Geometry of the black-to-white hole transition

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Based on work with
M. Han and C. Rovelli

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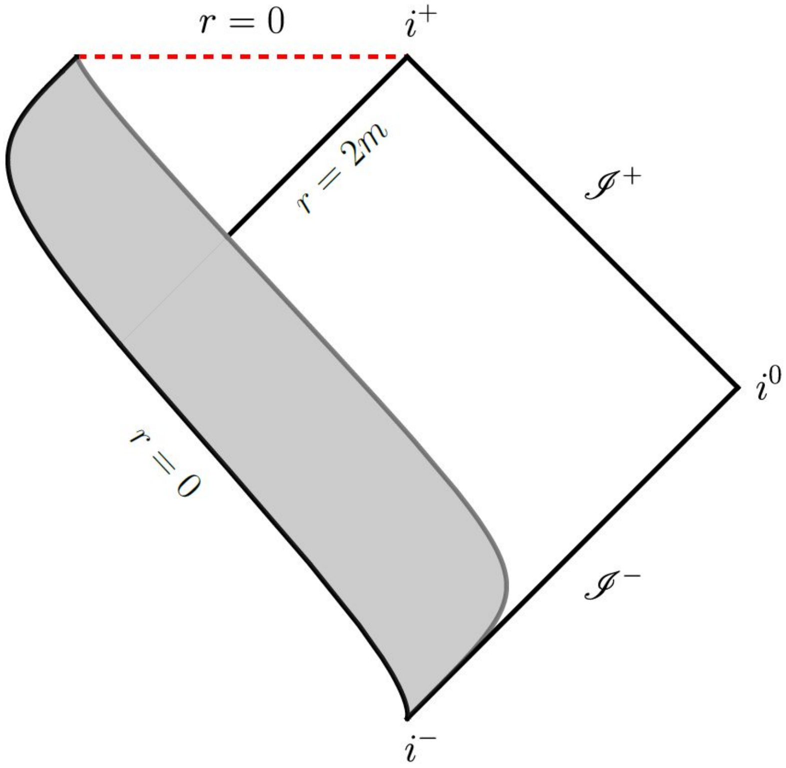
LOOPs'24, 7 May 2024

Oppenheimer–Snyder collapse



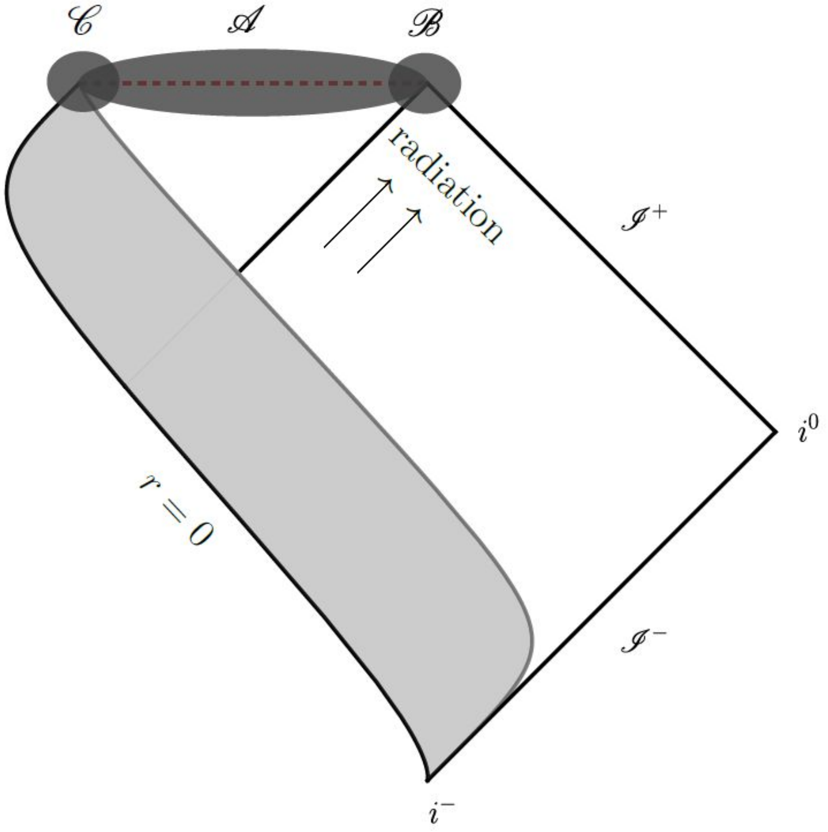
Interior of the star (FLRW metric):

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right]$$

Exterior of the star (Schwarzschild metric):

$$\mathrm{d}s^2 = -(1 - 2m/r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{(1 - 2m/r)} + r^2\mathrm{d}\Omega^2$$

Quantum region of a black hole spacetime



Quantum gravitational effects cannot be neglected in:

- Region A: Planckian curvature near classical singularity
- Region B: physics of the horizon at the end of the evaporation
- Region C: quantum gravity regime of the collapsing matter

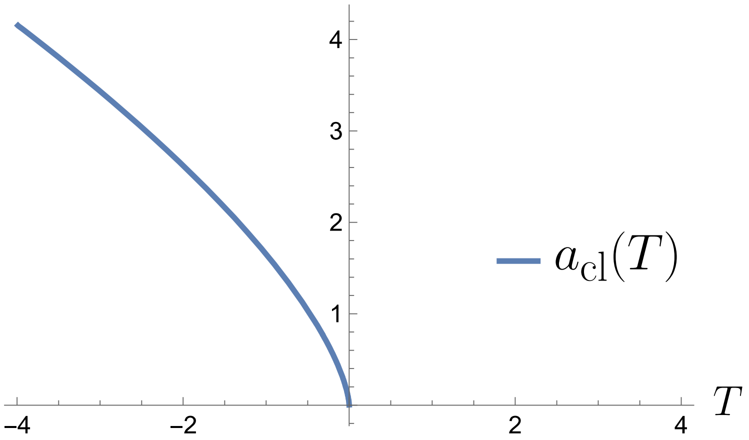
Interior of the star (region C)

Planck units $(c = G = \hbar = 1)$

Classical case

$$ds^2 = -\,dT^2 + a^2(T)\left(dR^2 + R^2\, d\Omega^2\right)$$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}\rho \quad \Rightarrow \quad a(T) = \left(\frac{9mT^2}{2R_{\text{star}}^3}\right)^{1/3}$$



Quantum case

Interior of the star (region C)

Planck units $(c = G = \hbar = 1)$

Classical case

$$\mathrm{d}s^2 = -\,\mathrm{d}T^2 + a^2(T)\left(\mathrm{d}R^2 + R^2\,\mathrm{d}\Omega^2\right)$$

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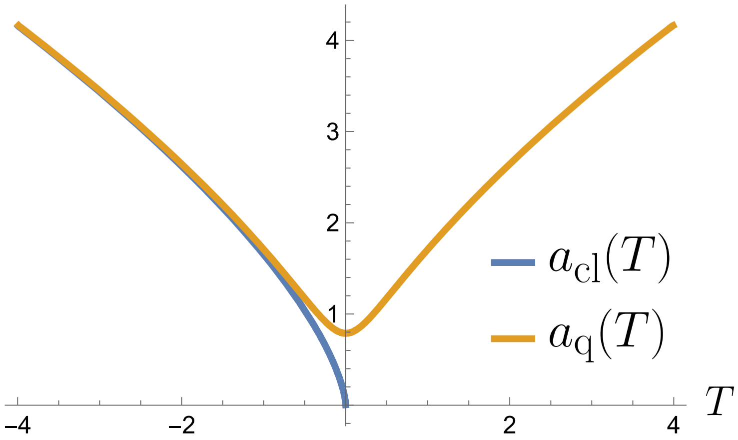


Quantum case

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$

$$A = \frac{3}{2\pi\rho_c}$$

$$a(T) = \left(\frac{9mT^2 + Am}{2R_{\text{star}}^3}\right)^{1/3}$$

Kelly, Santacruz, Wilson-Ewing (2020)

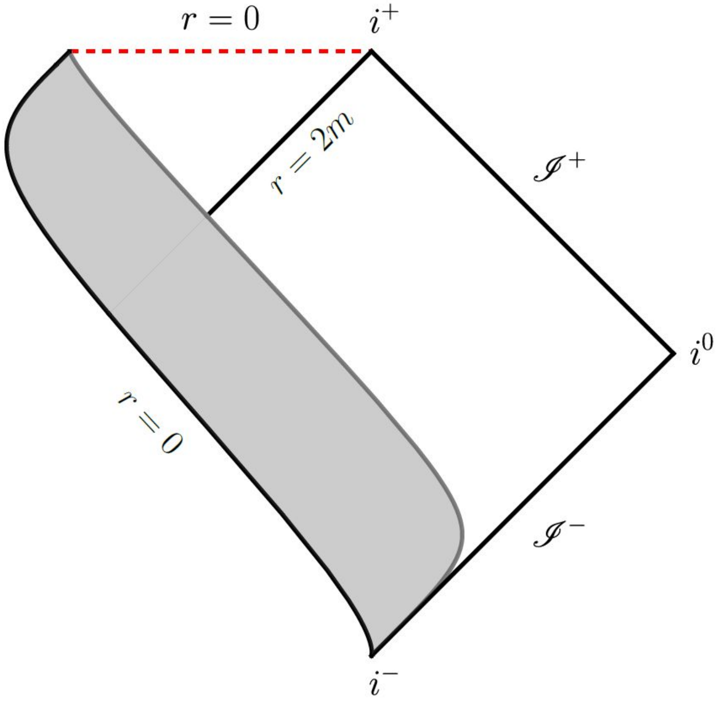
Exterior of the star (region A)

Classical case

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2\,d\Omega^2$$

$$f(r) = 1 - \frac{2m}{r}$$

$$r_{\rm h} = 2m$$



Beware: the isometry of the black hole interior with the Kantowski-Sachs spacetime cannot be used here!

Exterior of the star (region A)

Quantum case $\left(A = \dfrac{3}{2\pi\rho_c} \ll m^2\right)$

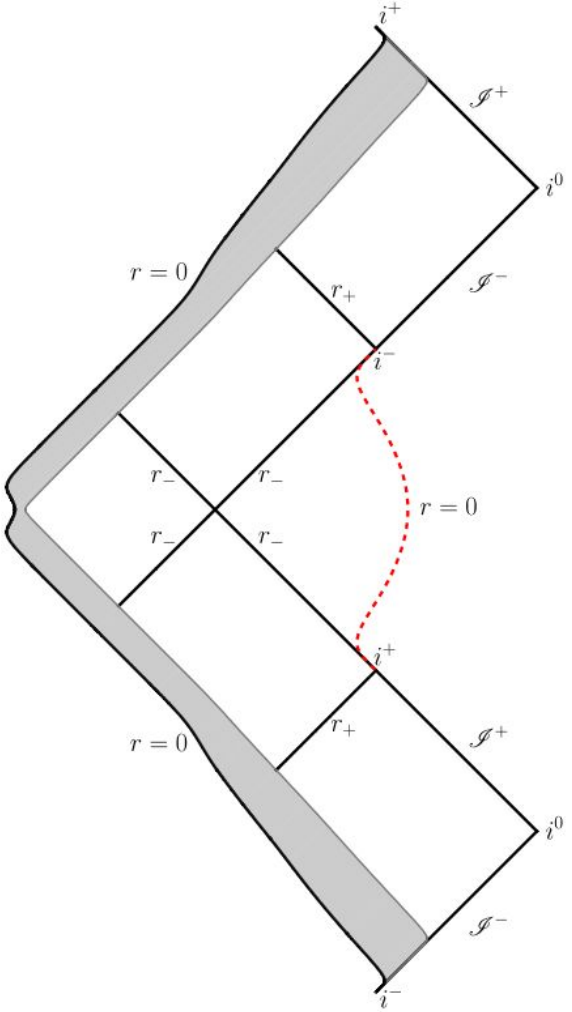
$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + f^{-1}(r)\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2$$

$$f(r) = 1 - \frac{2m}{r} + \frac{Am^2}{r^4}$$

$$r_+ = 2m + O(A/m)$$

$$r_- = \sqrt[3]{Am/2} + O\left(A^{2/3}/m^{1/3}\right)$$

Kelly, Santacruz, Wilson-Ewing (2020)
Lewandowski, Ma, Yang, Zhang (2023)
Bobula and Pawlowski (2023)



FS (2023)

Shockwave??

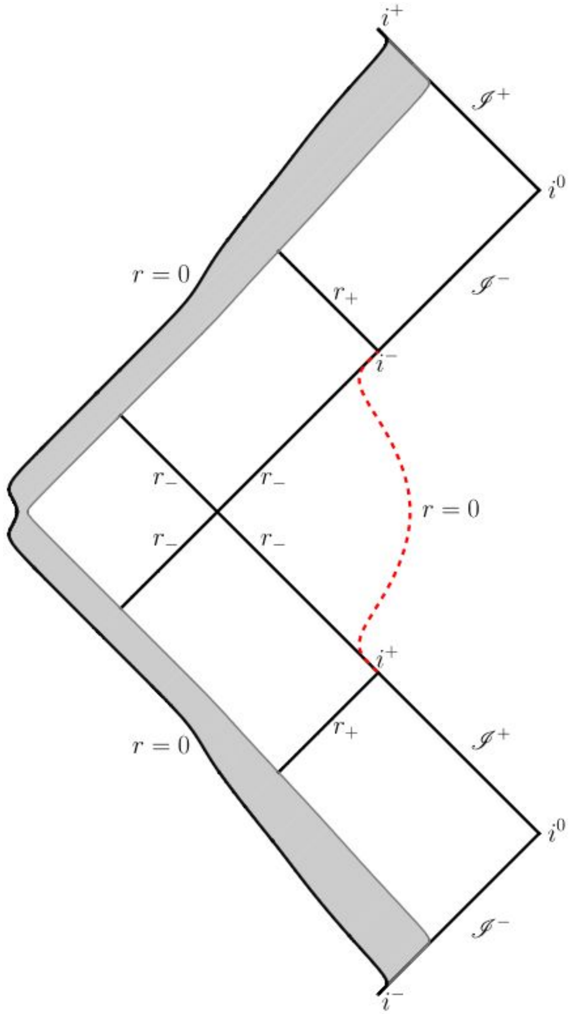
$$ds^2 = -dt_{\mathrm{PG}}^2 + (dr + N^r(t_{\mathrm{PG}}, r)\, dt_{\mathrm{PG}})^2 + r^2 d\Omega^2$$

$$N^r(t_{\mathrm{PG}}, r) = \begin{cases} -\frac{6r\, t_{\mathrm{PG}}}{9t_{\mathrm{PG}}^2 + A} & r \leq r_b \\ \sqrt{1 - f(r)} & r > r_b \end{cases}$$

Discontinuity

Physical shockwave

Kelly, Santacruz, Wilson-Ewing (2020)



FS (2023)

Shockwave??

$$ds^2 = -dt_{PG}^2 + (dr + N^r(t_{PG}, r)\, dt_{PG})^2 + r^2 d\Omega^2$$

$$N^r(t_{PG}, r) = \begin{cases} -\frac{6r\, t_{PG}}{9t_{PG}^2 + A} & r \leq r_b \\ \sqrt{1 - f(r)} & r > r_b \end{cases}$$

↑

Discontinuity

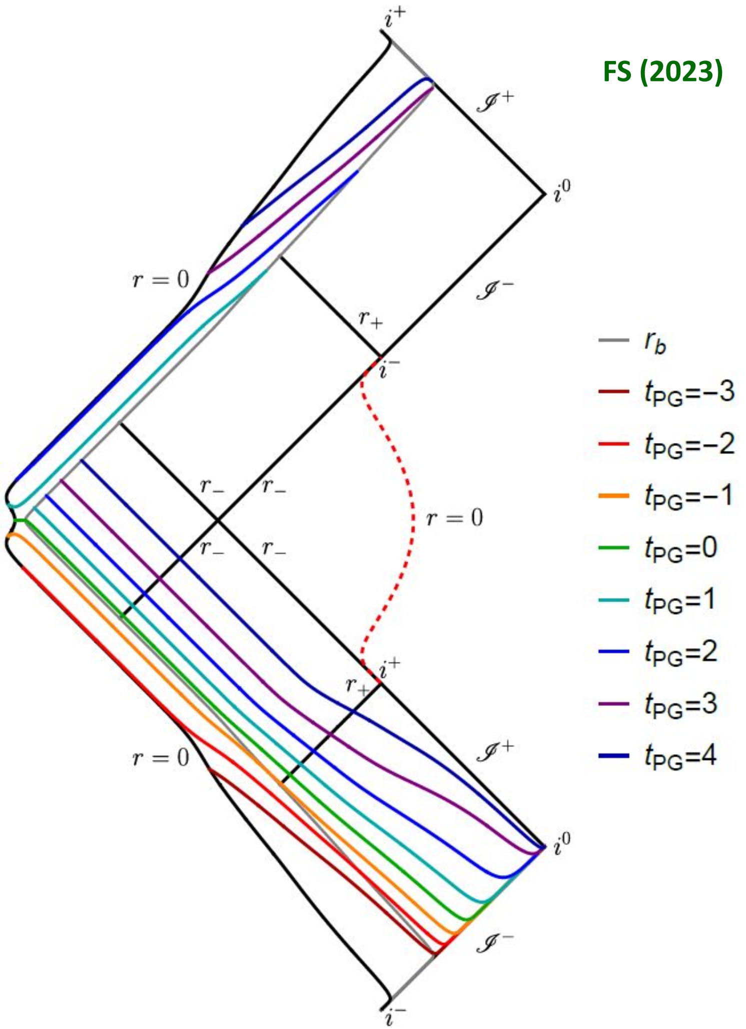
Physical shockwave

Kelly, Santacruz, Wilson-Ewing (2020)

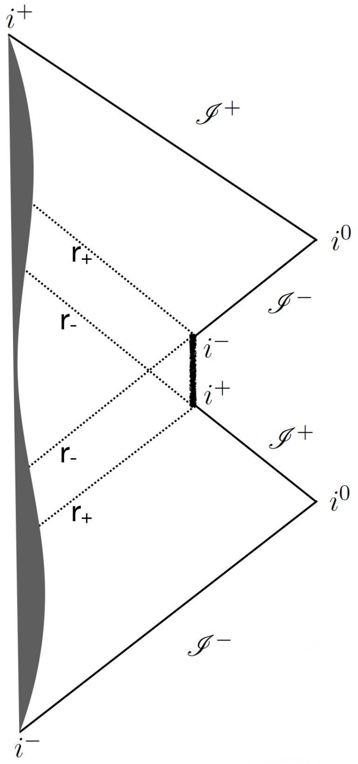
Coordinate artifact!!

Fazzini, Rovelli, FS (2023)

FS (2023)

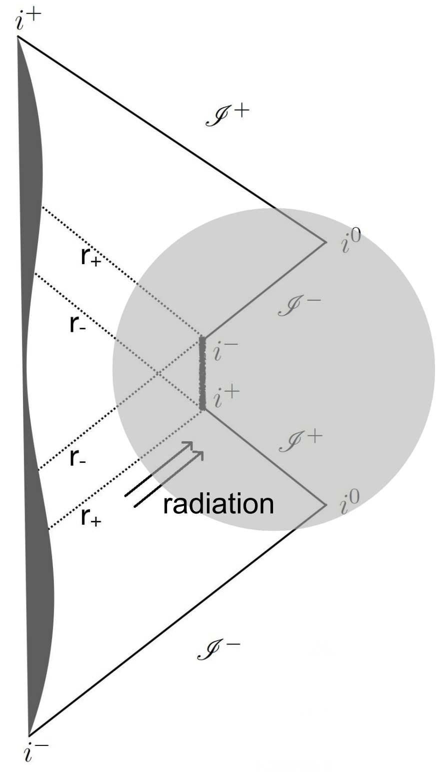


Physics of the horizon (region B)



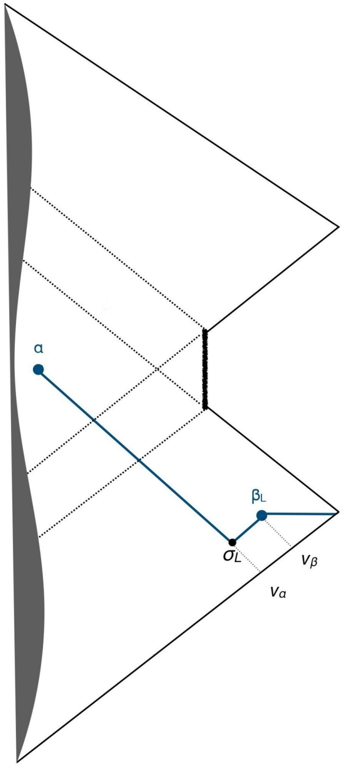
Physics of the horizon (region B)

Haggard and Rovelli (2015)



Physics of the horizon (region B)

Han, Rovelli, FS (2023)



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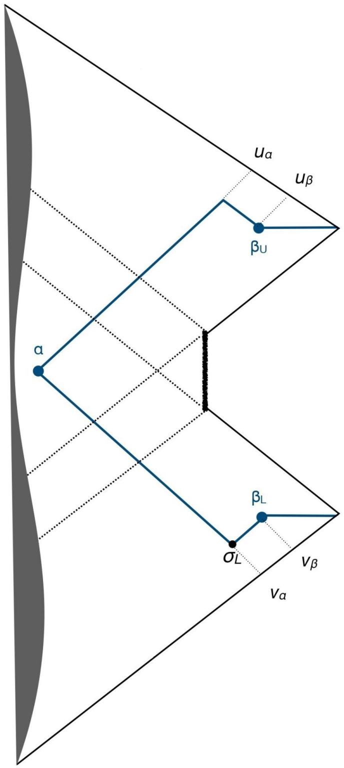

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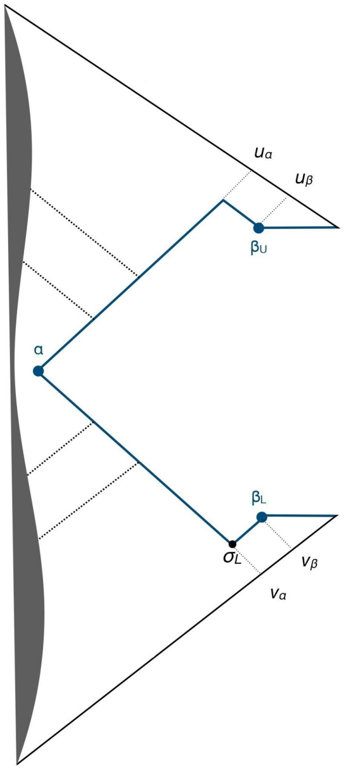

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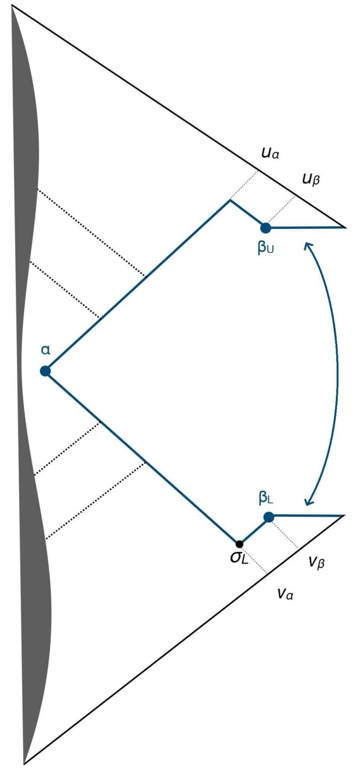

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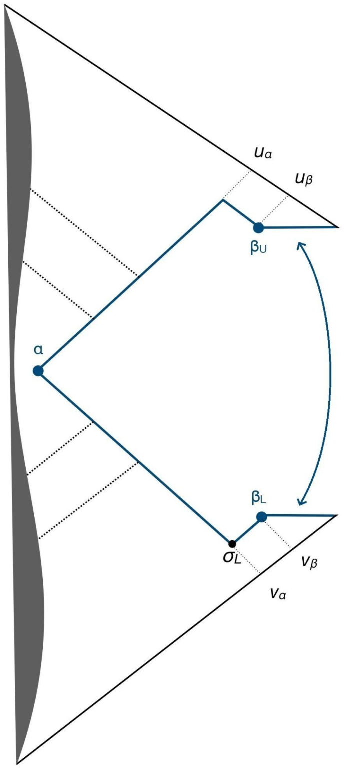
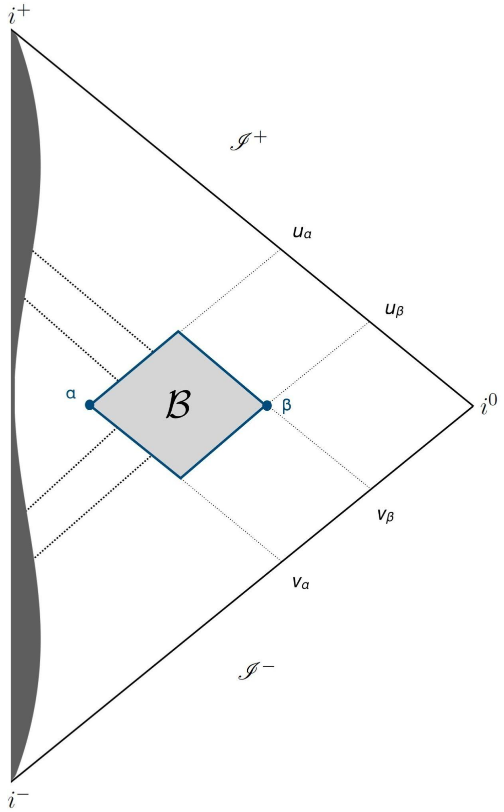
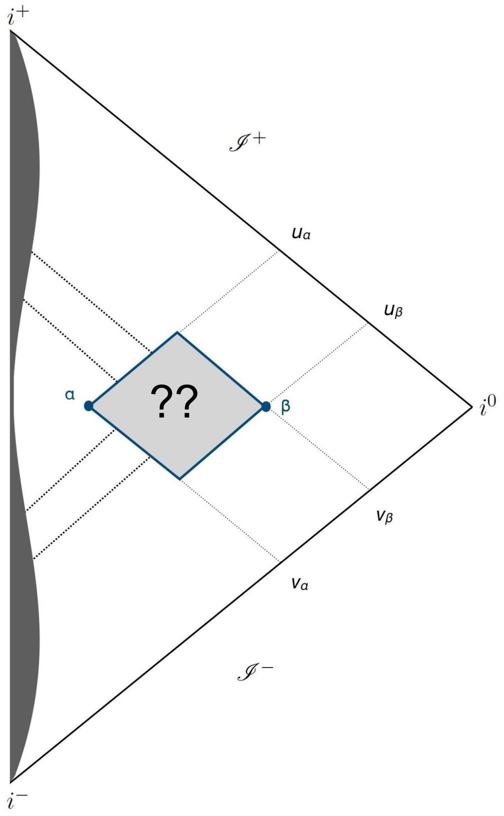




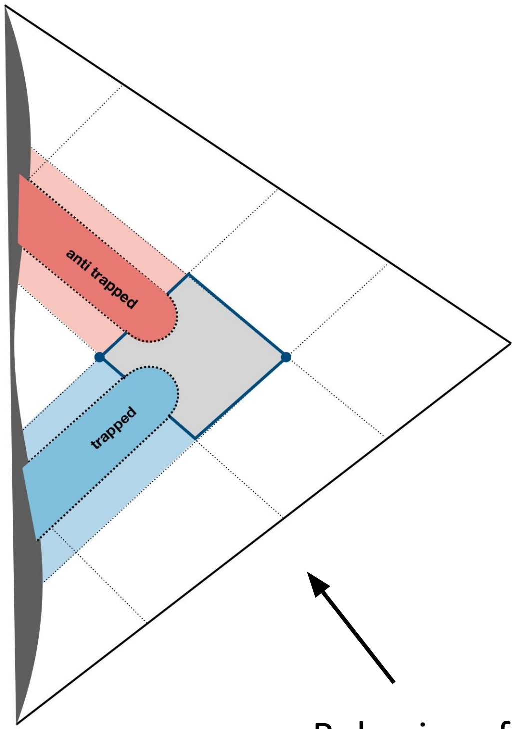
What happens inside region B?



What do we expect to find inside region B?

- Effective metric whose dynamics can be studied perturbatively
- Deep quantum geometry where classical concept of metric lose any meaning

Effective metric scenario

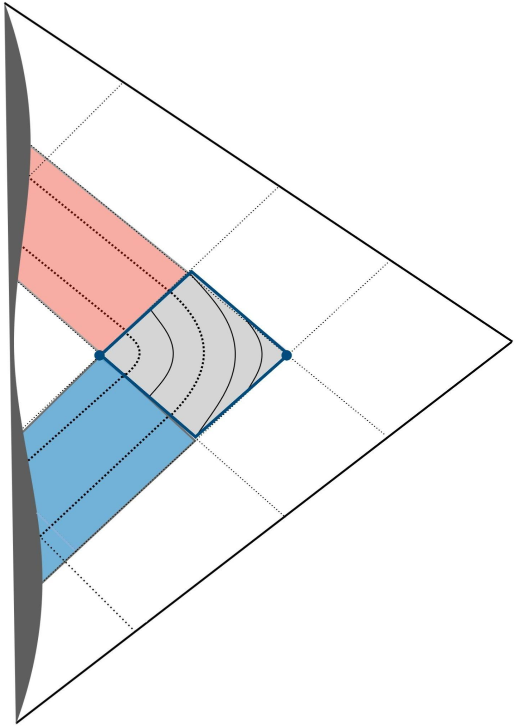


- There is a natural extension of the black-to-white metric inside of region B
  [Han, Rovelli, FS (2023)]

- It provides a proof of concept for the existence of a regular effective metric in region B

Behavior of trapped regions

Behavior of r=const. surfaces



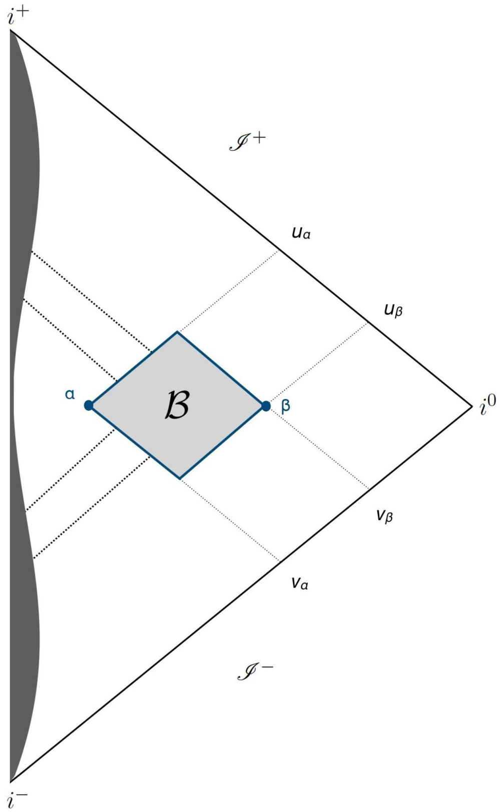
Conclusions

- The black-to-white hole transition is a natural scenario for the end of the evaporation of a black hole

- A concrete effective metric describing the black-to-white hole spacetime and its non-singular interior has been constructed

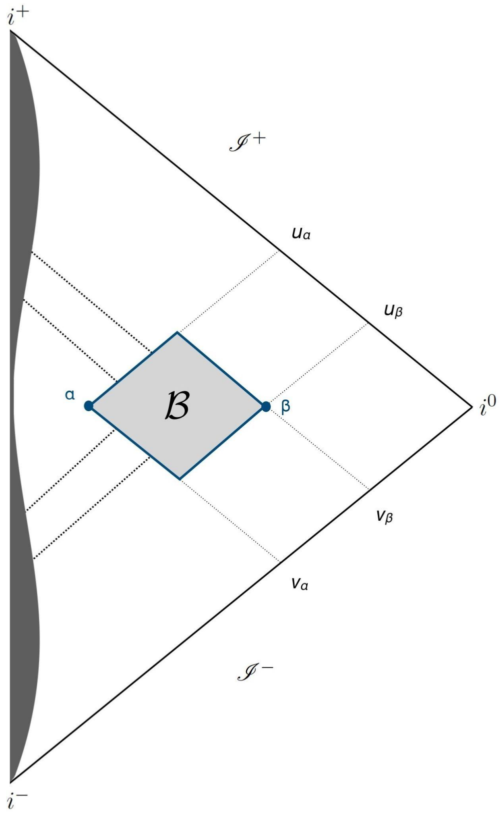
- The metric discontinuity in PG coordinates of the quantum OS spacetime can be seen as a coordinate artifact

- There is a natural extension of the black-to-white hole metric inside of region B



Spin foam framework

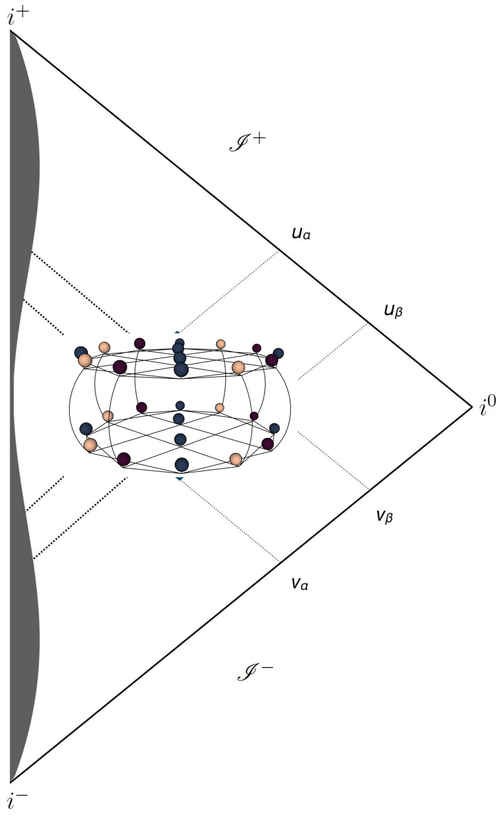
Han, Rovelli, FS (2023)



- The black-to-white hole geometry depends on 4 parameters: $(m, \mathcal{T}, v_\alpha, v_\beta)$

Spin foam framework

Christodoulou, D'Ambrosio, Martin-Dussaud, Rovelli, FS (2021)
FS, Rovelli, Martin-Dussaud (2021)



- The black-to-white hole geometry depends on 4 parameters: $(m, \mathcal{T}, v_\alpha, v_\beta)$
- A discretization Γ of the boundary can be defined starting from the 3d induced geometry and a Hilbert space $\mathcal{H}_\Gamma$ assigned to it
- A coherent state $\Psi(m, \mathcal{T}, v_\alpha, v_\beta)$ peaked on the boundary geometry can be defined in $\mathcal{H}_\Gamma$

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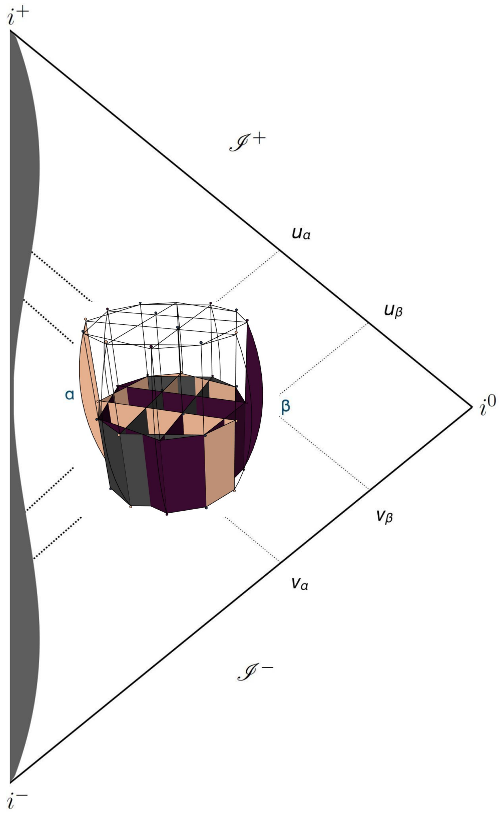

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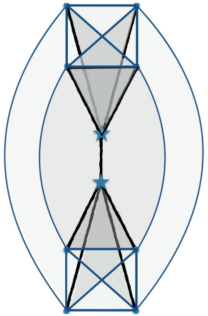
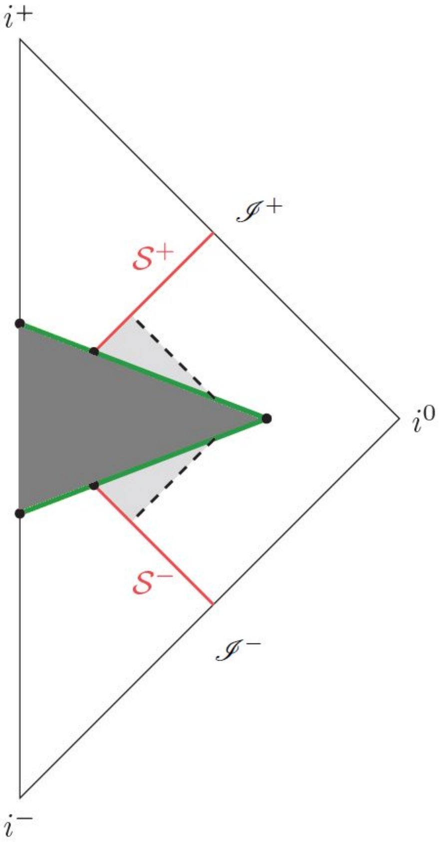
- A coherent state $\Psi(m, \mathcal{T}, v_\alpha, v_\beta)$ peaked on the boundary geometry can be defined in $\mathcal{H}_\Gamma$
- A spinfoam describing the quantum transition can be constructed
- The EPRL-KKL transition amplitude $W(m, \mathcal{T}, v_\alpha, v_\beta)$ can be computed

Investigations of the transition amplitude





- Analytical investigation of the EPRL transition amplitude for the Haggard-Rovelli spacetime gives

$$p \sim e^{-\alpha\, m^2/m_{\rm pl}^2}\,, \qquad \tau \sim m\, e^{\alpha\, m^2/m_{\rm pl}^2}$$

[Christodoulou and D'Ambrosio (2018)]
[Christodoulou, D'Ambrosio, Theofilis (2023)]

- These results have been recently confirmed numerically

[Frisoni (2023)]

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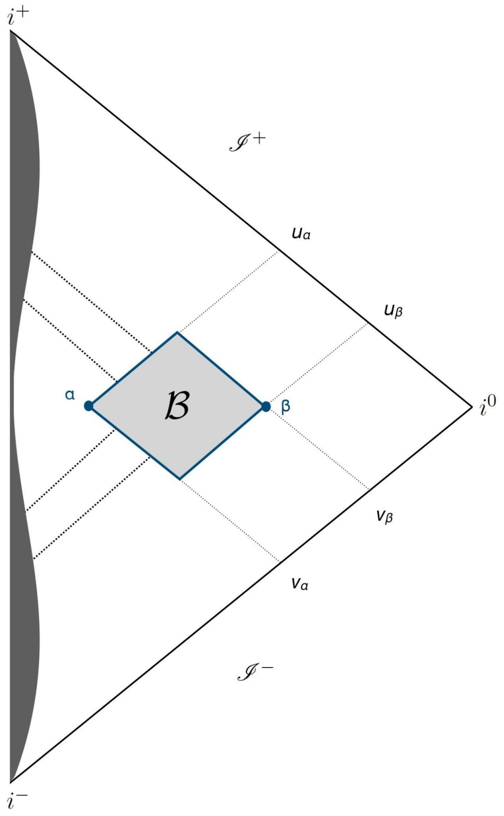

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- These results have been recently confirmed numerically

[Frisoni (2023)]

- Numerical investigation of the spin foam amplitude for only region B

[Han, Qu, Zhang (2024)]