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Purification of Hawking Radiation:
Lessons from a moving mirror analogy

Paula Ariela Calizaya Cabrera

Louisiana State University

in collaboration with Ivan Agullo & Beatriz Elizaga Navascués

The Moving Mirror Analogy

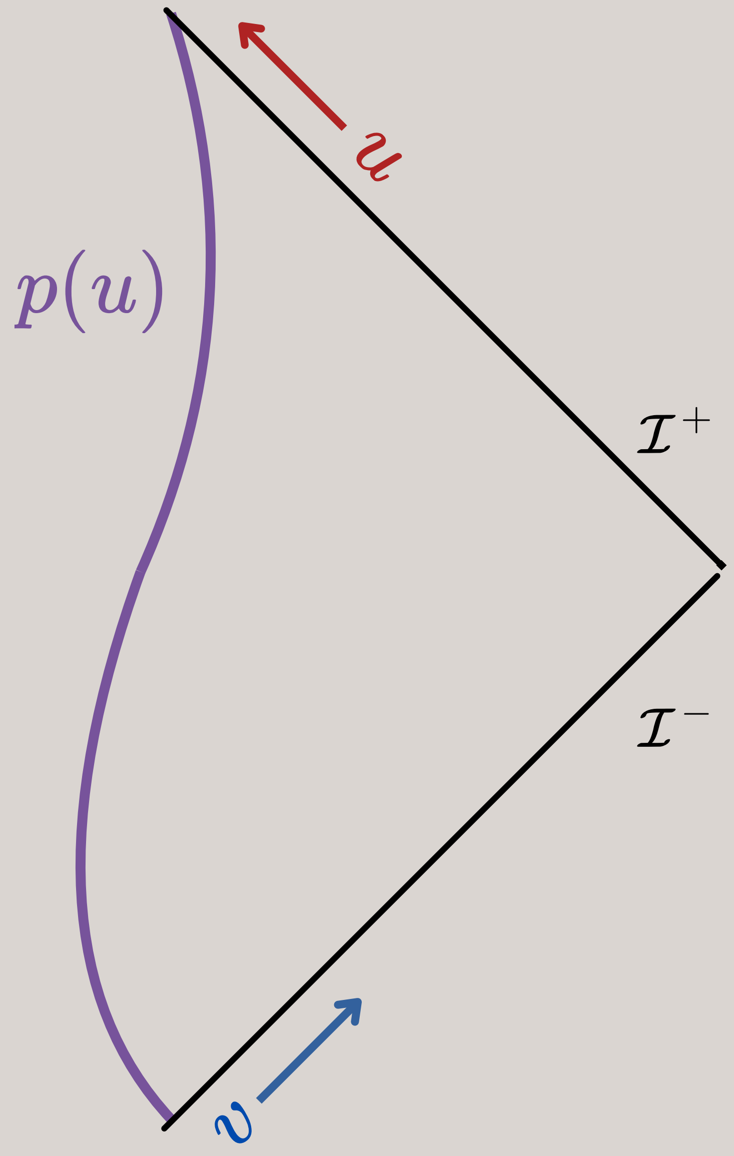
Massless Scalar field in a (1+1) Minkowski spacetime with reflecting boundary conditions

- Wave equation $\partial_u \partial_v \varphi(u,v) = 0$ (u, v null coords.)
- Mirror (Dirichlet boundary condition) follows a trajectory $v = p(u)$ such that $\varphi(u, p(u)) = 0$
- Give initial conditions such that

$$\varphi_\omega^{in}|_{\mathcal{I}^-} = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega v}$$ with only +ve frequencies

$$\varphi_\omega^{in}|_{\mathcal{I}^+} = -\frac{1}{\sqrt{4\pi\omega}} e^{-i\omega p(u)} = \int_0^\infty d\omega' (\alpha_{\omega\omega'} e^{-i\omega' u} + \beta_{\omega\omega'} e^{i\omega' u})$$

- 'in' vacuum at $\mathcal{I}^-$ transitions to an excited state state at $\mathcal{I}^+$ when $\beta_{\omega\omega'} \neq 0$



(Fulling & Davies '76):

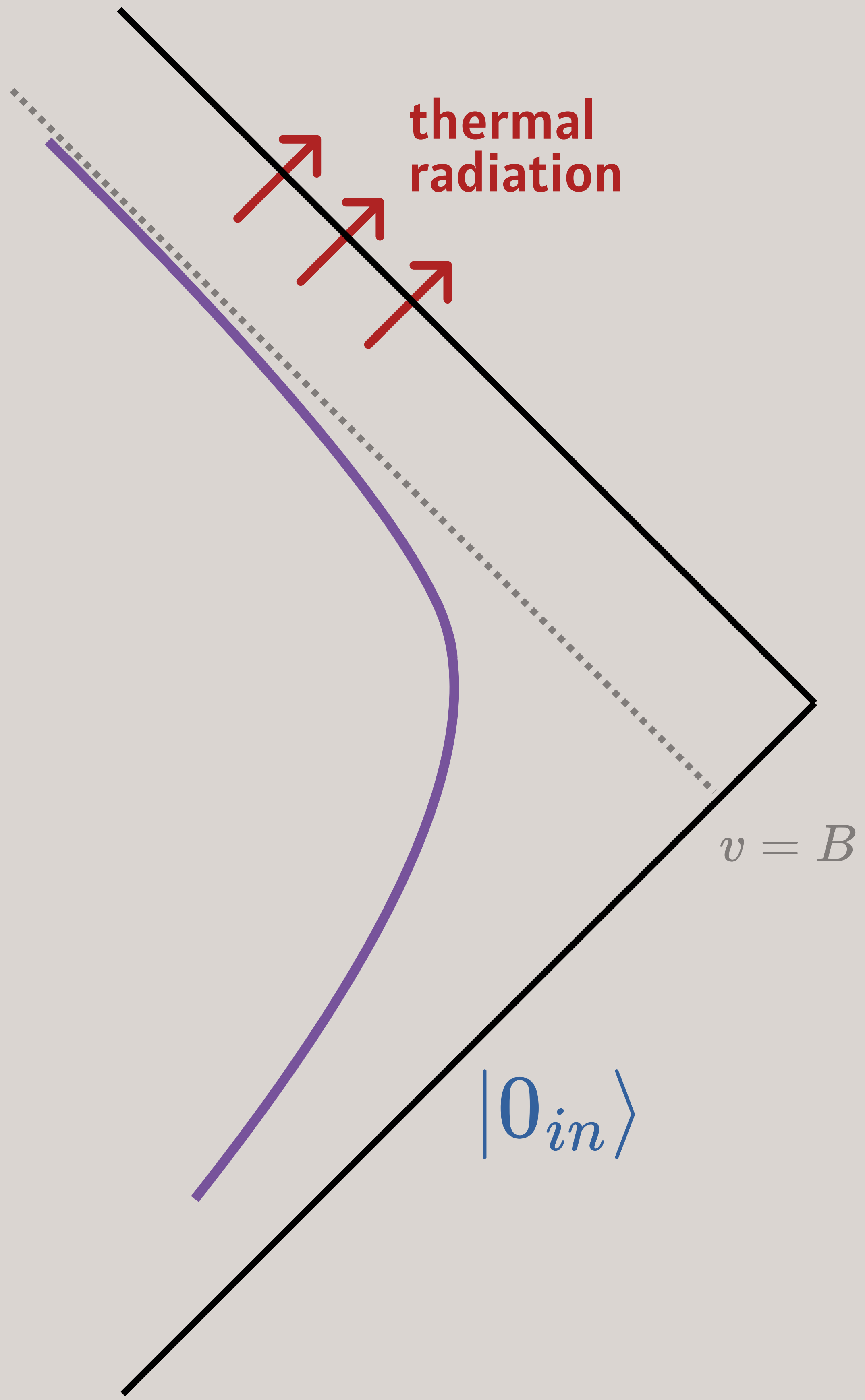
$$p(u) = B - Ae^{-\kappa u}$$ such that $$\frac{\ddot{p}(u)}{\dot{p}(u)} = -\kappa$$

mimics black hole

In this example there is thermal radiation arriving at $\mathcal{I}^+$

$$|\beta_{\omega\omega'}|^2 = \frac{1}{2\pi\kappa\omega'}\left(\frac{1}{e^{2\pi\omega/\kappa} - 1}\right)$$

with temperature given by $T = \frac{\kappa}{2\pi}$



Evaporation Model

Goal: Find the Moving Mirror Analog of an Evaporating Black Hole

- **Mass Loss Rate, as proposed by Hawking:**

$$\dot{M}(u) = -\alpha M(u)^{-2} \longrightarrow M(u) = M_0\left(1 - \frac{3\alpha}{M_0^3}(u - u_0)\right)^{1/3}$$

$\alpha \sim 10^{-3}$ planck units (Page '75)

- **Hypothesis:** $-\frac{\ddot{p}(u)}{\dot{p}(u)} = \kappa = \frac{1}{4M} \longrightarrow -\frac{\ddot{p}(u)}{\dot{p}(u)} = \kappa(u) = \frac{1}{4M(u)}$

$\kappa(u)$ changes adiabatically

- **These leads us to an expression of p(u) of the form**

$$p(u) = v_0 + 4\dot{v}_0\, e^{-M_0^2/8\alpha}\left\{M_0\, e^{M_0^2/8\alpha} - M(u)\, e^{M(u)^2/8\alpha} + \sqrt{2\pi\alpha}\left[\mathrm{erfi}\left(\frac{M(u)}{2\sqrt{2\alpha}}\right) - \mathrm{erfi}\left(\frac{M_0}{2\sqrt{2\alpha}}\right)\right]\right\}$$

exact solution up to two constants

Important Observation:

- p(u) locally exhibits an exponential form over significantly long intervals of u, due to the adiabatic nature of $\kappa(u)$

$$p_\star(u) = v_\star^{(H)} - \frac{\dot{v}_\star}{\kappa_\star}\, e^{\kappa_\star(u-u_\star)}$$

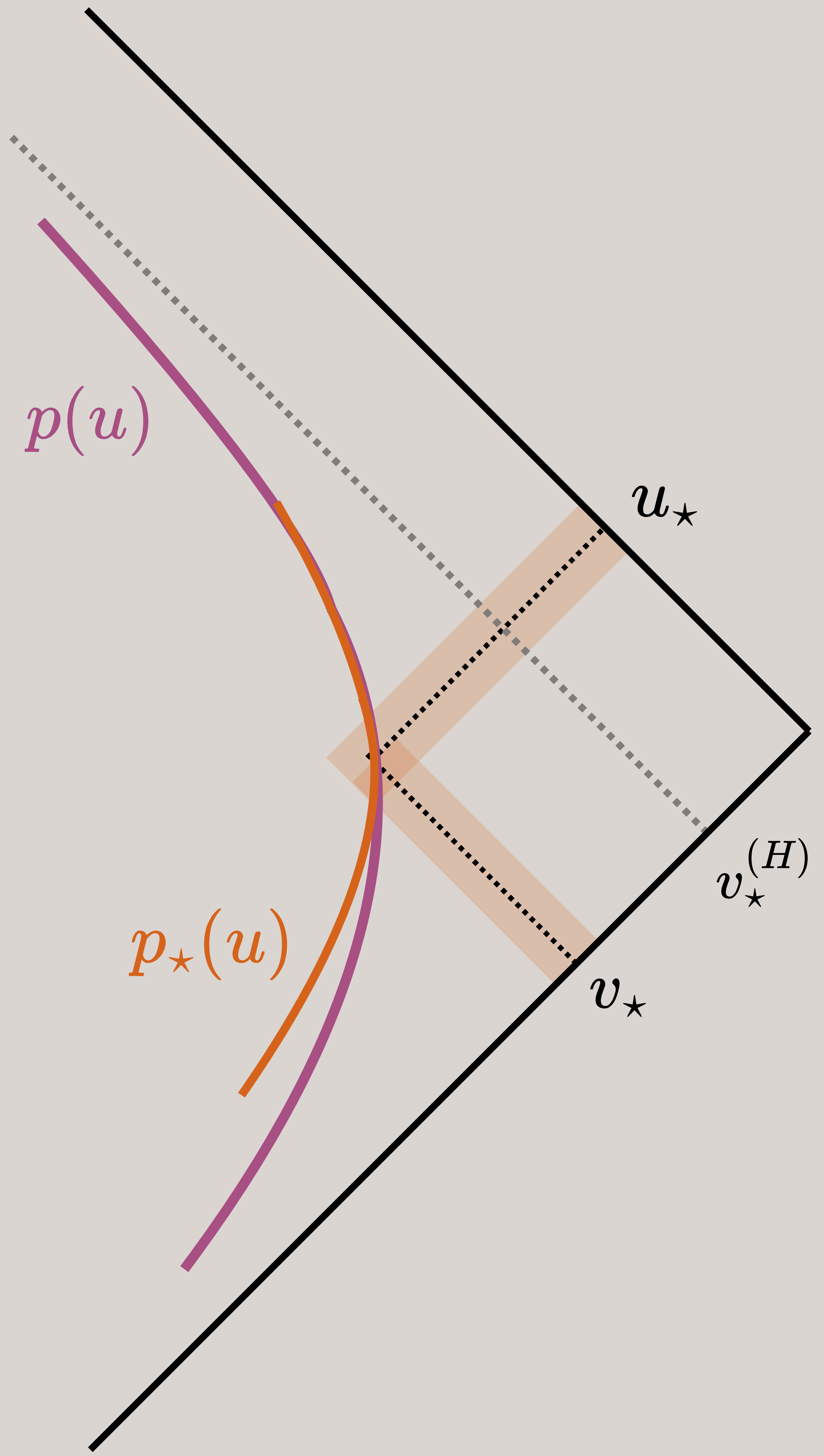
$$u \in [u_\star - \Delta u, u_\star + \Delta u]$$

$$\Delta u << M(u_\star)^2/\sqrt{\alpha}$$

with

$$v_\star^{(H)} \equiv v_\star + \kappa_\star^{-1}\, \dot{v}_\star$$

instantaneous
"would-be horizon"



- At every interval there is quasi-thermal radiation with adiabatically changing temperature

Caveat:

- We model full evaporation by letting the mirror trajectory become inertial when $M(u) = 0$

$$p(u) = \text{linear}, \quad u > u_{evp}$$

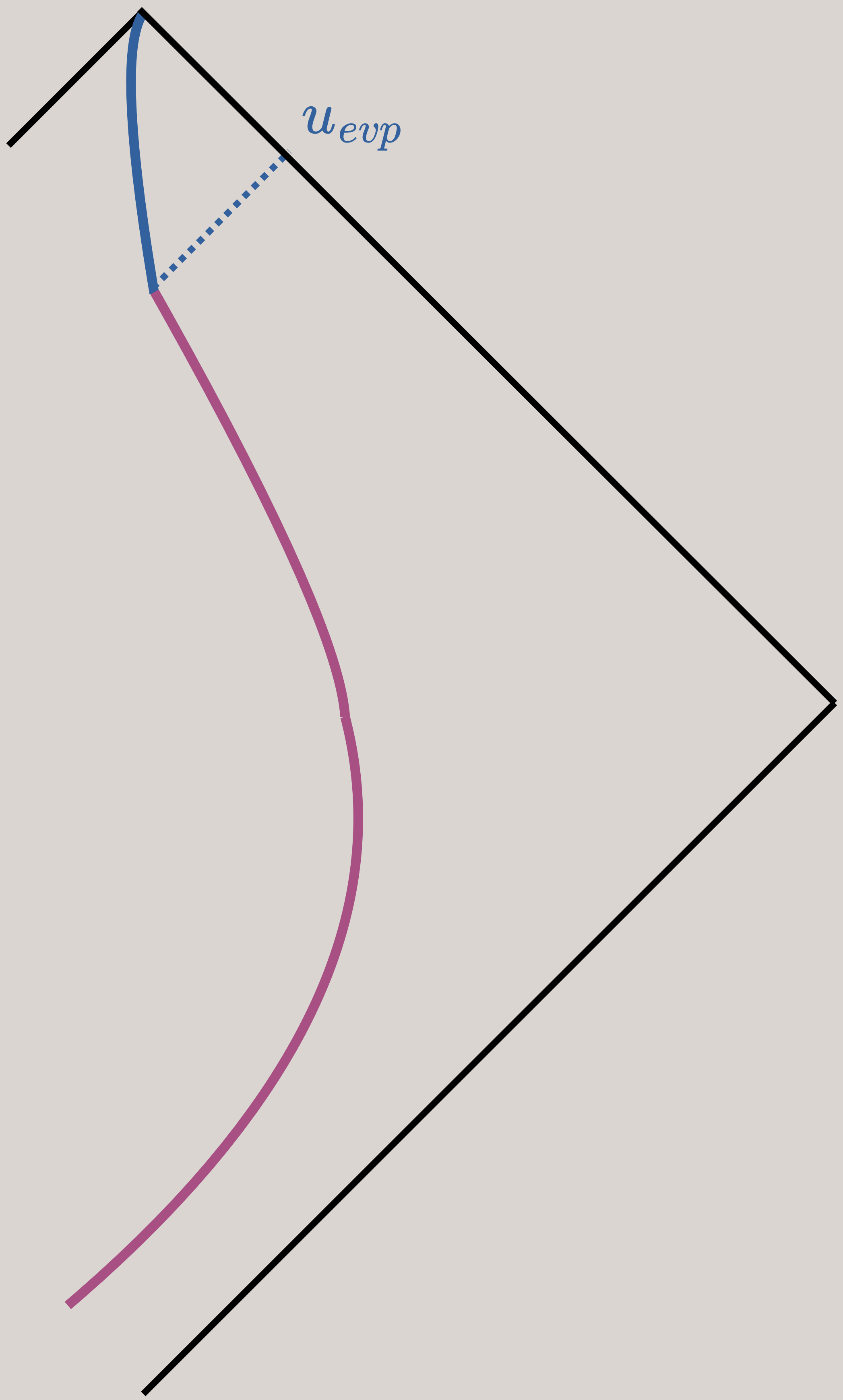
Evaporation a la Hawking ('75)

- But $\kappa(u) \to \infty$ for $M(u) \to 0$!
- Here we'll explore what happens in this, the most extremal case

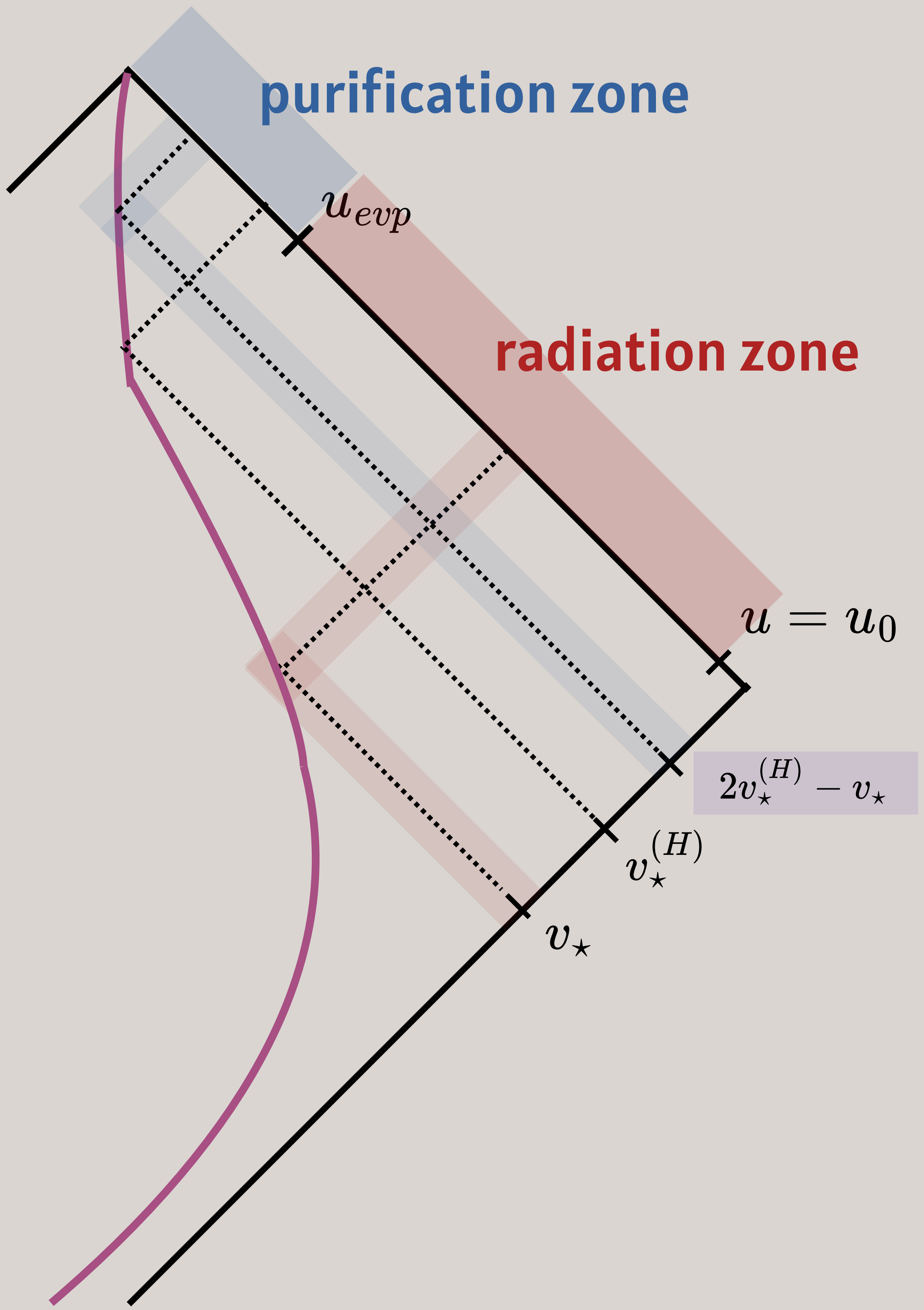
new physics to smoothen the transition?

- Imposing continuity, we find the value of the constants

$$p(u) = v_{evp} + 4\dot{v}_0 e^{-M_0/8\alpha}(u - u_{evp})$$



Purifiers/ Partner Modes



- **Purifier modes = Purify the thermal quanta arriving at $\mathcal{I}^+$**

  where are they?

- **When the mirror trajectory is exponential, we know where the partners will be** (Wald '75)

- **Purifier modes are centered at** $2v_\star^{(H)} - v_\star$

$$v_\star^{(H)} \equiv v_\star + \kappa_\star^{-1}\, \dot{v}_\star \longrightarrow \quad v_\star^{(H)} > v_{evp}, \quad u_\star^{(H)} > u_{evp}$$

instantaneous "would-be horizon"

- **All purifier modes get reflected only once the mirror trajectory is inertial**

This implies that:

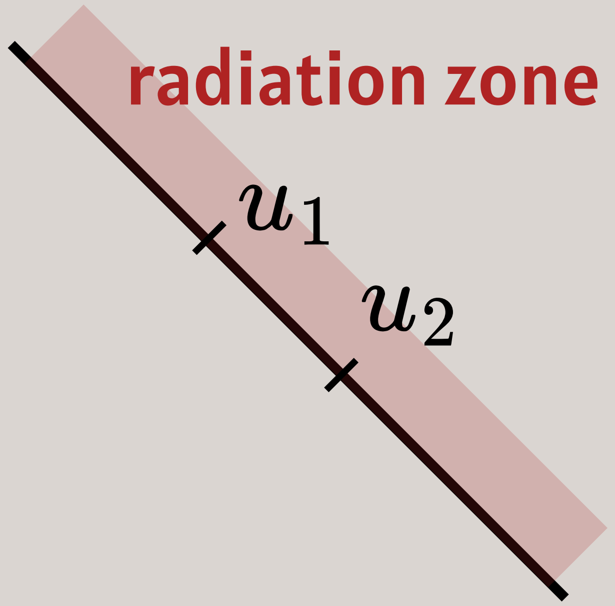
1. Late-time vacuum fluctuations purify the thermal radiation
2. Hawking radiation gets purified without energy costs!

Concrete example of ideas first posed by Hotta, Schützhold & Unruh in 2015

Two-Point Functions

(no wave-packets, no tails)

Two-Point Functions

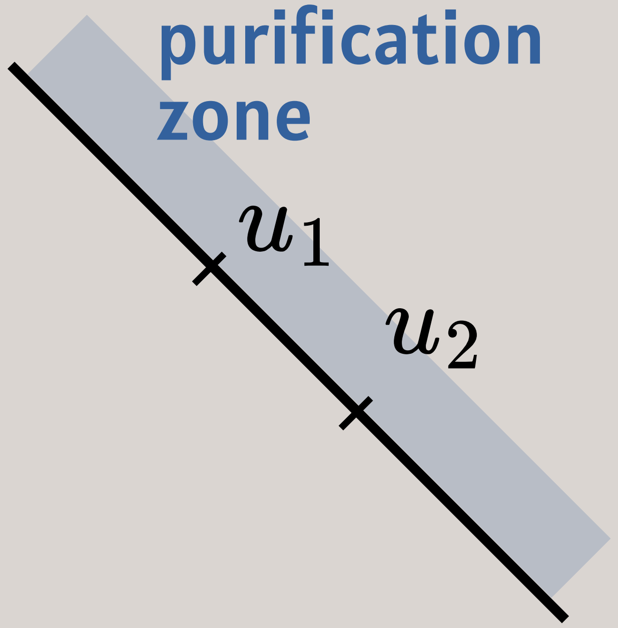


- For massless scalar field in (1+1) $\langle 0|\phi(x')\phi(x)|0\rangle$ is infrared divergent
- Meaningful interpretation in terms of $\partial_\mu \phi$

thermal two-point function

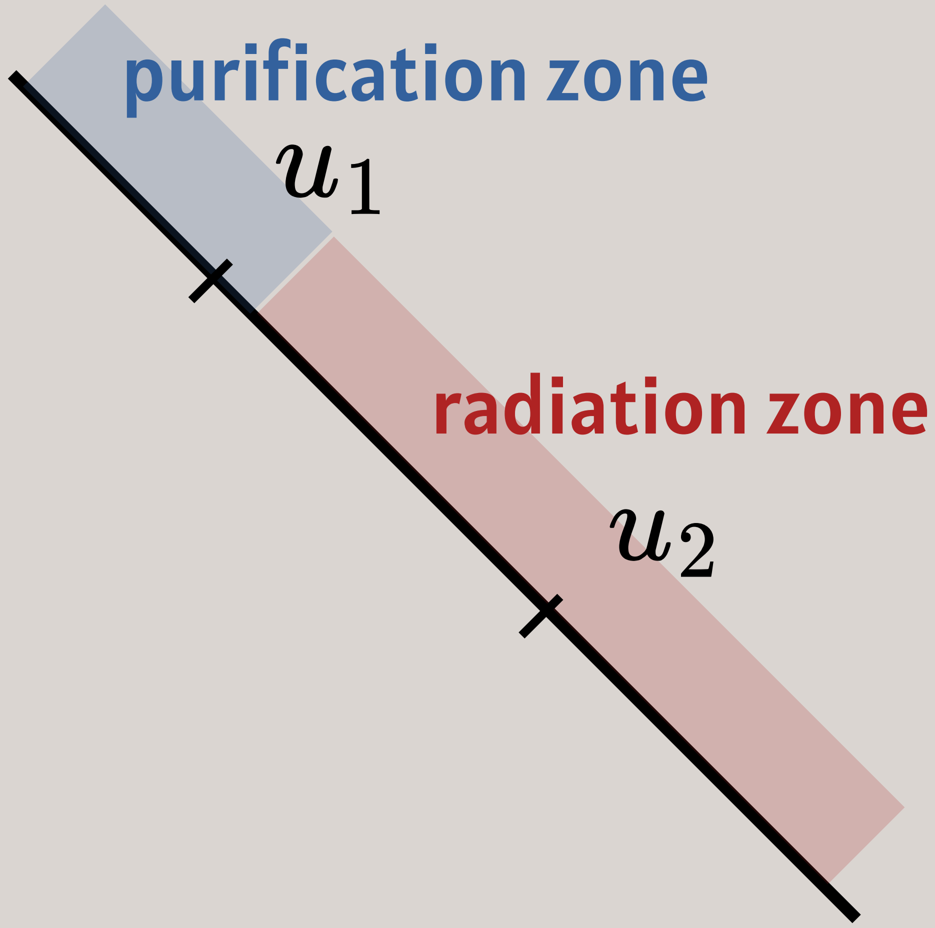
$$\langle 0_{in}|\partial_u\phi(u_1)\partial_u\phi(u_2)|0_{in}\rangle = \begin{cases} -\frac{1}{4\pi}\frac{(\kappa_\star/2)^2}{\sinh^2(\frac{\kappa_\star}{2}(u_1-u_2))} & u_1, u_2 < u_{evp} \text{ (locally)} \\ -\frac{1}{4\pi}\frac{1}{(u_1-u_2-i\epsilon)^2} & u_1, u_2 > u_{evp} \end{cases}$$

a local observer cannot distinguish this from the vacuum



We arrive to this without using wavepackets; therefore, we remove the issue of tails

- Using two-point functions, we can also show that indeed the purifier modes purify the "thermal" modes



$$|\langle 0_{in}|\partial_u\phi(u_1)\ \partial\phi(u_2)|0_{in}\rangle| >> |\langle 0_{out}|\partial_u\phi(u_1)\ \partial\phi(u_2)|0_{out}\rangle|$$

Correlations in the in vacuum are much stronger than they would be in the out state. These correlations purify the full state

Conclusions

1. This moving mirror trajectory produces, locally at $\mathcal{I}^+$, approximately thermal radiation with mass evaporation as proposed by Hawking.

$$T(u) = \frac{1}{8\pi M(u)} \qquad M(u) = M_0(1 - \frac{3\alpha}{M_0^3}(u - u_0))^{1/3}$$

2. We have found a simple model where Hawking radiation is purified by vacuum fluctuations

3. Purification with vacuum fluctuations indicates purification without energy costs

Poses exciting possibilities for Black Hole physics :)

- Can this specific model mimic a real Black Hole?

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Thanks!