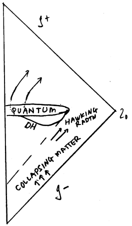
Spherical Collapse and Black Hole Evaporation

Madhavan Varadarajan

(Raman Research Institute)

Introductory Remarks

Resolution of black hole singularities by quantum gravity opens up a multitude of rich and fascinating physical possibilities. One of them (Ashtekar-Bojowald) is a mechanism to alleviate the Hawking Information Loss problem :



Classical singularity replaced by chunk of 'quantum spacetime'. A vast region opens up 'beyond' the singularity wherein correlations, which hitherto disappeared into the singularity, now emerge to purify the Hawking radiation.

It would be good to check how far this qualitative picture is realised and to endow it with quantitative precision in a simpler system than full blown gravity.

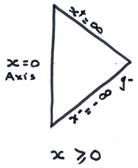
We consider spherically symmetric scalar field collapse such that:

- Geometry is that of Spherically symm GR.
  Ensures $T_H \sim 1/M$.
- Axis of symmetry is part of sptime.
  Ensures a single set of asymptotic regions rather than 'Kruskal' type situation.
- Matter coupling depends on the Areal Radius in a specific way.
  Ensures classical solvability (Vaidya).
  Ensures computability of matter stress energy exp value, and hence, formulation of semiclassical Einstein equations.
  Allows the contemplation of matter evolution 'through' the singularity.

I will set up the model, explore consequences of semiclassical equations. We will see that this leads to strong supporting evidence for the AB paradigm.

Kinematics

- Use conformal coordinates along radial light rays:
  $ds^2 = -e^{2\rho}(dx^+dx^-) + R^2(d\Omega)^2 \equiv e^{2\rho}(-(dt)^2 + (dx)^2)) + R^2(d\Omega)^2$
  where $x^\pm = t \pm x$.
- Axis is at $R = 0$. Restrict attention to timelike axis. Use conformal freedom to 'straighten' it out to the $x = 0$ line. Region of interest is $x \geq 0$ part of $x - t$ plane:



- Axis is part of sptime, geometry is non singular at axis:
  $R = 0, \partial_x R = 1, \rho = 0, \partial_x \rho = 1$.
  4d Differentiability of f at axis: $\partial_+ f = \partial_- f$ ("Reflecting Bdry Condts")
- Initial conditions at $\mathcal{I}^-$: Require metric to be asymptotically flat, matter field f to be of compact support (+ technical condtn for 'rapid collapse') .

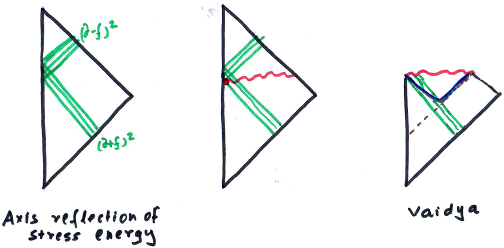
Dynamics

$$ds^2 = -e^{2\rho}(-(dt)^2 + (dx)^2)) + R^2(d\Omega)^2 \equiv {}^{(2)}ds^2 + R^2(d\Omega)^2$$

- $S_{geometry} = \frac{1}{8\pi G}\int d^4x\sqrt{-{}^{(4)}g}\,{}^{(4)}R$ + spherical symmetry
  Integrating over angles we obtain:
  $S_{geometry} = \frac{1}{2G}\int d^2x\sqrt{-{}^{(2)}g}[{}^{(2)}\mathcal{R}\ R^2 + 2(\nabla R)^2 + 2]$
- Matter coupling: The matter coupling is chosen to depend on the Areal Radius R so that the matter action is:
  $S_{matter} = -\frac{1}{8\pi}\int d^4x\sqrt{-{}^{(4)}g}\,{}^{(4)}g^{ab}\frac{1}{R^2}(\nabla_a f\nabla_b f)$
  Integrating over angles, we obtain:
  $S_{matter} = -\frac{1}{2}\int d^2x\sqrt{-{}^{(2)}g}(\nabla f)^2$.
  The coupling reduces to conformal coupling to the 2d metric
  ${}^{(2)}ds^2 = -e^{2\rho}(-(dt)^2 + (dx)^2))$
  f doesnt see conformal factor, satisfies 2d flat sptime wave eqn. $f(x^+, x^-) = f_+(x^+) + f_-(x^-)$
  Only nontrivial stress energy: $T_{\pm\pm} = \frac{(\partial_\pm f_\pm)^2}{4\pi R^2}$: ingoing and outgoing null dust streams.
  If only ingoing: Vaidya...

Classical Solution

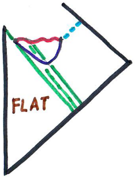
Despite both streams, classical solution *is* Vaidya!



QFT on this fixed classical geometry yields Hawking radiation at standard T_H.

Semiclassical Solution

Due to 2d conformal coupling can compute stress energy exp value on the spherical symm geometry.
Can then explicitly formulate 4d semiclassical Einstein eqns $G_{ab} = 8\pi G \langle \hat{T}_{ab} \rangle$.
Mix of mathematical analysis, physical arguments and prior numerical studies yields:



Asymptotic analysis as $x^+ \to \infty$:

- Certain well motivated assumptions on asymptotic flatness suggest that the metric takes the 'outgoing' Vaidya form as $x^+ \to \infty$. It is reasonable to assume that the semiclassical equations hold in this asymptotically flat region. Analysis of eqns suggests x^+ is good asymptotically flat coordinate for this (assumed) Vaidya metric but not necessarily x^-. Let asymp flat outgoing null coordinate be $\bar{u}(x^-)$.
- Semiclassical eqns imply a balance law at $\mathcal{I}^+$:
  $\frac{dM_B}{d\bar{u}} = -\frac{1}{2}R^2\langle \hat{T}_{\bar{u}\bar{u}}\rangle = -\frac{1}{4}(\partial_{\bar{u}} f)^2 +$ *quantum contribution*.
- Red term is not negative definite but we can pull part of it to the LHS, to obtain (similar to CGHS (AA et al)):
  $\frac{d}{d\bar{u}}[M_B + \frac{\hbar}{48}\frac{\bar{u}''}{(\bar{u}')^2}] = -\frac{1}{4}(\partial_{\bar{u}} f)^2 - \frac{\hbar}{96}\frac{(\bar{u}'')^2}{(\bar{u}')^4}$
- Assume: Flux vanishes when corrected Bondi mass exhausted. Black, Red flux=0. Latter implies $\bar{u} = \alpha x^- + \beta$.
  Implies that physical $\mathcal{I}^+$ is as long as the fiducial $\mathcal{I}^+$:
  Quantum Extension of the classical Vaidya $\mathcal{I}^+$.

Closing Remarks

The fact that the quantum scalar field lives on the fiducial flat sptime, sees no singularities and coordinatizes the true degrees of freedom of our system, together with our asymptotic analysis suggests the following picture: