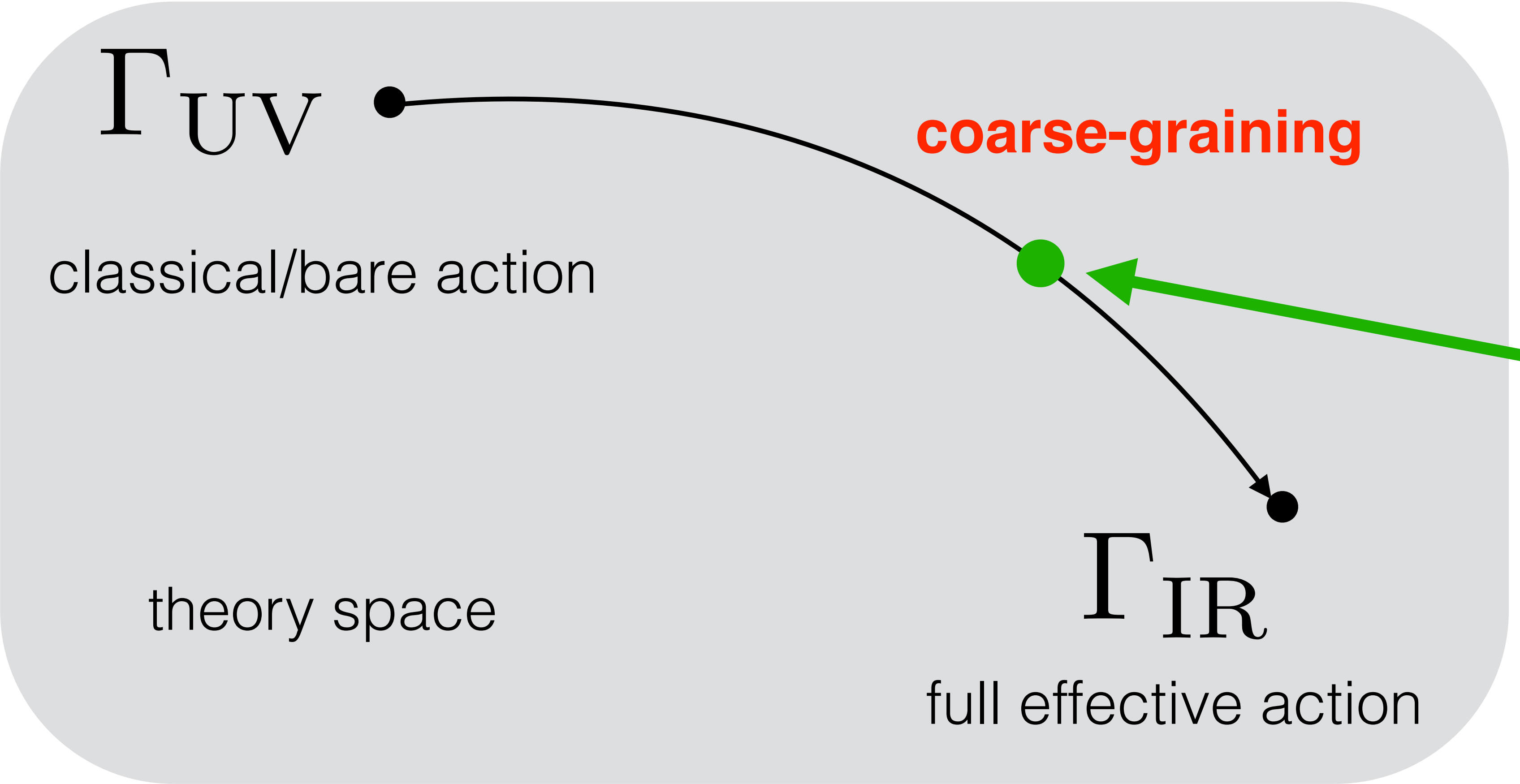


from Wilsonian perspective:

- choose phenomenological action S **between** UV and IR
- effective theory adapted to symmetries of the system at hand

- from action compute equation of motion
- compute uniform minimizers (background) φ_0
- linearize equation of motion $$\varphi(g_1, g_2, g_3, g_4) \;\; \rightarrow \;\; \underbrace{\varphi_0}_{\text{background}} + \underbrace{\delta\varphi(g_1, g_2, g_3, g_4)}_{\text{fluctuations}}$$
- extract correlation function and correlation length $C, \; \xi$
- compute **Ginzburg Q** (strength of fluctuations over φ_0) $$Q = \frac{\int_\xi (\mathrm{d}g)^4 C(g_1, g_2, g_3, g_4)}{\int_\xi (\mathrm{d}g)^4 \varphi_0^2}$$
  - should be small then mean-field theory self-consistent

Results: Ginzburg Q

(Marchetti, Oriti, Pithis Thürigen 2021; Marchetti, Oriti, Pithis Thürigen 2023; Marchetti, Oriti, Pithis Thürigen 2023; Dekhil, Jercher, Oriti, Pithis 2024)

exponential suppression due to hyperbolic domain

domain derived from Lorentz group $G = \mathrm{SL}(2,\mathbb{C}) \sim \mathbb{H}^3$

skirt radius: a

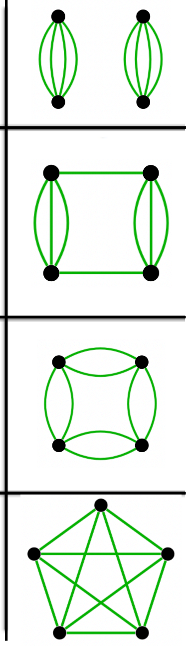
- results for spacelike quartic model + BC simplicity:

rank

$$Q \sim \underbrace{\lambda_\gamma}_{\text{coupling}} \xi^2 \mathrm{e}^{\boxed{-2(4-s_0)\frac{\xi}{a}}}$$

specifies non-local combinatorics

		graph γ	
$s_0 = 0$	double-trace melon		tensorial
$s_0 = 1$	quartic melonic		tensorial
$s_0 = 2$	quartic necklace		tensorial
$s_0 = 3$	simplicial		



$(s_0 < 4)$

- generalization to arbitrary interactions:

valency of interaction

$$Q \sim \lambda_\gamma^{\frac{2}{V_\gamma - 2}} \xi^{\frac{V_\gamma}{V_\gamma - 2}} \mathrm{e}^{-2(4-s_0)\frac{\xi}{a}} \xrightarrow[\text{at criticality}]{\xi \to \infty} 0$$

= Landau-Ginzburg theory self-consistent

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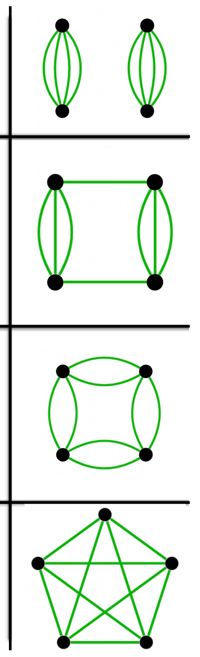

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- inclusion of free massless scalar matter on lattice ✓
- most realistic case so far: colored simplicial, BC simplicity, spacelike tetrahedra ✓

(Dekhil, Jercher, Pithis wip)

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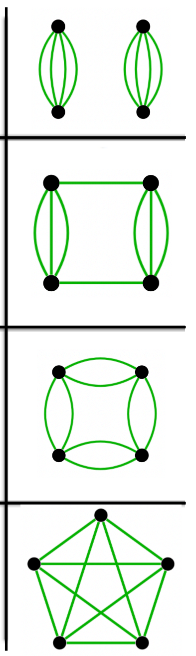

tensorial

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- **upshot: Ginzburg Q vanishes at criticality**
  - non-perturbative/condensate phase exists
    - towards tentative continuum geometric interpretation

(Marchetti, Oriti, Pithis Thürigen 2023)

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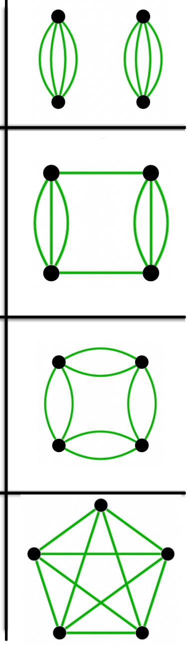

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(Marchetti, Oriti, Pithis Thürigen 2023)

- WIP: BC, timelike + lightlike building blocks **(Dekhil, Jercher, Pithis wip)**
- WIP: transfer to EPRL-model **(Diogo Simao, Jercher, Oriti, Pithis, Steinhaus wip)**

Conclusion and open problems…

Conclusion

- **GFT as tentative completions of BC, EPRL and other spin foam models**

- **discussing GFT renormalization = discussing spin foam renormalization**

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- **discussing GFT renormalization = discussing spin foam renormalization**

    - **step by step**, towards analysis of more realistic models in GFT renormalization

- **renormalization: pathway to extract continuum limit and macroscopic physics**

    - **cosmology and black holes**

(talks by Jercher, Marchetti, Oriti, Wilson-Ewing)

The road ahead…

- **main open issues:**
    - clarify further theory space: relation between full-fledged models for 4d Lorentzian QG (colored simplicial + simplicity constraints + non-trivial propagator) and tensor-invariant models

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    - when done, extension of pert. + non-pert. renormalization to such cases
        - analyse their phase structure
            - in particular: apply methods to models with **EPRL simplicity constraints**

(Diogo Simao, Jercher, Oriti, Pithis, Steinhaus wip)

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              (Diogo Simao, Jercher, Oriti, Pithis, Steinhaus wip)
    - devise **observables and tools** to characterize geometry of different phases
        - implement composite-operator renormalization scheme
    - connect with FRG analyses for first-order gravities in the continuum

      (e.g. Gies, Sabor Salek 2022)

Thank you for your attention!

Backup slides

GFT and renormalization (extra)

- **General idea:**
  - formal relation between GFT and spin foam models is intrinsically perturbative

$$Z = \sum_{\Gamma} \frac{\lambda^{n(\Gamma)}}{\mathrm{sym}(\Gamma)} \mathcal{A}_{\Gamma}$$

  - check that set of GFT interactions is stable under shifting the cut-off Λ
    - translates into formal stabilty of corresponding spin foams
  - stability only holds with finitely many GFT interactions turned on
- equivalent to perturbative or non-perturbative renormalizability of the GFT model
  - relevant GFT interactions give finitely many spin foam vertices dominant in pert. phase
  - relevant GFT interactions give finitely many spin foam vertices dominant in non-pert. phase

GFT and non-perturbative renormalization (extra)

- controlling the continuum limit ~ evaluating full GFT partition function
  - in non-perturbative regime
  - at least via approximation
  - corresponds to evaluating complete spin foam model (all complexes)
  - expect different phases and phase transitions as result of quantum dynamics
    - regime of many and interacting QG d.o.f.
    - identify relevant phase for effective continuum geometry
    - extract effective continuum dynamics and relate it to GR
- requires GFT non-perturbative renormalization
  - effort to map out non-perturbative IR fixed points (+ phase diagram) via FRG methods:

    ~ definition of full GFT path integral
    ~ full continuum limit (all dofs of spin foam model)

  - such analysis difficult for full-fledged Lorentzian models: resort to LG analysis

GFT and perturbative renormalization (extra)

- control over theory space: models with tensor-invariance
- many results
- **no closure:**
  - mostly groups U(1) or SU(2), homogeneous spaces SU(2)/U(1)
  - consistent perturbative renormalization scheme (Ben Geloun, Bonzom 2011; Ben Geloun, Rivasseau 2013; Rivasseau 2012; Ben Geloun, Samary 2013; Ben Geloun 2012; Ben Geloun, Livine 2013; Ben Geloun 2014; Lahoche, Oriti 2015; Ben Geloun, Toriumi 2018; Ben Geloun, Toriumi 2024)
    - closely analogous to large-N results of tensor models
      - melonic graphs most divergent
    - asymptotic freedom in UV for quartic models due to tensor-invariance (Rivasseau 2015)
    - also constructive renormalization applied (Gurau, Rivasseau 2015; Rivasseau 2018; Delpouve, Rivasseau 2016; Rivasseau, Vignes-Tourneret 2019; Rivasseau, Vignes-Tourneret 2021)
    - stochastic analysis (Chandra, Ferdinant 2023)
    - perturbative scheme fits into Connes-Kreimer Hopf-algebraic framework (Raasakka, Tanasa 2014; Avohou, Rivasseau, Tanasa 2015; Thürigen 2021; Thürigen 2021)
- perturbative renormalization analyses **with closure:**
  - UV divergences can be absorbed in effective interactions with tensor invariance (Carrozza, Oriti, Rivasseau 2014; Samary, Vignes-Tourneret 2014; Carrozza, Oritit, Rivasseau 2014; Carrozza 2014; Lahoche, Oriti, Rivasseau 2015; Carrozza 2015; Carrozza 2016)
  - corresponds to coarse graining of the lattice

“quantum theory” (dynamics):

GFT and BF theory

$$Z_{\mathrm{GFT}} = \int \mathcal{D}\varphi \mathcal{D}\bar{\varphi} e^{-S_{\mathrm{GFT}}[\varphi,\bar{\varphi}]} = \sum_{\Gamma} \frac{\lambda^{V_\Gamma}}{\mathrm{Sym}(\Gamma)} \mathcal{A}_\Gamma$$

# of vertices

GFT Feynman amplitude $\mathcal{A}_\Gamma$

Γ graph dual to triangulation

Curiously: Boulatov and Ooguri model provide GFT quantizations of BF-theory in 3d & 4d

$\longrightarrow$ **build models starting with BF-theory (TFT)**

BF-theory:

$$S[\omega, B] = \int B_{IJ} \wedge F^{IJ}(\omega)$$

$\mathfrak{g}-$ valued 2-form field strength: $F^{IJ}(\omega) = \mathrm{d}\omega^{IJ} + \omega^I_J \wedge \omega^{KJ}$ $\mathfrak{g}-$ valued connection 1-form

$$Z = \int \mathcal{D}\omega \mathcal{D}B e^{iS[\omega,B]} = \int \mathcal{D}\omega \delta(F(\omega))$$

(integral over flat connections, i.e. no local dof)

(=volume of space of flat connection, infinitely large!)

ill-defined in the continuum $\xrightarrow{\text{resort to}}$ **quantization on a regulating lattice structure**

- **quantisation of BF-theory on a lattice: “GFT does the job”** [Ooguri 9205090]
- **Improvements: coloured graphs to exclude topological singularities** [Gurau, Rivasseau, Bonzom,…]
- **note: quantization + BF + sc not necessarily equal to BF + sc + quantization**

Criticism against the BC model and alleviations

- BC vertex does not yield tensorial structure of lattice graviton propagator [Alesci, Rovelli 0708.0883]
    - Obvious mismatch of LQG boundary states and BC boundary states. [Baratin, Oriti 1108.1178]
- Area-length constraints are missing [Alexandrov 0802.3389]
    - Recently it was shown (on a hypercubical lattice) that the BC model is still viable and potentially lies in the same universality class as the EPRL model in an effective continuum limit. [Dittrich 2105.10808]
- What is the role of degenerate geometries in the BC model? [Barrett, Steele 0209023]
    - Need further analysis including timelike and lightlike configurations.
- Constraints are "too strongly" imposed [Engle, Pereira, Rovelli 0705.2388]
- Closure and simplicity are imposed in a non-covariant and non-commuting manner [Baratin, Oriti 1002.4723]
    - Both problems resolved in extended BC model. [Baratin, Oriti 1108.1178]
- EPRL model favored since boundary states are closer to canonical LQG, the Barbero-Immirzi parameter is incorporated
    - Absence of BI parameter does not rule out the BC model. At the same time, questions wrt the precise value and running of the BI parameter and parity violation issues of the EPRL model should be addressed. [Charles 1705.10984; Benedetti, Speziale 1111.0884]

For now, criticisms are not conclusive and the BC model deserves further attention.

Lorentzian GFT models…

Examples of GFT models for 4d Lorentzian QG

EPRL model $$S = \sum_{\substack{j_{v_a i} \\ m_{v_a i}, \iota_a}} \bar{\varphi}^{j_{v_1}\iota_1}_{m_{v_1}} \varphi^{j_{v_2}\iota_2}_{m_2} (\mathcal{K}_2)^{j_{v_1} j_{v_2} \iota_1 \iota_2}_{m_{v_1}, m_{v_2}} + V$$

$$V = \sum_{j_i, m_i, \iota_i} \left[\varphi^{j_1 j_2 j_3 j_4 \iota_1}_{m_1 m_2 m_3 m_4} \varphi^{j_4 j_5 j_6 j_7 \iota_2}_{m_4 m_5 m_6 m_7} \varphi^{j_7 j_3 j_8 j_9 \iota_3}_{m_7 m_3 m_8 m_9} \varphi^{j_9 j_6 j_2 j_{10} \iota_4}_{m_9 m_6 m_2 m_{10}} \varphi^{j_{10} j_8 j_5 j_1 \iota_5}_{m_{10} m_8 m_5 m_1} \times \tilde{\mathcal{V}}_5(j_1, \ldots, j_{10}; \iota_1, \ldots, \iota_5) \right]$$

$$\tilde{\mathcal{V}}_5(j_{ab}, i_a) = \sum_{n_a} \int d\rho_a (n_a^2 + \rho_a^2) \left(\bigotimes_a f^{i_a}_{n_a \rho_a}(j_{ab}) \right) 15j_{SL(2,\mathbb{C})} \left((2j_{ab}, 2j_{ab}\gamma); (n_a, \rho_a) \right)$$

$$f^i_{n\rho} := i^{m_1 \ldots m_4} \, \bar{C}^{n\rho}_{(j_1, m_1) \ldots (j_4, m_4)} \qquad \rho = \gamma n \quad n = 2j$$

SL(2,C) data mapped to SU(2) ones; almost SU(2) spin network states; Immirzi parameter

simplicity constraints can be encoded in kinetic term; various ambiguities; can be parametrised in kinetic term

(detailed derivation Diogo Simao, Jercher, Oriti, Pithis, Steinhaus wip)

BC model $$S = \left[\prod_i \int \mathrm{d}\rho_i \, 4\rho_i^2 \sum_{j_i m_i} \right] \bar{\varphi}^{\rho_i}_{j_i m_i} \varphi^{\rho_i}_{j_i m_i} \qquad + \frac{\lambda}{5} \left[\prod_{a=1}^{10} \int \mathrm{d}\rho_a \, 4\rho_a^2 \sum_{j_a m_a} \right]$$

irreps of SL(2,C) $$\left(\prod_{a=1}^{10} (-1)^{-j_a - m_a} \right) \{10\rho\}_{\mathrm{BC}} \varphi^{\rho_1 \rho_2 \rho_3 \rho_4}_{j_1 m_1 j_2 m_2 j_3 m_3 j_4 m_4} \varphi^{\rho_4 \rho_5 \rho_6 \rho_7}_{j_4 - m_4 j_5 m_5 j_6 m_6 j_7 m_7}$$

$$\varphi^{\rho_7 \rho_3 \rho_8 \rho_9}_{j_7 - m_7 j_3 - m_3 j_8 m_8 j_9 m_9} \varphi^{\rho_9 \rho_6 \rho_2 \rho_{10}}_{j_9 - m_9 j_6 - m_6 j_2 - m_2 j_{10} m_{10}} \varphi^{\rho_{10} \rho_8 \rho_5 \rho_1}_{j_{10} - m_{10} j_8 - m_8 j_5 - m_5 j_1 - m_1} + c.c$$

continuous SL(2,C) data; covariant "spin networks" states; no Immirzi parameter

(see also Jercher, Oriti, Pithis 2021; Jercher, Oriti, Pithis 2022)

(Oriti ILQGS talk 2020)