DYNAMICAL SYMMETRIES FOR COSMOLOGY AND BLACK HOLES

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Based on: **arXiv: 2205.02615** w/ M. Geiller and E.R. Livine

arXiv: 2401.16036 w/ S. Ribisi

Motivations

What is the role of symmetries in (quantum) gravity?

- Asymptotic symmetries, soft theorems and memory effects
- Non-perturbative handle on quantization
- Structure of solutions to Einstein's equation

- **Reduced models**
  - Lower dimensional gravity (d<4)
    - Simpler dynamics (no local degrees of freedom)
    - Known solution space

  - **Existence of spacetime Killing vectors**
    - Very simple toy models
    - Insights on phenomenology (cosmology & black holes)

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Symmetries in reduced models

(Classical) Symmetries: phase space and dynamics

Unconstrained field space $\xrightarrow{\text{Constraints}}$ Physical solutions

$\{q_{\alpha\beta}, \Pi^{\gamma\delta}\}$

$\{A^i_\alpha, E^\beta_j\}$

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Physical solutions $\circlearrowleft$ Large diffeomorphisms
NJA, …

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Homogenous models

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$\{q_i, p^j\}$

$\xrightarrow{\text{Solve equations of motion}}$

Trajectories ↺ Dynamical symmetries

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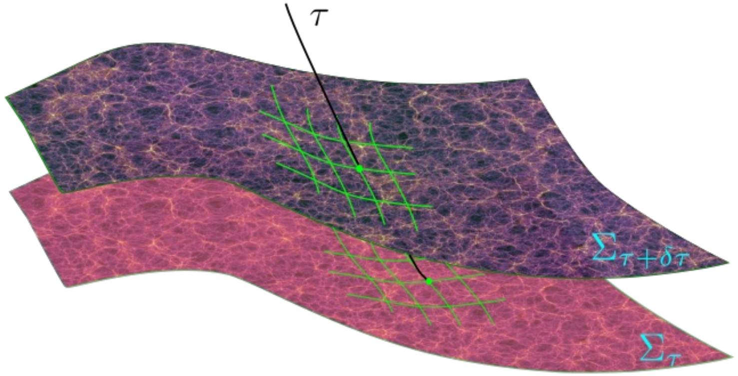
Minisuperspaces and 2nd geometrization

Finite number of degrees of freedom ➡️ Mechanical model



Einstein's field equations

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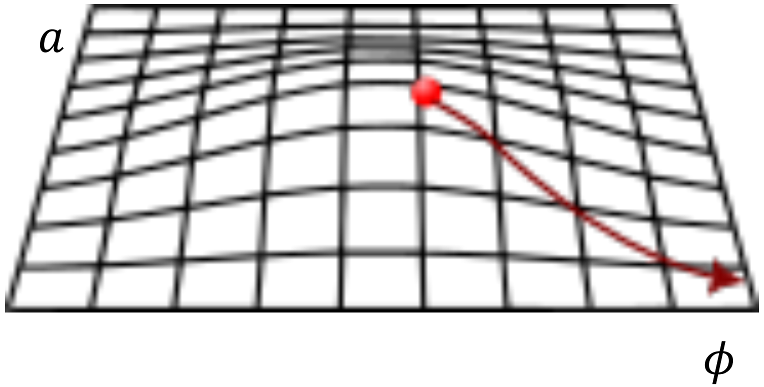

Geodesics on field space

Minisuperspaces and 2nd geometrization

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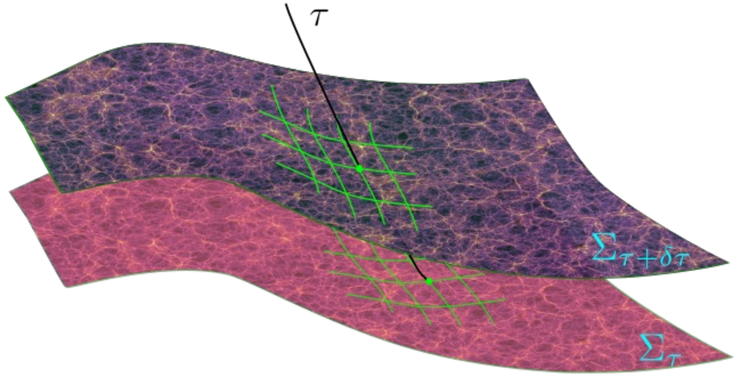

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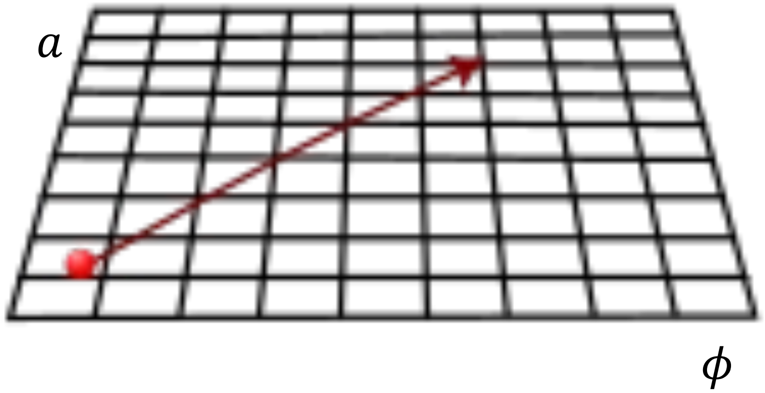

Geodesics on field space

Dynamical symmetries from field space isometries

For cosmology and static black holes: **Schrödinger algebra** [Ben Achour, Livine, Oriti, Piani '22] [Geiller, Livine, FS '22]

- Cosmology and hydrodynamics [D. Oriti's talk]
- Group quantization [Ben Achour, Livine '19, FS '21]
- Perturbation theory [Ben Achour, Livine, Mukohyama, Uzan '21]

Vaidya field space and covariance

Extension to infinite dimensional field spaces [Ribisi, FS ‘24]

Vaidya: Null shell collapse or evaporating black hole

$$ds^2 = -\frac{B}{X}dv^2 + 2N\,drdv + X^2\,d\Omega^2$$

Non-trivial action of the Schrödinger group on **solution space**

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Schrödinger symmetry

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Schrödinger symmetry

Doesn't *commute* with residual diffeomorphisms

- Usual covariant phase space in 2d → **Non-integrable charges**
- Completely gauge fixed phase space → **Integrable charges**

Vaidya phase space

"Edge modes" vs Dynamical symmetries

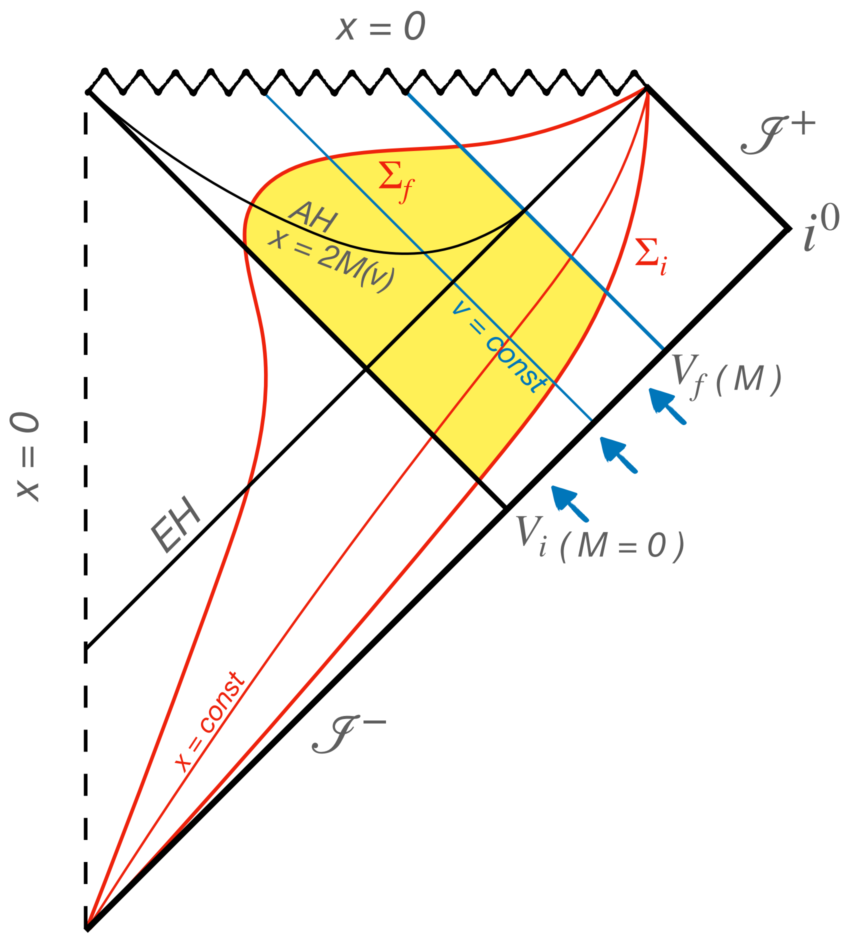
$$ds^2 = \frac{B}{X}dv^2 + 2N\,drdv + X^2\,d\Omega^2$$

- Covariant phase space

$$S_{EH} = \int N - \frac{X'(B' - 2\partial_v(NX))}{N}$$

- Complete gauge fixing

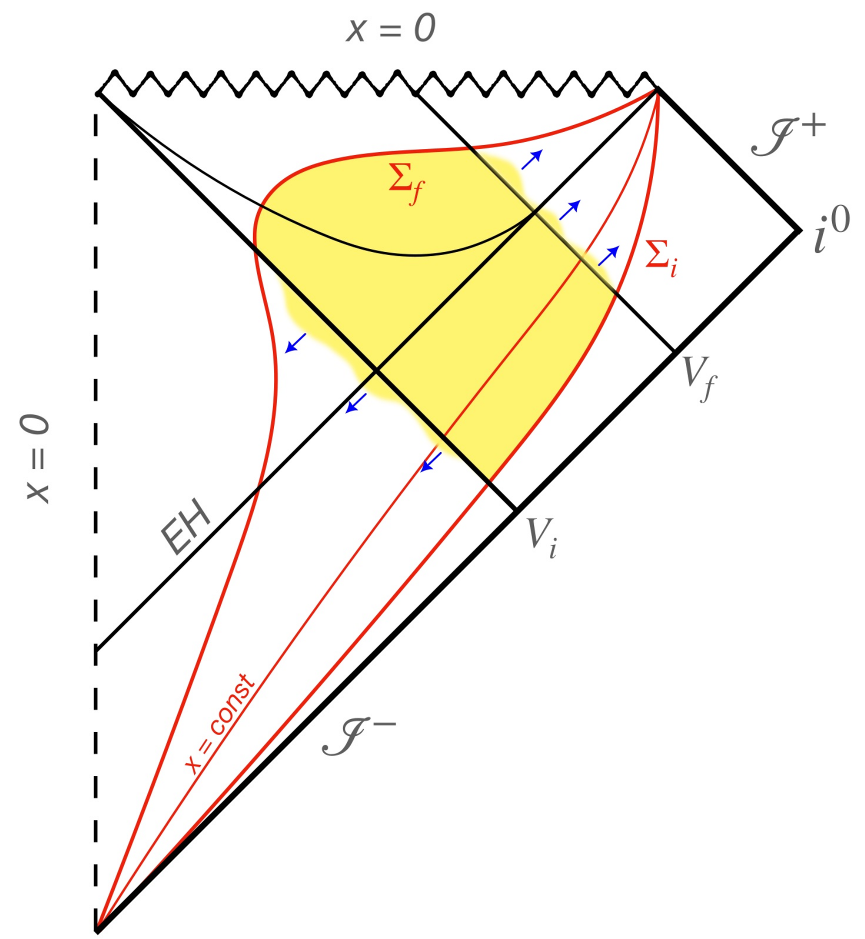
$$S_M = -\int X'\mathfrak{B}' \,, \qquad \mathfrak{B} = B - 2rN\dot{X} \,, \qquad N' = 0$$



Vaidya phase spaces

“Edge modes” vs Dynamical symmetries

- Covariant phase space
  - Boundary diffeomorphisms appear as physical,
  - Charges living at the corner



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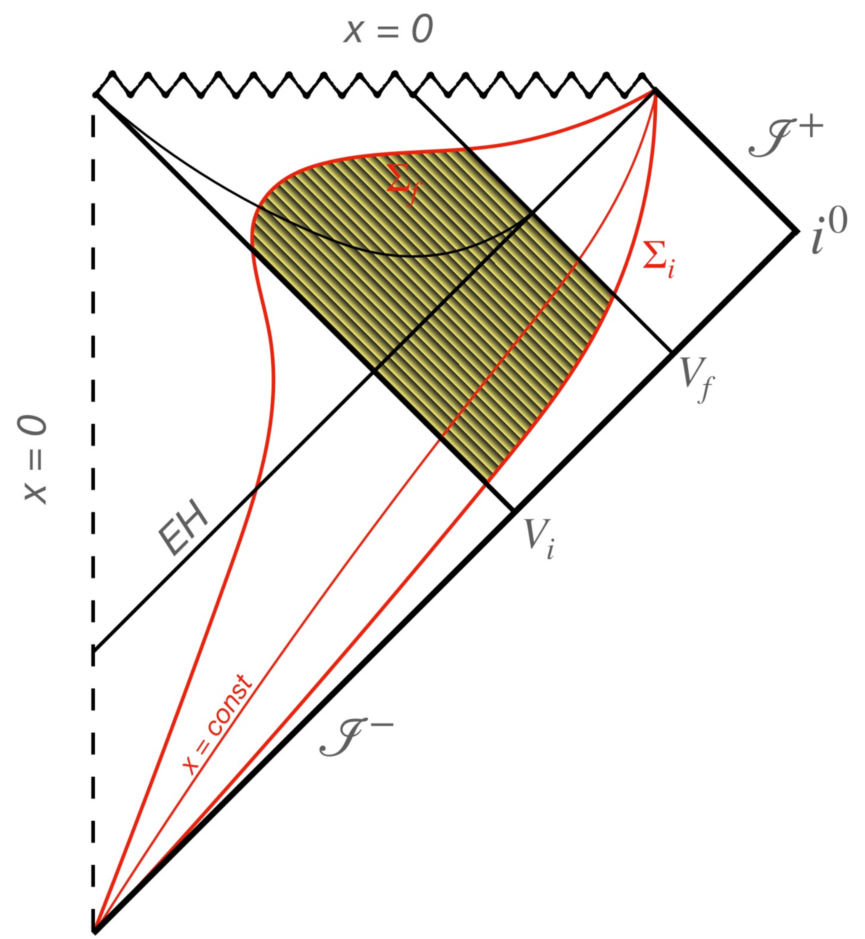
  - Charges on a slice at constant radius
  - Decoupled mechanical models



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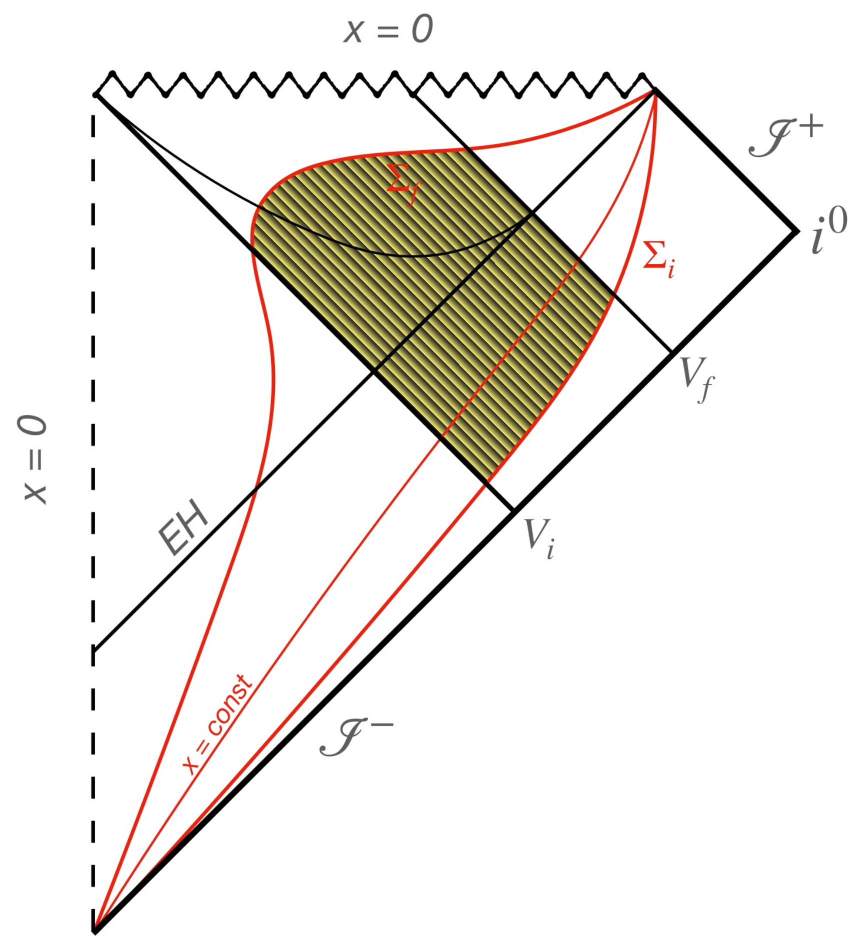
Different phase space, same solutions



Conclusions & Outlooks

Dynamical symmetries

Minisuperspaces:

- "Second geometrization" and symmetries
- Hydrodynamics and quantization

Beyond minisuperspace:

- Complementarity with corner algebra

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Future perspectives

- Extend to other midisuperspaces (e.g. Kerr)
- Quantization of Vaidya

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Thank you for your attention !