Carrollian Stretched Horizons

Geometry, Dynamics, and Phase Space

(2211.06415, 2405.xxxxx with Laurent Freidel)

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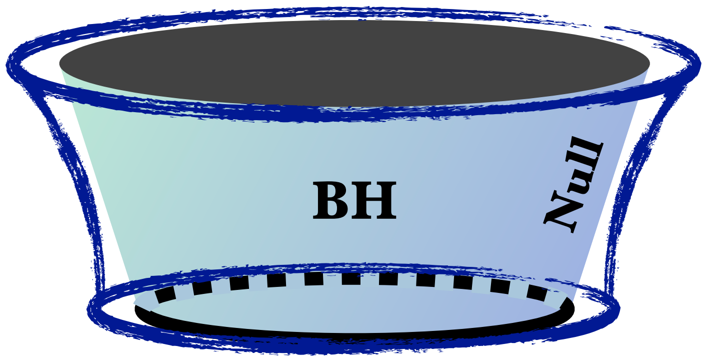

[Damour '78, Thorne-Price '86]

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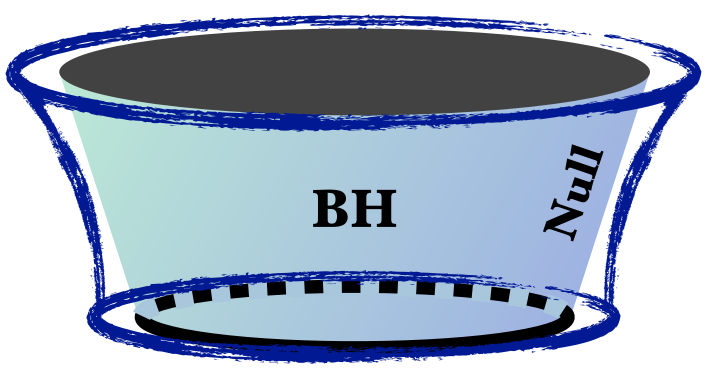

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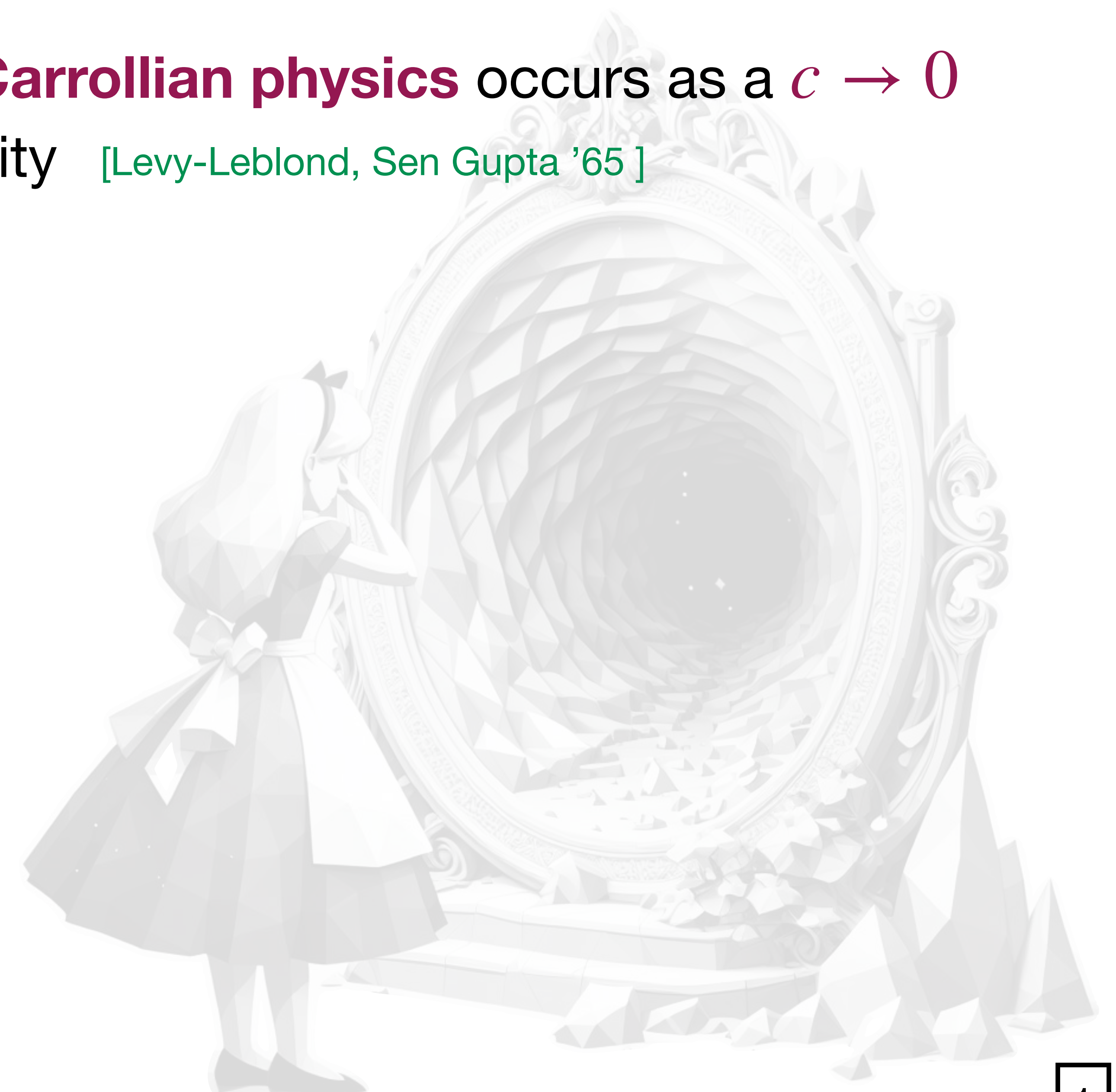

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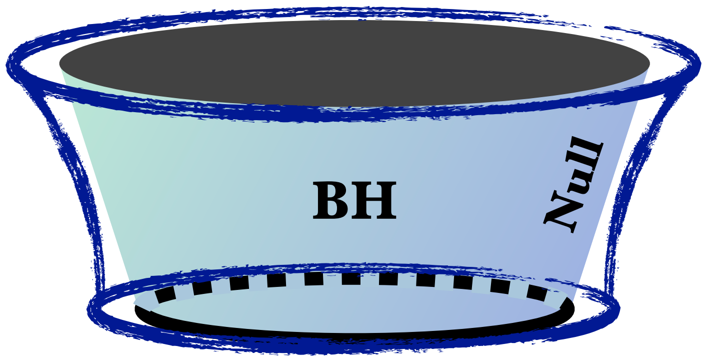

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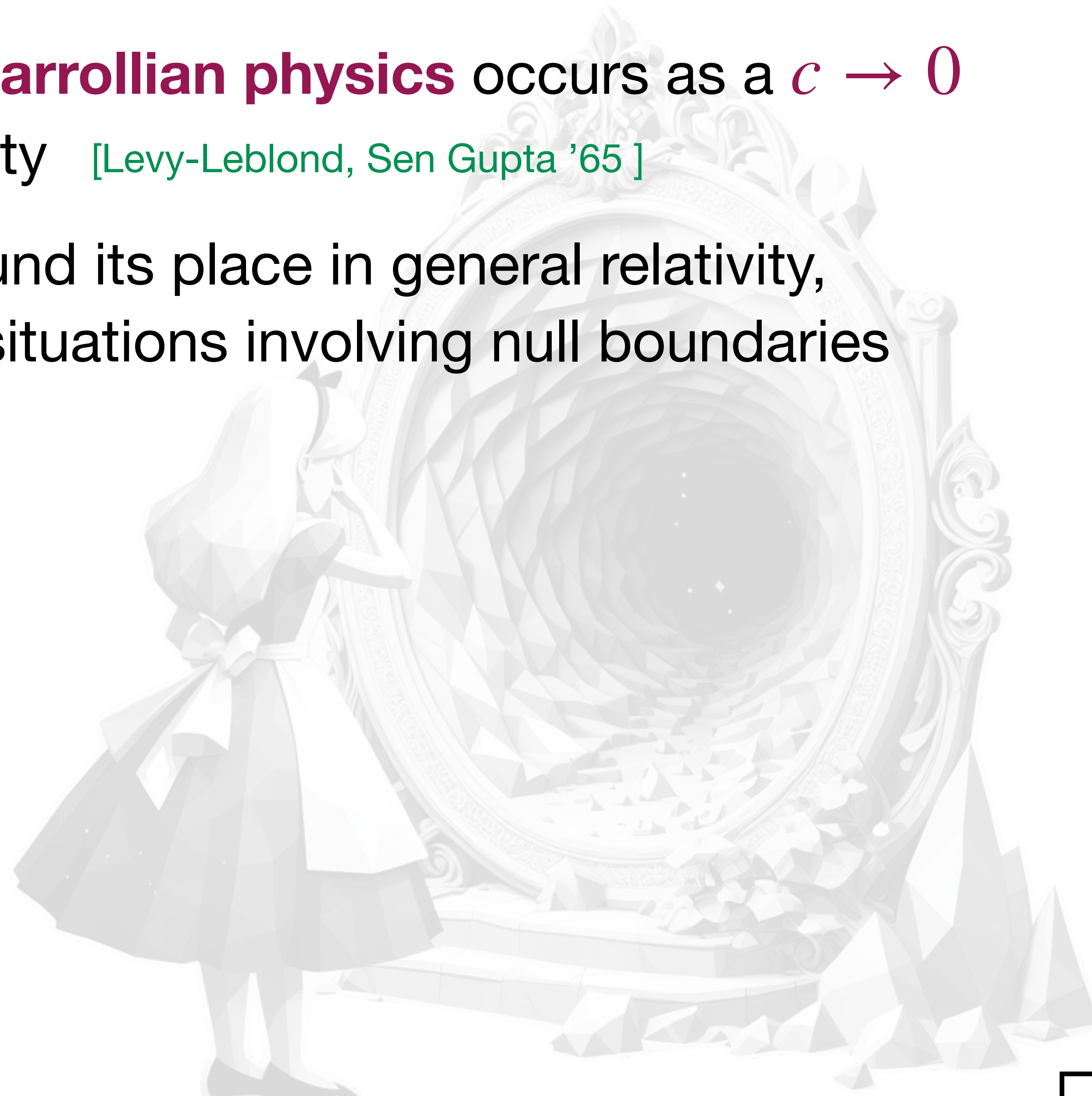

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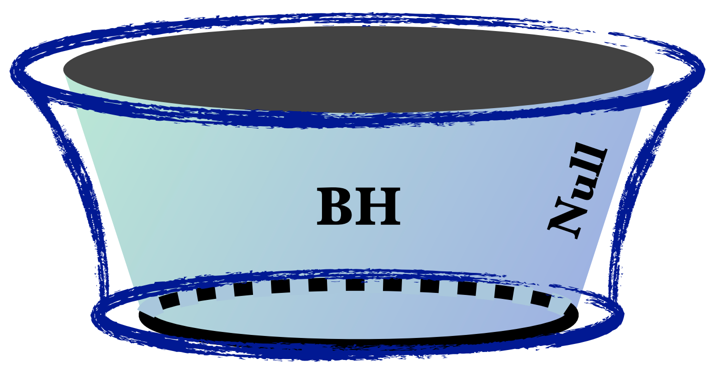

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This talk

- Carrollian Stretched Horizon
- Embedding in a spacetime
- Dynamics
- Symplectic structure
- Symmetries and Charges

Carrollian Stretched Horizon

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$\mathscr{H}$ = 3D timelike surface with a *Carrollian structure* ($\pi : \mathscr{H} \to \mathcal{S}$ *where* $\mathcal{S}$ *is a 2-sphere)*

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Stretched Carrollian Structure (sCarroll): (v^i, k_j, h_{ij}, ρ)

- Vertical vector v^i pointing along fibers
- Metric h_{ij}
- Ruling k_i with $v^i k_i = 1$
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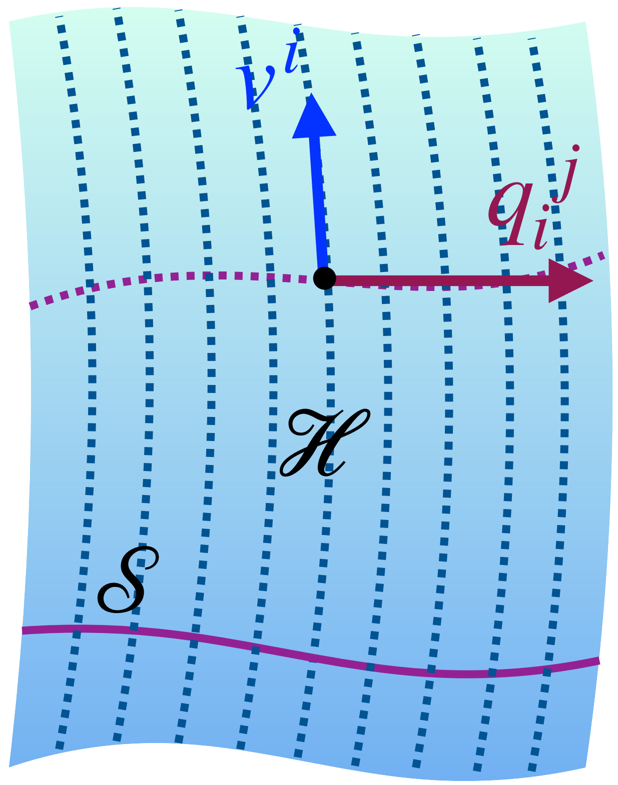
[Timelike generalization and modern language of Levy-Leblond '64, Ashtekar '78 -'24, Henneaux '81, Dautcourt '97, Duval-Gibbons-Horvarthy '14, and others]



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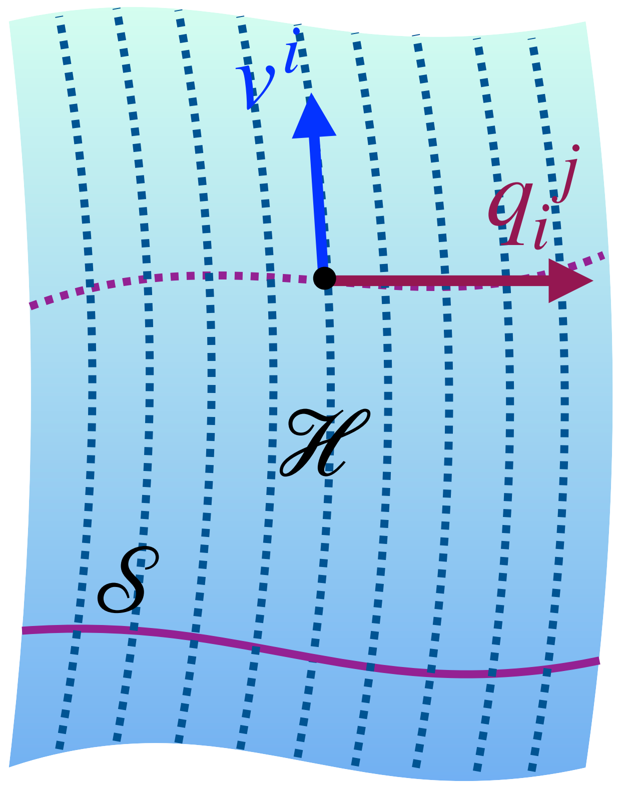

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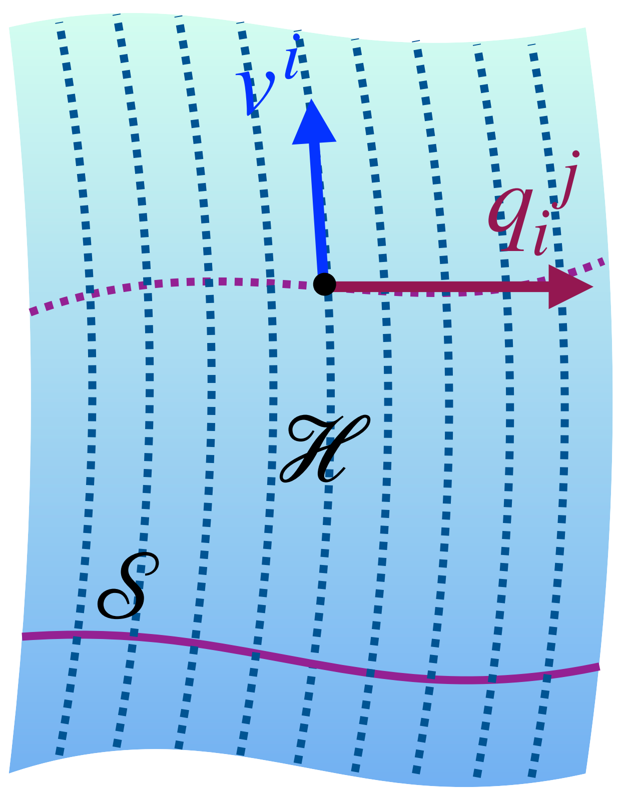

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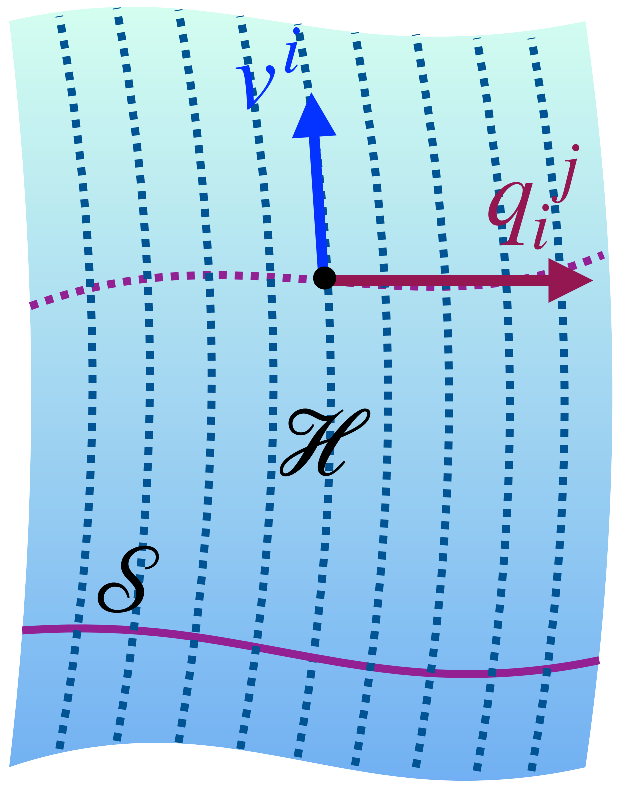

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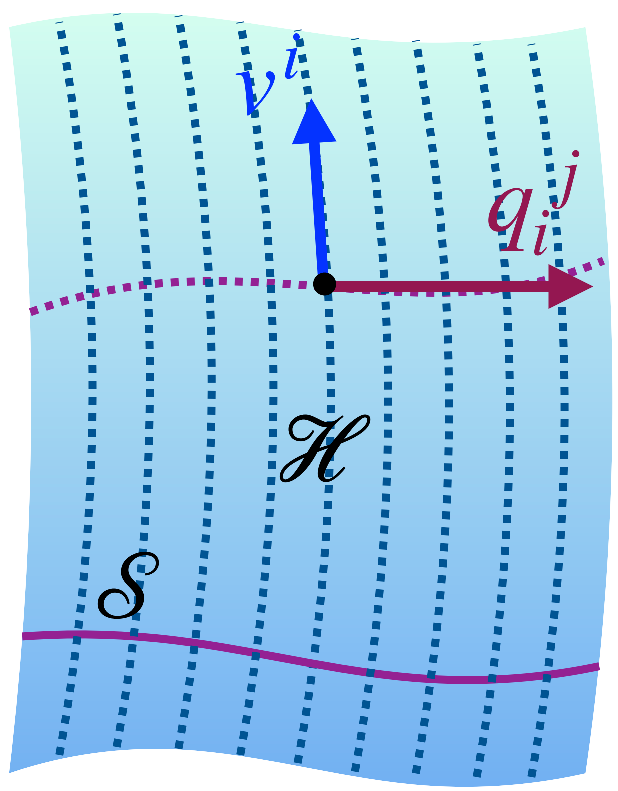

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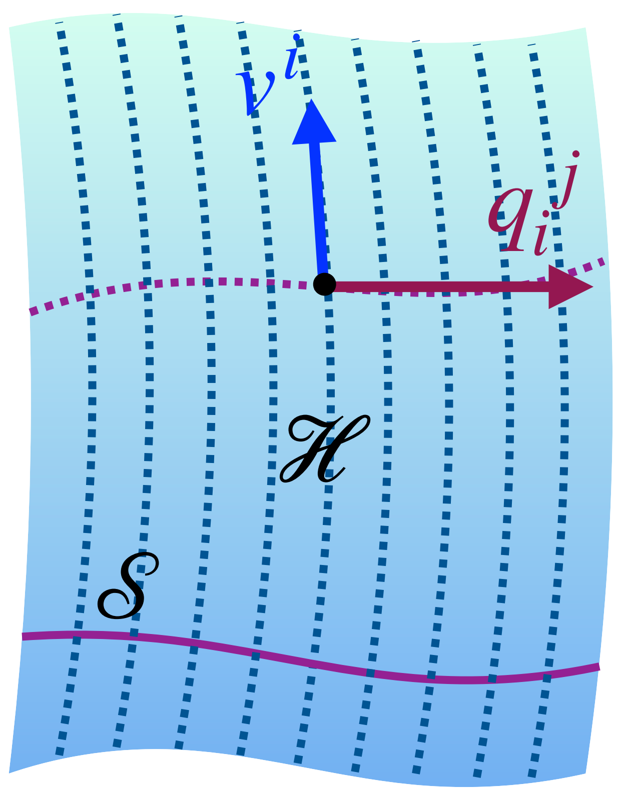

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Frame $q_{ij} = e_i{}^A e_j{}^B q_{AB}$

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energy heat momentum viscous stress pressure

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- *The quantities $\left(\mathsf{E}, \mathsf{P}, \mathsf{J}^i, \mathsf{p}_i, \mathsf{S}_i{}^j, \bar{\theta}\right)$ are conjugate canonical momenta to the sCarroll structure of the Einstein gravity phase space when pulled-back to $\mathscr{H}$*

Embedding $\mathscr{H}$ in Spacetime

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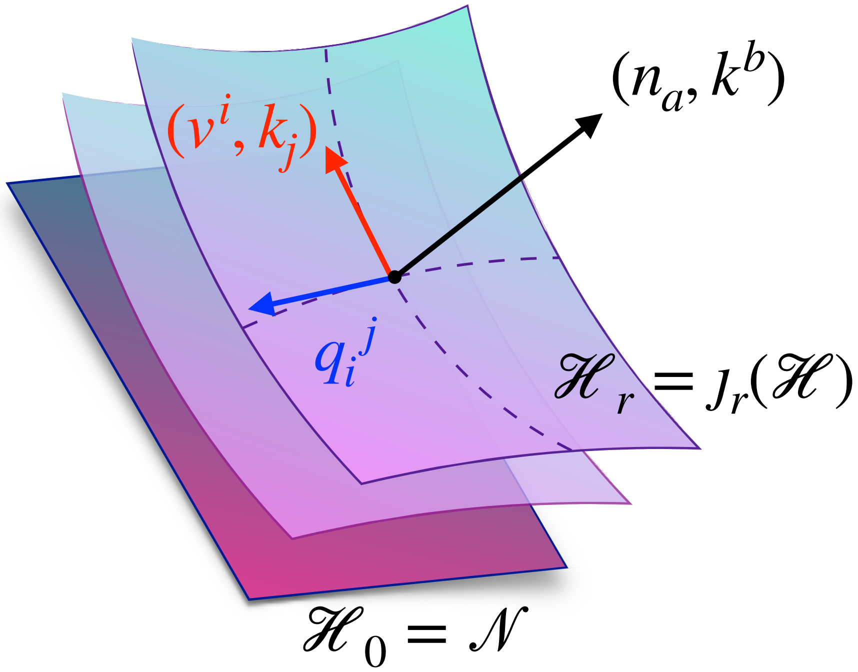

A family of sCarroll structures $\left(v^i(r), k_j(r), h_{ij}(r), \rho(r)\right)$

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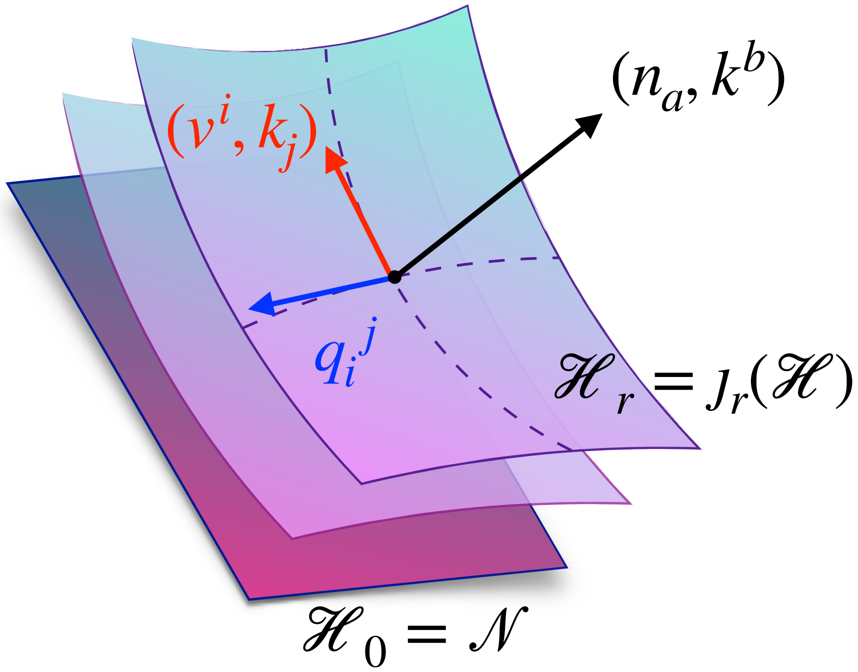

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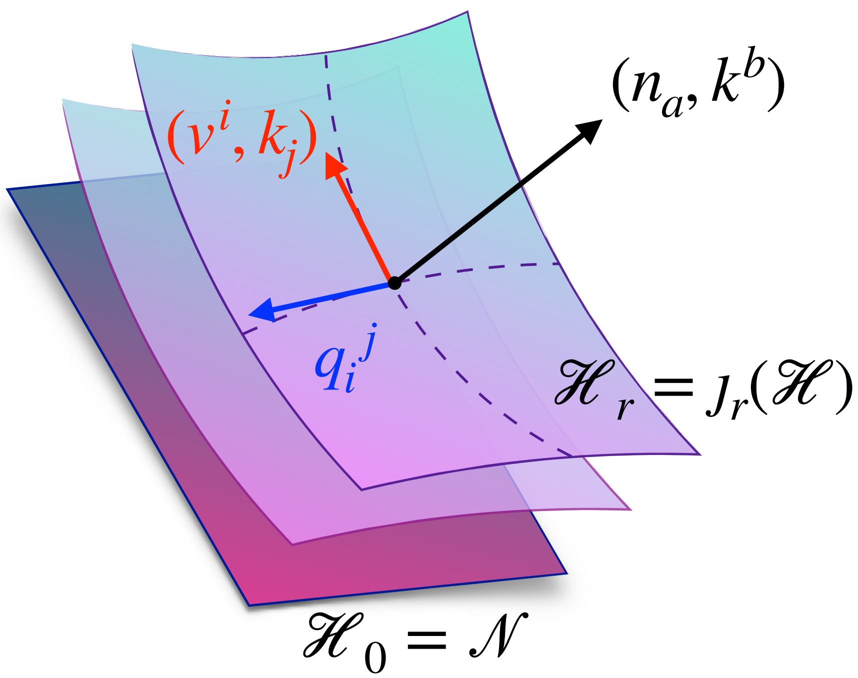

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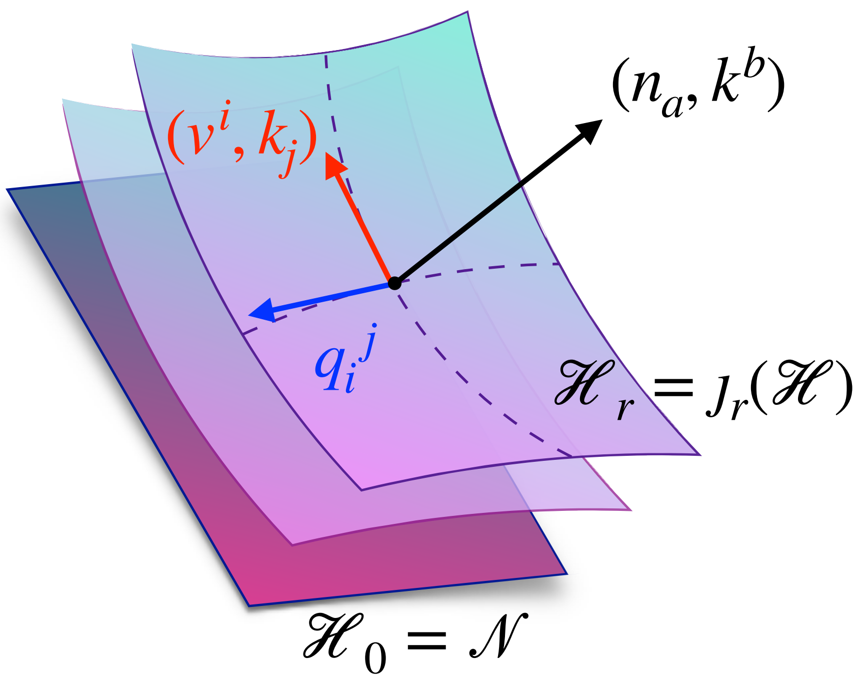

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Equivalence between ***Stretched Horizon*** perspective and ***sCarrollian*** perspective

$$(n_a, k^a, g_{ab}) \Longrightarrow (v^a, k_a, h_{ab}, \rho)$$

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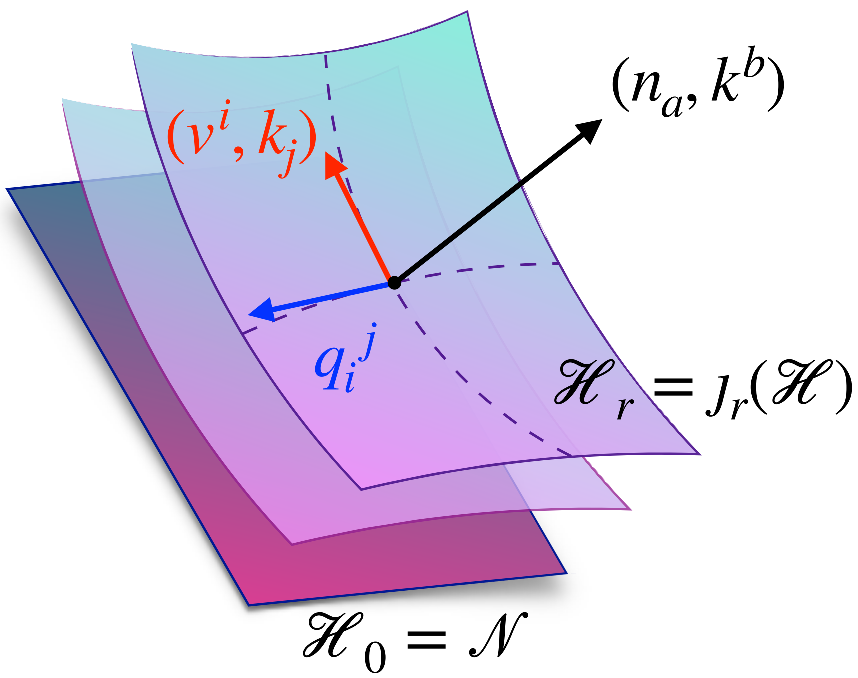

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$$(n_a, k^a, g_{ab}) \Longrightarrow (v^a, k_a, h_{ab}, \rho)$$

When the stretching $\rho = \text{constant}$, which is always possible by rescaling: $n_a \to \mathrm{e}^{\phi} n_a$ and $k^a \to \mathrm{e}^{-\phi} k^a$, we have the relation

$$\Pi_i{}^a G_a{}^b n_b = D_j \mathsf{T}_i{}^j$$

The Einstein equations projected onto $\mathscr{H}$ is the conservation laws of sCarrollian stress tensor

[Donnay-Marteau '19, Chandrasekaran-Speranza '20, Chandrasekaran-Flanagan-Shehzad-Speranza '21, PJ-Freidel '22]

Symplectic Potential

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$$\mathsf{T}_i{}^j = -k_i\left(\mathsf{E}v^j + \mathsf{J}^j\right) + \mathsf{p}_i v^j + \left(\mathsf{S}_i{}^j + \mathsf{P}q_i{}^j\right) \quad \text{and} \quad \bar{\theta} = q^{ij}D_i k_j = \partial_r \ln\sqrt{q}$$

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$$\Theta^{\text{can}}_{\mathscr{H}} = -\int_{\mathscr{H}} \left(\left(\mathsf{E} v^a + \mathsf{J}^a\right) \delta k_a + \mathsf{p}_a \delta v^a - \frac{1}{2}(\mathsf{S}^{ab} + \mathsf{P} q^{ab}) \delta q_{ab} + \overline{\theta} \delta \rho \right) \epsilon_{\mathscr{H}}$$

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Intrinsic but derivable from the Einstein-Hilbert gravity, by fixing the rigging structure to be a background structure ($\delta n_a = 0,\ \delta k^a = 0$)

$$\Theta^{\text{EH}}_{\mathscr{H}} = \Theta^{\text{can}}_{\mathscr{H}} + \delta \mathsf{B}_{\mathscr{H}} + \vartheta_{\partial\mathscr{H}}$$

Boundary Action: $\mathsf{B}_{\mathscr{H}} = \int_{\mathscr{H}} \mathsf{N}\epsilon_{\mathscr{H}}$

Corner Potential: $\vartheta_{\partial\mathscr{H}} = -\frac{1}{2}\int_{\partial\mathscr{H}} \delta v^a \iota_a \epsilon_{\mathscr{H}}$

[sCarrollian generalization to many works: Hayward ’93, Ashtekar ‘00, Lewandowski ’04, Lehner-Myers-Poisson-Sorkin ‘16, Parattu-Chakraborty-Padmanabhan ’16, De Paoli-Speziale ’17, Freidel-Hopfmuller ’16-’18, Adami-Grumiller-Sheikh-Jabbari-Taghiloo-Yavartanoo-Zwikel ’21, Chandrasekaran-Speranza ‘21, Chandrasekaran-Flanagan-Shehzad-Speranza ’21, Freidel-PJ ’22, Odak-Rignon-Bret-Speziale ’23, Chandrasekaran-Flanagan ’23, Ciambelli-Freidel-Leigh ’23, Freidel-Riello ’24]

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Symmetries of sCarroll structure, preserving h_{ij}

$$\delta_W v^a = W v^a \quad \delta_W k_a = -W k_a \quad \delta_W \rho = 2W\rho$$

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- Transformations of under (R, W, Z^A) are anomalous: $\delta_{(R,W,Z)} \neq \mathscr{L}_{(R,W,Z)}$

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[Null case: Adami-Grumiller-Sheikh-Jabbari-Taghiloo-Yavartanoo-Zwikel '21, BMSW: Freidel-Oliveri-Pranzetti-Speziale '21, Geiller-Zwikel '22 & '24]

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Energy & Momentum evolutions:

$$-G_{vn} = \mathscr{L}_v \mathsf{E} + (\mathsf{E} + \mathsf{P})\theta + (\mathscr{D}_A + 2\varphi_A)\mathsf{J}^A + \mathsf{S}^{AB}\sigma_{AB}$$

$$G_{An} = \mathscr{L}_v \mathsf{p}_A + \theta \mathsf{p}_A + \mathsf{E}\varphi_A + \mathsf{J}^B w_{BA} + \left(\mathscr{D}_B + \varphi_B\right)(\mathsf{S}_A{}^B + \mathsf{P}\delta_A^B)$$

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Transverse Evolution:

$$-\mathrm{Ric}_{kn} = \frac{1}{2} q^{AB} G_{AB} - \rho G_{kk} = \mathscr{L}_k \mathsf{N} + (\mathsf{E} + \mathsf{P})\bar{\theta} - (\mathscr{D}_A + 2\mathsf{p}_A + 2\varphi_A)\mathsf{p}^A + \mathsf{S}^{AB}\bar{\sigma}_{BA}$$

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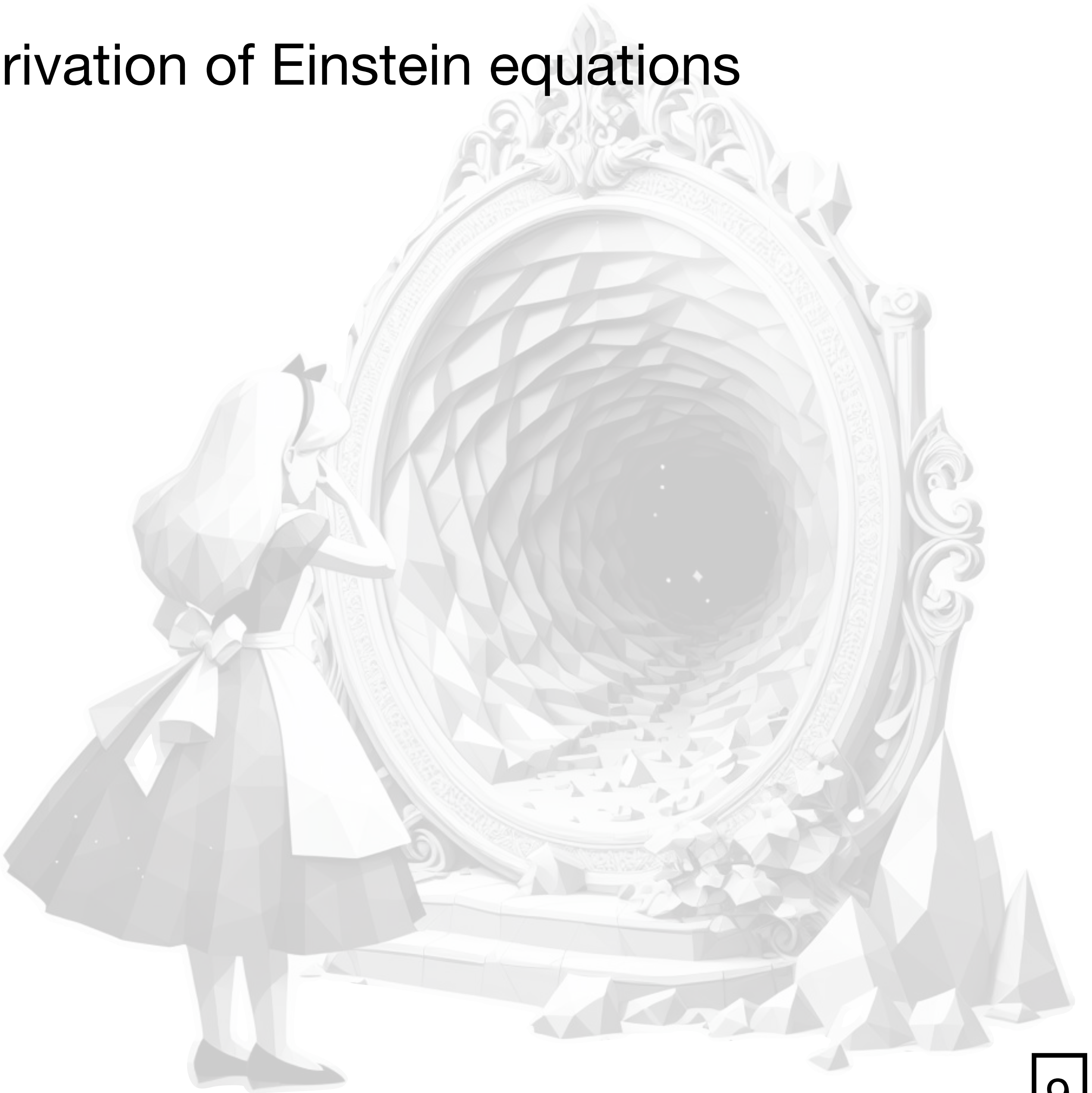

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✤ Quantization?

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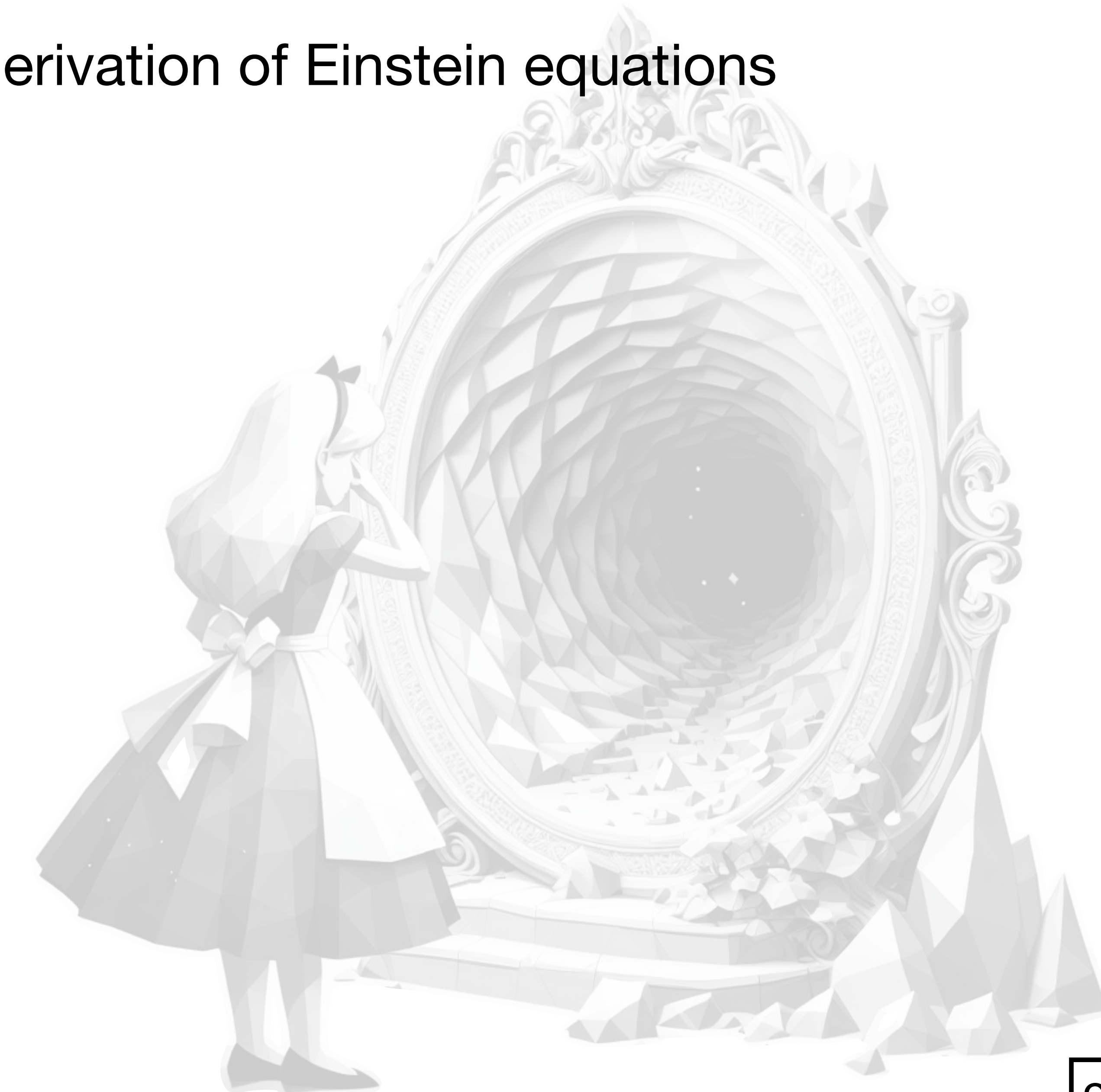

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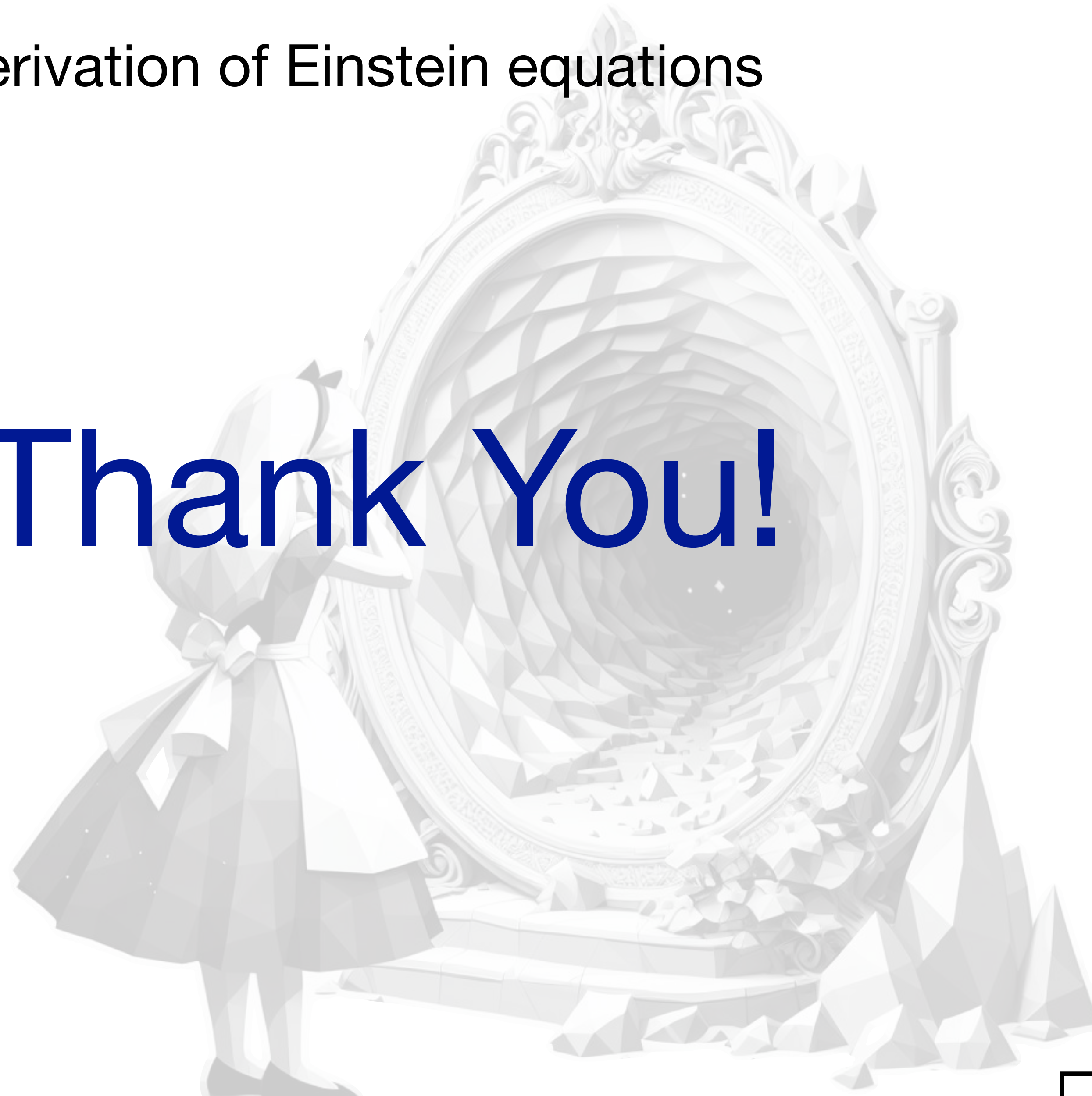

Thank You!

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This mechanism can be traced back to the **Bianchi identity** $\nabla_b G_a{}^b = 0$

What does it mean? Can we think of this mechanism as the action of some symmetries? Relation with asymptotic infinities?