The role of discreteness in the information loss paradox: insights from a toy model

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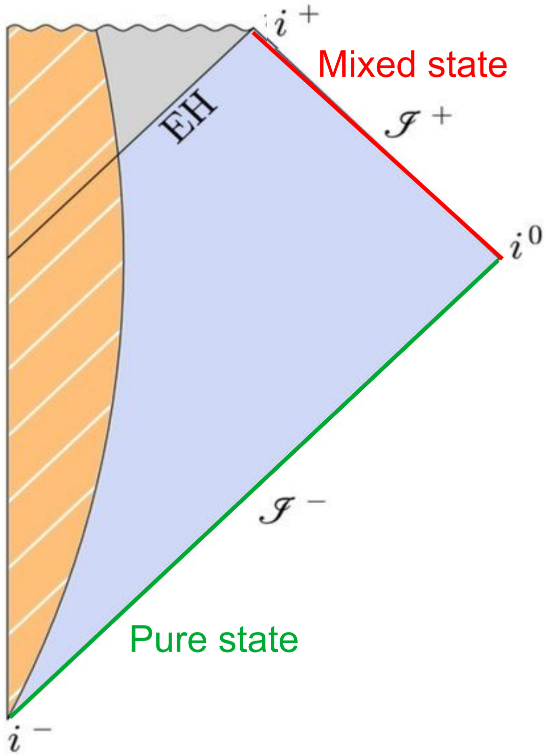
The paradox



A pure state evolves to a mixed state [S.Hawking ‘75 and R.Wald ‘75]

It is expected that all the initial energy will be recovered in the final mixed state (apart from a Planckian remnant)

-Where did the information go ?

-How can all the information be conserved with so little energy available ?

Past results

[Banks, Peskin and Susskind ‘83] : Decoherence $\Longrightarrow$ dissipation.
No dissipation $\Longrightarrow$ No decoherence.

[Wald and Unruh ‘95] and others contested the argument.

[Unruh ‘12] suggested a model of a particle interacting with exterior decrease of freedoms, destroying its purity while conserving its energy.

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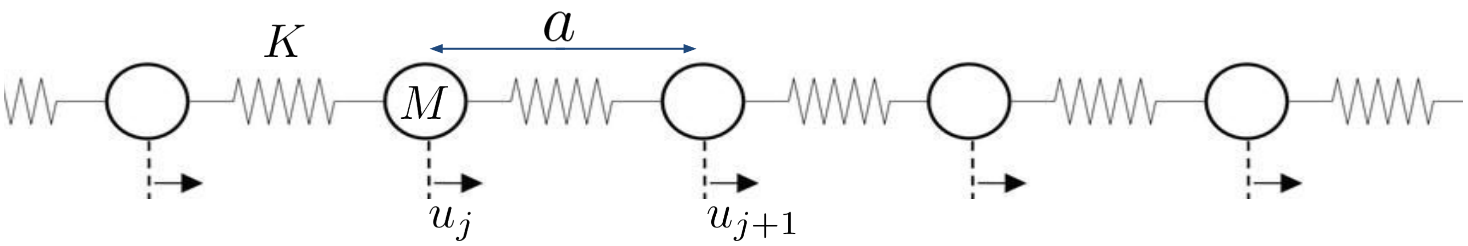
We want to construct a field theory version of this model, i.e a model of a quantum field interacting with a bath such that it decoheres while conserving its energy

LQG predict the existence of local d.o.f at the Planck scale, these are good candidates to play the role of the bath

The model



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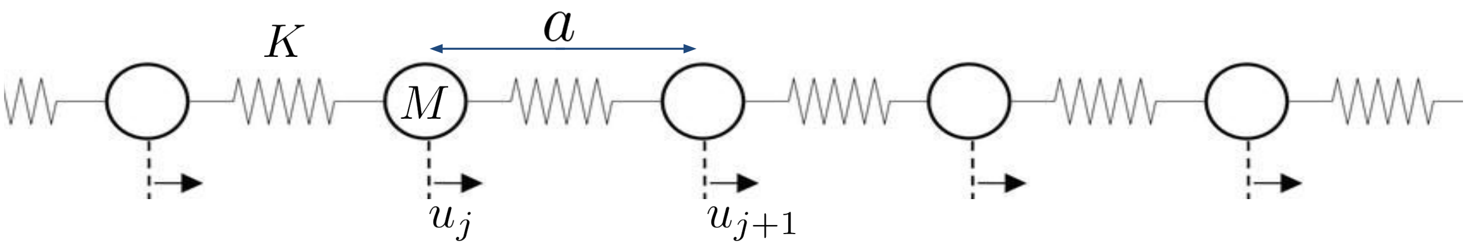

$$H = \sum_{j=1}^{N} \left[\frac{p_j^2}{2M} + \frac{K}{2}(u_j - u_{j+1})^2 \right] \xrightarrow{\text{F.T}: j \to q} H = \sum_q \left[\frac{p_q p_{-q}}{2M} + \frac{M}{2} u_q u_{-q} \omega_q^2 \right]$$

Where $\omega_q = 2\sqrt{\frac{K}{M}} \left| \sin\left(\frac{qa}{2}\right) \right|$ implying that there is a maximal frequency $\omega_q^{\max} = 2\sqrt{\frac{K}{M}}$

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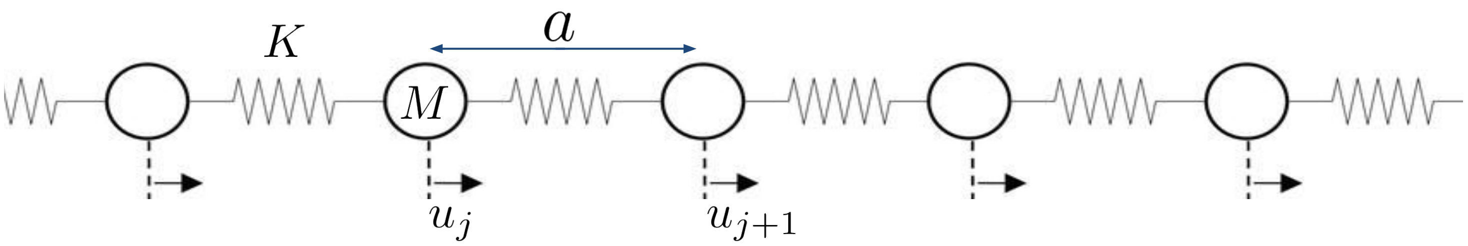

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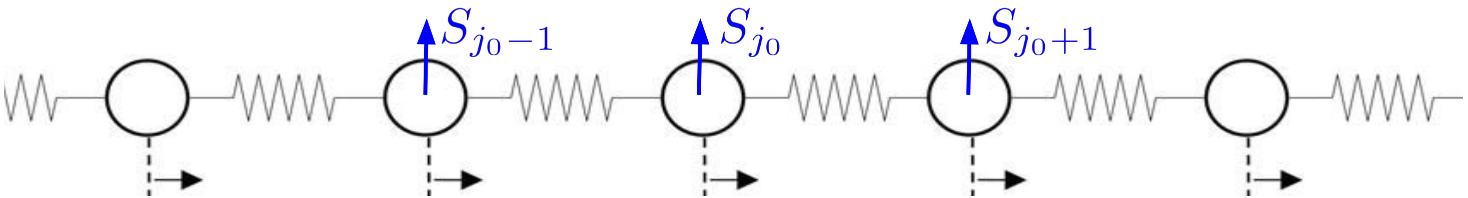
In the limit $a \to 0$ this is equivalent to a 1+1D massless scalar field

The Hilbert space is $\mathcal{H}_{\text{Matter}} = \bigotimes_q \mathcal{H}_q$ where $\mathcal{H}_q$ is the Fock space of the q^{th} H.O.

The model



$$H = \sum_q \left[\frac{p_q p_{-q}}{2M} + \frac{M}{2} u_q u_{-q} \omega_q^2 \right] + \alpha \sum_{j=j_0-n}^{j_0+n} S_{z_j} \left[\frac{p_j^2}{2M} + \frac{M}{2N} \left(\sum_q \omega_q u_q e^{-iqja} \right)^2 \right]$$

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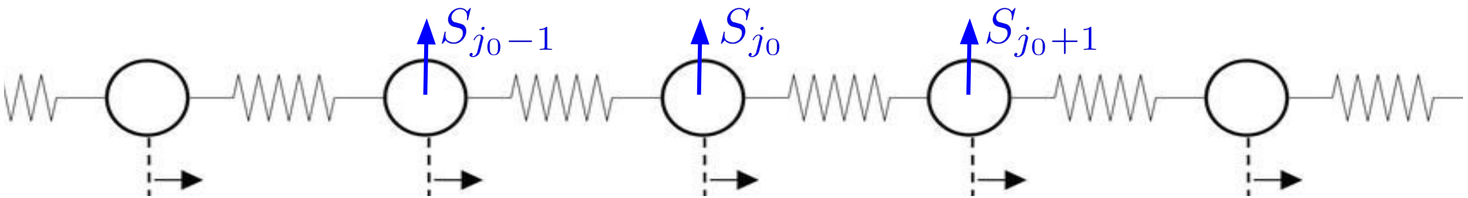

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The total Hilbert space becomes $\mathcal{H} = \mathcal{H}_{\text{Matter}} \otimes \mathcal{H}_{\text{Spins}}$ with $\mathcal{H}_{\text{Spins}} = \bigotimes_{j=1}^{N} \mathcal{H}_{\text{Spin,j}}$

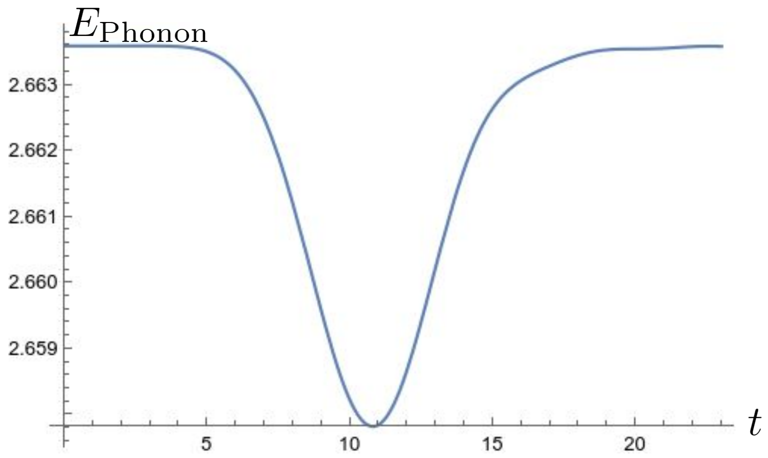
-Can an initially pure excitation of the matter field become entangled while retaining its energy ?

-How does this process depend on the energy of the excitation ?

Results

A perturbative resolution plus a numerical analysis leads to the following results :

The interaction with the spins is local
and the phonons preserve their energy.



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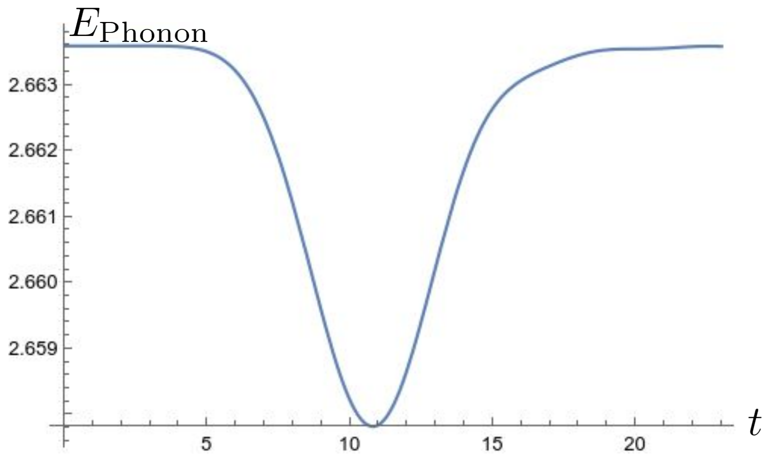

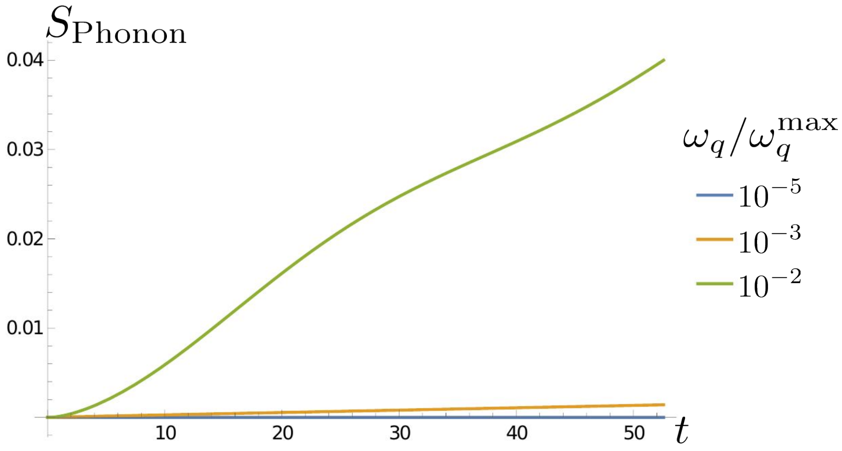

The entropy of the phonons increases during its interaction with the spins.
This effect is negligible for frequencies $\omega_q \ll \omega_q^{\max}$

Conclusion

- The model shows that decoherence can occur without dissipation in QFT.
- Local d.o.f, as those suggested by LQG, can constitute an environment allowing this mechanism.
- This process is significant only for modes of wavelength $\lambda \sim a$ and thus has no impact on the physics we have access to today.