Spinfoams' Bulk-Boundary

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Loops' 24 Conference - Fort Lauderdale

Why Bulk-Boundary ?

Because of Holographic behaviour of Quantum Gravity

Dynamics of region of space-time region faithfully represented by a theory living on the boundary

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BH entropy,
entropy bounds
and area law

No screening of gravity,
Effects always propagate
to boundary… but BHs ??

Diffeomorphism inv,
delocalised observables

Thermodynamics of spacetime,
Einstein eqns from entropy flux

Asymptotic symmetry alg,
boundary symmetry alg

AdS/CFT-like correspondences,
holographic dualities

Why Bulk-Boundary ?

Because of Holographic behaviour of Quantum Gravity

Dynamics of region of space-time region faithfully
represented by a theory living on the boundary

But also simply because

- ↳ Bulk <-> Boundary = basic propagation in (quantum) theory
- ↳ Bulk <-> Boundary = pb of boundary conditions & quantum states
- ↳ Bulk <-> Boundary = basic setting of path integral
- ↳ Bulk <-> Boundary = natural experimental setting

3d Quantum Gravity as Test Ground

Because Topological Field Theory $\longrightarrow$ Naturally Holographic

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Because Topological Field Theory ⟶ Naturally Holographic

↳ No local d.o.f. in bulk ⟶ "Global" d.o.f. in topology and on boundary

Thus perfect to study structure and role of boundary state

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↳ Exactly quantizable → Convergence of approaches to 3d quantum gravity

Thus can work in fully-defined theory and make exact computations for probability amplitudes

Work in Ponzano-Regge model, to learn about spinfoams in general

Ponzano-Regge Path Integral

↳ Discrete path integral: Spinfoams

Prescribes a probability amplitude to every 3d triangulation

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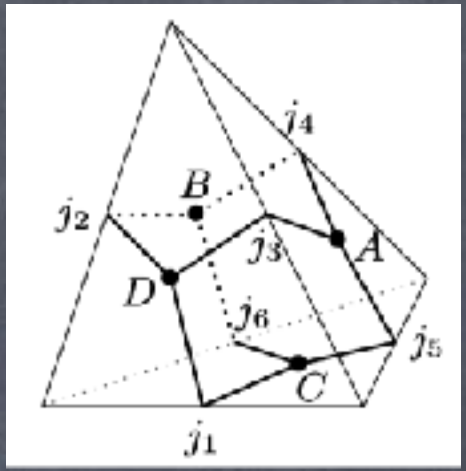
- Extensions: **Versatile formalism** — Lorentzian signature, q-deformation, supergravities, 3d GFTs

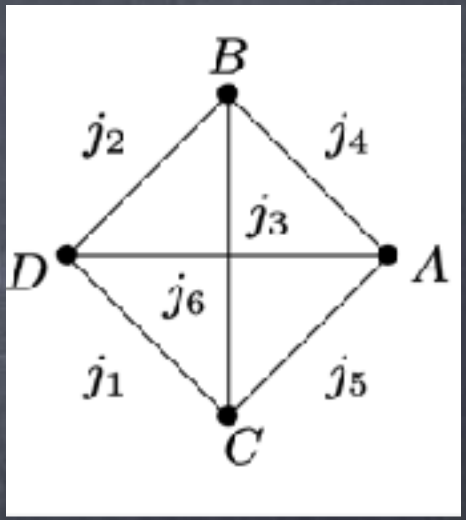
Ponzano-Regge Path Integral in details

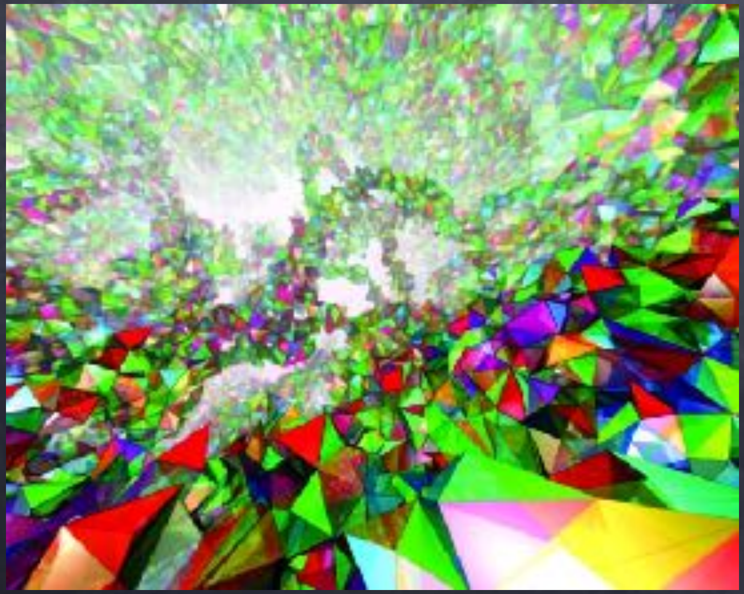
Prescribes a probability amplitude to every 3d triangulation

$$\mathcal{Z}_\Delta = \sum_{\{j_\ell\}} \prod_\ell (-1)^{2j_\ell}(2j_\ell+1) \prod_t (-1)^{\sum_{\ell\in t} j_\ell} \prod_T \{6j\}_T$$



- Spin j on edge $\qquad L = \sqrt{j(j+1)}\, l_{Planck}$
- Intertwiner I on each triangle





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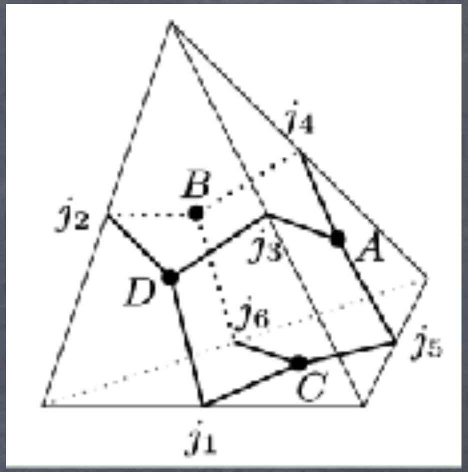

Spin j on edge $L = \sqrt{j(j+1)}\, l_{Planck}$

Defines path integral for quantized Regge calculus

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix} \underset{j_k \gg 1}{\sim} \frac{1}{\sqrt{12\pi V}} \cos\left(S_R[\{j_k\}] + \frac{\pi}{4}\right)$$

Regge action for discrete gravity

$$S_R = \sum_{k=1}^{6} (j_k + \tfrac{1}{2})\theta_k$$

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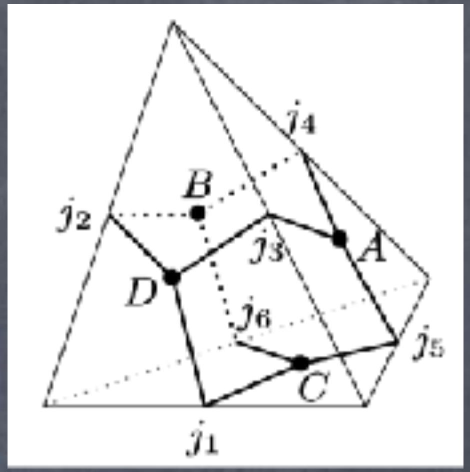

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A LOCALLY HOLOGRAPHIC ANSATZ

$$S_R = \sum_{k=1}^{6} (j_k + \tfrac{1}{2})\theta_k$$

Regge action for discrete gravity

=

Discretization of boundary action !

$\int_{2d} \sqrt{h}\, K = S_{GHY}(3\text{-cell})$

(Regge 1961)

Ponzano-Regge Path Integral in details

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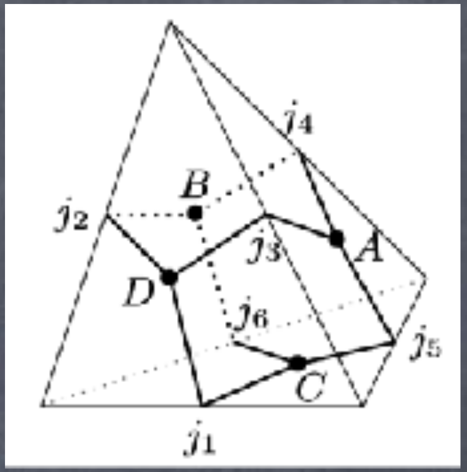

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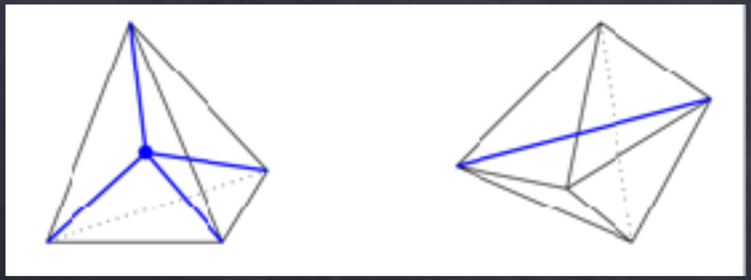
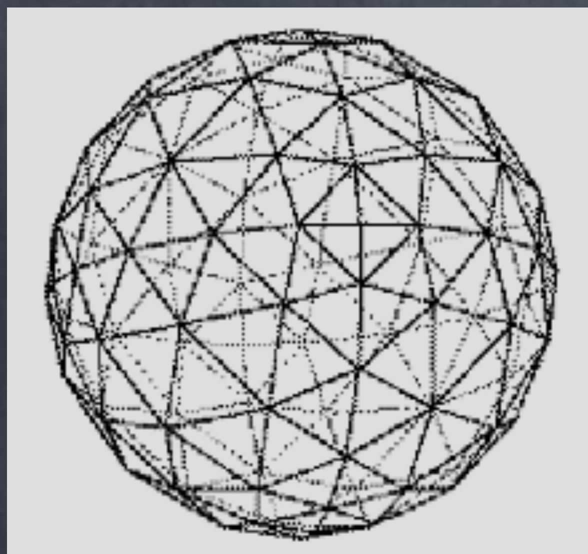
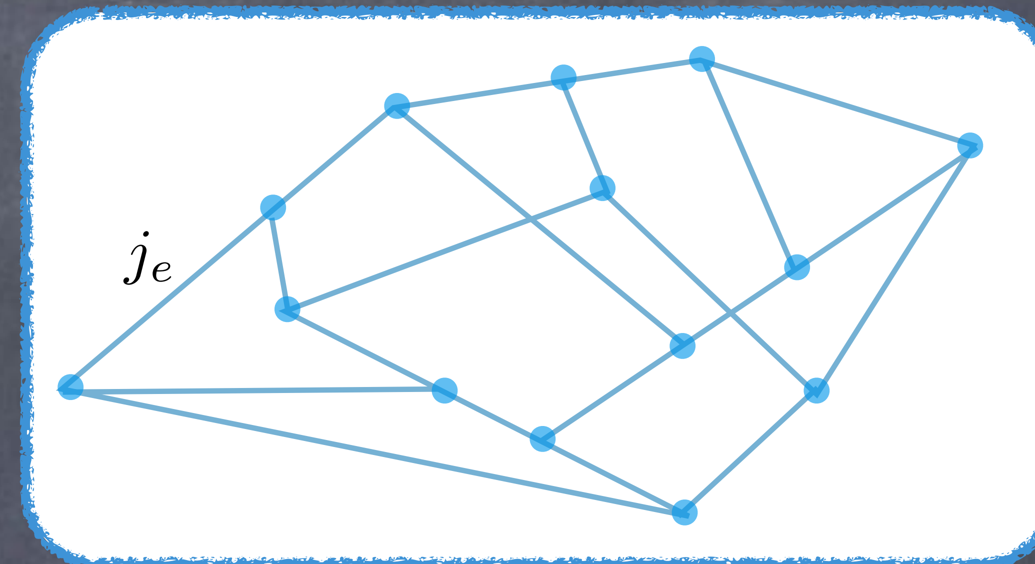
Does not depend on details of triangulation, but only on boundary state !

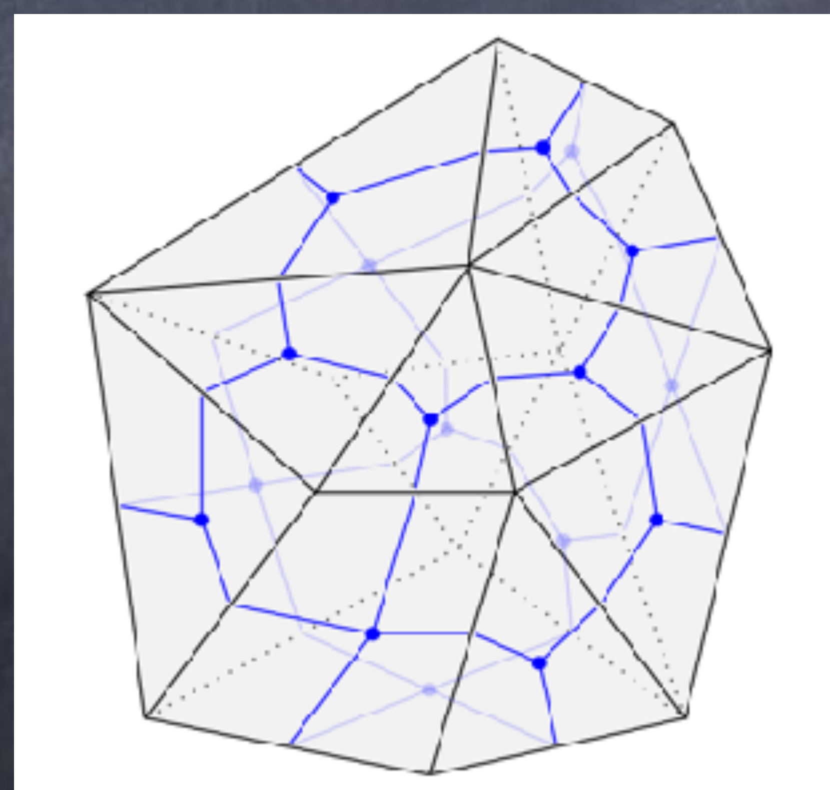


Quantum boundary conditions on the 2-sphere



Boundary state
=
Spin Network
=
Quantum Boundary
Conditions





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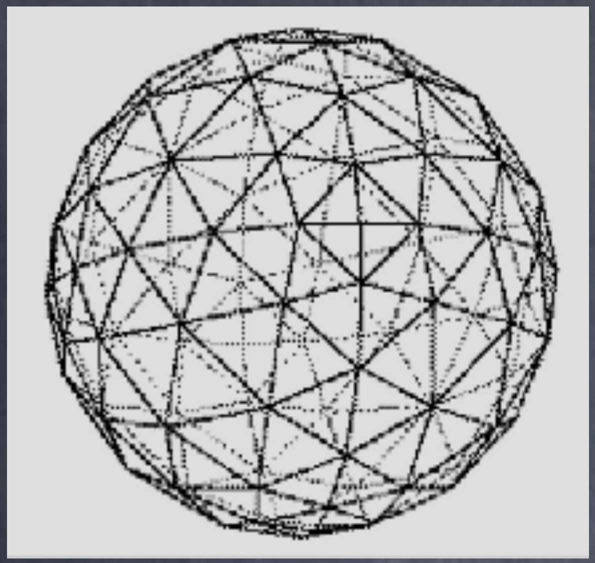

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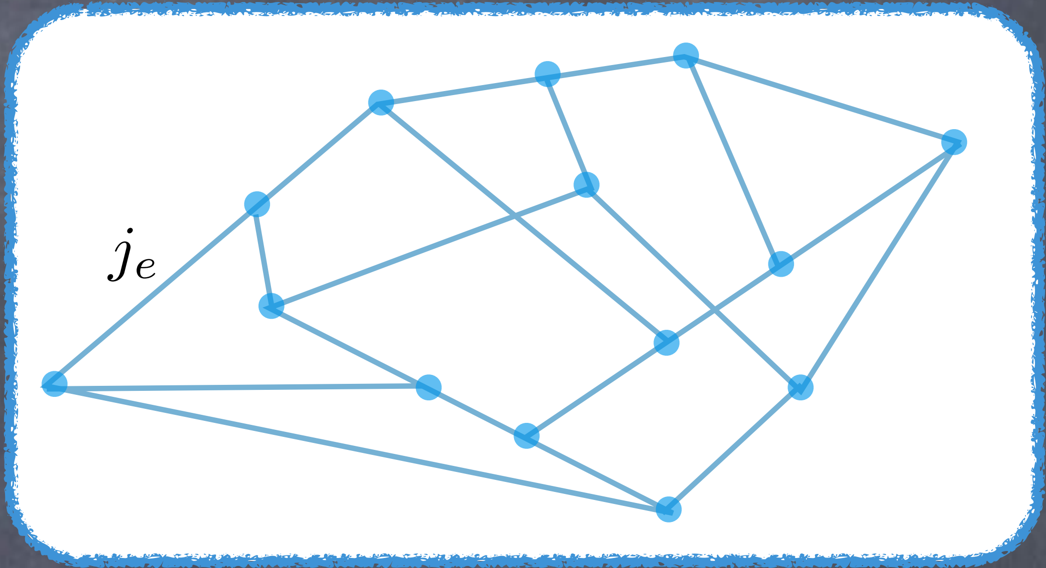

PR Amplitude projects onto physical state = flat connection

So it is given by spin network evaluation :

$$s^\Gamma(\{j_e\}) = \psi^\Gamma_{\{j_e\}}(\mathbb{1}) = \sum_{\{m_e\}} \prod_e (-1)^{j_e - m_e} \prod_v \begin{pmatrix} j_{e_1^v} & j_{e_2^v} & j_{e_3^v} \\ \epsilon^v_{e_1} m_{e_1^v} & \epsilon^v_{e_2} m_{e_2^v} & \epsilon^v_{e_3} m_{e_3^v} \end{pmatrix}$$

Straightforward contraction of Clebsh-Gordan coefficients

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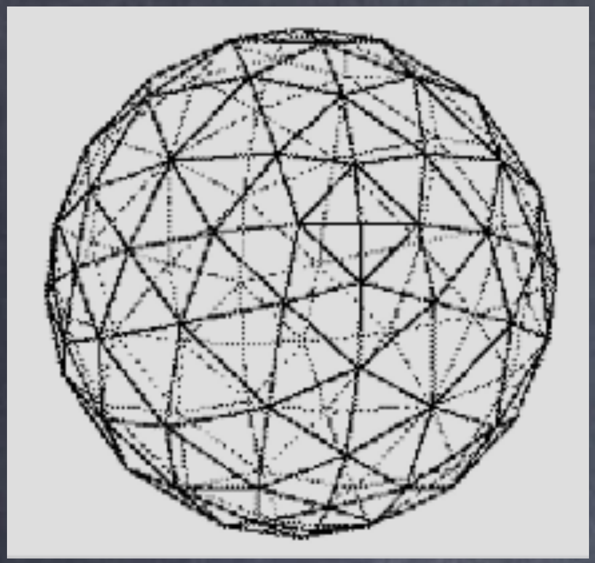

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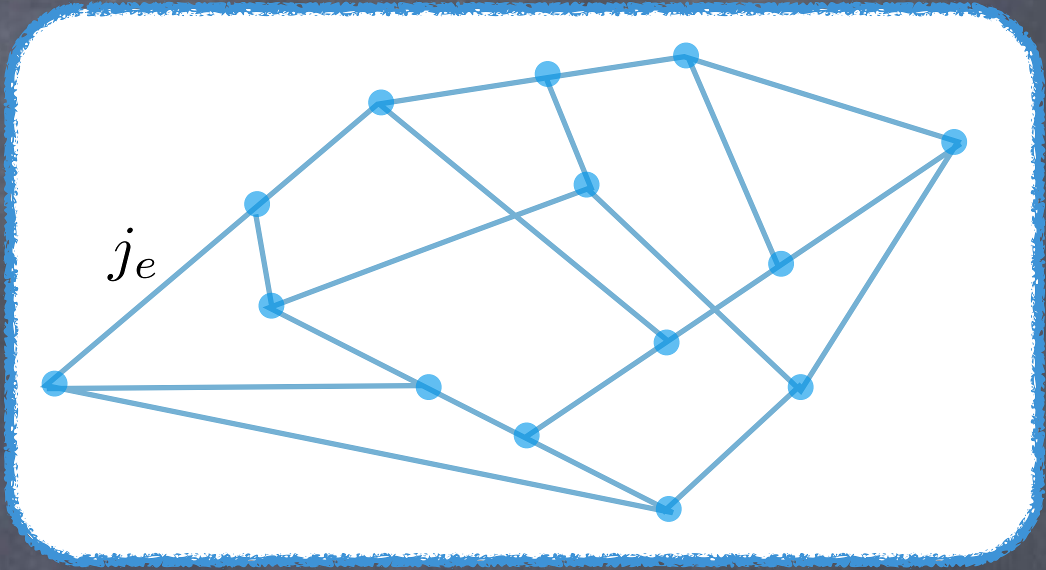
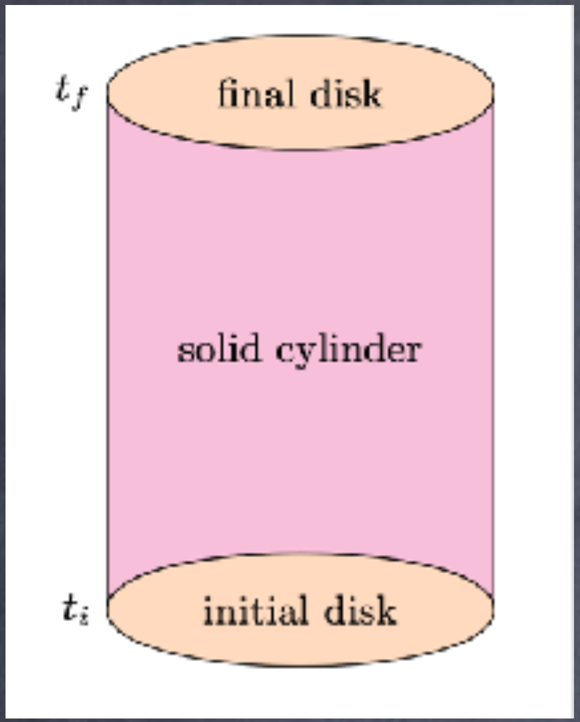


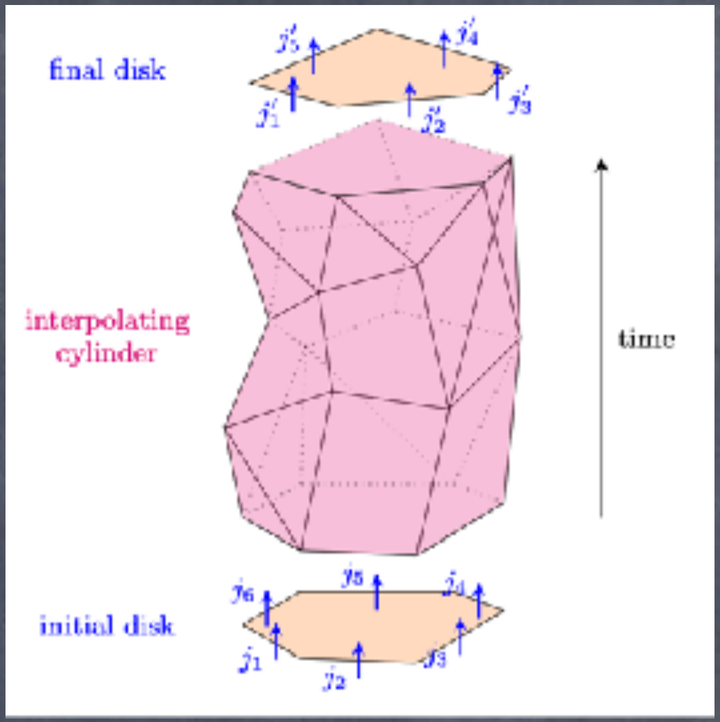
Boundary state does not need to be pure spin networks

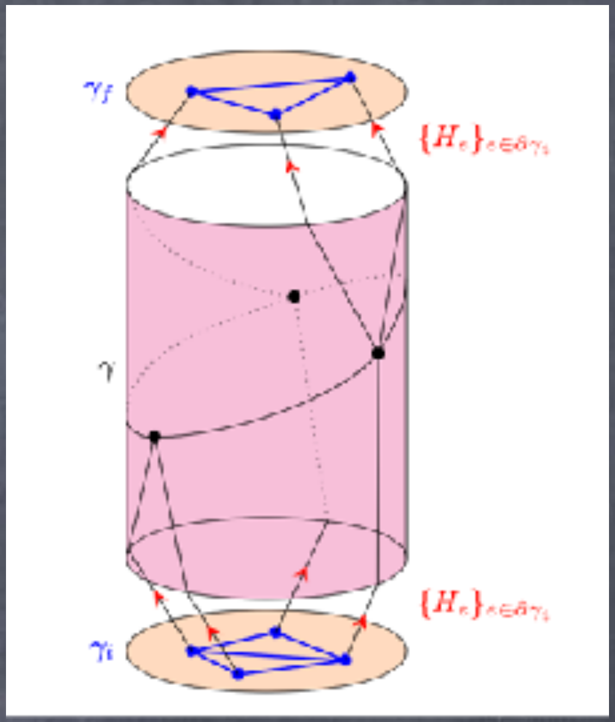
But one can use arbitrary (coherent) superpositions of spins

↳ Possibility to explore how QG amplitudes depend on (type of) quantum boundary conditions

The Ponzano-Regge Propagator







The Ponzano–Regge cylinder and propagator for 3d quantum gravity
2107.03264 [L]

Quasi-local holographic dualities in non-perturbative 3d quantum gravity I
-Convergence of multiple approaches and examples of Ponzano–Regge statistical duals
1710.04202 [Dittrich, Goeller, L, Riello]

Quasi-local holographic dualities in non-perturbative 3d quantum gravity II
– From coherent quantum boundaries to BMS3 characters
1710.04237 [Dittrich, Goeller, L, Riello]

Non-Perturbative 3D Quantum Gravity: Quantum Boundary States
and Exact Partition Function 1912.01968 [Goeller, L, Riello]

Spin selection, transfer matrix, quantum gates

Mapping to 6-vertex model

Recovers BMS3 characters, link with flat space holography

But let's stay on the sphere today

and leave the torus for
future exploration

3d Quantum Gravity - 2d Ising duality

Introduce interesting class of coherent states:

$$Z_\Gamma^{PR}[\{Y_e\}] = \sum_{\{j_e\}} \sqrt{\frac{\prod_v (J_v+1)!}{\prod_{e,v}(J_v-2j_e)!}} \prod_e Y_e^{2j_e}\, s_\Gamma(\{j_e\})$$

Coherent state parameters

Combinatorial weight

Controls Poisson-like distribution for spins

PR amplitude for pure spin networks

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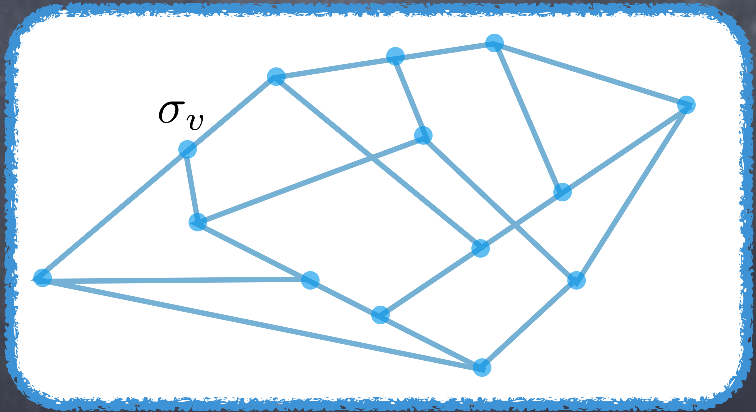
Compute PR amplitude:

$$Z_\Gamma^{PR}[\{Y_e\}] \propto \frac{1}{Z^{Ising}[\{y_e\}]^2}$$

$$Y_e = \tanh y_e$$



$$\sigma_v = \pm 1 \in \mathbb{Z}_2$$

$$Z_\Gamma^{Ising}(\{y_e\}) = \sum_\sigma \exp\left(\sum_e y_e \sigma_{s(e)}\sigma_{t(e)}\right)$$

2d Ising zeroes from Quantum Gravity

Get Ising zeroes (critical couplings) from (quantum) geometry

as 2d triangulations embedded in 3d flat space

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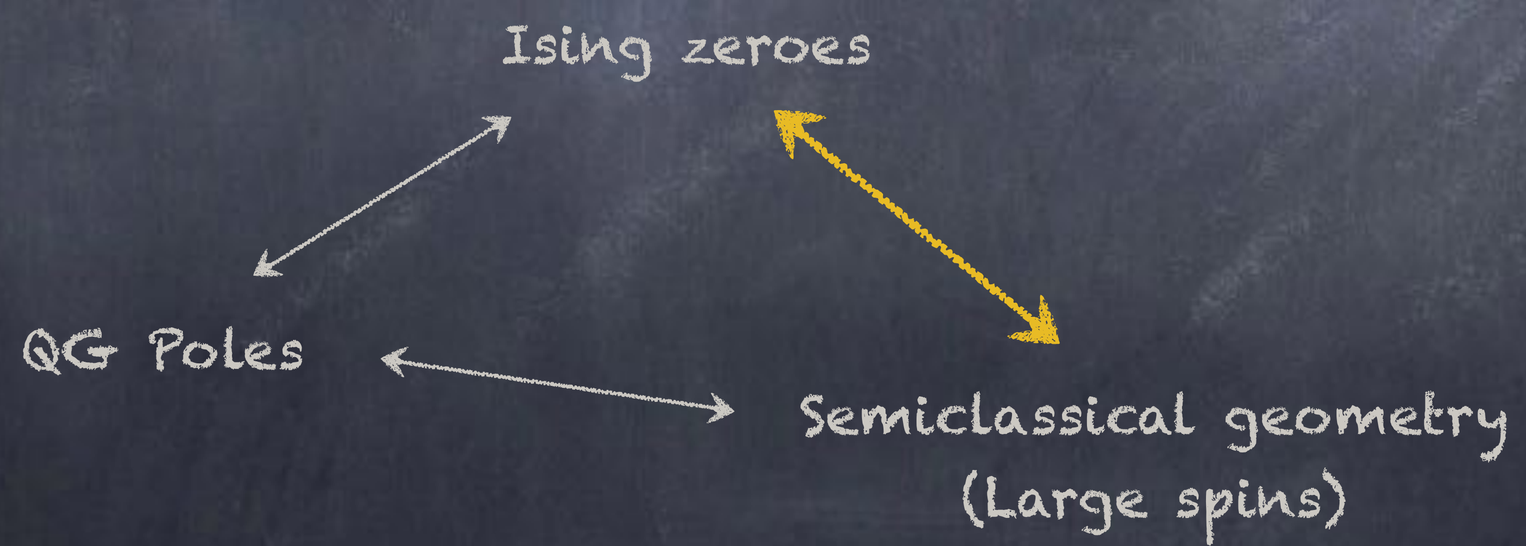

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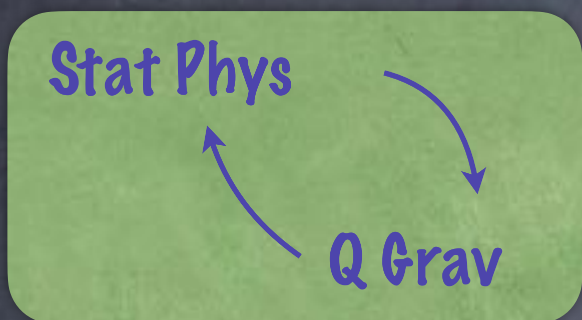
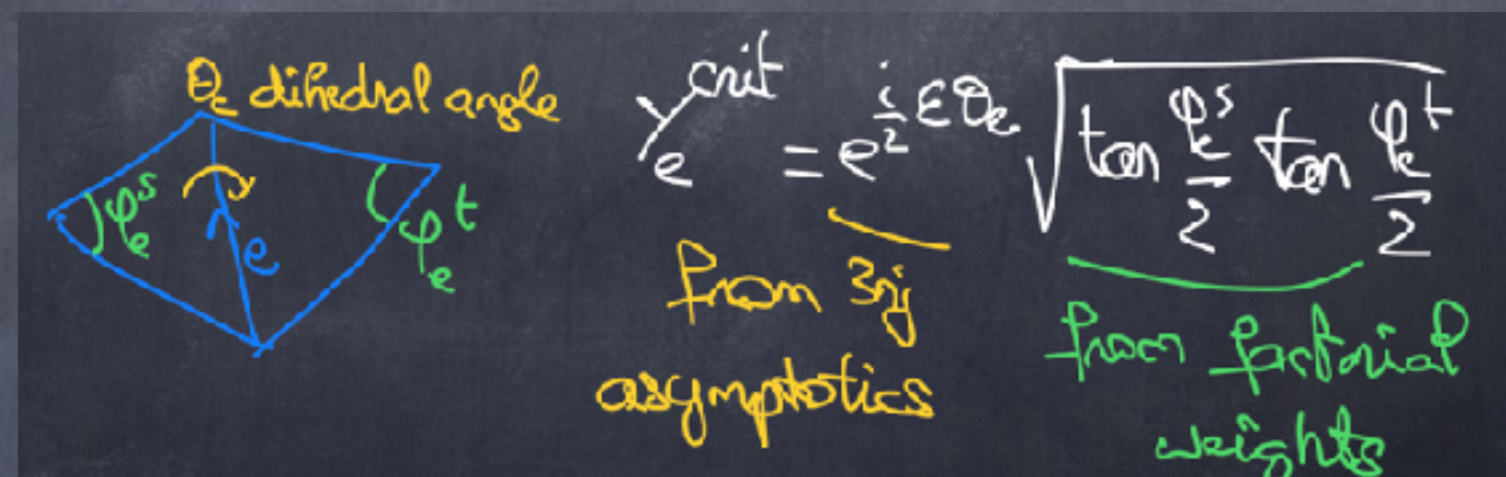
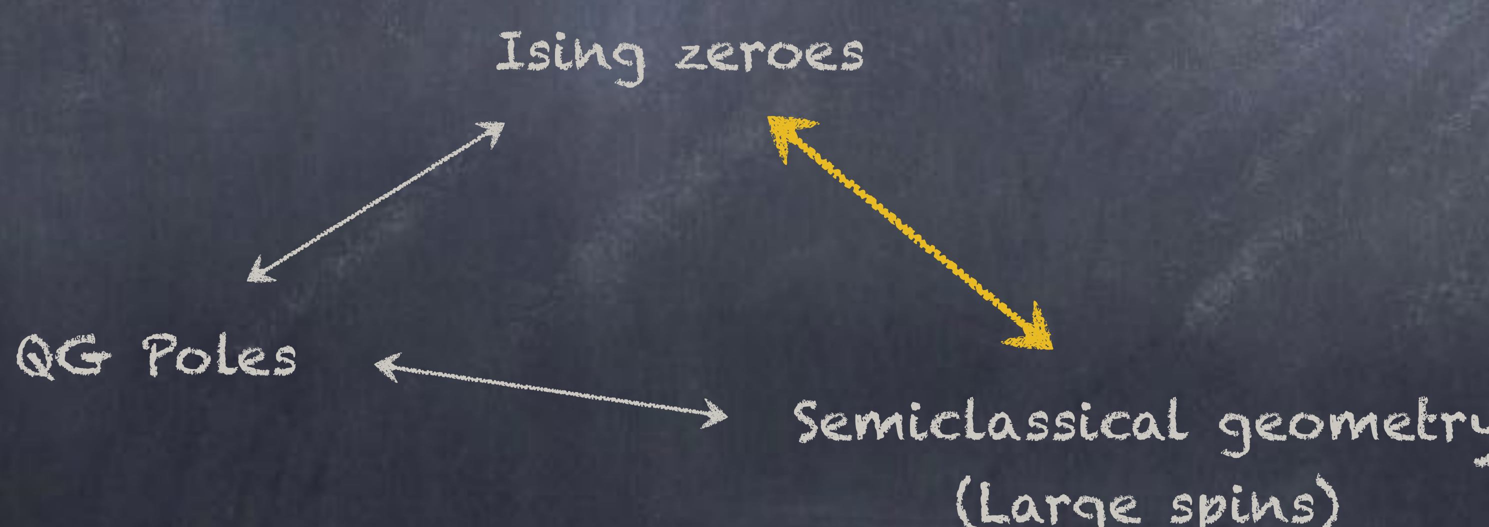


2405.01253 [Bonzom, L]





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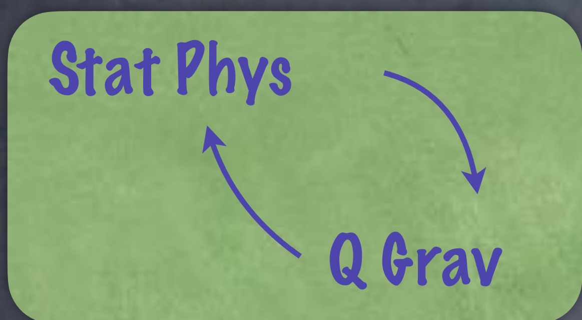

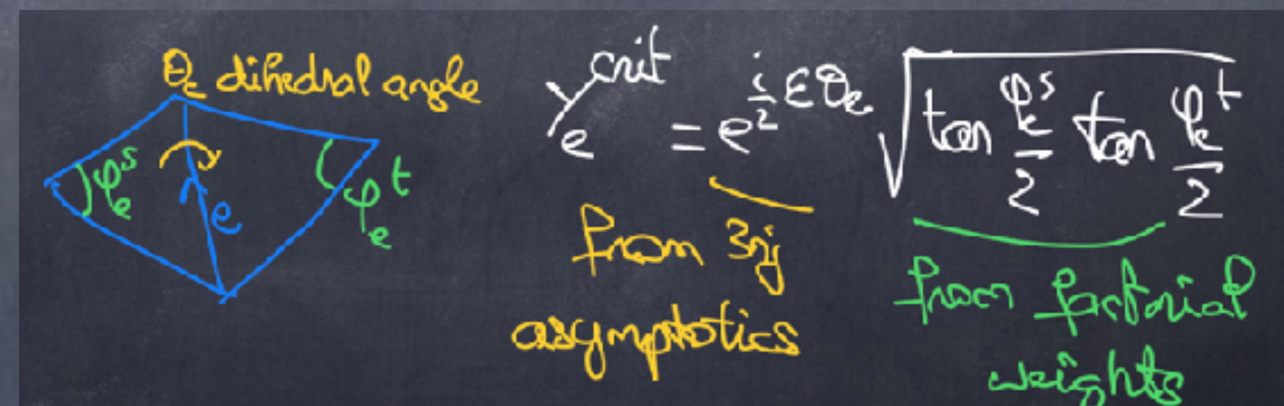

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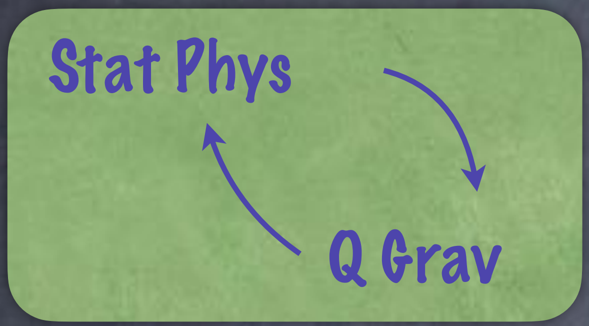
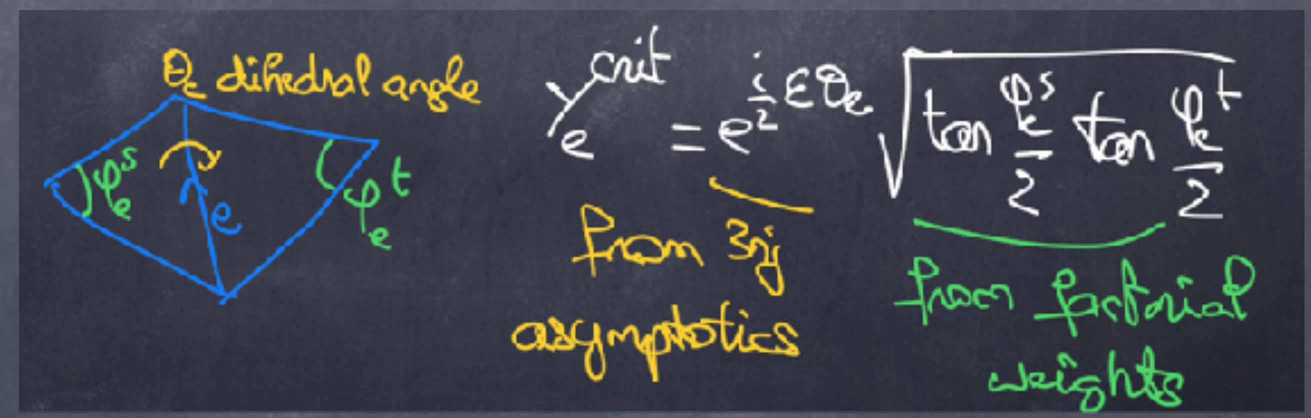
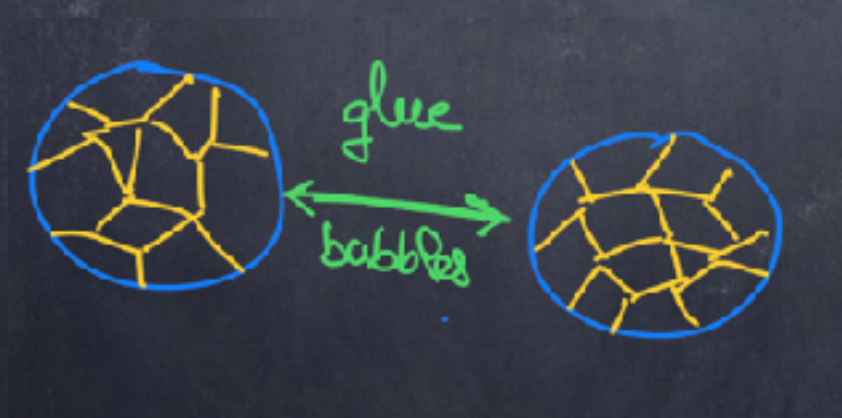




BUT MORE … 3d QG from fusion of non-critical conformal blocks



As topological network of Ising bubbles !

↓

Always remains a boundary theory

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Get Ising zeroes (critical couplings) from (quantum) geometry

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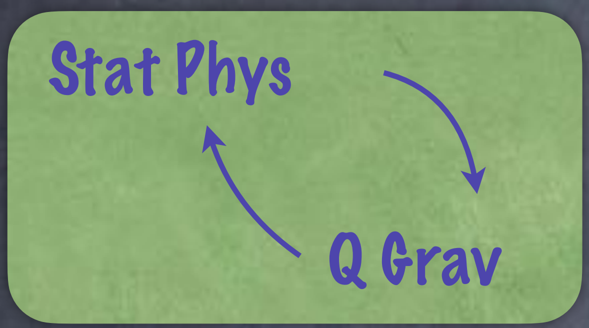

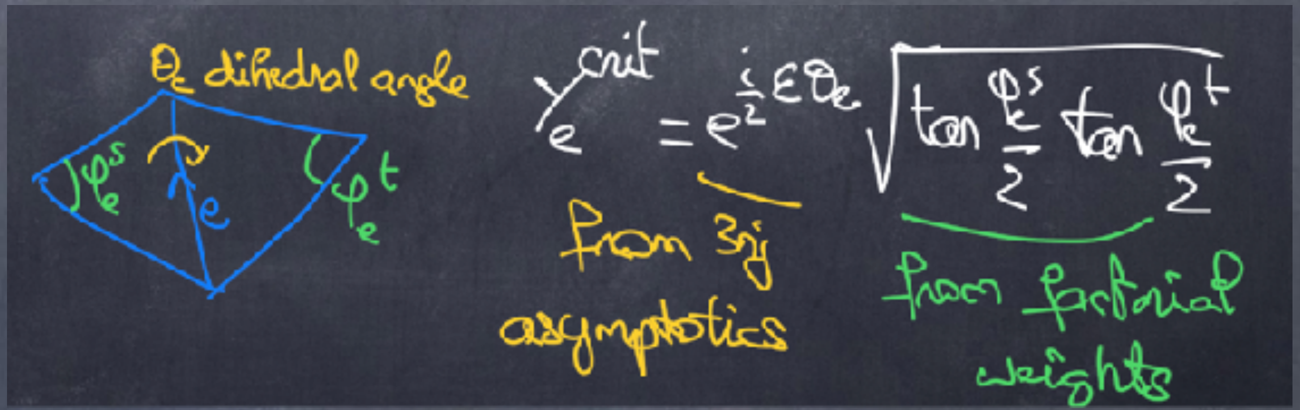

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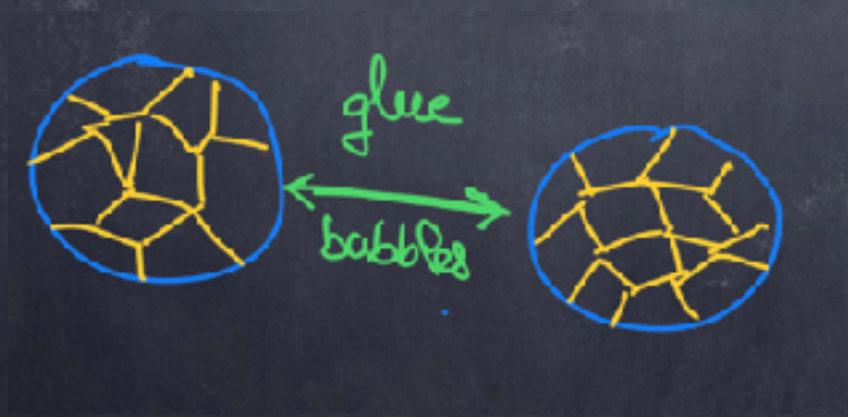

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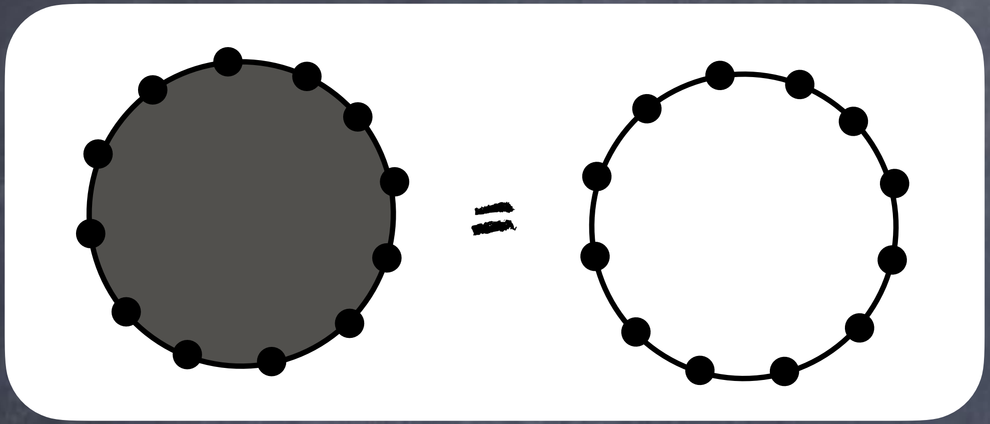
Bulk Coarse-graining -> Boundary refinement & renormalization

What's going on ?

Bulk from Boundary - Boundary from Bulk

Can build TQFT from boundary theory: Example of 1d Ising -> 2d



$$\mathcal{Z}[y_1, y_2, ..] \equiv Z_\partial[y_1, y_2, ..] = \sum_{\{\sigma_v = \pm\}} e^{\sum_k y_k \sigma_k \sigma_{k+1}}$$

Bulk theory

Boundary data

Boundary theory

Boundary couplings

"Edge modes"

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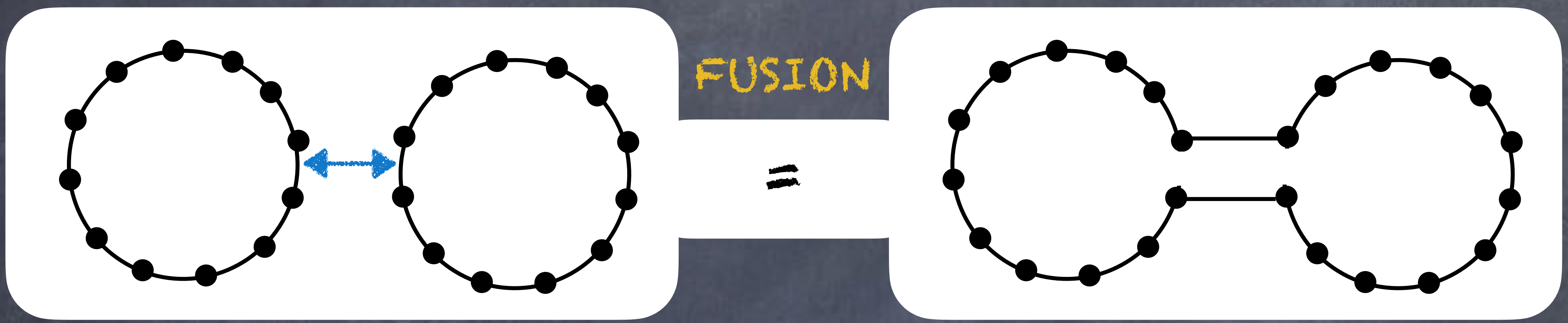

$$Z_\partial[y_1, y_2, ..] = \sum_{\{\sigma_v = \pm\}} e^{\sum_k y_k \sigma_k \sigma_{k+1}} = \prod_k \cosh y_k + \prod_k \sinh y_k$$

$$Z_\partial[y_1, y_2, ..] \# Z_\partial[y_1, \tilde{y}_2, ..] = Z_\partial[y_2, .., \tilde{y}_2, ..]$$

$$Z_\partial[y_1, y_2, ..] \# Z_\partial[y_1, \tilde{y}_2, ..] = \oint \cos 2\theta \, \mathrm{d}\theta \, Z_\partial[e^{i\theta}, y_2, ..] \# Z_\partial[e^{i\theta}, \tilde{y}_2, ..]$$

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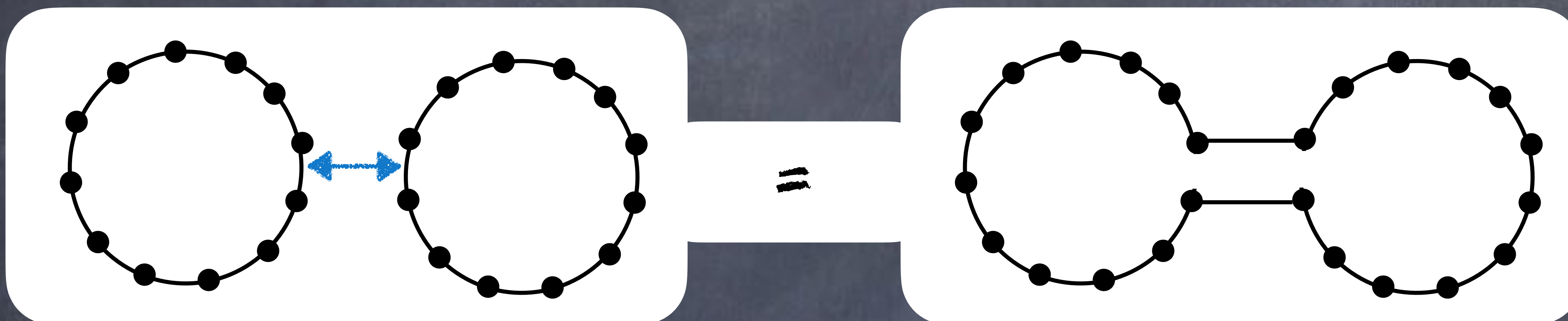
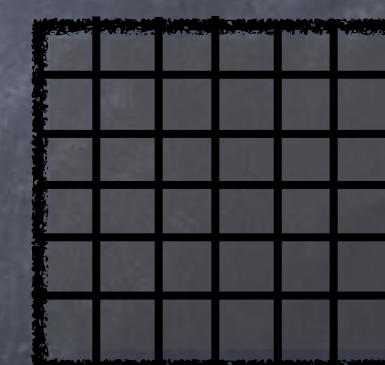


Coarse-grain to compute bulk amplitude from global boundary

FUSE

DE-FUSE



$\mathcal{Z} = Z_\partial$

Refine to get bulk as network of boundary theories

$$\mathcal{Z} = Z_\partial \# Z_\partial \# Z_\partial \# Z_\partial \# \dots$$

$$= \int \prod_e \mathrm{d}y_e \sum_{\{\sigma_v, f\}} \dots$$

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Can have non-trivial fusion: Add coupling to boundary theory

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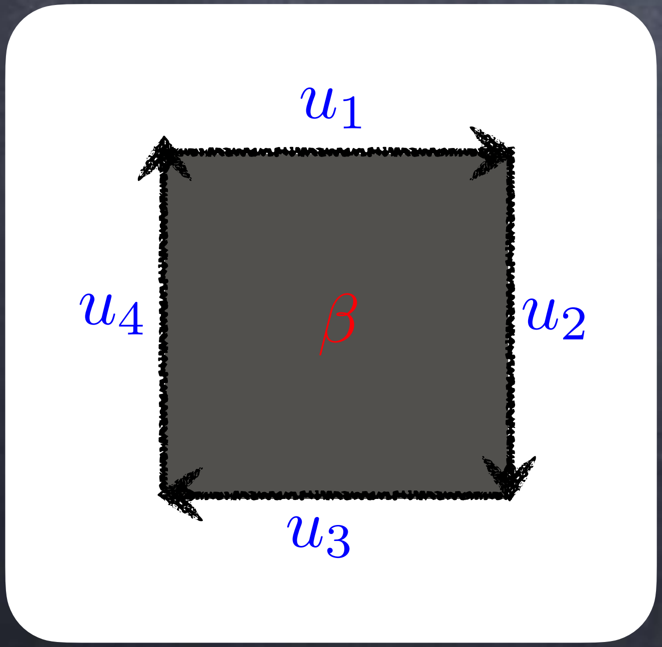
$$\mathcal{Z}^{(\beta)}[\{u_k\}] = Z_\partial^{(\beta)}[\{u_k\}] = e^{-\beta(\overrightarrow{\sum_k} u_k)^2}$$



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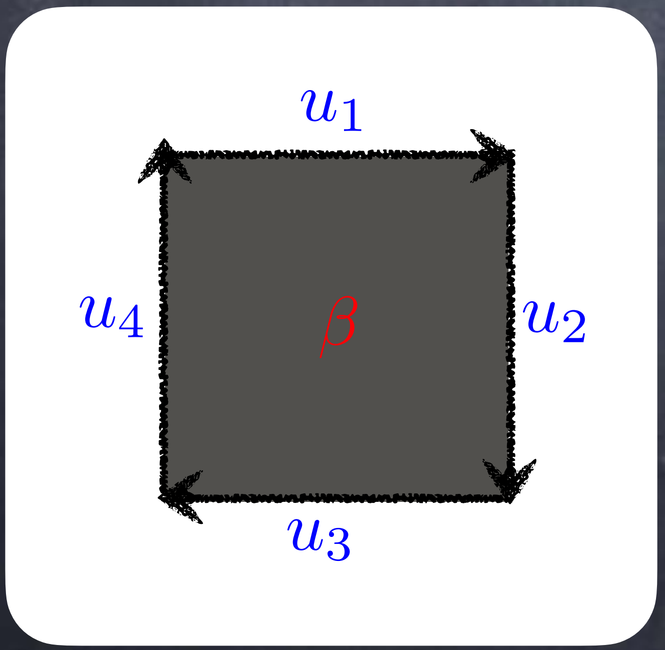

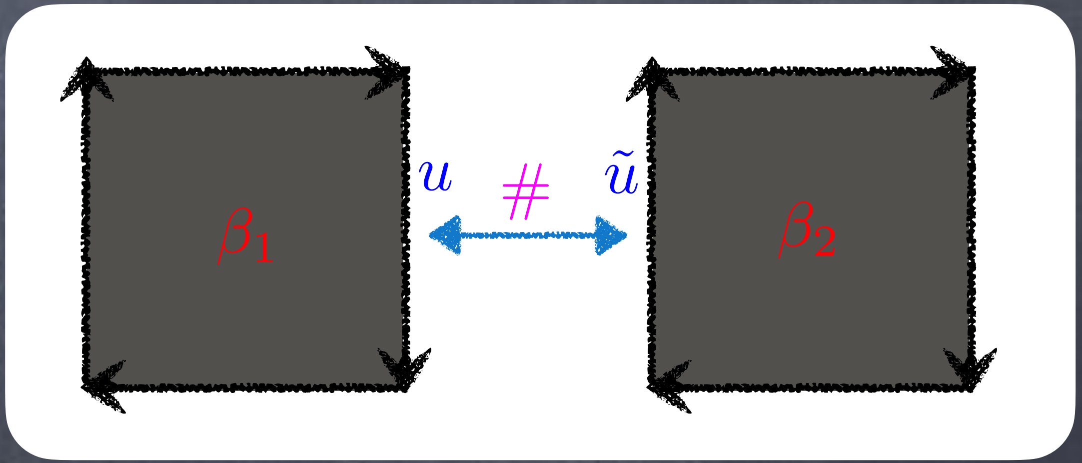

$$Z_\partial^{(\beta_1)} \#_{u\leftrightarrow\tilde{u}} Z_\partial^{(\beta_2)} = \int \mathrm{d}u\, Z_\partial^{(\beta_1)}[u,\{u_k\}] Z_\partial^{(\beta_2)}[u,\{\tilde{u}_k\}]$$

$$= Z_\partial^{(\beta)}[\{u_k,\tilde{u}_k\}]$$

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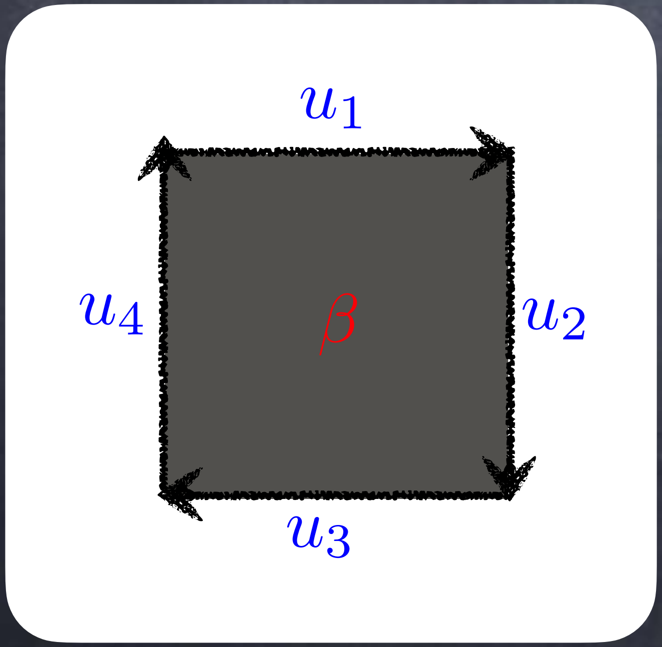

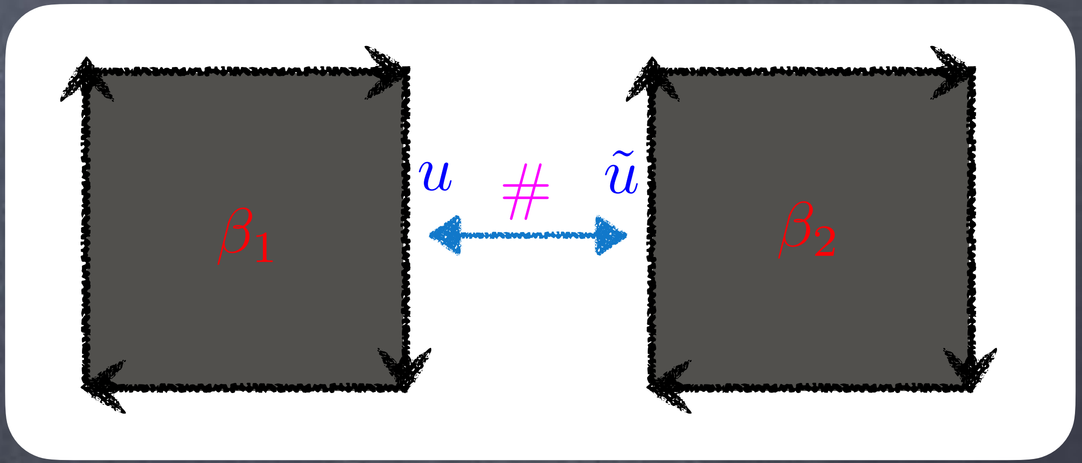

$$Z_\partial^{(\beta_1)} \# Z_\partial^{(\beta_2)} = Z_\partial^{(\beta)} \quad \text{with} \quad \frac{1}{\beta} = \frac{1}{\beta_1} + \frac{1}{\beta_2}$$

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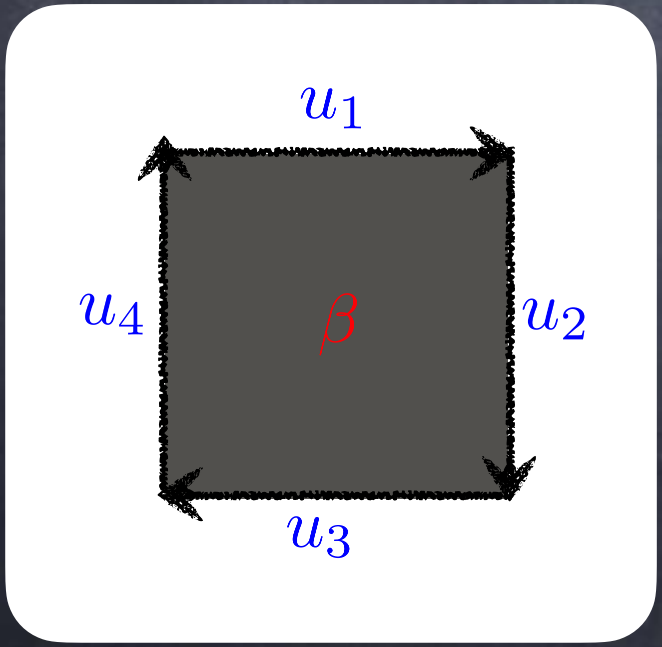

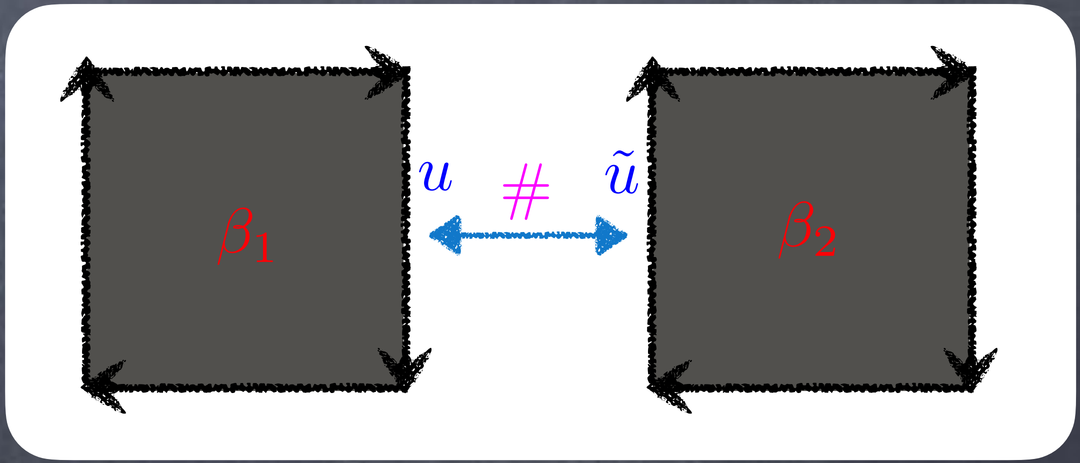

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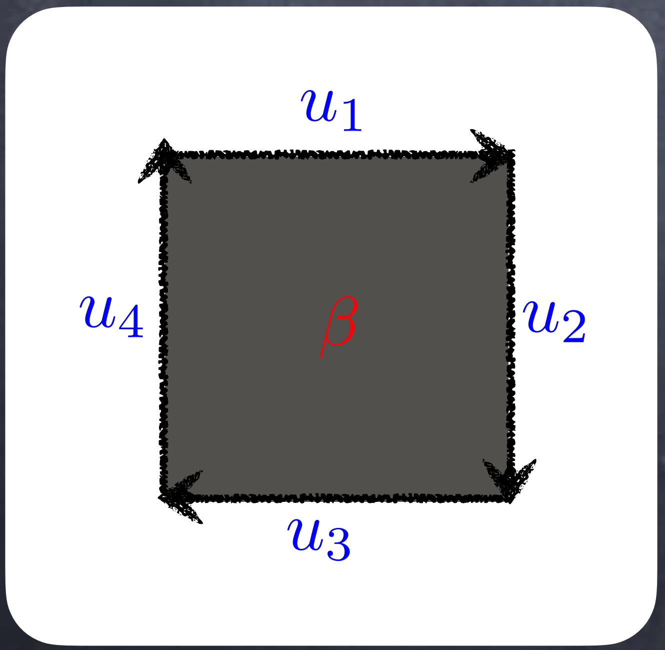

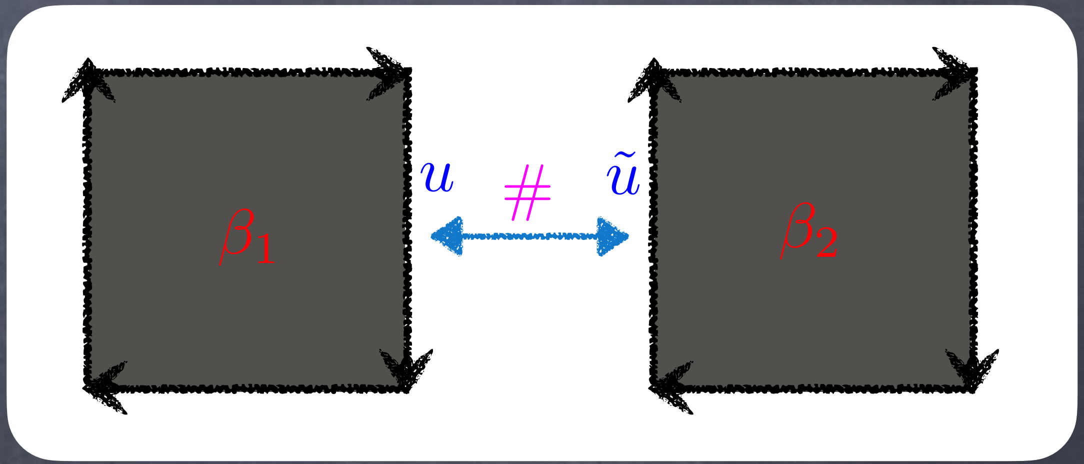

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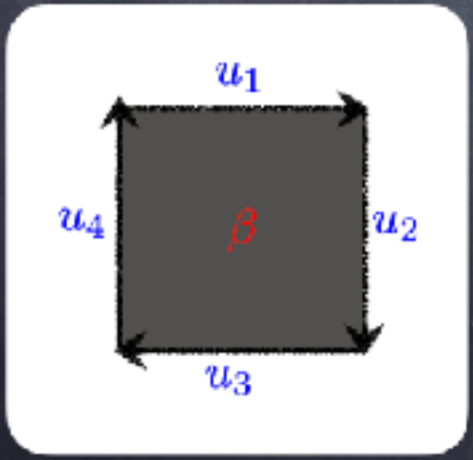

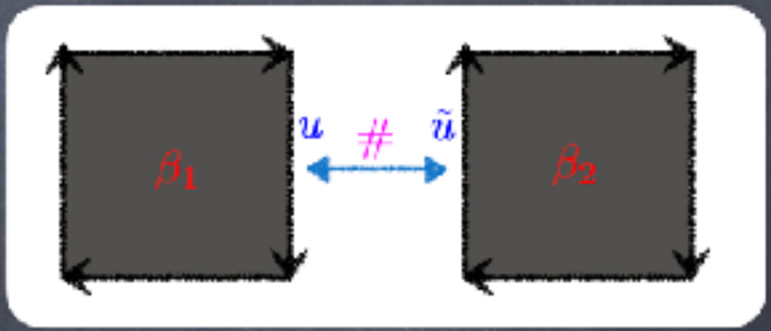

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BTW that was 2d BF with B^2 potential

Simplest example of spinfoam renormalisation

Lessons

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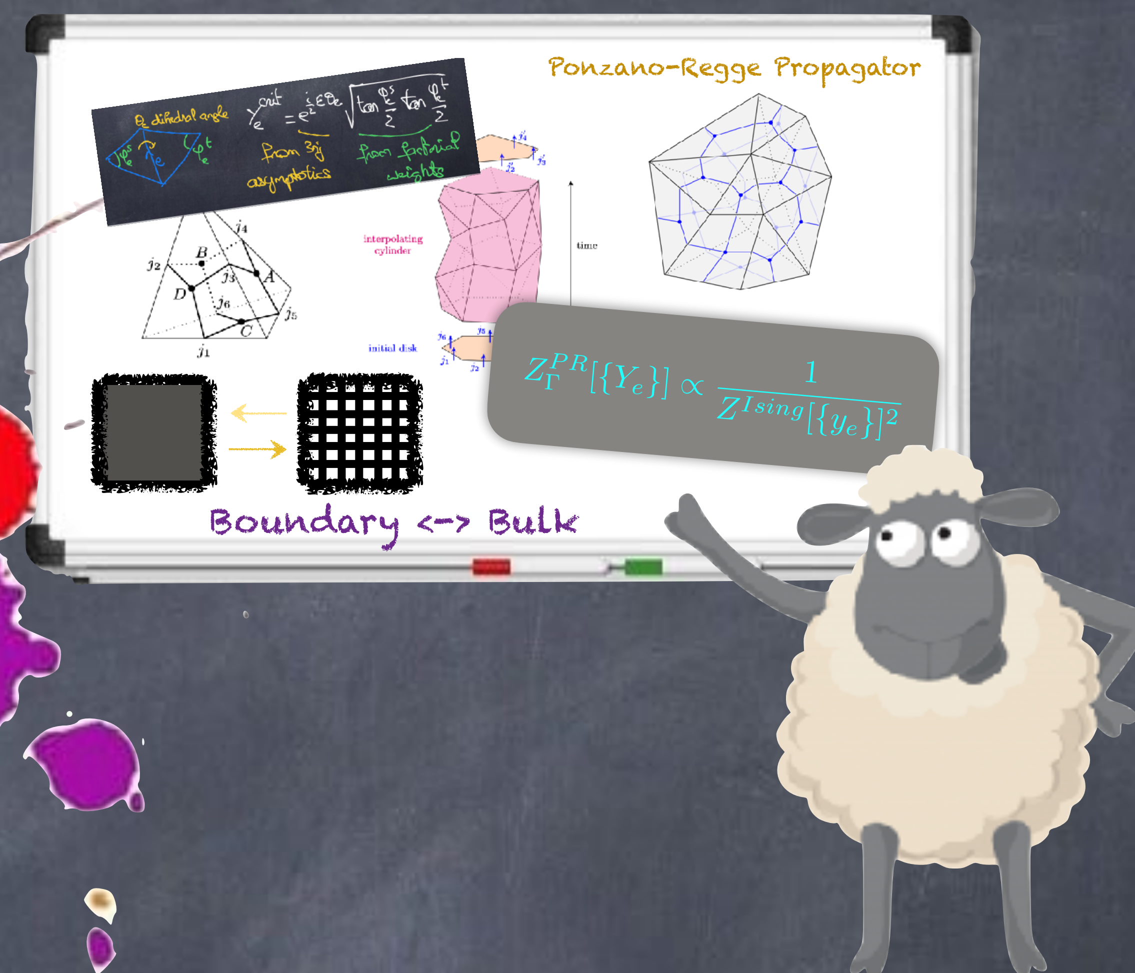
Now extend PR work to higher dimensions (3d, 4d)

To study interplay between
renormalisation & holography
in spinfoam models


Merci!


Ponzano-Regge Propagator
$Z_\Gamma^{PR}[\{Y_e\}] \propto \frac{1}{Z^{Ising}[\{y_e\}]^2}$
Boundary <-> Bulk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LOOPs