Nonsingular Spherical BHs with Holonomy corrections

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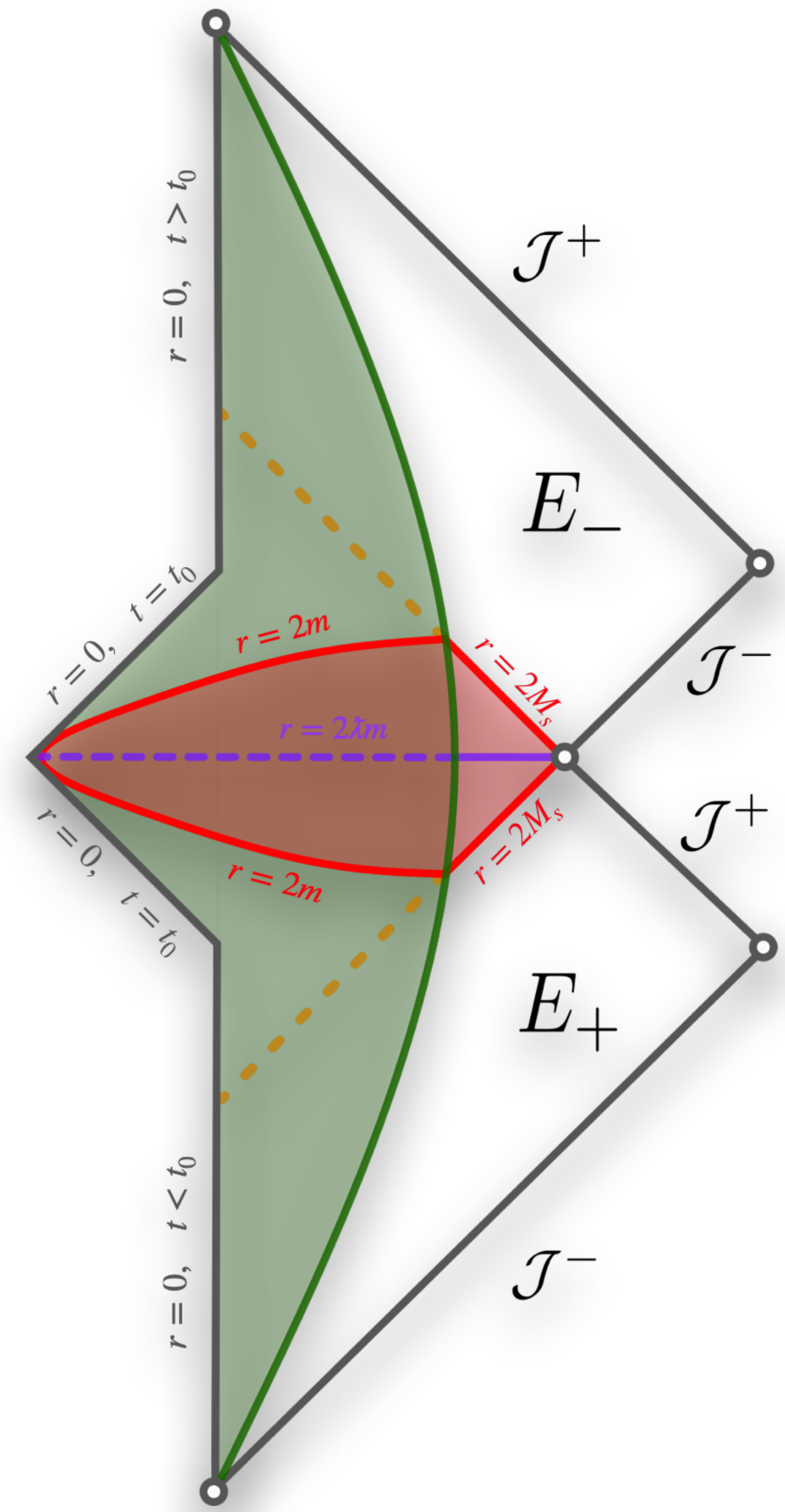
Outline

Modified Hamiltonians: The issue of covariance

Nonsingular static BHs (M, Q, Λ)

Coupling matter: The issue of covariance *again*

Dynamical BHs: Gravitational collapse



Modified Hamiltonians

Our goal:

$$H_T = \int (N\mathcal{H} + N^x \mathcal{D}) dx,$$

i. Gauge transformations on phase space = coordinate transformations

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Assumptions:

ii. Lapse and Shift are defined as in GR

iii. Corrections preserve the area of the *SO(3)* orbits

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Hamiltonian ⇔ Metric

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$$H_T = \int (N\mathcal{H} + N^x\mathcal{D})dx,$$

iv. Derivatives: linear in second order and quadratic in first order derivatives of momenta

v. There exists a continuous limit to GR

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$$H_T = \int (N\mathcal{H} + N^x\mathcal{D})dx,$$

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Closure of the hypersurface deformation algebra

$$\{D[s_1], D[s_2]\} = D\left[s_1 s_2' - s_1' s_2\right],$$
$$\{D[s_1], H[s_2]\} = H\left[s_1 s_2'\right],$$
$$\{H[s_1], H[s_2]\} = D\left[F(s_1 s_2' - s_1' s_2)\right],$$

and $F = q^{xx}$

Spacetime embeddability

Modified Hamiltonians

Hamiltonian ⇔ Metric

No propagating degrees of freedom

$$H_T = \int (N\mathcal{H} + N^x\mathcal{D})dx,$$

Two pairs of conjugate variables $\{q_i(x_1), p_j(x_2)\} = \delta_i^j \delta(x_1 - x_2)$

$q_1 = K_x \quad p_1 = E^x$

$q_2 = K_\varphi \quad p_2 = E^\varphi$

$\mathcal{D} = -q_1 p_1' + q_2' p_2$ *classical diffeomorphism constraint*

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Six free functions of p_1 only $\mathfrak{g},\ V,\ A,\ W,\ \omega,$ and φ

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$$\mathcal{H} = \mathfrak{g}\left[p_2 V - p_2 \frac{A}{\omega^2}\sin^2(\omega q_2)\frac{\partial}{\partial p_1}\left[\log\left(\frac{A\,W}{\omega^2}\right)\right] + \frac{1}{2}\left(\frac{p_1''}{p_2} - \frac{p_1' p_2'}{p_2^2} + \frac{\partial \log(W)}{\partial p_1}\frac{(p_1')^2}{2p_2}\right)\cos^2(\omega q_2 + \varphi)\right.$$
$$\left. - \left(q_1 + q_2 p_2 \frac{\partial \log(\omega)}{\partial p_1}\right)\frac{A}{\omega}\sin(2\omega q_2) - \left(\omega q_1 + q_2 p_2 \frac{\partial \omega}{\partial p_1} + p_2 \frac{\partial \varphi}{\partial p_1}\right)\left(\frac{p_1'}{2p_2}\right)^2 \sin(2(\omega q_2 + \varphi))\right],$$

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Trigonometric functions ⇔ covariance

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Six free functions of p_1 only $\mathfrak{g},\ V,\ A,\ W,\ \omega,$ and φ

Holonomy corrections appear naturally!

[A-B, Brizuela (2024)]

$$\mathcal{D} = -q_1 p_1' + q_2' p_2$$ *classical diffeomorphism constraint*

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Modified Hamiltonians

Hamiltonian ⇔ Metric

Construction of the model such that it obeys the *hypersurface deformation algebra* with a suitable structure function to embed it in (M, g)

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$$\{H[s_1], H[s_2]\} = D[F(s_1 s_2' - s_1' s_2)],$$

$$F = \frac{F_s}{(E^\varphi)^2} \quad \text{with}$$

$$F_s := \mathfrak{g}^2 \cos(\omega K_\varphi + \varphi) \left(A \cos(\omega K_\varphi - \varphi) + \left(\frac{E^{x\prime}}{2E^\varphi} \right)^2 \omega^2 \cos(\omega K_\varphi + \varphi) \right)$$

Modified Hamiltonians

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$$ds^2 = \sigma N^2 dt^2 + \frac{1}{|F|}(dx + N^x dt)^2 + r^2 d\Omega^2,$$

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Generic features

Curvature in terms of ∇r, $\nabla^2 r$, and ${}^{(2)}R$

Existence of a KVF ξ orthogonal to ∇r [A-B, Brizuela (2024)]

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Special surfaces

Vanishing of $g(\xi,\xi)$, $g(\nabla r, \nabla r)$, and F_s

Modified Hamiltonians

Hamiltonian ⇔ Metric

Special cases

Trigonometric functions ⇔ covariance

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$$\mathcal{H} = \mathfrak{g}\Bigg[p_2 V - p_2 \frac{A}{\omega^2} \sin^2(\omega q_2) \frac{\partial}{\partial p_1}\left[\log\left(\frac{A\,W}{\omega^2}\right)\right] + \frac{1}{2}\left(\frac{p_1''}{p_2} - \frac{p_1' p_2'}{p_2^2} + \frac{\partial \log(W)}{\partial p_1}\frac{(p_1')^2}{2p_2}\right)\cos^2(\omega q_2 + \varphi)$$

$$- \left(q_1 + q_2 p_2 \frac{\partial \log(\omega)}{\partial p_1}\right)\frac{A}{\omega}\sin(2\omega q_2) - \left(\omega q_1 + q_2 p_2 \frac{\partial \omega}{\partial p_1} + p_2 \frac{\partial \varphi}{\partial p_1}\right)\left(\frac{p_1'}{2p_2}\right)^2 \sin(2(\omega q_2 + \varphi))\Bigg],$$

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i. Constant ω: the Hamiltonian is bounded in q_2

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Trigonometric functions ⇔ covariance

i. Constant ω: the Hamiltonian is bounded in q_2

ii. $\varphi = 0$ and $A \geq 0$: spacetime is Lorentzian (no signature change)

Holonomy corrections appear naturally!

$$\mathcal{H} = \mathfrak{g}\Bigg[p_2 V - p_2 \frac{A}{\omega^2}\sin^2(\omega q_2)\frac{\partial}{\partial p_1}\left[\log\left(\frac{AW}{\omega^2}\right)\right] + \frac{1}{2}\left(\frac{p_1''}{p_2} - \frac{p_1' p_2'}{p_2^2} + \frac{\partial \log(W)}{\partial p_1}\frac{(p_1')^2}{2p_2}\right)\cos^2(\omega q_2 + \varphi)$$

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iii. $W = c\sqrt{p_1}$ and $V = -1/(2p_1)$: $\exists\, m : m' \approx 0$

$$\mathcal{H} = \mathfrak{g}\left[p_2 V - p_2 \frac{A}{\omega^2} \sin^2(\omega q_2) \frac{\partial}{\partial p_1}\left[\log\left(\frac{AW}{\omega^2}\right)\right] + \frac{1}{2}\left(\frac{p_1''}{p_2} - \frac{p_1' p_2'}{p_2^2} + \frac{\partial \log(W)}{\partial p_1}\frac{(p_1')^2}{2p_2}\right)\cos^2(\omega q_2 + \varphi)\right.$$

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Is this too restrictive? **NO**

GR satisfies all these conditions… and more

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We find a *one-parameter* family of theories with $\omega = \lambda \in \mathbb{R}$,
such that GR is $\lambda = 0$. This is the effective covariant polymerization!

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$$\mathcal{H} = -\frac{E^\varphi}{2\sqrt{E^x}\sqrt{1+\lambda^2}}\left(1+\frac{\sin^2(\lambda K_\varphi)}{\lambda^2}\right) - \sqrt{E^x}K_x\frac{\sin(2\lambda K_\varphi)}{\lambda\sqrt{1+\lambda^2}}\left(1+\left(\frac{\lambda E^{x\prime}}{2E^\varphi}\right)^2\right)$$
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[A-B, Brizuela, Vera (2022)]

Nonsingular BHs: **vacuum**

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$$ds^2 = -\left(1 - \frac{2m}{\tilde{r}}\right)d\tilde{t}^2 + \left(1 - \frac{r_0}{\tilde{r}}\right)^{-1}\left(1 - \frac{2m}{\tilde{r}}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2 d\Omega^2,$$

$$r_0 := 2m\frac{\lambda^2}{1+\lambda^2}$$

$$\mathcal{H} = -\frac{E^\varphi}{2\sqrt{E^x}\sqrt{1+\lambda^2}}\left(1 + \frac{\sin^2(\lambda K_\varphi)}{\lambda^2}\right) - \sqrt{E^x}K_x\frac{\sin(2\lambda K_\varphi)}{\lambda\sqrt{1+\lambda^2}}\left(1 + \left(\frac{\lambda E^{x\prime}}{2E^\varphi}\right)^2\right) + \frac{\cos^2(\lambda K_\varphi)}{2\sqrt{1+\lambda^2}}\left(\frac{E^{x\prime}}{2E^\varphi}\left(\sqrt{E^x}\right)' + \sqrt{E^x}\left(\frac{E^{x\prime}}{E^\varphi}\right)'\right),$$

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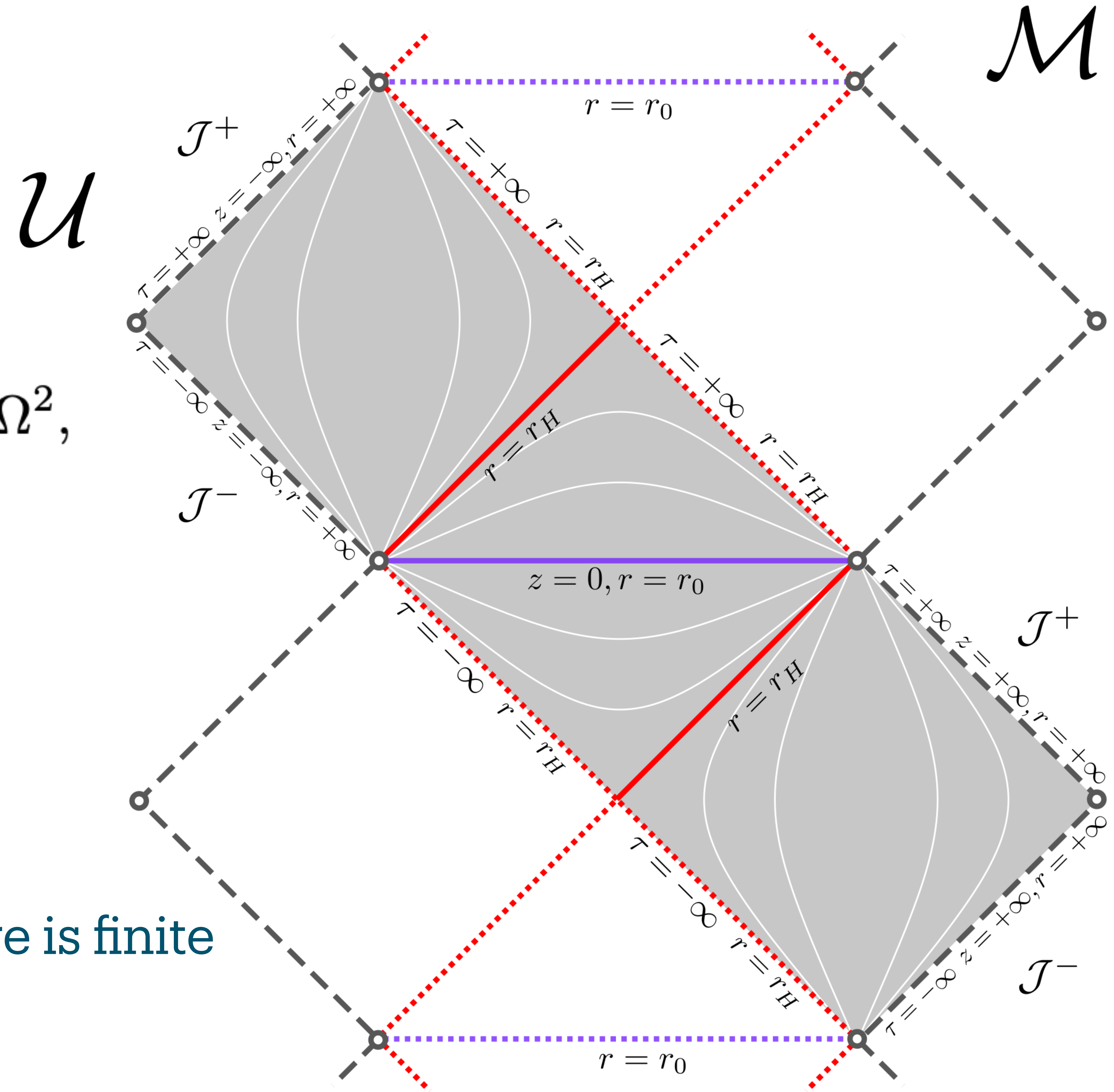

The singularity is replaced with a transition surface $r = r_0$ of positive radius towards a time-reversed (WH) region

The surface is always inside the *trapped* region and curvature is finite

The spacetime is geodesically complete

[A-B, Brizuela, Vera (2022)]

Nonsingular BHs: **Q** and **Λ**

$$\mathcal{H} = -\frac{E^\varphi}{2\sqrt{E^x}\sqrt{1+\lambda^2}}\left(1+\frac{\sin^2(\lambda K_\varphi)}{\lambda^2}\right) - \sqrt{E^x}K_x\frac{\sin(2\lambda K_\varphi)}{\lambda\sqrt{1+\lambda^2}}\left(1+\left(\frac{\lambda E^{x\prime}}{2E^\varphi}\right)^2\right)$$
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Nonsingular BHs: **Q** and **Λ**

$$ds^2 = -\left(1 - \frac{2m(r)}{r}\right) d\tilde{t}^2 + r^2 d\Omega^2$$

$$+ \left(1 - \frac{2\bar{\lambda} m(r)}{r}\right)^{-1} \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2$$

$$\boxed{\bar{\lambda} := \frac{\lambda^2}{1+\lambda^2}}$$

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Nonsingular BHs: **Q and Λ**

$$ds^2 = -\left(1-\frac{2m(r)}{r}\right)d\tilde{t}^2 + r^2 d\Omega^2 + \left(1-\frac{2\bar{\lambda}m(r)}{r}\right)^{-1}\left(1-\frac{2m(r)}{r}\right)^{-1}dr^2$$

$$m \to m(r) = M - \frac{Q^2}{2r^2} + \frac{\Lambda}{6}r^3$$

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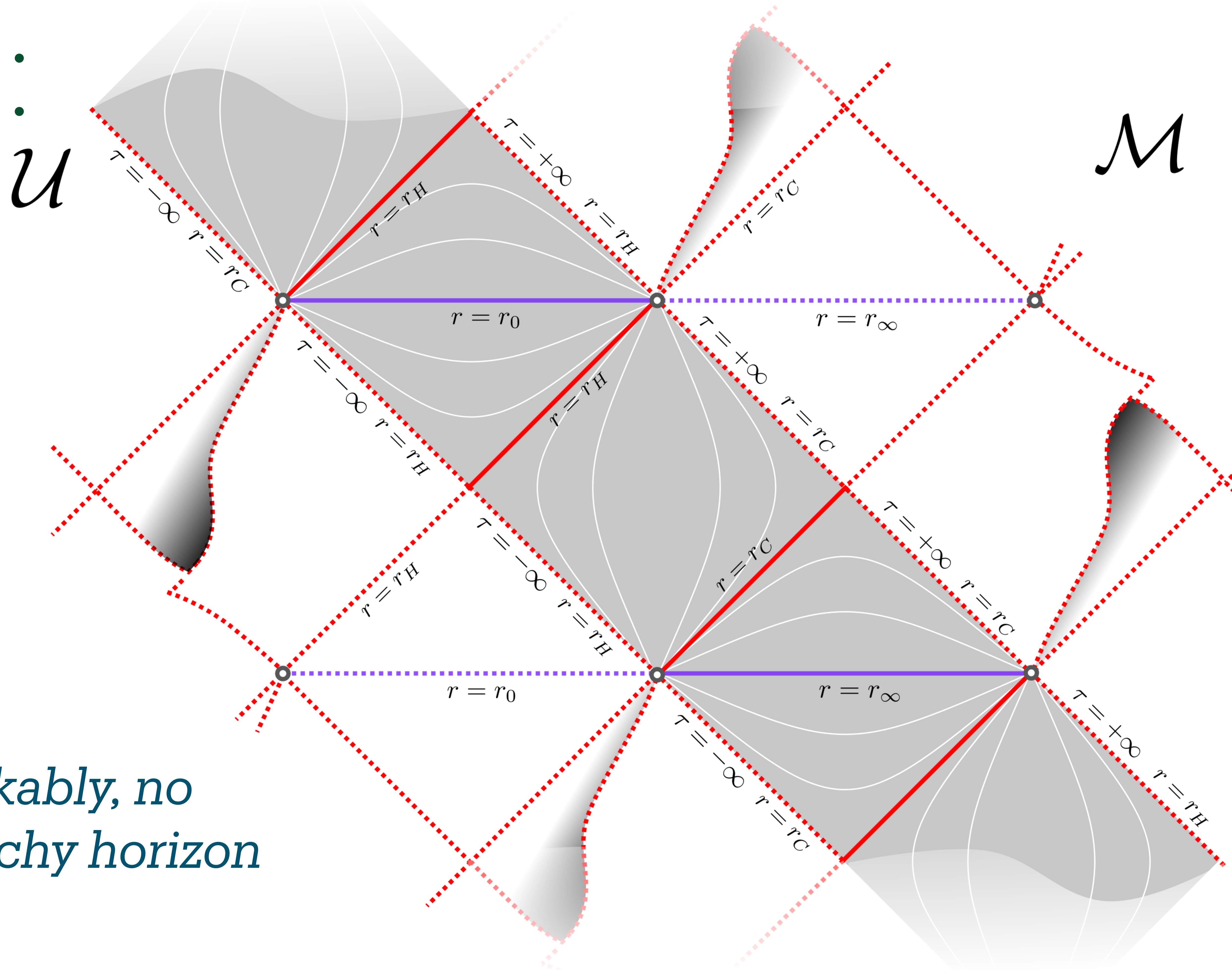
Remarkably, no inner Cauchy horizon



Singularity resolution in a nutshell: positive **M**, small **Q**, and positive **Λ**

[A-B, Brizuela, Vera (2023)]

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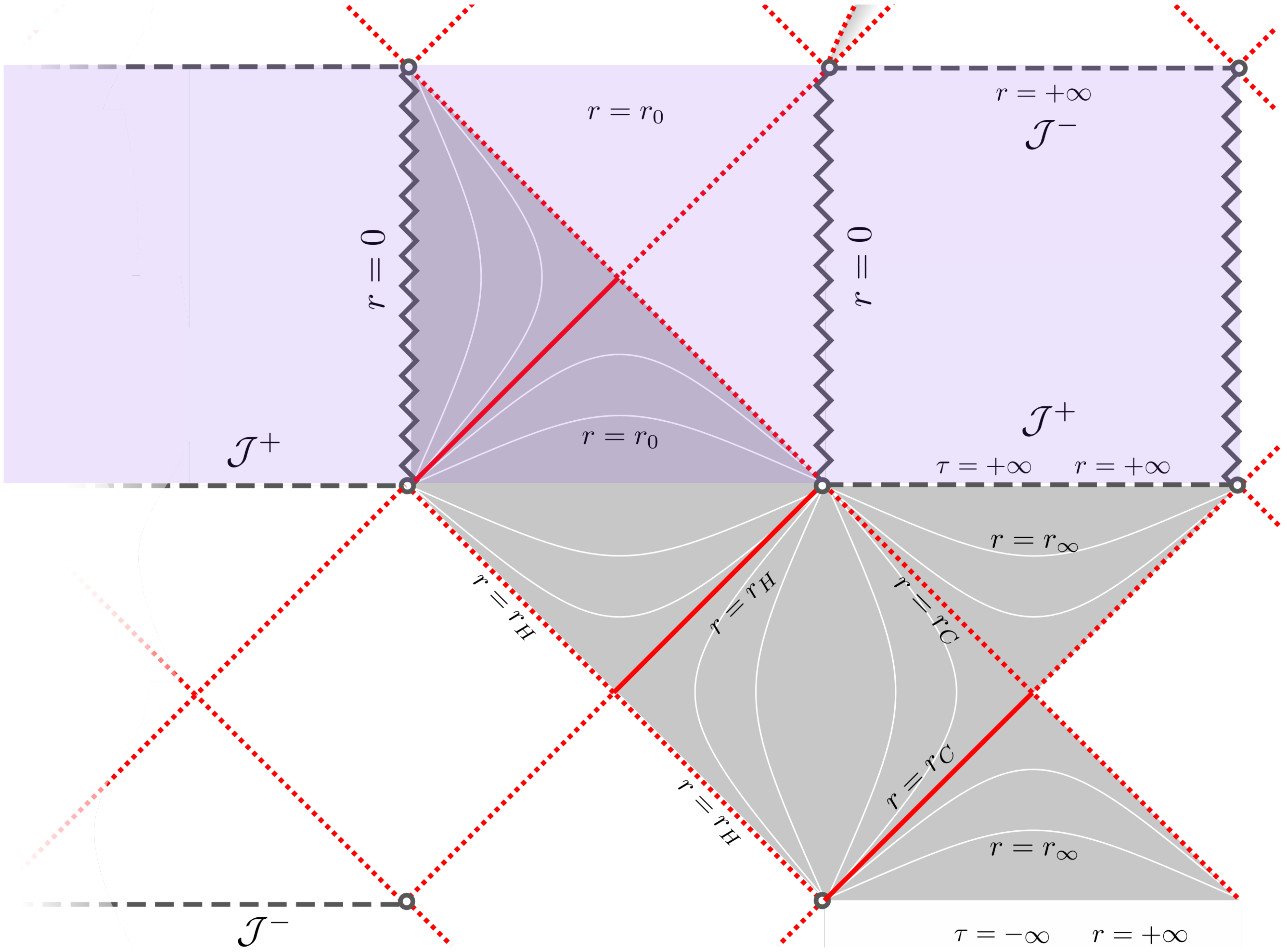

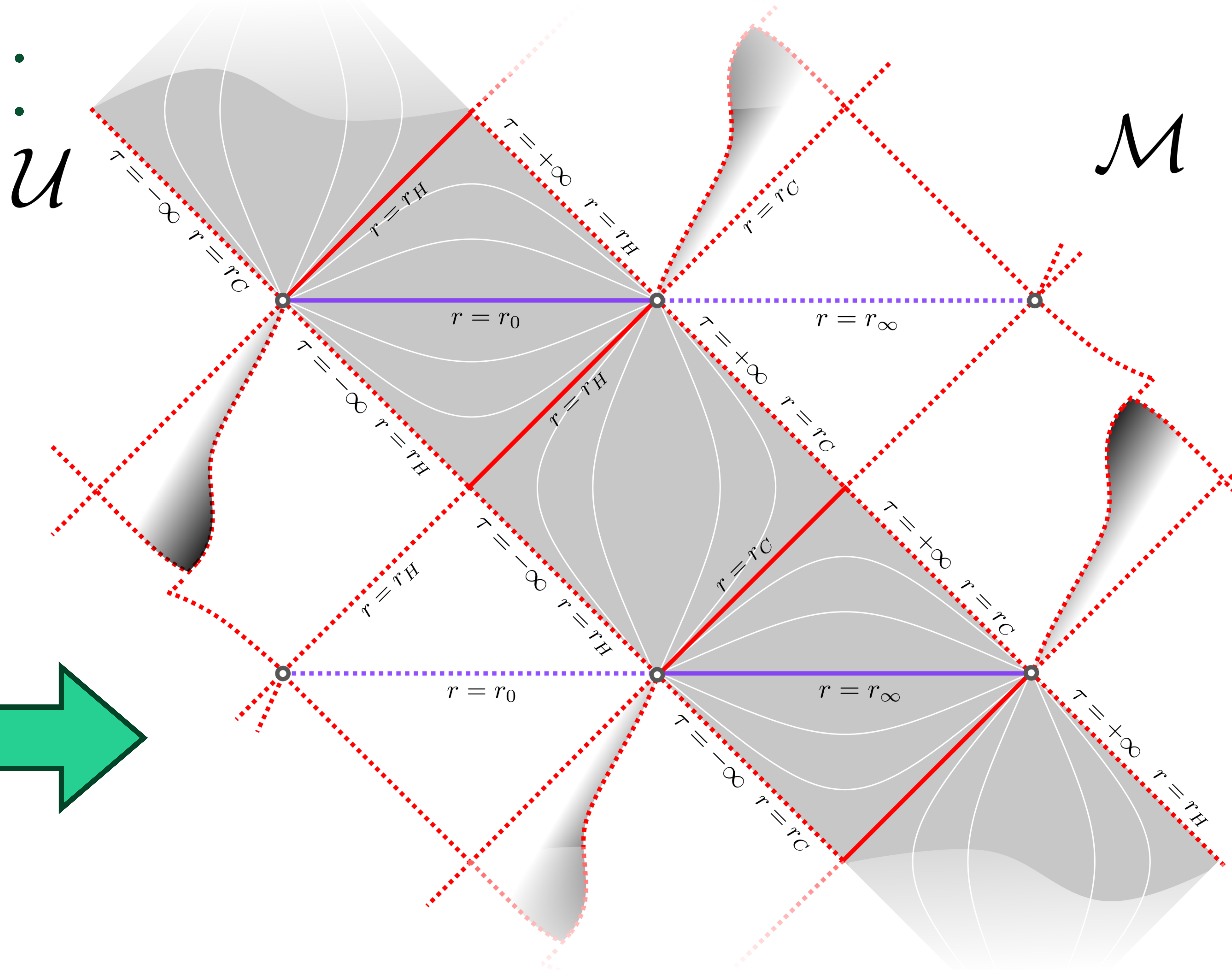

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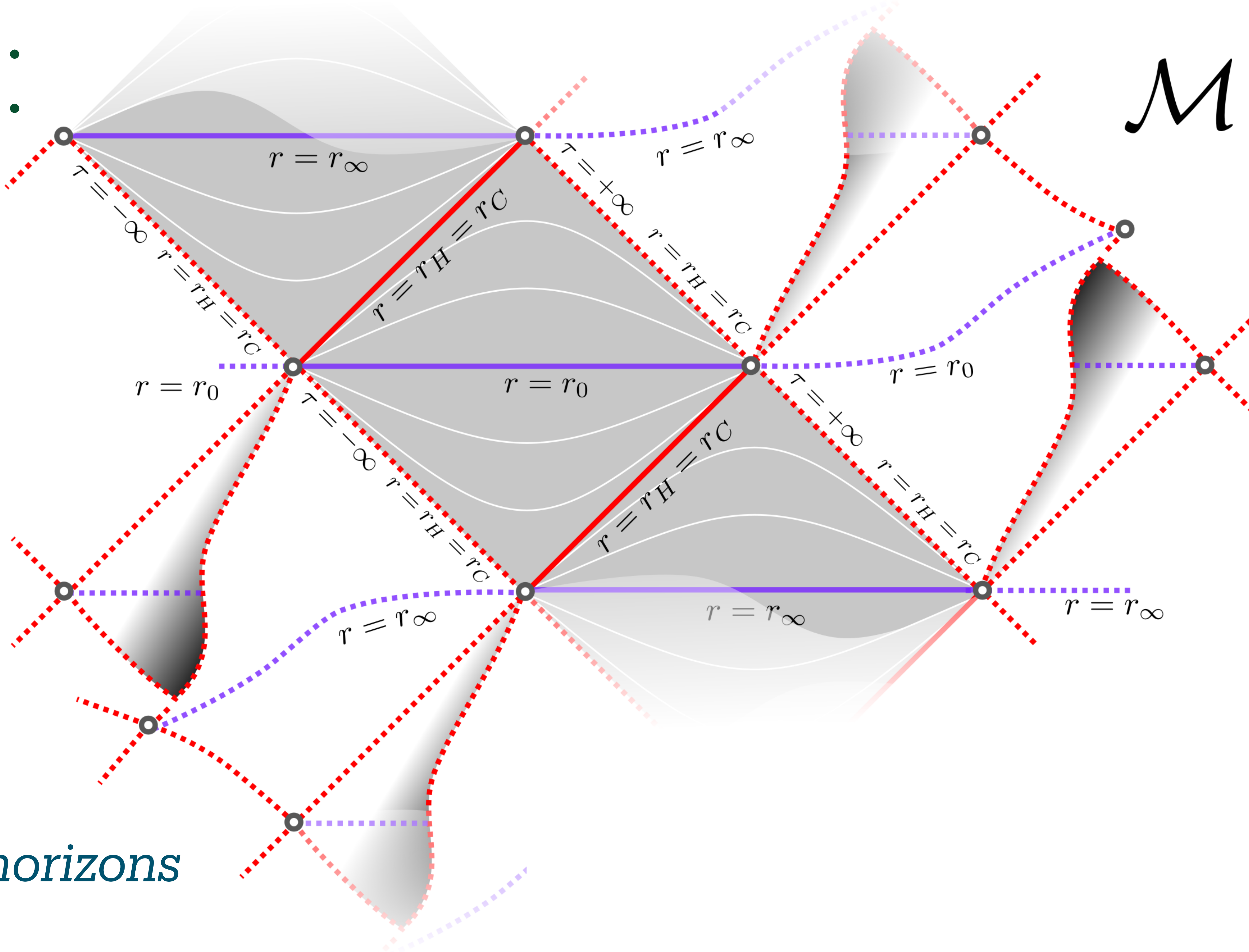
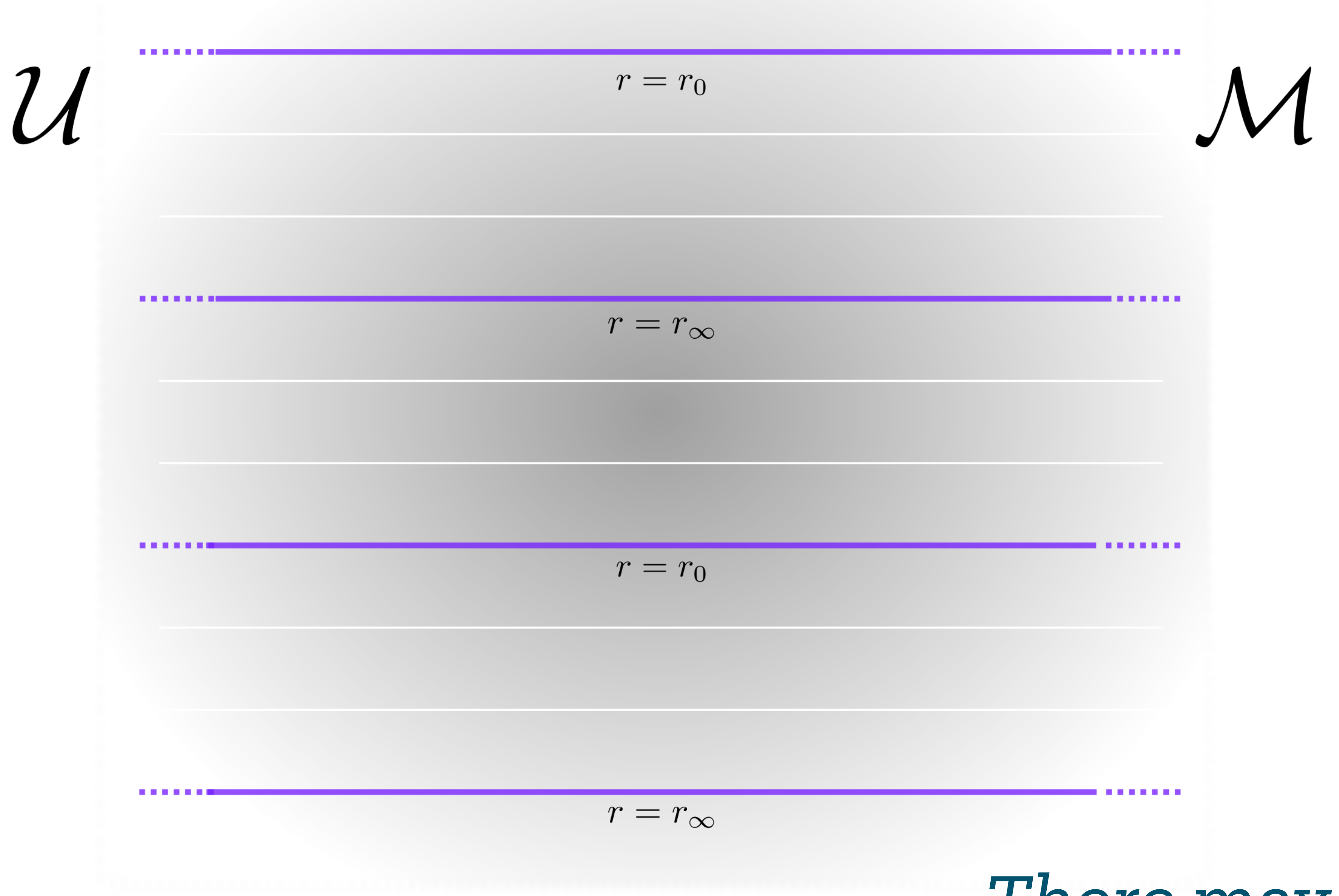
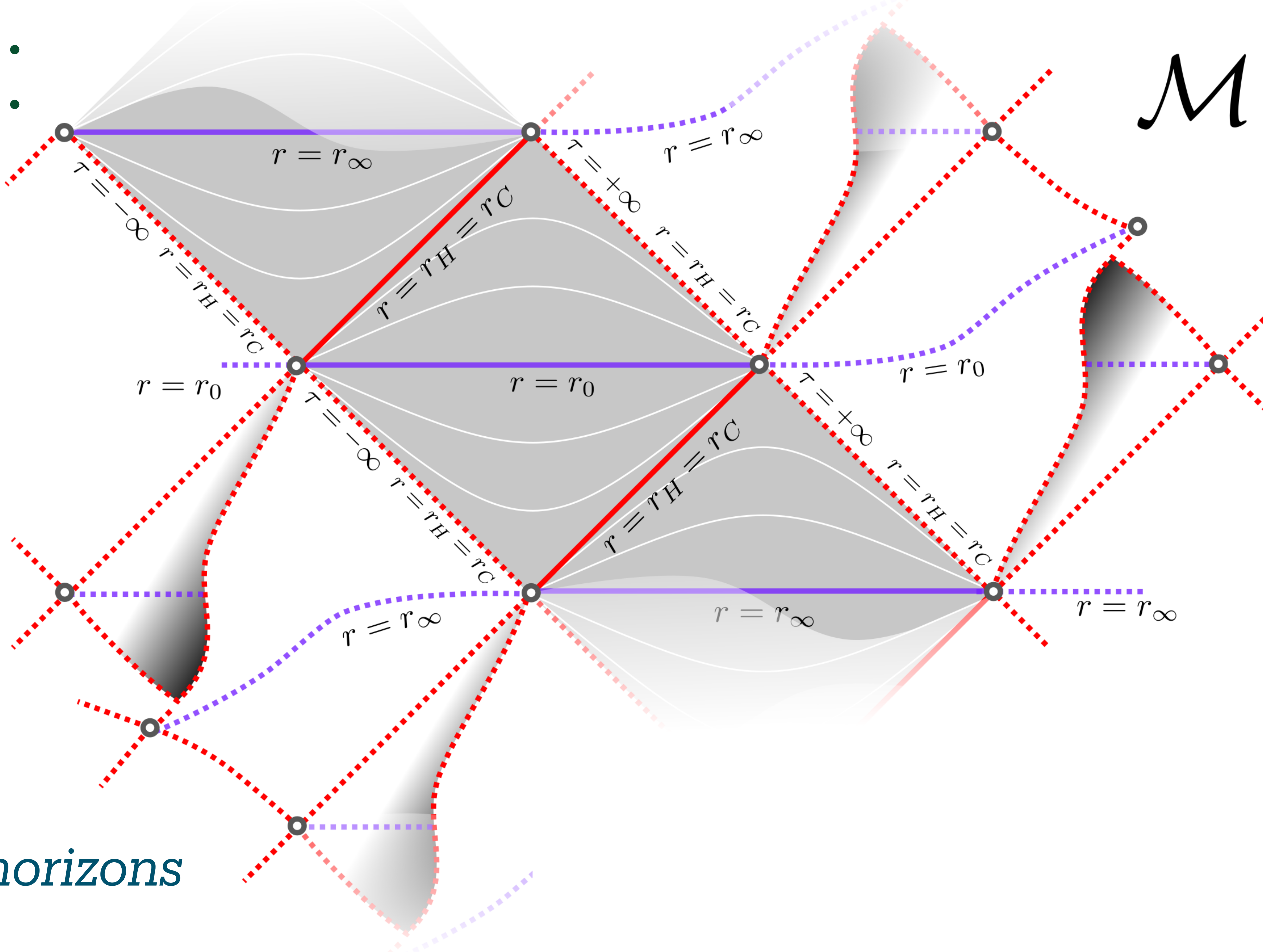


There may be 2, 1, or 0 horizons

Singularity resolution in a nutshell: positive **M**, small **Q**, and positive **Λ** [A-B, Brizuela, Vera (2023)]

Nonsingular BHs:





There may be 2, 1, or 0 horizons

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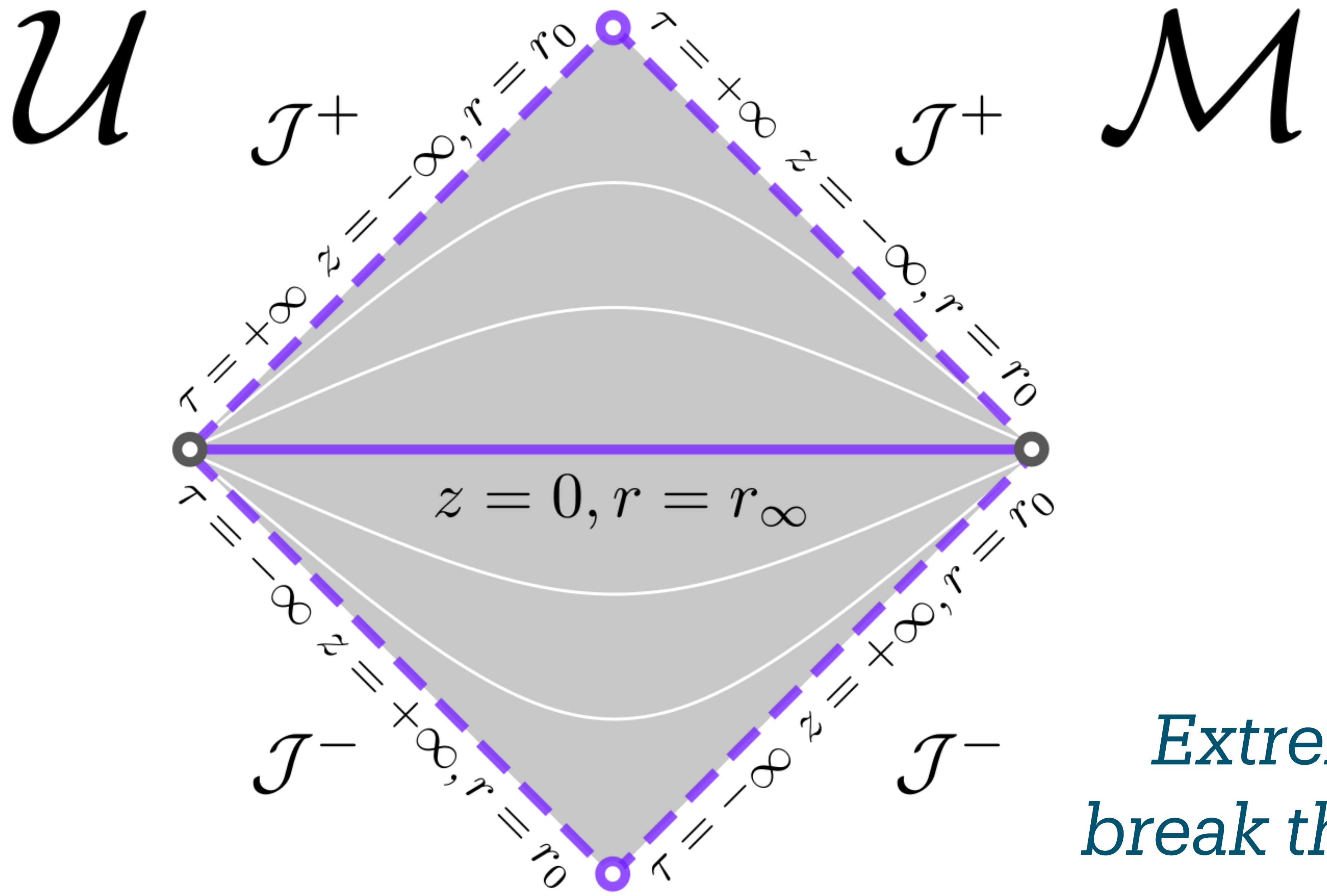

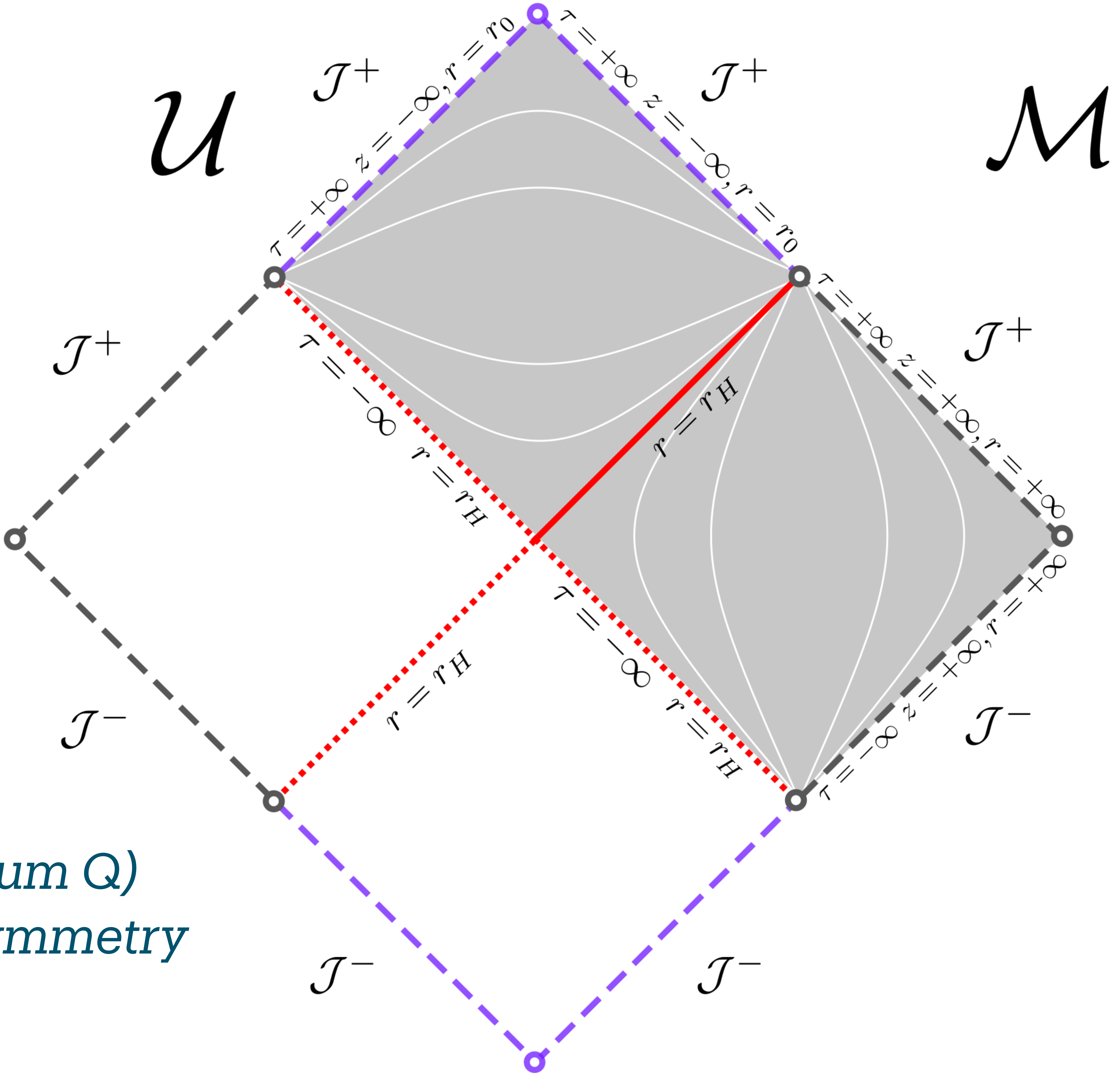

Extremal cases (maximum Q)
break the time-reversal symmetry

Singularity resolution in a nutshell: positive **M**, small **Q**, and positive **Λ**

[A-B, Brizuela, Vera (2023)]

Dynamical BHs: **dust**

$$ds^2 = \sigma N^2 dt^2 + \frac{1}{|F|}(dx + N^x dt)^2 + r^2 d\Omega^2,$$

$$\{D[s_1], D[s_2]\} = D[s_1 s_2' - s_1' s_2],$$
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$$\mathcal{H} = -\frac{E^\varphi}{2\sqrt{E^x}\sqrt{1+\lambda^2}}\left(1 + \frac{\sin^2(\lambda K_\varphi)}{\lambda^2}\right) - \sqrt{E^x} K_x \frac{\sin(2\lambda K_\varphi)}{\lambda\sqrt{1+\lambda^2}}\left(1 + \left(\frac{\lambda E^{x\prime}}{2E^\varphi}\right)^2\right)$$
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[A-B, Brizuela (2024)]

Minimal coupling

$$\mathcal{H} = -\frac{E^\varphi}{2\sqrt{E^x}\sqrt{1+\lambda^2}}\left(1 + \frac{\sin^2(\lambda K_\varphi)}{\lambda^2}\right) - \sqrt{E^x} K_x \frac{\sin(2\lambda K_\varphi)}{\lambda\sqrt{1+\lambda^2}}\left(1 + \left(\frac{\lambda E^{x\prime}}{2E^\varphi}\right)^2\right)$$
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Dynamical BHs: **dust**

To solve Hamiltonian equations: same procedure as in GR

1. Choose the dust frame: $\phi = t$
2. Conservation of gauge: $N = 1$
3. Dust momentum: $P_\phi = E$
4. New variables: $(E^x, E^\varphi, K_\varphi) \to (r, m, \kappa)$

[Solve diffeomorphism for K_x]

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Minimum for collapsing branch

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Maximum for expanding branch
ONLY when $\kappa < 0$

Minimum for collapsing branch

Periodic oscillations with: $T = 2\pi m(1 + \lambda|\kappa|)|\kappa|^{-3/2}$

Dynamical BHs: **dust**

[A-B, Brizuela (2024)]

Still, one gauge freedom to fix

We choose $m = m(x) \geq 0$

Conservation: $N^x = 0$

$$\dot{r} = N^x r' - \epsilon \sqrt{1 - \frac{2\lambda m}{r}} \sqrt{\kappa + \frac{2m}{r}},$$

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$$t - t_0 = -\epsilon \sqrt{\frac{2r^3}{9m}} \sqrt{1 - \frac{2\lambda m}{r}} \left(1 + \frac{4\lambda m}{r}\right),$$

$$t - t_0 = -\epsilon \frac{r}{\kappa} \sqrt{1 - \frac{2\lambda m}{r}} \sqrt{\frac{2m}{r} + \kappa} + \epsilon \frac{2m}{\kappa^{3/2}} (1 - \lambda\kappa) \operatorname{artanh} \sqrt{\frac{\kappa(r - 2\lambda m)}{2m + \kappa r}},$$

$$t - t_0 = +\epsilon \frac{r}{|\kappa|} \sqrt{1 - \frac{2\lambda m}{r}} \sqrt{\frac{2m}{r} - |\kappa|} - \epsilon \frac{2m}{|\kappa|^{3/2}} (1 + \lambda|\kappa|) \arctan \sqrt{\frac{|\kappa|(r - 2\lambda m)}{2m - |\kappa| r}},$$

Maximum for expanding branch ONLY when $\kappa < 0$

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- Trajectories on phase space describe a smooth bounce of dust
- The radius of each dust-shell has a positive infimum, achieved in finite proper time
- However… curvature scalars diverge there. *Singularity resolution is not complete*

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What happens here?

Maximum for expanding branch ONLY when $\kappa < 0$

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Dynamical BHs: **dust**

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- We can build a whole family of regular metrics covariantly associated with the dynamics

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Curvature scalars are bounded for $n \geq 1$

[A-B, Brizuela (2024)]

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Dust is now coupled to a conformal fiducial metric

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Dust is now coupled to a conformal fiducial metric

It is still possible to couple it to the physical metric…

$$\dot{r} = N^x r' - \epsilon\sqrt{1-\frac{2\lambda m}{r}}\sqrt{\kappa+\frac{2m}{r}},$$

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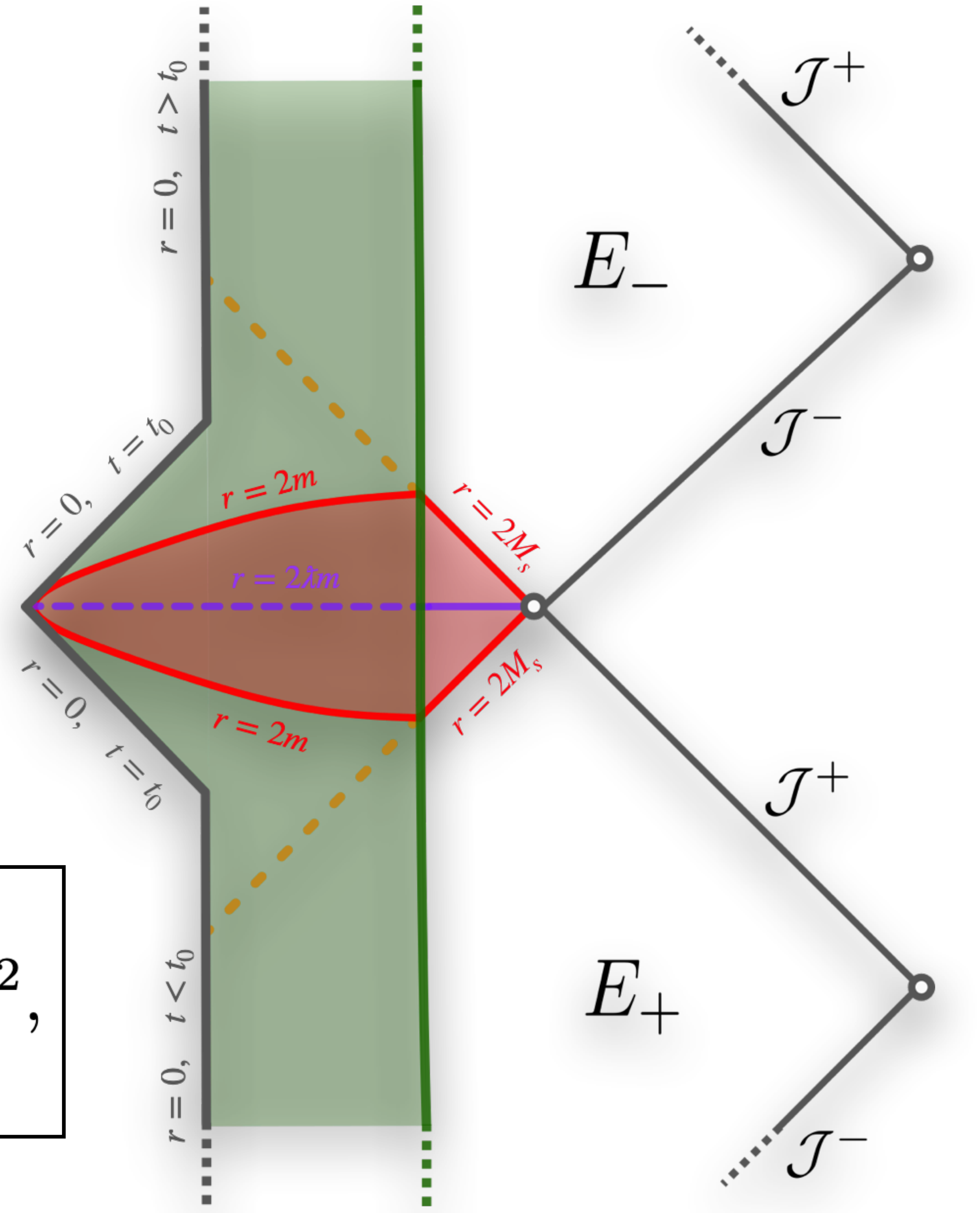


$$d\tilde{s}^2 = -\left(1-\frac{2\lambda m(x)}{r(t,x)}\right)^{-n} dt^2 + \left(1-\frac{2\lambda m(x)}{r(t,x)}\right)^{-(n+1)} \frac{\left(r'(t,x)\right)^2}{1+\kappa(x)} dx^2 + r(t,x)^2 d\sigma^2,$$

$$\dot{r} = -\epsilon\sqrt{1-\frac{2\lambda m}{r}}\sqrt{\kappa+\frac{2m}{r}}, \qquad \mathcal{E} := \sqrt{1-\lambda}\frac{m'}{r^2 r'}\left(1-\frac{2\lambda m}{r}\right)^{\frac{n+1}{2}},$$

Curvature scalars are bounded for $n \geq 1$

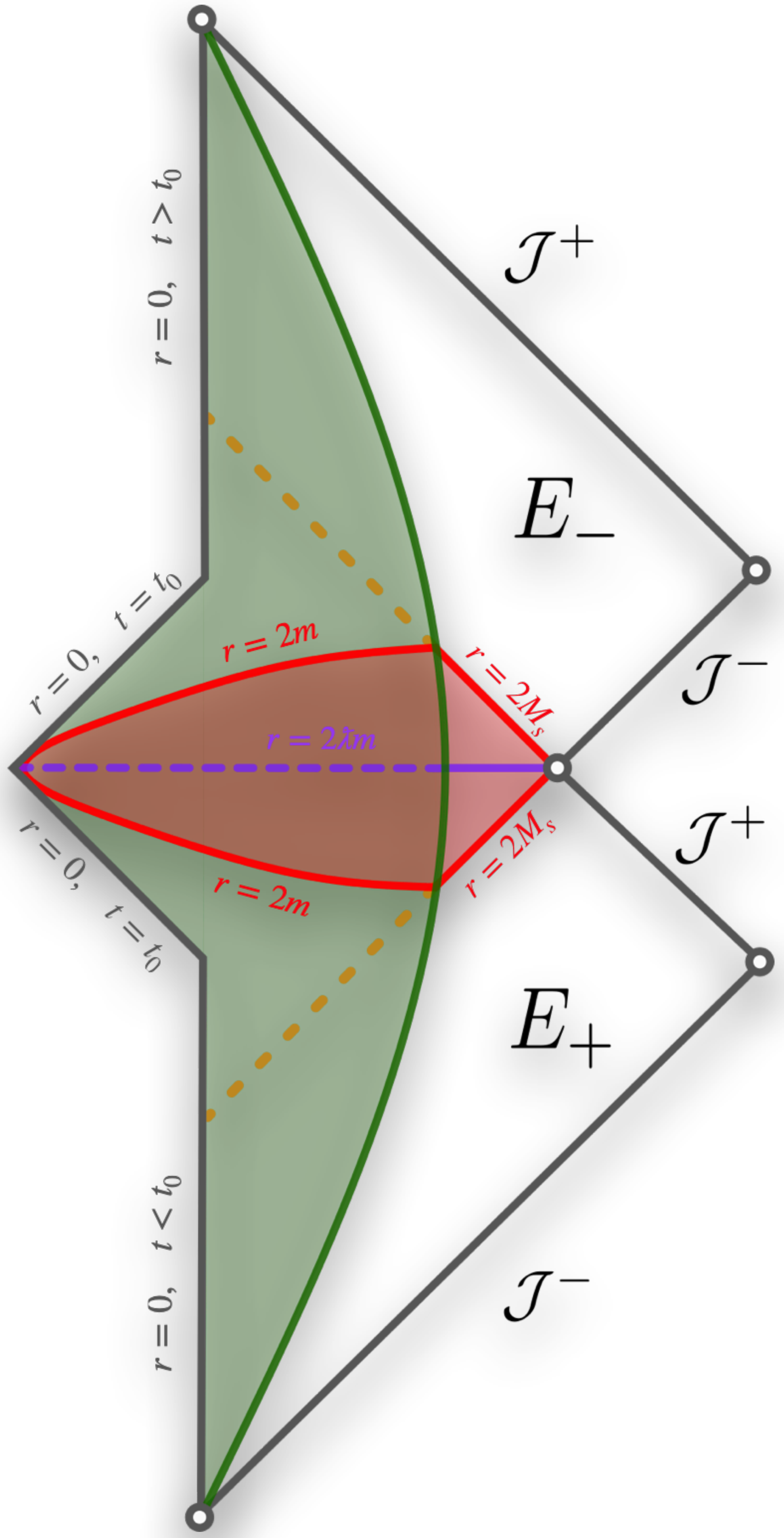
[A-B, Brizuela (2024)]

Summary & Outlook

We found an effective Hamiltonian theory with a well-defined geometric description, and such that GR is a *singular* limit.

Static and dynamical solutions are under control in this effective description. Physically reasonable cases are **free of singularities**,

- Modelling numerical collapse
- Understanding effective corrections from full LQG
- Homogeneous reduction: Is it possible to find FLRW?
- Less symmetric scenarios… effective Kerr BHs



Modified Hamiltonians

Hamiltonian ⇔ Metric

$$H_T = \int (N\mathcal{H} + N^x\mathcal{D})dx,$$

Our goal:

i. Gauge transformations on phase space = coordinate transformations

Assumptions:

ii. Lapse and Shift are defined as in GR

iii. Corrections preserve the area of the *SO(3)* orbits

Modified Hamiltonians

Hamiltonian ⇔ Metric

Further Assumptions:

$$H_T = \int (N\mathcal{H} + N^x \mathcal{D}) dx,$$

iv. Derivatives: linear in second order and quadratic in first order derivatives of momenta

v. There exists a continuous limit to GR

Closure of the hypersurface deformation algebra

$$\begin{aligned}
\{D[s_1], D[s_2]\} &= D\left[s_1 s_2' - s_1' s_2\right], \\
\{D[s_1], H[s_2]\} &= H\left[s_1 s_2'\right], \\
\{H[s_1], H[s_2]\} &= D\left[F(s_1 s_2' - s_1' s_2)\right],
\end{aligned}$$

and $F = q^{xx}$

Spacetime embeddability

Modified Hamiltonians

Hamiltonian ⇔ Metric

No propagating degrees of freedom

$$H_T = \int (N\mathcal{H} + N^x\mathcal{D})dx,$$

Two pairs of conjugate variables $\{q_i(x_1), p_j(x_2)\} = \delta_i^j \delta(x_1 - x_2)$

$q_1 = K_x \quad p_1 = E^x$

$q_2 = K_\varphi \quad p_2 = E^\varphi$

Six free functions of p_1 only $\mathfrak{g},\ V,\ A,\ W,\ \omega,$ and φ

$$\mathcal{D} = -q_1 p_1' + q_2' p_2$$

classical diffeomorphism constraint

$$\mathcal{H} = \mathfrak{g}\Bigg[p_2 V - p_2 \frac{A}{\omega^2}\sin^2(\omega q_2)\frac{\partial}{\partial p_1}\left[\log\left(\frac{AW}{\omega^2}\right)\right] + \frac{1}{2}\left(\frac{p_1''}{p_2} - \frac{p_1' p_2'}{p_2^2} + \frac{\partial \log(W)}{\partial p_1}\frac{(p_1')^2}{2p_2}\right)\cos^2(\omega q_2 + \varphi)$$

$$- \left(q_1 + q_2 p_2 \frac{\partial \log(\omega)}{\partial p_1}\right)\frac{A}{\omega}\sin(2\omega q_2) - \left(\omega q_1 + q_2 p_2 \frac{\partial \omega}{\partial p_1} + p_2 \frac{\partial \varphi}{\partial p_1}\right)\left(\frac{p_1'}{2p_2}\right)^2 \sin(2(\omega q_2 + \varphi))\Bigg],$$

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Trigonometric functions ⇔ covariance

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Holonomy corrections appear naturally!

[A-B, Brizuela (2024)]

$\mathcal{D} = -q_1 p_1' + q_2' p_2$ *classical diffeomorphism constraint*

$$\mathcal{H} = \mathfrak{g}\left[p_2 V - p_2 \frac{A}{\omega^2} \sin^2(\omega q_2) \frac{\partial}{\partial p_1}\left[\log\left(\frac{A\,W}{\omega^2}\right)\right] + \frac{1}{2}\left(\frac{p_1''}{p_2} - \frac{p_1' p_2'}{p_2^2} + \frac{\partial \log(W)}{\partial p_1}\frac{(p_1')^2}{2p_2}\right)\cos^2(\omega q_2 + \varphi)\right.$$

$$\left. - \left(q_1 + q_2 p_2 \frac{\partial \log(\omega)}{\partial p_1}\right)\frac{A}{\omega}\sin(2\omega q_2) - \left(\omega q_1 + q_2 p_2 \frac{\partial \omega}{\partial p_1} + p_2 \frac{\partial \varphi}{\partial p_1}\right)\left(\frac{p_1'}{2p_2}\right)^2 \sin(2(\omega q_2 + \varphi))\right],$$

Modified Hamiltonians

Hamiltonian ⇔ Metric

Construction of the model such that it obeys the *hypersurface deformation algebra* with a suitable structure function to embed it in (M, g)

Trigonometric functions ⇔ covariance

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$$F = \frac{F_s}{(E^\varphi)^2} \quad \text{with}$$

$$F_s := \mathfrak{g}^2 \cos(\omega K_\varphi + \varphi) \left(A \cos(\omega K_\varphi - \varphi) + \left(\frac{E^{x\prime}}{2E^\varphi} \right)^2 \omega^2 \cos(\omega K_\varphi + \varphi) \right)$$

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Generic features

Curvature in terms of ∇r, $\nabla^2 r$, and ${}^{(2)}R$

Existence of a KVF ξ orthogonal to ∇r [A-B, Brizuela (2024)]

Special surfaces

Vanishing of $g(\xi, \xi)$, $g(\nabla r, \nabla r)$, and F_s

Modified Hamiltonians

Hamiltonian ⇔ Metric

Special cases

Trigonometric functions ⇔ covariance

i. Constant ω: the Hamiltonian is bounded in q_2

ii. $\varphi = 0$ and $A \geq 0$: spacetime is Lorentzian (no signature change)

iii. $W = c\sqrt{p_1}$ and $V = -1/(2p_1)$: $\exists\, m : m' \approx 0$

Holonomy corrections appear naturally!

$$\mathcal{H} = \mathfrak{g}\left[p_2 V - p_2 \frac{A}{\omega^2}\sin^2(\omega q_2)\frac{\partial}{\partial p_1}\left[\log\left(\frac{AW}{\omega^2}\right)\right] + \frac{1}{2}\left(\frac{p_1''}{p_2} - \frac{p_1' p_2'}{p_2^2} + \frac{\partial \log(W)}{\partial p_1}\frac{(p_1')^2}{2p_2}\right)\cos^2(\omega q_2 + \varphi)\right.$$
$$\left. - \left(q_1 + q_2 p_2 \frac{\partial \log(\omega)}{\partial p_1}\right)\frac{A}{\omega}\sin(2\omega q_2) - \left(\omega q_1 + q_2 p_2 \frac{\partial \omega}{\partial p_1} + p_2 \frac{\partial \varphi}{\partial p_1}\right)\left(\frac{p_1'}{2p_2}\right)^2 \sin(2(\omega q_2 + \varphi))\right],$$

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Is this too restrictive? **NO**

GR satisfies all these conditions... and more

Trigonometric functions ⇔ covariance

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We find a *one-parameter* family of theories with $\omega = \lambda \in \mathbb{R}$, such that GR is $\lambda = 0$. This is the effective covariant polymerization!

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[A-B, Brizuela, Vera (2022)]

Nonsingular BHs: **vacuum**

$$ds^2 = -\left(1 - \frac{2m}{\tilde r}\right) d\tilde t^2 + \left(1 - \frac{r_0}{\tilde r}\right)^{-1} \left(1 - \frac{2m}{\tilde r}\right)^{-1} d\tilde r^2 + \tilde r^2 d\Omega^2,$$

$$r_0 := 2m \frac{\lambda^2}{1+\lambda^2}$$

$$\mathcal{H} = -\frac{E^\varphi}{2\sqrt{E^x}\sqrt{1+\lambda^2}} \left(1 + \frac{\sin^2(\lambda K_\varphi)}{\lambda^2}\right) - \sqrt{E^x} K_x \frac{\sin(2\lambda K_\varphi)}{\lambda\sqrt{1+\lambda^2}} \left(1 + \left(\frac{\lambda E^{x\prime}}{2E^\varphi}\right)^2\right)$$
$$+ \frac{\cos^2(\lambda K_\varphi)}{2\sqrt{1+\lambda^2}} \left(\frac{E^{x\prime}}{2E^\varphi}\left(\sqrt{E^x}\right)' + \sqrt{E^x}\left(\frac{E^{x\prime}}{E^\varphi}\right)'\right),$$

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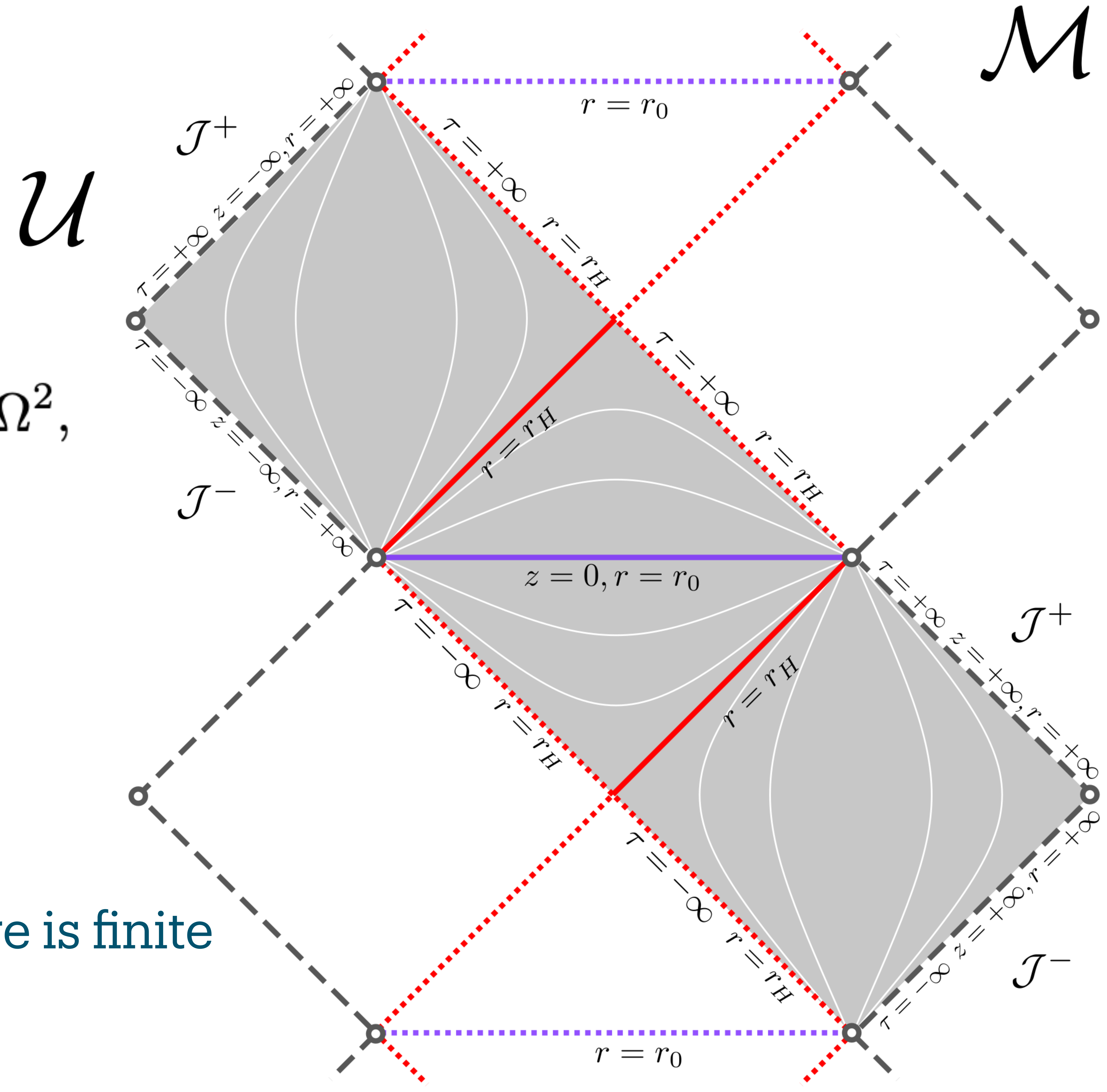


The singularity is replaced with a transition surface $r = r_0$ of positive radius towards a time-reversed (WH) region

The surface is always inside the *trapped* region and curvature is finite

The spacetime is geodesically complete

[A-B, Brizuela, Vera (2022)]

Nonsingular BHs: **Q** and **Λ**

$$ds^2 = -\left(1 - \frac{2m(r)}{r}\right) d\tilde{t}^2 + r^2 d\Omega^2$$

$$+ \left(1 - \frac{2\bar{\lambda} m(r)}{r}\right)^{-1} \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2$$

$$m \to m(r) = M - \frac{Q^2}{2r^2} + \frac{\Lambda}{6} r^3$$

$$\bar{\lambda} := \frac{\lambda^2}{1+\lambda^2}$$

$$\mathcal{H} = -\frac{E^\varphi}{2\sqrt{E^x}\sqrt{1+\lambda^2}}\left(1 + \frac{\sin^2(\lambda K_\varphi)}{\lambda^2}\right) - \sqrt{E^x} K_x \frac{\sin(2\lambda K_\varphi)}{\lambda\sqrt{1+\lambda^2}}\left(1 + \left(\frac{\lambda E^{x\prime}}{2E^\varphi}\right)^2\right)$$

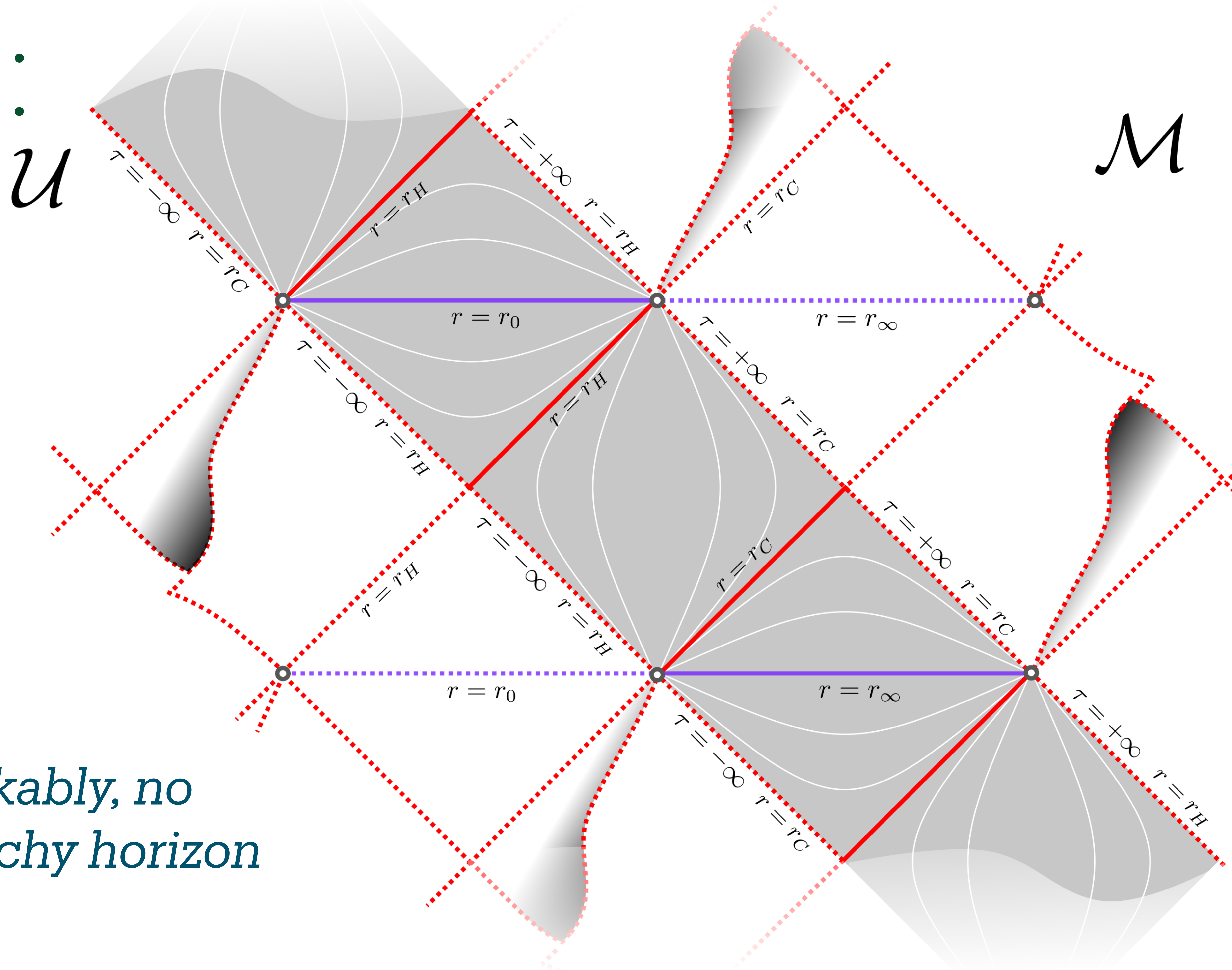
$$+ \frac{\cos^2(\lambda K_\varphi)}{2\sqrt{1+\lambda^2}}\left(\frac{E^{x\prime}}{2E^\varphi}\left(\sqrt{E^x}\right)' + \sqrt{E^x}\left(\frac{E^{x\prime}}{E^\varphi}\right)'\right) + \frac{\sqrt{E^x} E^\varphi}{2\sqrt{1+\lambda^2}}\left(\Lambda + \left(\frac{Q}{E^x}\right)^2\right)$$

Nonsingular BHs:

$$ds^2 = -\left(1 - \frac{2m(r)}{r}\right) d\tilde{t}^2 + r^2 d\Omega^2$$
$$+ \left(1 - \frac{2\bar{\lambda} m(r)}{r}\right)^{-1} \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2$$

$$m \to m(r) = M - \frac{Q^2}{2r^2} + \frac{\Lambda}{6} r^3$$

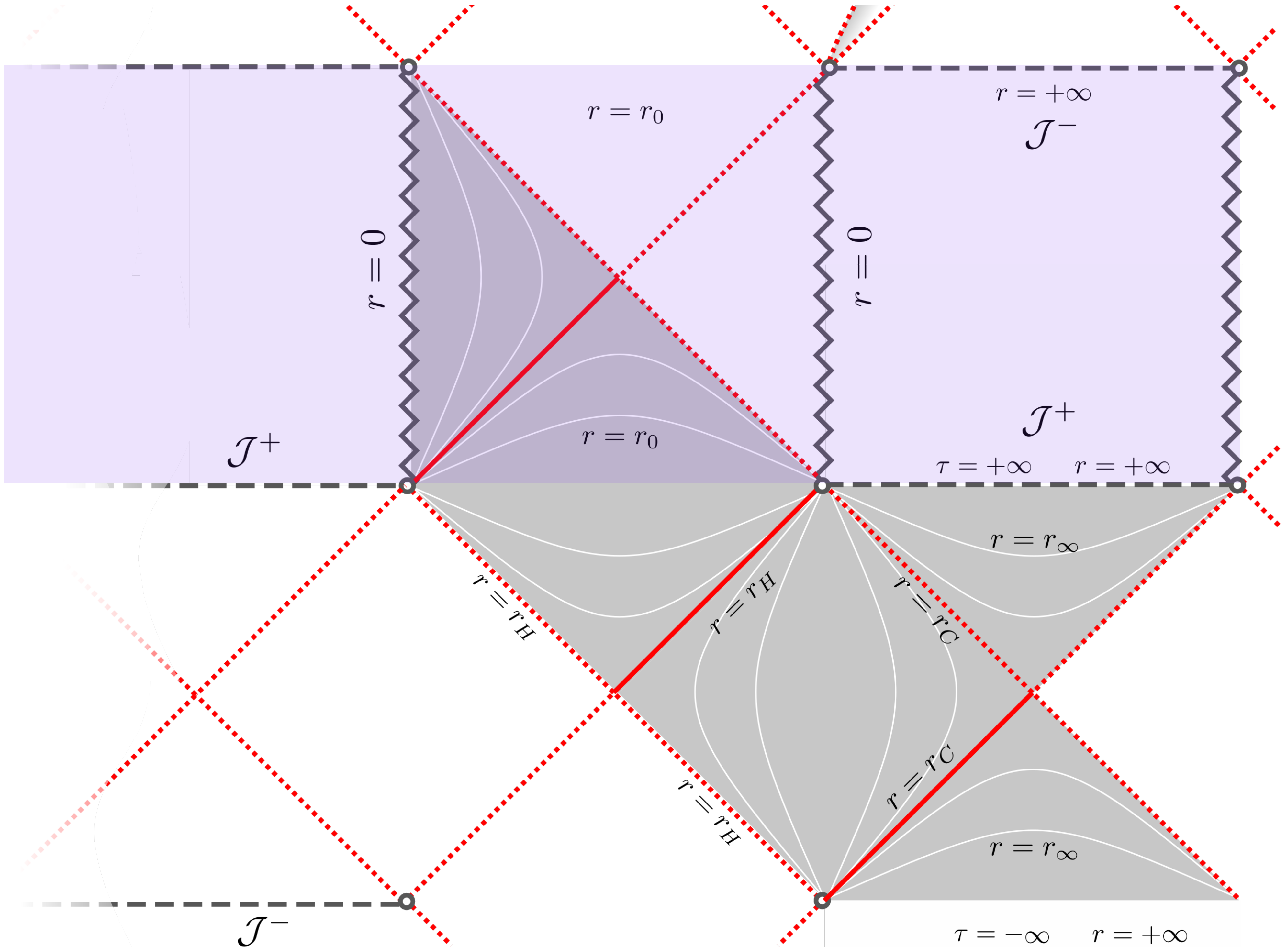
Remarkably, no inner Cauchy horizon

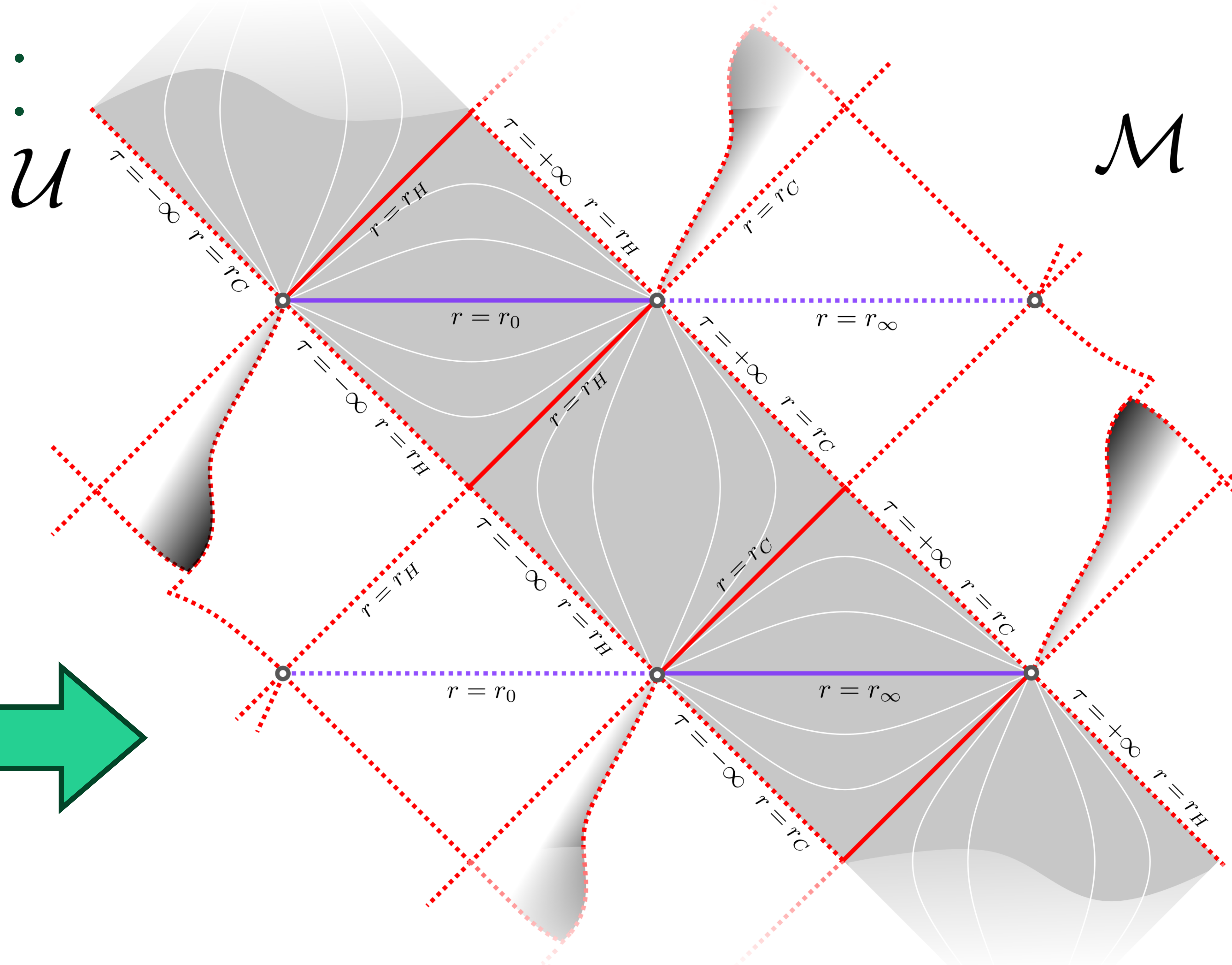


Singularity resolution in a nutshell: positive **M**, small **Q**, and positive **Λ**

[A-B, Brizuela, Vera (2023)]

Nonsingular BHs:





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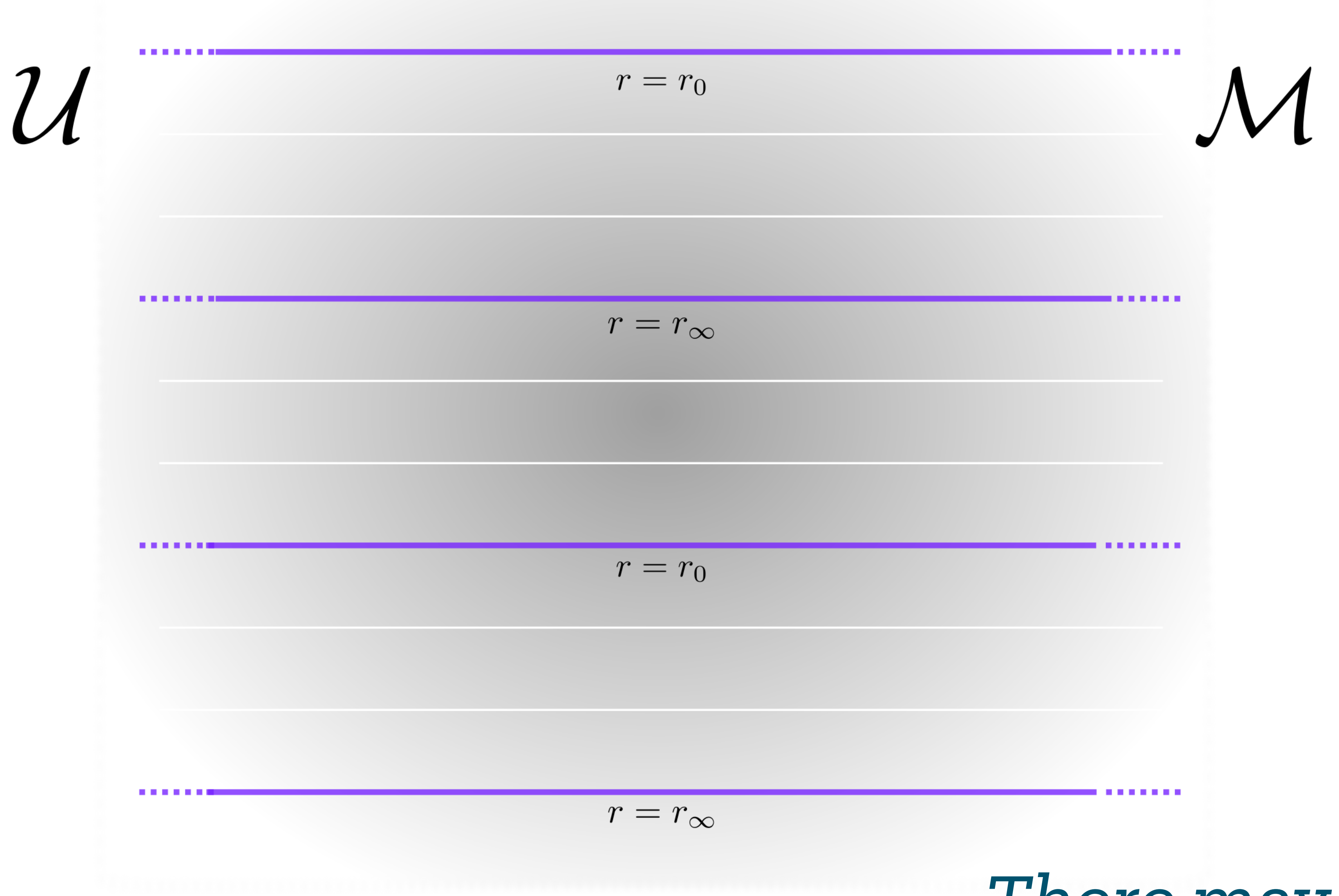
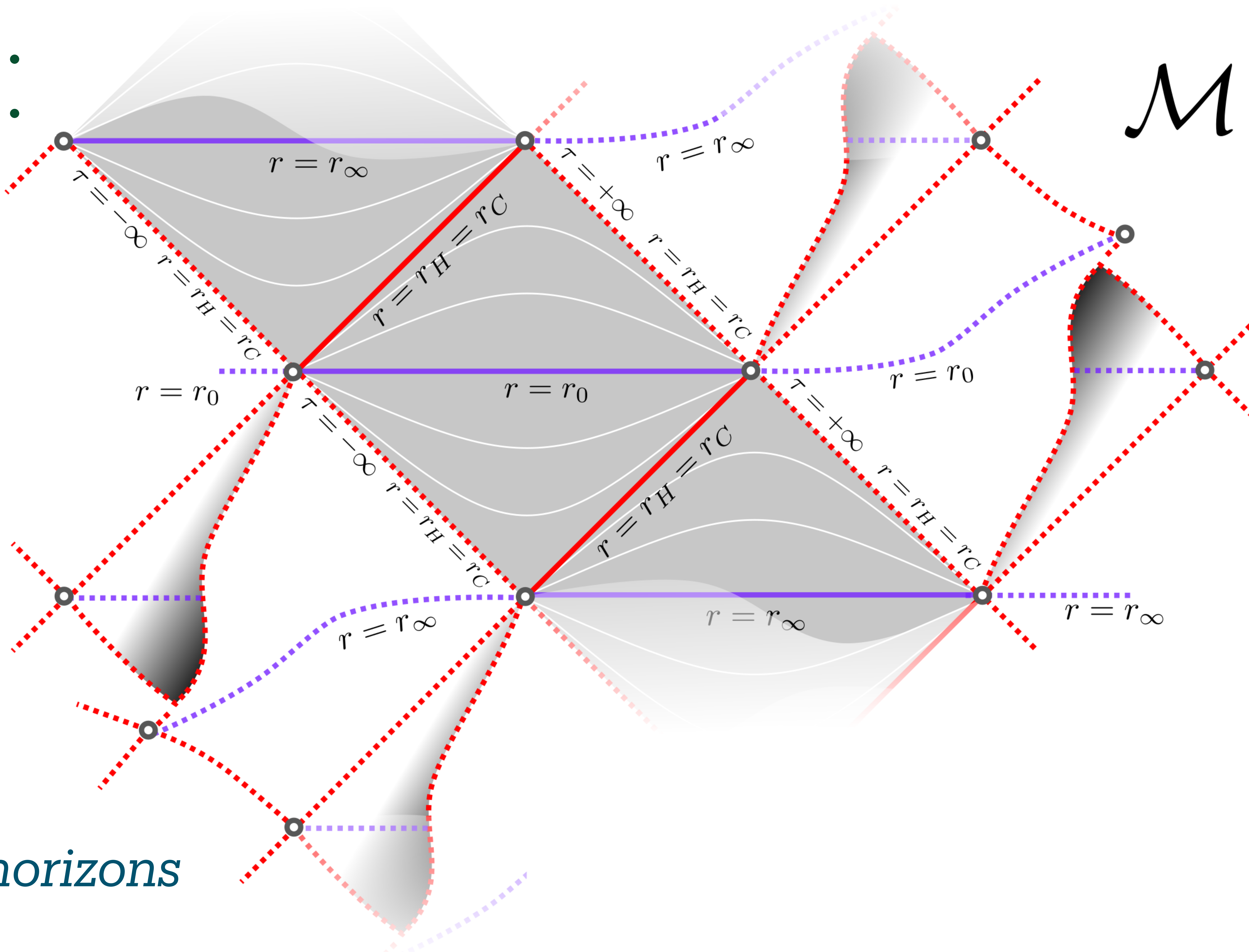
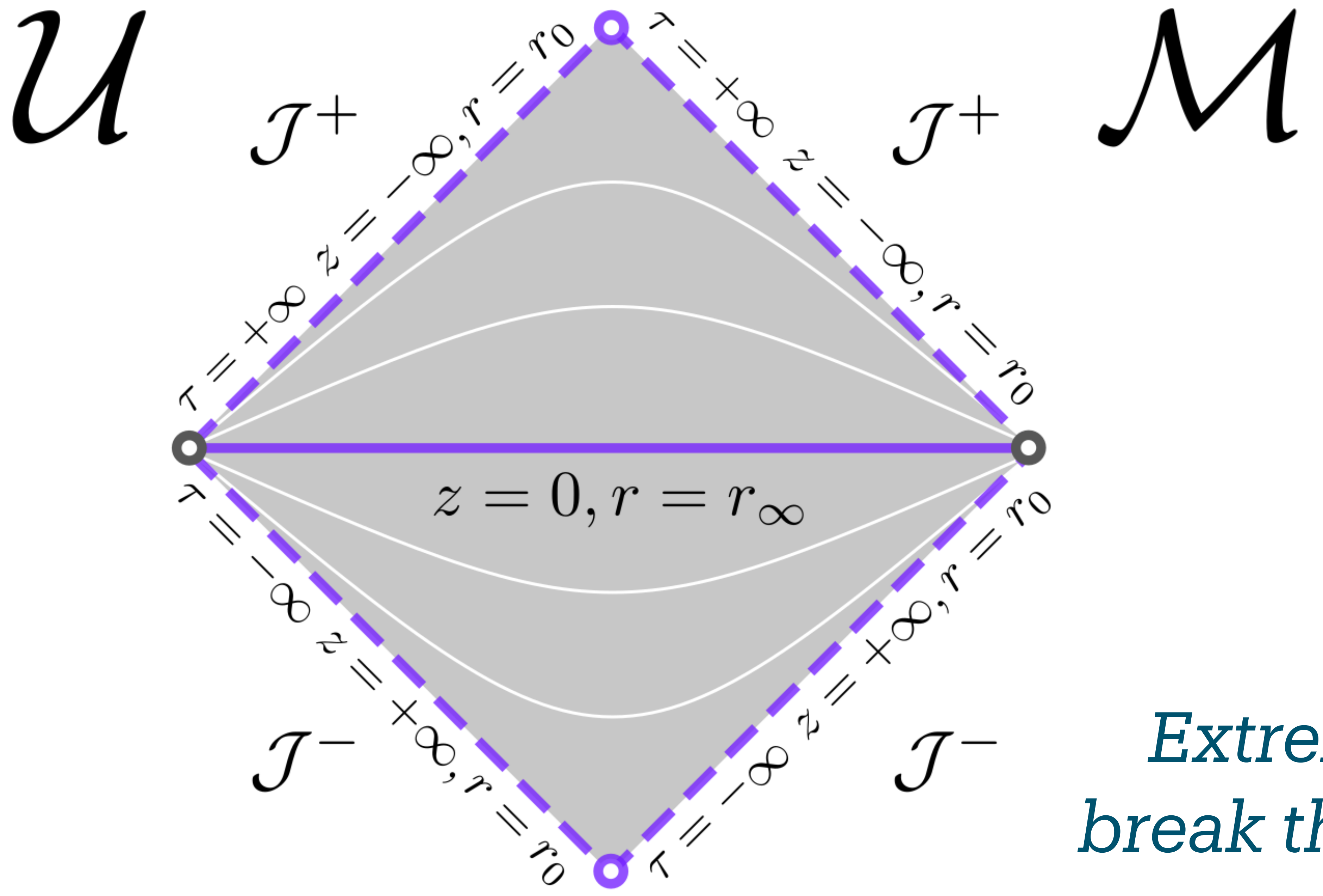


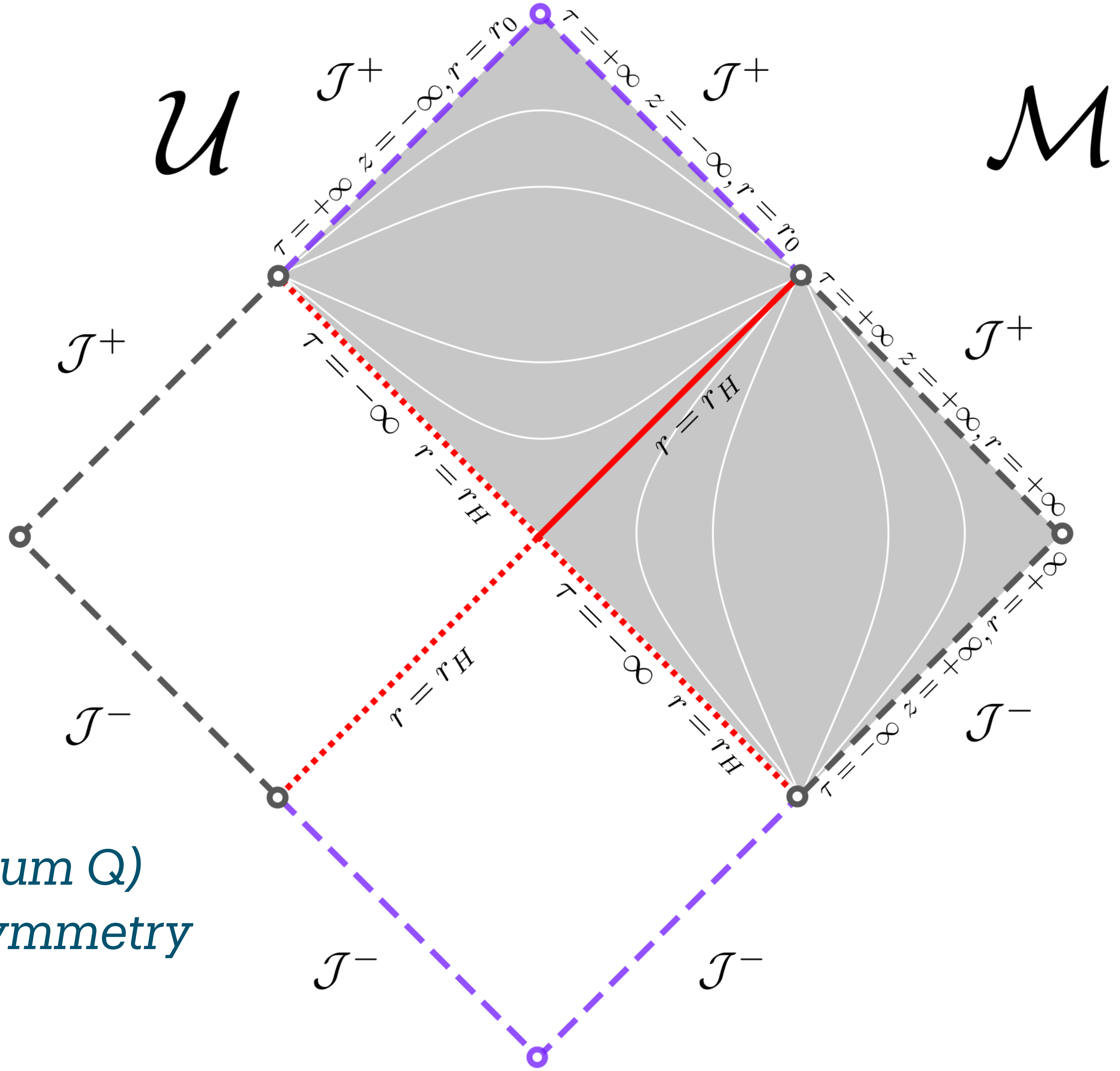


There may be 2, 1, or 0 horizons

Singularity resolution in a nutshell: positive **M**, small **Q**, and positive **Λ**   [A-B, Brizuela, Vera (2023)]

Nonsingular BHs:





Extremal cases (maximum Q) break the time-reversal symmetry

Singularity resolution in a nutshell: positive **M**, small **Q**, and positive **Λ**

[A-B, Brizuela, Vera (2023)]

Dynamical BHs: **dust**

$$ds^2 = \sigma N^2 dt^2 + \frac{1}{|F|}(dx + N^x dt)^2 + r^2 d\Omega^2,$$

$$\{D[s_1], D[s_2]\} = D[s_1 s_2' - s_1' s_2],$$
$$\{D[s_1], H[s_2]\} = H[s_1 s_2'],$$
$$\{H[s_1], H[s_2]\} = D[F(s_1 s_2' - s_1' s_2)],$$

[A-B, Brizuela (2024)]

Minimal coupling

$$\mathcal{H} = -\frac{E^\varphi}{2\sqrt{E^x}\sqrt{1+\lambda^2}}\left(1 + \frac{\sin^2(\lambda K_\varphi)}{\lambda^2}\right) - \sqrt{E^x} K_x \frac{\sin(2\lambda K_\varphi)}{\lambda\sqrt{1+\lambda^2}}\left(1 + \left(\frac{\lambda E^{x\prime}}{2E^\varphi}\right)^2\right)$$
$$+ \frac{\cos^2(\lambda K_\varphi)}{2\sqrt{1+\lambda^2}}\left(\frac{E^{x\prime}}{2E^\varphi}\left(\sqrt{E^x}\right)' + \sqrt{E^x}\left(\frac{E^{x\prime}}{E^\varphi}\right)'\right) + P_\phi\sqrt{1 + \frac{\cos^2(\lambda K_\varphi)}{1+\lambda^2}\left(1 + \left(\frac{\lambda E^{x\prime}}{2E^\varphi}\right)^2\right)\frac{E^x}{E^{\varphi 2}}(\phi')^2},$$

Dynamical BHs: **dust**

To solve Hamiltonian equations: same procedure as in GR

1. Choose the dust frame: $\phi = t$
2. Conservation of gauge: $N = 1$
3. Dust momentum: $P_\phi = E$
4. New variables: $(E^x, E^\varphi, K_\varphi) \to (r, m, \kappa)$

[Solve diffeomorphism for K_x]

$$\mathcal{H} = -\frac{E^\varphi}{2\sqrt{E^x}\sqrt{1+\lambda^2}}\left(1+\frac{\sin^2(\lambda K_\varphi)}{\lambda^2}\right) - \sqrt{E^x}K_x\frac{\sin(2\lambda K_\varphi)}{\lambda\sqrt{1+\lambda^2}}\left(1+\left(\frac{\lambda E^{x\prime}}{2E^\varphi}\right)^2\right)$$
$$+\frac{\cos^2(\lambda K_\varphi)}{2\sqrt{1+\lambda^2}}\left(\frac{E^{x\prime}}{2E^\varphi}\left(\sqrt{E^x}\right)' + \sqrt{E^x}\left(\frac{E^{x\prime}}{E^\varphi}\right)'\right) + P_\phi\sqrt{1+\frac{\cos^2(\lambda K_\varphi)}{1+\lambda^2}\left(1+\left(\frac{\lambda E^{x\prime}}{2E^\varphi}\right)^2\right)\frac{E^x}{E^{\varphi 2}}(\phi')^2},$$

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To solve Hamiltonian equations: same procedure as in GR

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3. Dust momentum: $P_\phi = E$
4. New variables: $(E^x, E^\varphi, K_\varphi) \to (r, m, \kappa)$

[Solve diffeomorphism for K_x]

$$\dot{r} = N^x r' - \epsilon \sqrt{1 - \frac{2\lambda m}{r}} \sqrt{\kappa + \frac{2m}{r}},$$

$$\dot{m} = N^x m',$$

$$\dot{\kappa} = N^x \kappa',$$

Maximum for expanding branch ONLY when $\kappa < 0$

Minimum for collapsing branch

Periodic oscillations with: $T = 2\pi m(1 + \lambda|\kappa|)|\kappa|^{-3/2}$

Dynamical BHs: **dust**

[A-B, Brizuela (2024)]

Still, one gauge freedom to fix

We choose $m = m(x) \geq 0$

Conservation: $N^x = 0$

$$\dot{r} = N^x r' - \epsilon\sqrt{1 - \frac{2\lambda m}{r}}\sqrt{\kappa + \frac{2m}{r}},$$

$$\dot{m} = N^x m',$$

$$\dot{\kappa} = N^x \kappa',$$

$$t - t_0 = -\epsilon\sqrt{\frac{2r^3}{9m}}\sqrt{1 - \frac{2\lambda m}{r}}\left(1 + \frac{4\lambda m}{r}\right),$$

$$t - t_0 = -\epsilon\frac{r}{\kappa}\sqrt{1 - \frac{2\lambda m}{r}}\sqrt{\frac{2m}{r} + \kappa} + \epsilon\frac{2m}{\kappa^{3/2}}(1 - \lambda\kappa)\text{artanh}\sqrt{\frac{\kappa(r - 2\lambda m)}{2m + \kappa r}},$$

$$t - t_0 = +\epsilon\frac{r}{|\kappa|}\sqrt{1 - \frac{2\lambda m}{r}}\sqrt{\frac{2m}{r} - |\kappa|} - \epsilon\frac{2m}{|\kappa|^{3/2}}(1 + \lambda|\kappa|)\arctan\sqrt{\frac{|\kappa|(r - 2\lambda m)}{2m - |\kappa| r}},$$

Maximum for expanding branch
ONLY when $\kappa < 0$

Minimum for collapsing branch

Periodic oscillations with: $T = 2\pi m(1 + \lambda|\kappa|)|\kappa|^{-3/2}$

Dynamical BHs: **dust**

- Trajectories on phase space describe a smooth bounce of dust
- The radius of each dust-shell has a positive infimum, achieved in finite proper time
- However… curvature scalars diverge there. *Singularity resolution is not complete*

$$\dot{r} = N^x r' - \epsilon \sqrt{1 - \frac{2\lambda m}{r}} \sqrt{\kappa + \frac{2m}{r}},$$
$$\dot{m} = N^x m',$$
$$\dot{\kappa} = N^x \kappa',$$

What happens here?

Maximum for expanding branch ONLY when $\kappa < 0$

Minimum for collapsing branch

Periodic oscillations with: $T = 2\pi m(1 + \lambda|\kappa|)|\kappa|^{-3/2}$

Dynamical BHs: **dust**

- Trajectories on phase space describe a smooth bounce of dust
- There is a conformal freedom that we omitted at the beginning...
- We can build a whole family of regular metrics covariantly associated with the dynamics

Dust is now coupled to a conformal fiducial metric

It is still possible to couple it to the physical metric...

$$\dot{r} = N^x r' - \epsilon\sqrt{1 - \frac{2\lambda m}{r}}\sqrt{\kappa + \frac{2m}{r}},$$

$$\dot{m} = N^x m',$$

$$\dot{\kappa} = N^x \kappa',$$

$$d\tilde{s}^2 = \Omega^{-2}\left[-N^2 dt^2 + \frac{1}{F}\left(dx + N^x dt\right)^2\right] + r^2 d\sigma^2,$$

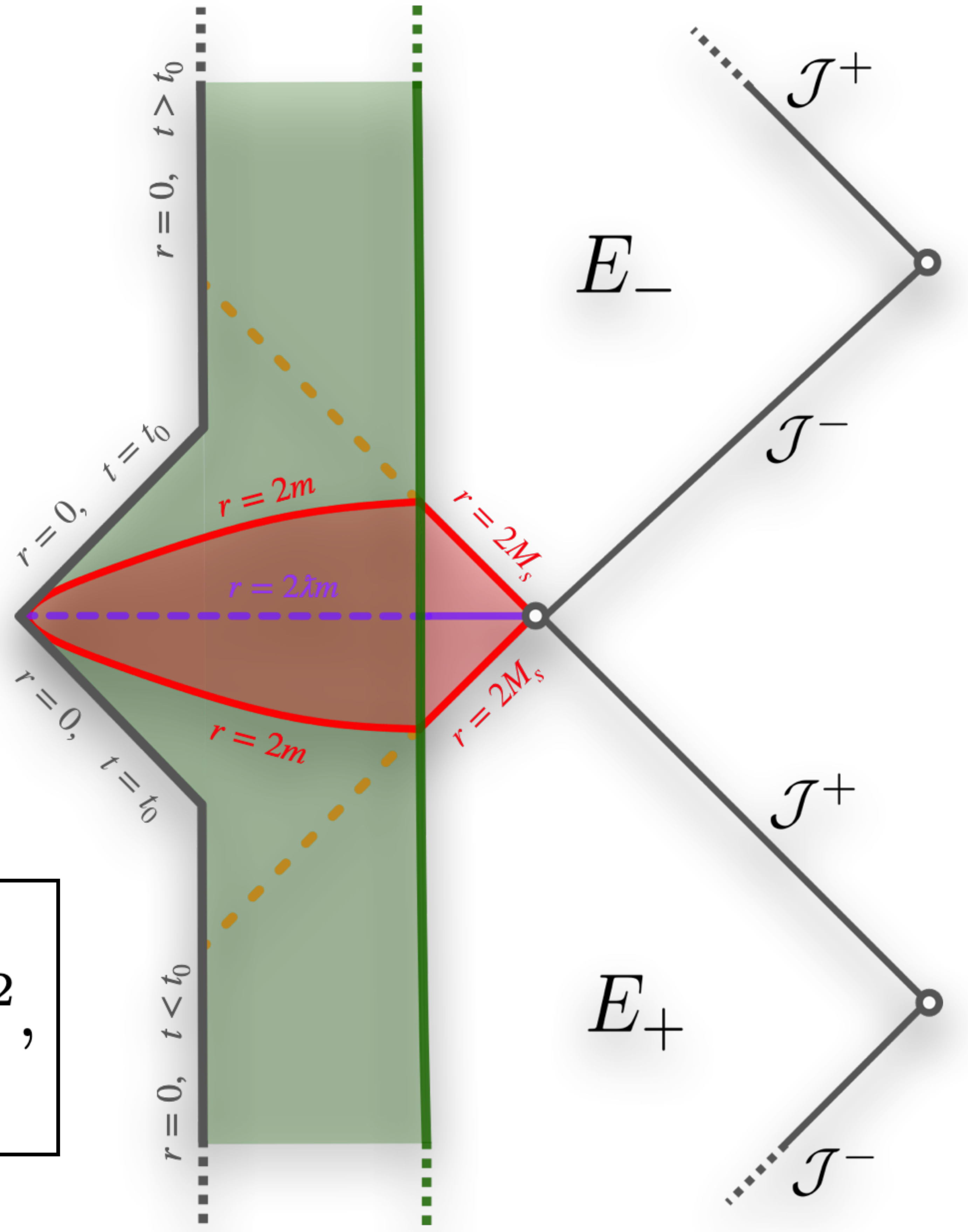
$$\Omega^{-2} = \left(1 - \frac{2\lambda m}{r}\right)^{-n}$$

Curvature scalars are bounded for $n \geq 1$

[A-B, Brizuela (2024)]

LOOPs

Dynamical BHs: **dust**



- Trajectories on phase space describe a smooth bounce of dust
- There is a conformal freedom that we omitted at the beginning…
- We can build a whole family of regular metrics covariantly associated with the dynamics

$$d\tilde{s}^2 = -\left(1-\frac{2\lambda m(x)}{r(t,x)}\right)^{-n} dt^2 + \left(1-\frac{2\lambda m(x)}{r(t,x)}\right)^{-(n+1)} \frac{\left(r'(t,x)\right)^2}{1+\kappa(x)} dx^2 + r(t,x)^2 d\sigma^2,$$

$$\dot{r} = -\epsilon\sqrt{1-\frac{2\lambda m}{r}}\sqrt{\kappa+\frac{2m}{r}}, \qquad \mathcal{E} := \sqrt{1-\lambda}\,\frac{m'}{r^2 r'}\left(1-\frac{2\lambda m}{r}\right)^{\frac{n+1}{2}},$$

Curvature scalars are bounded for $n \geq 1$

[A-B, Brizuela (2024)]

Summary & Outlook

We found an effective Hamiltonian theory with a well-defined geometric description, and such that GR is a *singular* limit.

Static and dynamical solutions are under control in this effective description. Physically reasonable cases are **free of singularities**,

- Modelling numerical collapse
- Understanding effective corrections from full LQG
- Homogeneous reduction: Is it possible to find FLRW?
- Less symmetric scenarios… effective Kerr BHs