Model metrics for quantum black hole evolution: Gravitational collapse, singularity resolution, and transient horizons

Samantha Hergott, Viqar Hussain, Saeed Rastgoo

Samantha Hergott

PERIMETER INSTITUTE
UNIVERSITY OF ALBERTA
YORK UNIVERSITÉ UNIVERSITY

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Overview

- Introduce a class of metrics that:
  - Model gravitational collapse
  - Have the quantum gravity feature of singularity resolution
  - Have a matter bounce like in many LQG theories
- Study horizon formation and properties
- Calculate BH lifetime

Motivation

- Spacetime itself ceases to exist at singularities
- Singularity theorems predict all (physical) spacetimes are singular

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- A quantum gravity model is expected to have singularity resolution
- Design a spacetime metric that has these features built into it

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The Metric

- Start with a generic spherically symmetric metric

$$ds^2 = -N^2(r,t)dt^2 + f^2(r,t)\left(dr + N^r(r,t)dt\right)^2 + g^2(r,t)d\Omega^2$$

- Use Painleve-Gullstrand coordinates of a freely falling observer starting from rest at $r = \infty$ by setting

$$N(r,t) = 1 \qquad f(r,t) = 1$$

$$g(r,t) = r \qquad N^r(r,t) = \sqrt{\frac{2GM(r,t)}{r}}$$

the metric is then

$$ds^2 = -\left(1 - \frac{2GM(r,t)}{r}\right)dt^2 + 2\sqrt{\frac{2GM(r,t)}{r}}dtdr + dr^2 + r^2 d\Omega^2$$

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The Mass

- M(r,t) is the mass contained within radius r at time t and related to density via

$$M(r,t) = 4\pi \int_0^r r^2 \rho(r,t) dr$$

- Ensure

$$\lim_{r\to\infty} M(r,t) = M_{ADM}$$

- Ensure $\rho(r,t)$ is bounded as $r \to 0$
- To model the collapse and the bounce, $M(r,t)$ should initially move toward $r = 0$, reverse direction at some small r, and then move outward

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The Mass

- The proposed mass is

$$M(r,t) = M_0 \left[1 + \tanh\left(\frac{r - r_0 - v(r,t)t}{l_0}\right)\right]^a \tanh\left(\frac{r^b}{l_0^b}\right)$$

- a and b are chosen such that

$$\rho(r,t) = \frac{1}{4\pi r^3}\frac{\partial M}{\partial r}$$

  is bounded as $r \to 0$
- $v(r,t)$ is a time-dependent velocity of the center of the collapsing shell given by

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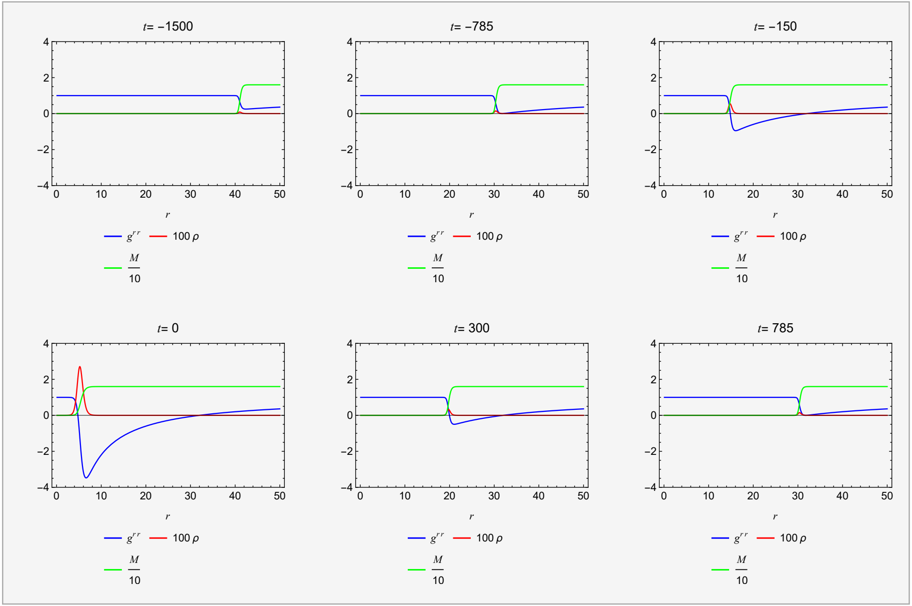
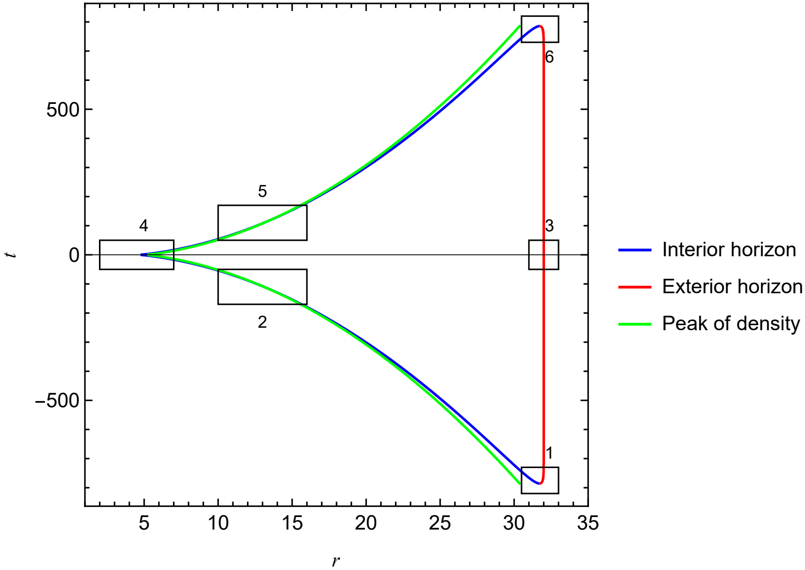
Horizons

- Look for horizon(s) by finding the roots of g^{rr}

$$\Theta(r,t) = g^{rr} = 1 - \frac{2GM(r,t)}{r} = 0$$

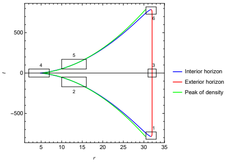

t= −1500
t= −785
t= −150
t= 0
t= 300
t= 785
r
g^{rr}
100 ρ
M/10

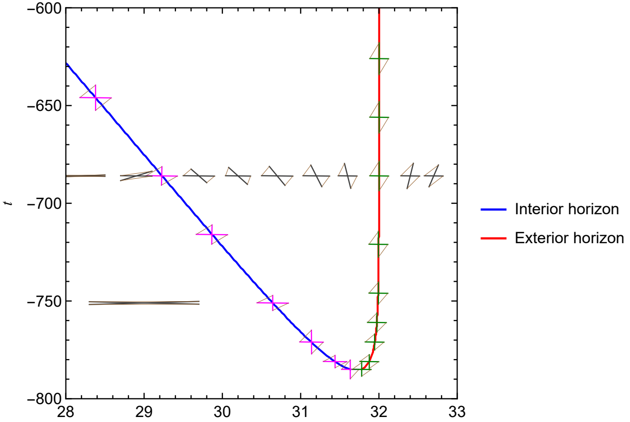
Horizons



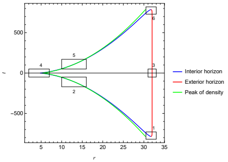
500
0
−500
t
5
10
15
20
25
30
35
r
1
2
3
4
5
6
Interior horizon
Exterior horizon
Peak of density

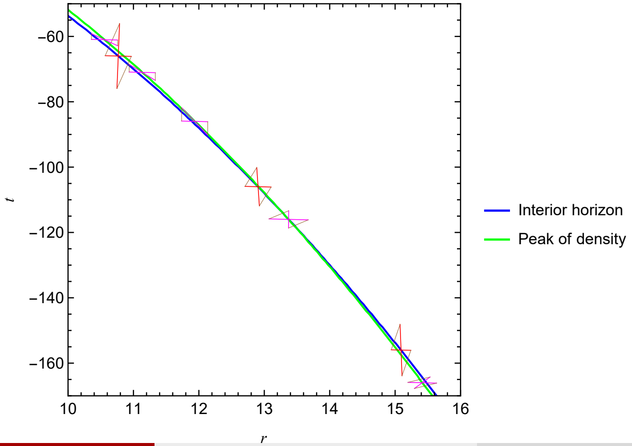
Causal Structure- Region 1



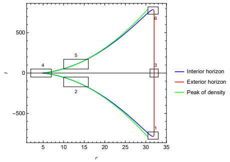


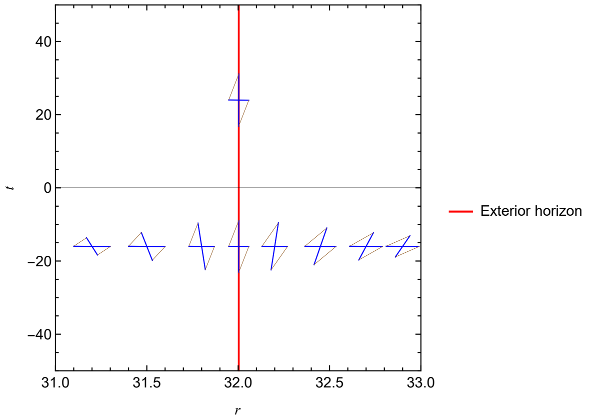
Causal Structure- Region 2



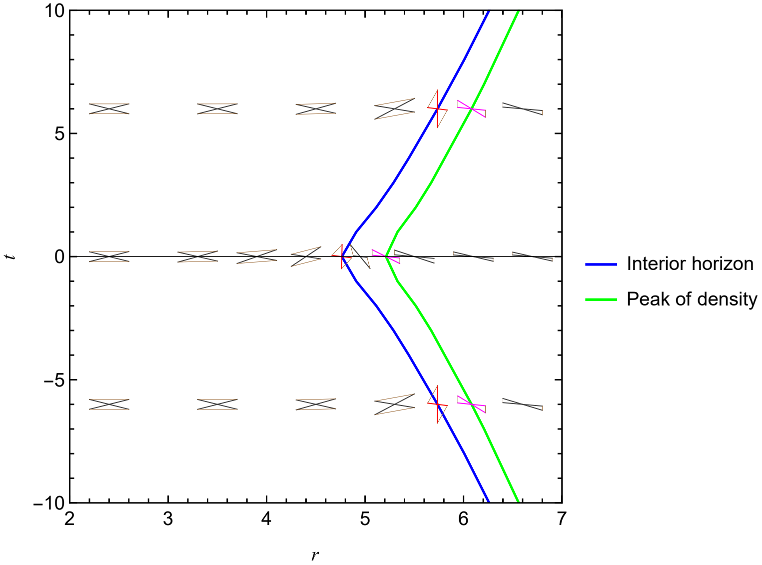


Causal Structure- Region 3

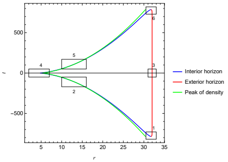


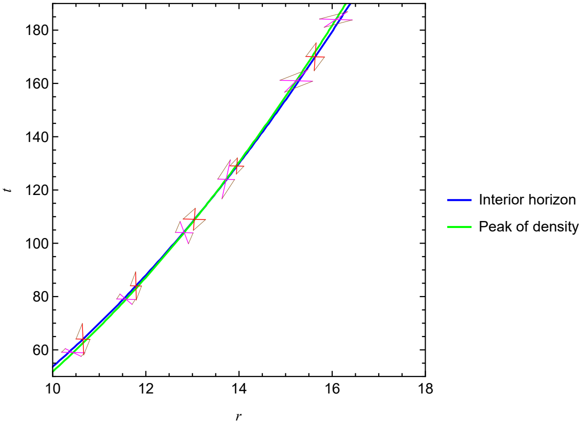


Causal Structure- Region 4

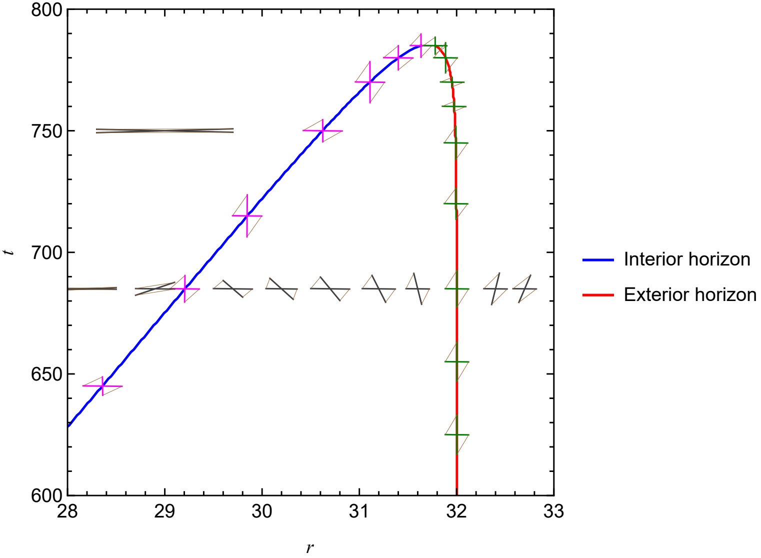


Causal Structure- Region 5

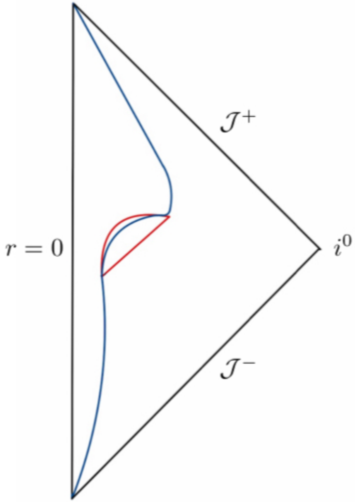




Causal Structure- Region 6



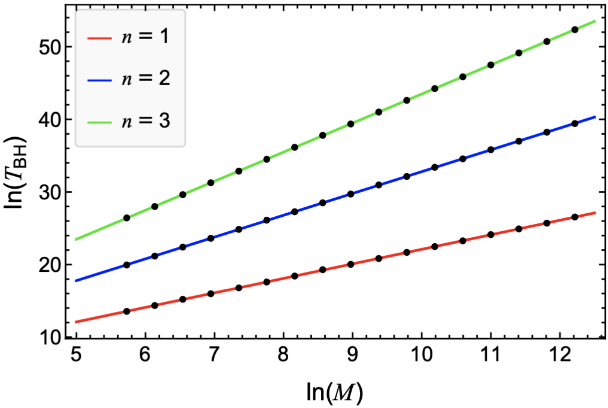
Conformal Diagram



Black Hole Lifetime

- Interval between formation and annihilation $T_{BH} = t_f - t_i$
- Recall

$$v(r,t) = \frac{A}{(1+r)^n} \tanh\left(\frac{t-t_0}{l_0}\right)$$



Black Hole Lifetime

- Find the relationship

$$T_{BH} \approx 2^{n+2} M_{ADM}^{n+1}$$

- $n = 1$, $T_{BH} \propto M^2$ which is that of gravitational dust collapse proposed by Husain et al. (arXiv:2109.08667)
- $n = 2$, $T_{BH} \propto M^3$ which is that of Hawking evaporation time

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Conclusion

- Designed a spacetime metrics that
  - Models gravitational matter collapse
  - Resolves the singularity as expected from a theory of quantum gravity
  - Exhibits horizon formation and evaporation with a temporary event horizon
  - Provides parameter dependent formula for black hole lifetime

THANK YOU