Pilot-wave approach to key issues in non-perturbative quantum gravity

Dr. Indrajit Sen

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1. I. Sen, S. Alexander and J. Dressel. A realist interpretation of unitarity in quantum gravity. *Class. Quantum Grav.* 41, 115005 (2024).
2. I. Sen. Physical interpretation of non-normalizable harmonic oscillator states and relaxation to pilot-wave equilibrium. *Nat. Sci. Rep.* 14, 669 (2024).

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1 The problem of time

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4. The problem of reality conditions.

All major approaches to resolving these problems based on orthodox quantum mechanics.

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2. The problem of non-normalizability.
3. The problem of classical limit.
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New conceptual approach based on pilot-wave theory.

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Pilot-wave theory

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Conceptual approach to key issues

1. The problem of time $\longleftrightarrow$ t parameterizes variation of the configuration.

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4. The problem of reality conditions $\longleftrightarrow$ implement at the level of guidance equation.

Pilot-wave non-perturbative quantum gravity

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Total constraint for interacting gravitational-fermionic system[1]:

$$\int_{\mathcal{M}} (\tilde{N}\hat{\mathcal{H}} + N^a \hat{\mathcal{V}}_a)\Psi[A, \xi] = 0$$

[1] S. Alexander et al., *Phys. Rev. D* **2022**, *106*, 106012.

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Total constraint for interacting gravitational-fermionic system:

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Ashtekar connection

2-component Weyl Spinor

Shift vector

Lapse function

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$$\hat{\mathcal{H}} = \frac{1}{2}\epsilon_{ijk}\hat{E}^{bj}\hat{E}^{ai}\left(F^k_{ab} + \frac{\Lambda}{3}\epsilon_{abc}\hat{E}^{ck}\right) + (\widehat{\mathcal{D}}_a\xi)_A\sigma_i^{AB}\hat{E}^{ai}\widehat{\Pi}_B + \hat{E}^{ai}(\widehat{\mathcal{D}}_a\xi)_A\sigma_i^{AB}\widehat{\Pi}_B$$

$$\hat{\mathcal{V}}_a = \frac{i}{2}F^k_{ab}\hat{E}^b_k + (\widehat{\mathcal{D}}_a\xi)_B\widehat{\Pi}^B$$

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Quantization scheme:

$$\hat{E}^{ai} \to \frac{\delta}{\delta A_{ai}}, \qquad \widehat{\Pi}_A \to -i\frac{\delta}{\delta \xi^A}$$

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$$\int_{\mathcal{M}} \tilde{N} \frac{\delta}{\delta A_{ai}} \left[\frac{\epsilon_{ijk}}{2} \frac{\delta}{\delta A_{bj}} \left(F^k_{ab} + \frac{\Lambda}{3} \epsilon_{abc} \frac{\delta}{\delta A_{ck}} \right) - 2(\widehat{\mathcal{D}}_a \xi)_A \sigma_i^{AB} i \frac{\delta}{\delta \xi^B} \right] \Psi[A, \xi]$$
$$+ \int_{\mathcal{M}} i N_b \frac{\delta}{\delta A_{ai}} \left[\frac{F^b_{ai}}{2} \Psi[A, \xi] \right] - \int_{\mathcal{M}} i N_b \frac{\Psi[A, \xi]}{2} \frac{\delta F^b_{ai}}{\delta A_{ai}}$$
$$= \int_{\mathcal{M}} N^b (\widehat{\mathcal{D}}_b \xi)^B i \frac{\delta}{\delta \xi^B} \Psi[A, \xi]$$

Where is time?

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Use ansatz $\Psi[A, \xi] = \Psi_K[A] \Phi[A, \xi]$

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$$\int_{\mathcal{M}} \frac{1}{\Psi_K} \frac{\delta}{\delta A_{ai}} \left\{ \left[\tilde{N} \frac{\epsilon_{ijk}}{2} \frac{\delta}{\delta A_{bj}} \left(F^k_{ab} + \frac{\Lambda}{3} \epsilon_{abc} \frac{\delta}{\delta A_{ck}} \right) + i N_b \frac{F^b_{ai}}{2} \right] \Psi_K \Phi[A,\xi] \right\}$$
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$$\frac{\delta \xi^B}{\delta t} \equiv N^b (\widehat{\mathcal{D}}_b \xi)^B + 2\ell_{\mathrm{Pl}}^2 \frac{\delta \ln \Psi_K}{\delta A_{ai}} \tilde{N} (\widehat{\mathcal{D}}_a \xi)_A \sigma_i^{AB}$$

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$$\frac{\partial \Phi[A, \xi]}{\partial t} \equiv \int_{\mathcal{M}} \frac{\delta \xi^B}{\delta t} \frac{\delta}{\delta \xi^B} \Phi[A, \xi]$$

implicitly depends on $\Psi[A, \xi]$

defines absolute simultaneity

$$\frac{\delta \xi^B}{\delta t} \equiv N^b(\widehat{\mathcal{D}}_b \xi)^B + 2\ell^2_{\mathrm{Pl}} \frac{\delta \ln \Psi_K}{\delta A_{ai}} \tilde{N}(\widehat{\mathcal{D}}_a \xi)_A \sigma^{AB}_i$$

Continuity equation

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$$\frac{\partial(|\Psi|^2\Omega)}{\partial t} + \nabla^{ai} J_{ai} + \overline{\nabla}^{ai} \overline{J}_{ai} = 0$$

$$\nabla^{ai} \equiv \int_{\mathcal{M}} \; \delta/\delta A_{ai}$$

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Constraint on Ω:

$$-\left[\frac{\delta^2\Omega}{\delta A_{ai}\delta A_{bj}} \frac{i\tilde{N}\epsilon_{ijk}}{2} F^k_{ab} + c.c \right] + \left[\frac{\delta^3\Omega}{\delta A_{ai}\delta A_{bj}\delta A_{ck}} \frac{i\tilde{N}\epsilon_{ijk}}{2} \frac{\Lambda}{3} \epsilon_{abc} + c.c \right]$$
$$- \left[N_b \frac{\delta\Omega}{\delta A_{ai}} \frac{F^b_{ai}}{2} + c.c \right] = 0$$

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Implications for Unitarity:

Ψ_K naturally factored out $\rightarrow$ $\Phi[A,\xi] = \frac{\Psi[A,\xi]}{\Psi_K[A,\xi]}$ may be normalizable.

Non-perturbative inner product.

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$$+ \frac{i\tilde{N}\ell_{\mathrm{Pl}}^2}{2}\epsilon_{ijk}\left(2F_{ab}^k\frac{\delta \ln \Phi}{\delta A_{bj}} + \frac{\ell_{\mathrm{Pl}}^2\Lambda}{3}\epsilon_{abc}\left[2\frac{\delta \ln \Phi}{\delta A_{bj}}\frac{\delta \ln \Psi_K}{\delta A_{ck}} + \frac{1}{\Phi}\frac{\delta^2\Phi}{\delta A_{ck}\delta A_{bj}}\right]\right)$$

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Classical solution:

$$\dot{A}_{ai} = \frac{i\tilde{N}}{2}E^{bj}\epsilon_{ijk}\left(2F^k_{ab} + \Lambda\epsilon_{abc}E^{ck}\right) - N_b F^b_{ai} + 2\tilde{N}(\widehat{\mathcal{D}}_a\xi)_A\sigma_i^{AB}\Pi^B$$

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For self-dual solutions,

$$F^j_{ik} = -\frac{\Lambda}{3}\epsilon_{ikb}E^{bj}$$

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Classical limit is obtained when $\Phi = \Phi_{SC}[\xi]$.

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Reality Conditions $(A_{ai} + \overline{A}_{ai} = 2\Gamma_{ai},\ E_{ai} = \overline{E}_{ai})$ implemented at the level of guidance equation.

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$$\Gamma_{ai} = \frac{1}{2}\epsilon_{ijk}E^{bk}\left(E^j_{a,b} - E^j_{b,a} + E^c_j E^l_a E^l_{c,b}\right) + \frac{1}{4}\epsilon_{ijk}E^{bk}\left(2E^j_a\frac{\mathbf{E}_{,b}}{\mathbf{E}} - E^j_b\frac{\mathbf{E}_{,a}}{\mathbf{E}}\right)$$

Continuity equation

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Probabilities stay normalized in minisuperspace.

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THANK YOU!

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Quantization due to decoherence.

Equilibrium density?

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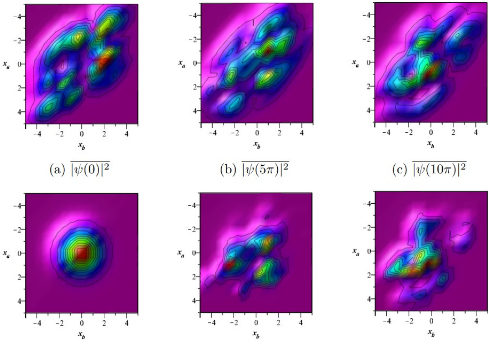

(a) $\overline{|\psi(0)|^2}$ (b) $\overline{|\psi(5\pi)|^2}$ (c) $\overline{|\psi(10\pi)|^2}$

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Normalizable case: $\overline{\rho} \longrightarrow \overline{|\psi|^2}$

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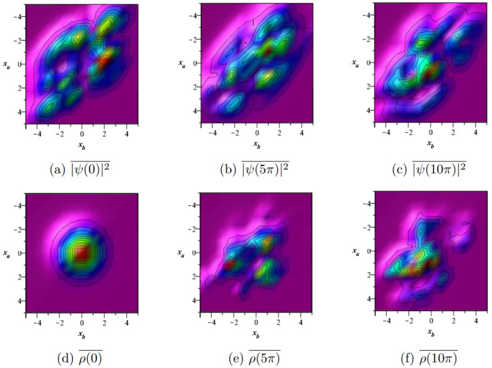

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Non-normalizable case: $\overline{\rho} \longrightarrow$ **?**

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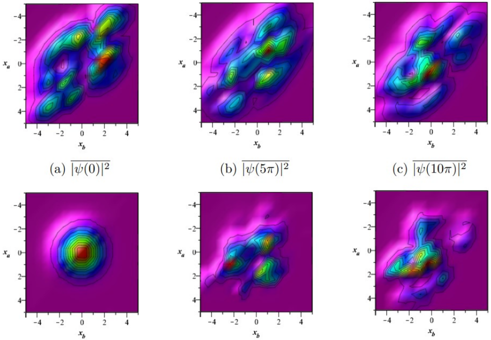

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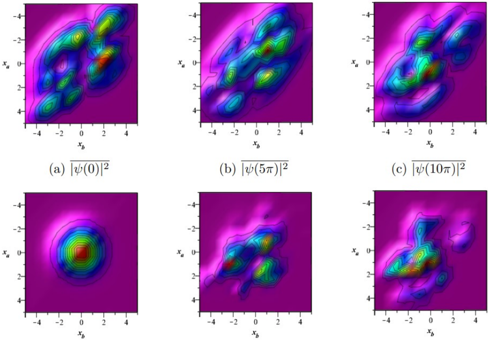

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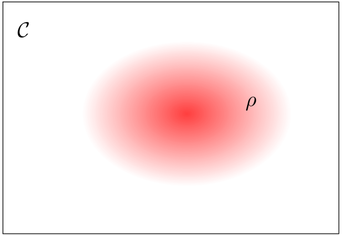

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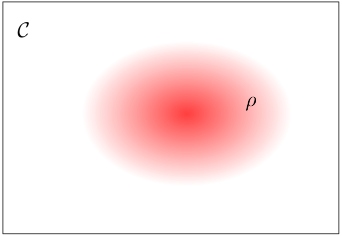

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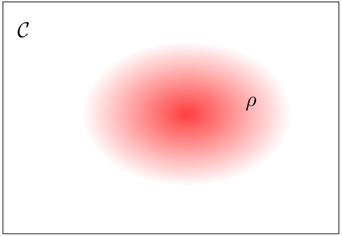

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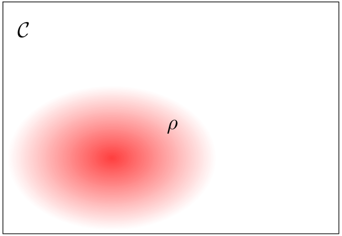

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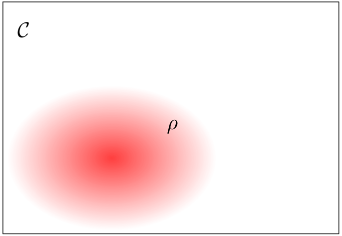

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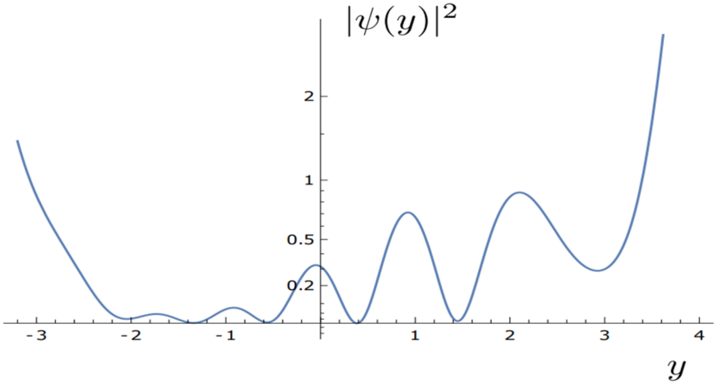

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