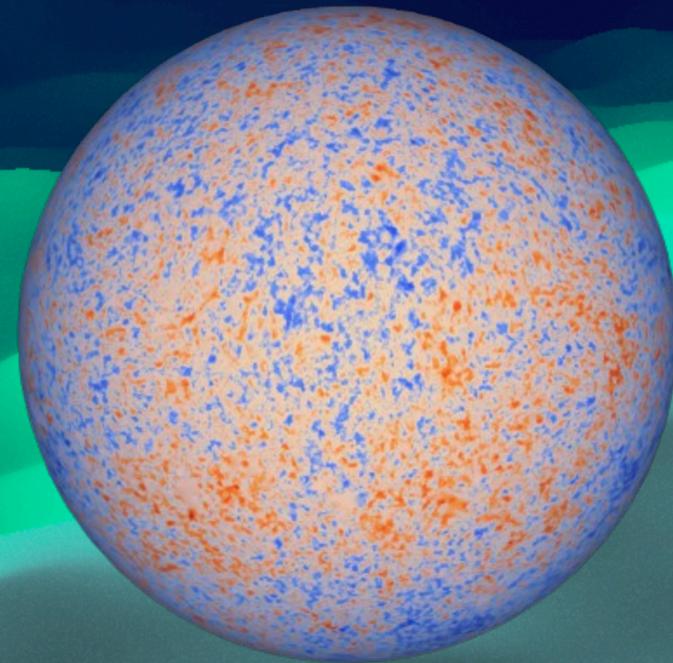


PRECISION COSMOLOGY AS A PROBE OF QUANTUM GRAVITY

Mauricio Gamonal
Physics PhD student at Penn State

Based on work in collaboration with Eugenio Bianchi [arXiv: 2405.03157]



Loops' 24 Conference - Florida Atlantic University, Fort Lauderdale, FL — May 7, 2024



PennState



QISS

THE QUANTUM INFORMATION
STRUCTURE OF SPACETIME



FULBRIGHT
Chile

THE BIG PICTURE

[MG PhD Thesis '26?]

Loop Quantum Cosmology

[Agullo, Ashtekar & Nelson '12]

[Fernandez-Mendez, Mena & Olmedo '12]

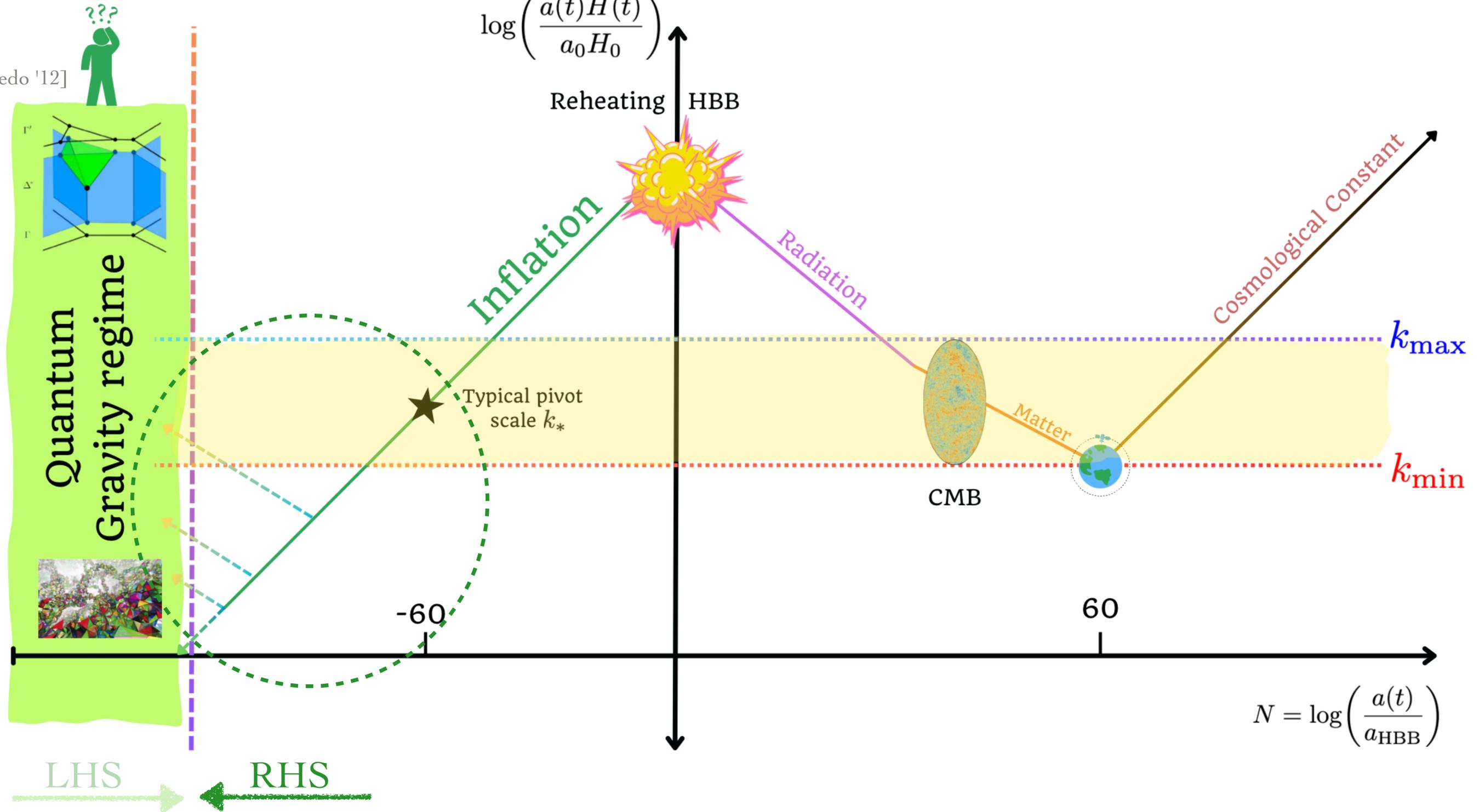
Spinfoam Cosmology

[Bianchi, Rovelli & Vidotto '10]

Cosmology from GFT

[Gielen, Oriti & Sindoni '13]

⋮



THE FRAMEWORK: A LARGE CLASS OF EFFECTIVE THEORIES OF INFLATION

Ingredients: Action Principle

1. A gravitational action (matter not required):

$$S = S[g_{\mu\nu}, \chi] \quad \text{with} \quad \begin{aligned} g_{\mu\nu}(\mathbf{x}, t) &= \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(\mathbf{x}, t) \\ \chi(\mathbf{x}, t) &= \bar{\chi}(t) + \delta\chi(\mathbf{x}, t) \end{aligned}$$

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2. Small perturbations (on a flat FLRW background):

Ψ : Any dynamical SVT mode

$$S_{\Psi}^{(2)}[\Psi] = \frac{1}{2} \int d^4x \, Z_{\Psi}(t) a(t)^3 \left(\dot{\Psi}^2 - \frac{c_{\Psi}(t)^2}{a(t)^2} (\partial_i \Psi)^2 \right)$$

Kinetic amplitude

Scale factor

Speed of sound

c.f. single scalar field: $Z_s(t) = \frac{\epsilon_{1H}(t)}{4\pi G}$, $Z_t(t) = \frac{1}{64\pi G}$, $c_t(t) = c_s(t) = 1$

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Theory	$Z_s(t)$	$c_s(t)$	$Z_t(t)$	$c_t(t)$
Single-field [22]	Eq. (74)	1	Eq. (75)	1
$R + \alpha R^2$ [23]	Eq. (96)	1	Eq. (99)	1
K -inflation [24]	✓	✓	✓	1
LQC+inflaton [25, 26]	✓	✓	✓	✓
$f(\varphi)$ -Gauss Bonnet [27]	✓	✓	✓	✓
$f(\varphi)$ -Chern Simons [28, 29]	✓	✓	×	×
General scalar-tensor [23]	✓	✓	✓	✓
Goldston mode EFT [20]	✓	✓	✓	✓
Multifield EFT [30]	✓	✓	✓	✓
Minimally broken CFT [31]	✓	✓	✓†	✓†
Weinberg's EFT [32]	✓	✓	✓†	✓†

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$$\epsilon_{1Z}(t) \equiv -\frac{\dot{Z}_{\psi}(t)}{H(t)Z_{\psi}(t)}, \quad \epsilon_{2Z}(t) \equiv -\frac{\dot{\epsilon}_{1Z}(t)}{H(t)\epsilon_{1Z}(t)} \dots$$

⋮

Accelerated expansion: $\epsilon_{1H}(t) < 1$

$$\ddot{a}(t) = (1 - \epsilon_{1H}(t)) a(t) H(t)^2 > 0$$

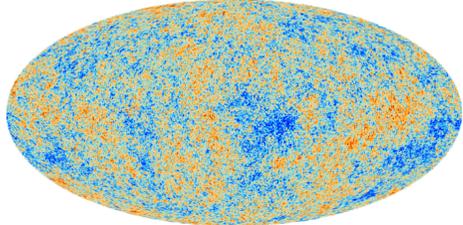
During inflationary epoch: $\epsilon_{n\rho*} \ll 1$

N3LO: $\mathcal{O}(\epsilon_*^3)$

THE FRAMEWORK: A LARGE CLASS OF EFFECTIVE THEORIES OF INFLATION

Methods: Green's function (a systematic expansion computable order-by-order)

[Gong & Stewart '01; Auclair & Ringeval '22; Bianchi & Gamonal '24]

Recall: 

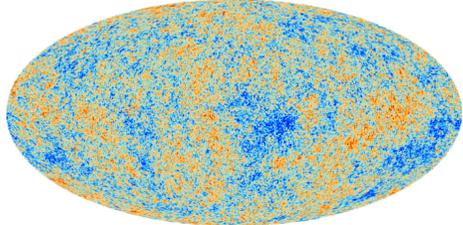
$$\langle 0 | \hat{\Psi}_f(t) \hat{\Psi}_f(t) | 0 \rangle = \int_0^\infty \frac{dk}{k} \frac{k^3}{2\pi^2} |u(k, t)|^2 |\tilde{f}(k)|^2 \longrightarrow u(t) \rightarrow w(y)$$

(Mukhanov-Sasaki+time reparametrization)

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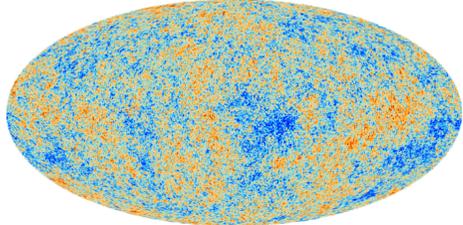
$$w''(y) + \left[1 - \frac{2}{y^2} \right] w(y) = \frac{q(y)}{y^2} w(y)$$

$$q(x) \equiv q_1 + q_2 \ln(x) + q_3 \ln(x)^2$$

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$$\left\{ \begin{array}{l} G(y, s) = \frac{i}{2} (w_0(y)w_0^*(s) - w_0(s)w_0^*(y)) \Theta(s - y) \\ w_0(y) = \left(1 + \frac{i}{y} \right) e^{iy} \quad [\text{Bunch \& Davies '78}] \end{array} \right.$$

$$w(y) = w_0(y) + \int_y^\infty \frac{q(y)}{s^2} w(s) G(y, s) ds = w_0(y) + w_1(y) + w_2(y) + w_3(y) + \mathcal{O}(\epsilon^4)$$

$$w_1(y) = g_1 \int_y^\infty \frac{G(y, s)}{s^2} w_0(s) ds$$

$$w_2(y) = g_2 \int_y^\infty \frac{G(y, s)}{s^2} \ln(s) w_0(s) ds + g_1 \int_y^\infty \frac{G(y, s)}{s^2} w_1(s) ds$$

$$w_3(y) = g_3 \int_y^\infty \frac{G(y, s)}{s^2} \ln(s)^2 w_0(s) ds + g_2 \int_y^\infty \frac{G(y, s)}{s^2} \ln(s) w_1(s) ds + g_1 \int_y^\infty \frac{G(y, s)}{s^2} w_2(s) ds$$

THE RESULTS: PRIMORDIAL POWER SPECTRUM AT N₃LO

$$\mathcal{P}_0^{(\psi)}(k) = \frac{\hbar H_*^2}{4\pi^2 c_{\psi_*}^3 Z_{\psi_*}} \left[1 + p_{0*} + p_{1*} \ln\left(\frac{k}{k_*}\right) + p_{2*} \ln\left(\frac{k}{k_*}\right)^2 + p_{3*} \ln\left(\frac{k}{k_*}\right)^3 \right]$$

starts at: $\mathcal{O}(\epsilon_*)$ $\mathcal{O}(\epsilon_*^2)$ $\mathcal{O}(\epsilon_*^3)$

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$$\mathcal{P}_0^{(\psi)}(k) = \frac{\hbar H_*^2}{4\pi^2 c_{\psi_*}^3 Z_{\psi_*}} \left[1 + \underbrace{p_{0*}}_{\substack{\text{starts at:} \\ \mathcal{O}(\epsilon_*)}} + \underbrace{p_{1*}}_{\substack{\text{starts at:} \\ \mathcal{O}(\epsilon_*)}} \ln\left(\frac{k}{k_*}\right) + \underbrace{p_{2*}}_{\substack{\text{starts at:} \\ \mathcal{O}(\epsilon_*^2)}} \ln\left(\frac{k}{k_*}\right)^2 + \underbrace{p_{3*}}_{\substack{\text{starts at:} \\ \mathcal{O}(\epsilon_*^3)}} \ln\left(\frac{k}{k_*}\right)^3 \right]$$

Amplitude: $\mathcal{A}_* \equiv \mathcal{P}_0(k_*)$

Tilt: $\theta_* \equiv k \frac{d}{dk} \ln(\mathcal{P}_0(k)) \Big|_{k=k_*}$, $(n_s \equiv 1 + \theta_s \quad n_t \equiv \theta_t)$

Running of the tilt: $\alpha_* \equiv k \frac{d}{dk} \left[k \frac{d}{dk} \ln(\mathcal{P}_0(k)) \right] \Big|_{k=k_*}$

Running of the running of the tilt: $\beta_* \equiv k \frac{d}{dk} \left\{ k \frac{d}{dk} \left[k \frac{d}{dk} \ln(\mathcal{P}_0(k)) \right] \right\} \Big|_{k=k_*}$

Quantity	Order	Expression
	NLO :	$-2\epsilon_{1H*} + \epsilon_{1Z*} + 3\epsilon_{1c*}$
	N2LO :	$-2\epsilon_{1H*}^2 + 2(1+C)\epsilon_{1H*}\epsilon_{2H*} + \epsilon_{1Z*}(\epsilon_{1H*} - C\epsilon_{2Z*}) + \epsilon_{1c*}(5\epsilon_{1H*} - 3\epsilon_{1c*} - \epsilon_{1Z*}) - (2+3C)\epsilon_{1c*}\epsilon_{2c*}$
	N3LO :	$-2\epsilon_{1H*}^3 + (14+6C-\pi^2)\epsilon_{1H*}^2\epsilon_{2H*} + \frac{1}{12}(-24-24C-12C^2+\pi^2)\epsilon_{1H*}\epsilon_{2H*}^2$ $+ \frac{1}{12}(-24-24C-12C^2+\pi^2)\epsilon_{1H*}\epsilon_{2H*}\epsilon_{3H*} + \epsilon_{1H*}^2\epsilon_{1Z*} + \frac{1}{2}(-10-2C+\pi^2)\epsilon_{1H*}\epsilon_{1Z*}\epsilon_{2H*}$ $+ \frac{1}{2}(-8-4C+\pi^2)\epsilon_{1H*}\epsilon_{1Z*}\epsilon_{2Z*} + \frac{1}{4}(8-\pi^2)\epsilon_{1Z*}^2\epsilon_{2Z*} + \frac{1}{24}(12C^2-\pi^2)\epsilon_{1Z*}\epsilon_{2Z*}^2$ $+ \frac{1}{24}(12C^2-\pi^2)\epsilon_{1Z*}\epsilon_{2Z*}\epsilon_{3Z*} + 3\epsilon_{1c*}^3 - 8\epsilon_{1c*}^2\epsilon_{1H*} + 7\epsilon_{1c*}\epsilon_{1H*}^2 + \epsilon_{1c*}^2\epsilon_{1Z*} - 2\epsilon_{1c*}\epsilon_{1H*}\epsilon_{1Z*}$ $+ \frac{1}{4}(100+36C-9\pi^2)\epsilon_{1c*}^2\epsilon_{2c*} + \frac{1}{2}(-36-16C+3\pi^2)\epsilon_{1c*}\epsilon_{1H*}\epsilon_{2c*} + \frac{1}{4}(28+4C-3\pi^2)\epsilon_{1c*}\epsilon_{1Z*}\epsilon_{2c*}$ $+ \frac{1}{8}(16+16C+12C^2-\pi^2)\epsilon_{1c*}\epsilon_{2c*}^2 + \frac{1}{2}(-38-14C+3\pi^2)\epsilon_{1c*}\epsilon_{1H*}\epsilon_{2H*}$ $+ \frac{1}{4}(24+8C-3\pi^2)\epsilon_{1c*}\epsilon_{1Z*}\epsilon_{2Z*} + \frac{1}{8}(16+16C+12C^2-\pi^2)\epsilon_{1c*}\epsilon_{2c*}\epsilon_{3c*}$
$\theta_*^{(\psi)}$	N2LO :	$2\epsilon_{1H*}\epsilon_{2H*} - \epsilon_{1Z*}\epsilon_{2Z*} - 3\epsilon_{1c*}\epsilon_{2c*}$
	N3LO :	$+6\epsilon_{1H*}^2\epsilon_{2H*} - 2(1+C)\epsilon_{1H*}\epsilon_{2H*}^2 - 2(1+C)\epsilon_{1H*}\epsilon_{2H*}\epsilon_{3H*} - \epsilon_{1H*}\epsilon_{2H*}\epsilon_{1Z*} - 2\epsilon_{1H*}\epsilon_{1Z*}\epsilon_{2Z*}$ $+ C\epsilon_{1Z*}\epsilon_{2Z*}^2 + C\epsilon_{1Z*}\epsilon_{2Z*}\epsilon_{3Z*} + 9\epsilon_{1c*}^2\epsilon_{2c*} - 8\epsilon_{1c*}\epsilon_{1H*}\epsilon_{2c*} + \epsilon_{1c*}\epsilon_{1Z*}\epsilon_{2c*} + (2+3C)\epsilon_{1c*}\epsilon_{2c*}^2$ $- 7\epsilon_{1c*}\epsilon_{1H*}\epsilon_{2H*} + 2\epsilon_{1c*}\epsilon_{1Z*}\epsilon_{2Z*} + (2+3C)\epsilon_{1c*}\epsilon_{2c*}\epsilon_{3c*}$
$\alpha_*^{(\psi)}$	N3LO :	$-2\epsilon_{1H*}\epsilon_{2H*}(\epsilon_{2H*} + \epsilon_{3H*}) + \epsilon_{1Z*}\epsilon_{2Z*}(\epsilon_{2Z*} + \epsilon_{3Z*}) + 3\epsilon_{1c*}\epsilon_{2c*}(\epsilon_{2c*} + \epsilon_{3c*})$
$\beta_*^{(\psi)}$	N3LO :	

THE IMPLEMENTATION: STAROBINSKY INFLATION IN THE GEOMETRIC FRAME

[Starobinsky '79-'80; Vilenkin '85]

Modified Friedmann equation:

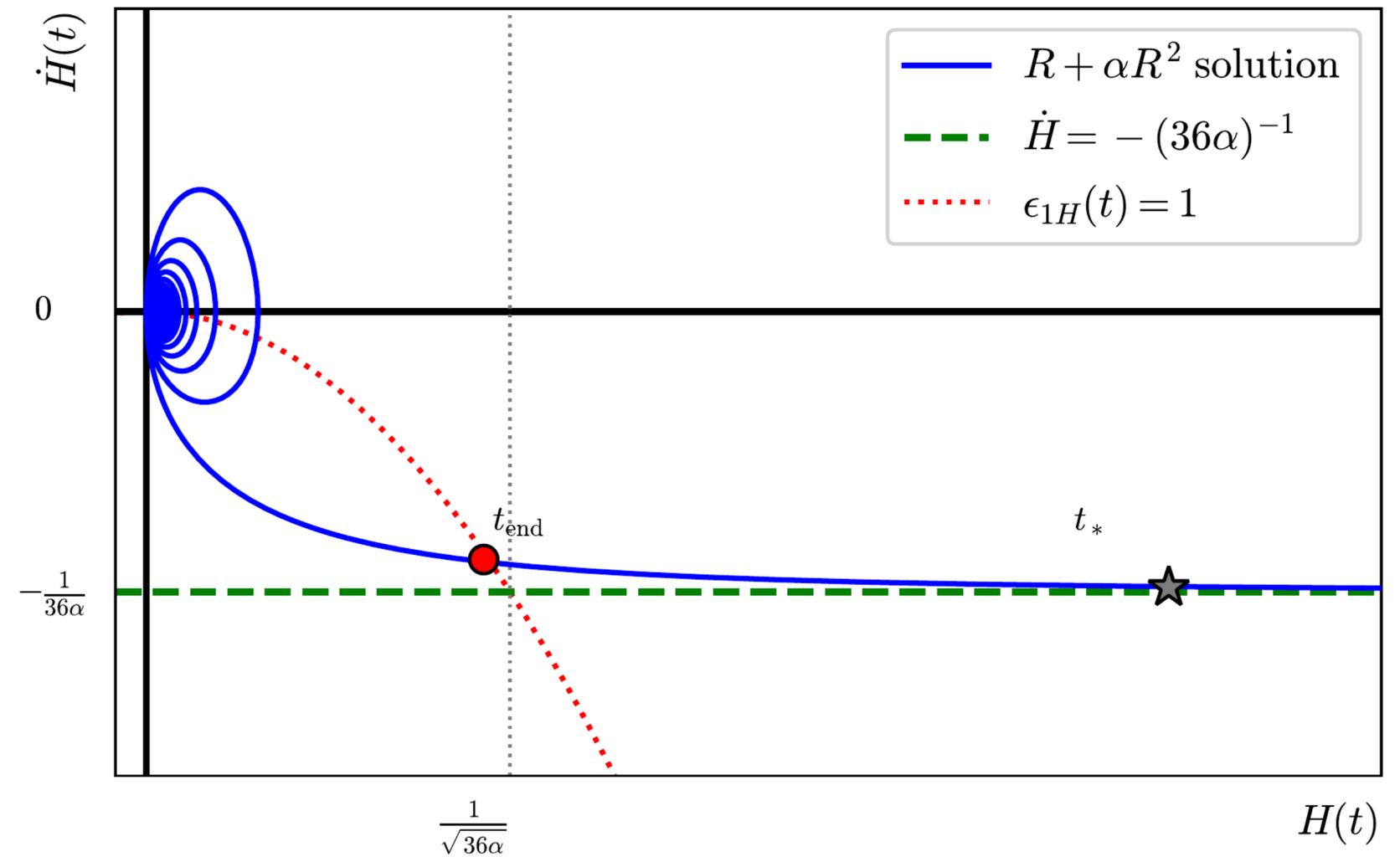
$$S[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R + \alpha R^2) \longrightarrow H(t)^2 + 6\alpha H(t)^4 \epsilon_{1H}(t) (3\epsilon_{1H}(t) + 2\epsilon_{2H}(t) - 6) = 0$$

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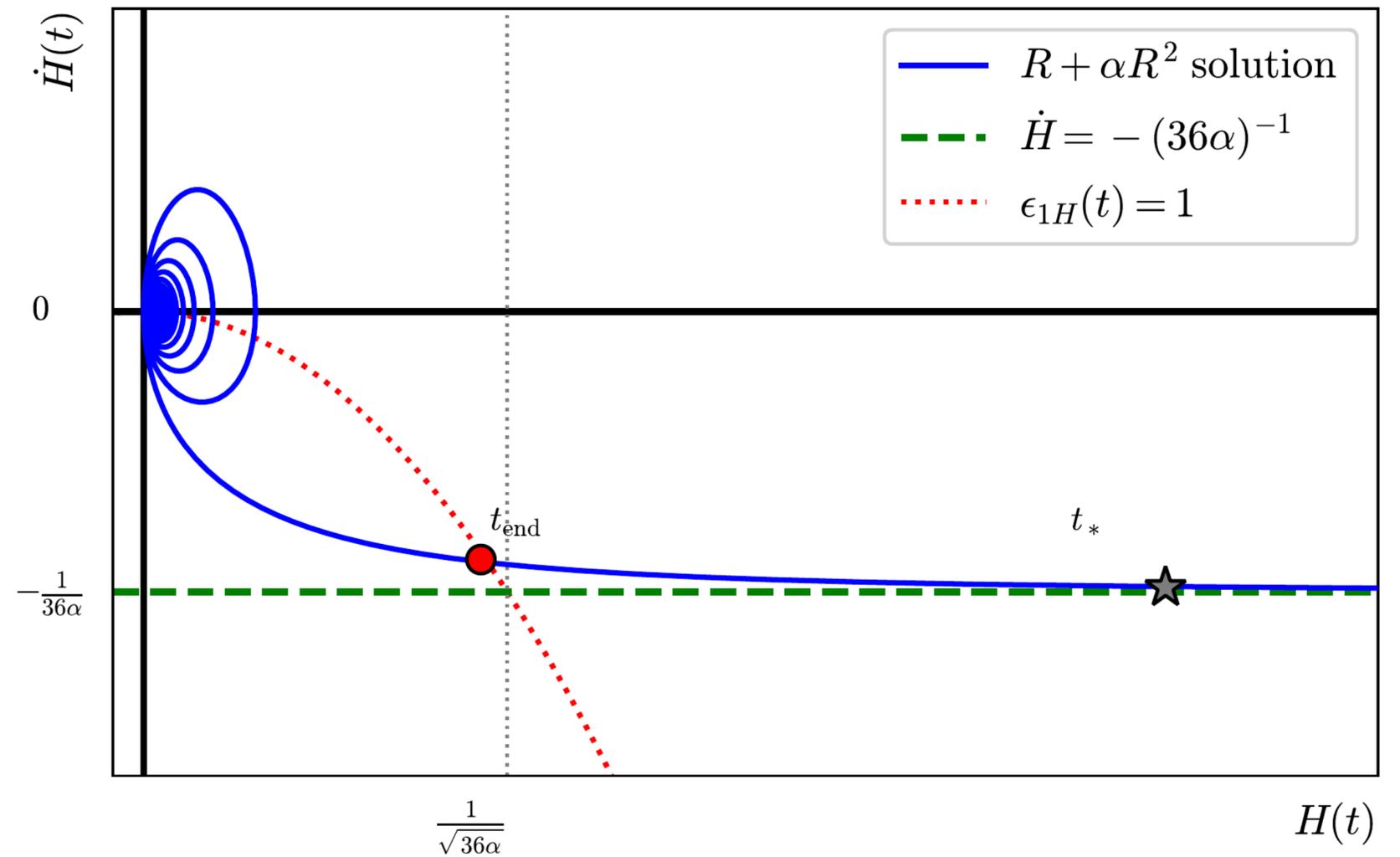
From quadratic action of cosmological perturbations ($c_s = c_t = 1$):

$$Z_s(t) = \frac{3\bar{\chi}(t)}{16\pi G_N} \left(\frac{\epsilon_\chi(t)}{1 + \frac{1}{2}\epsilon_\chi(t)} \right)^2, \quad Z_t(t) = \frac{\bar{\chi}(t)}{64\pi G_N}$$

with

$$\epsilon_\chi(t) = -\frac{\dot{\bar{\chi}}(t)}{H(t)\bar{\chi}}$$

$$\bar{\chi}(t) = 1 + 2\alpha\bar{R} = 1 - 12\alpha H(t)^2 (\epsilon_{1H}(t) - 2)$$



THE IMPLEMENTATION: STAROBINSKY INFLATION IN THE GEOMETRIC FRAME

Amplitudes of the primordial power spectrum:

$$\mathcal{A}_t = \frac{2G\hbar}{3\pi\alpha}(1 + \dots) \quad \mathcal{A}_s = \frac{G\hbar N_*^2}{18\pi\alpha}(1 + \dots)$$

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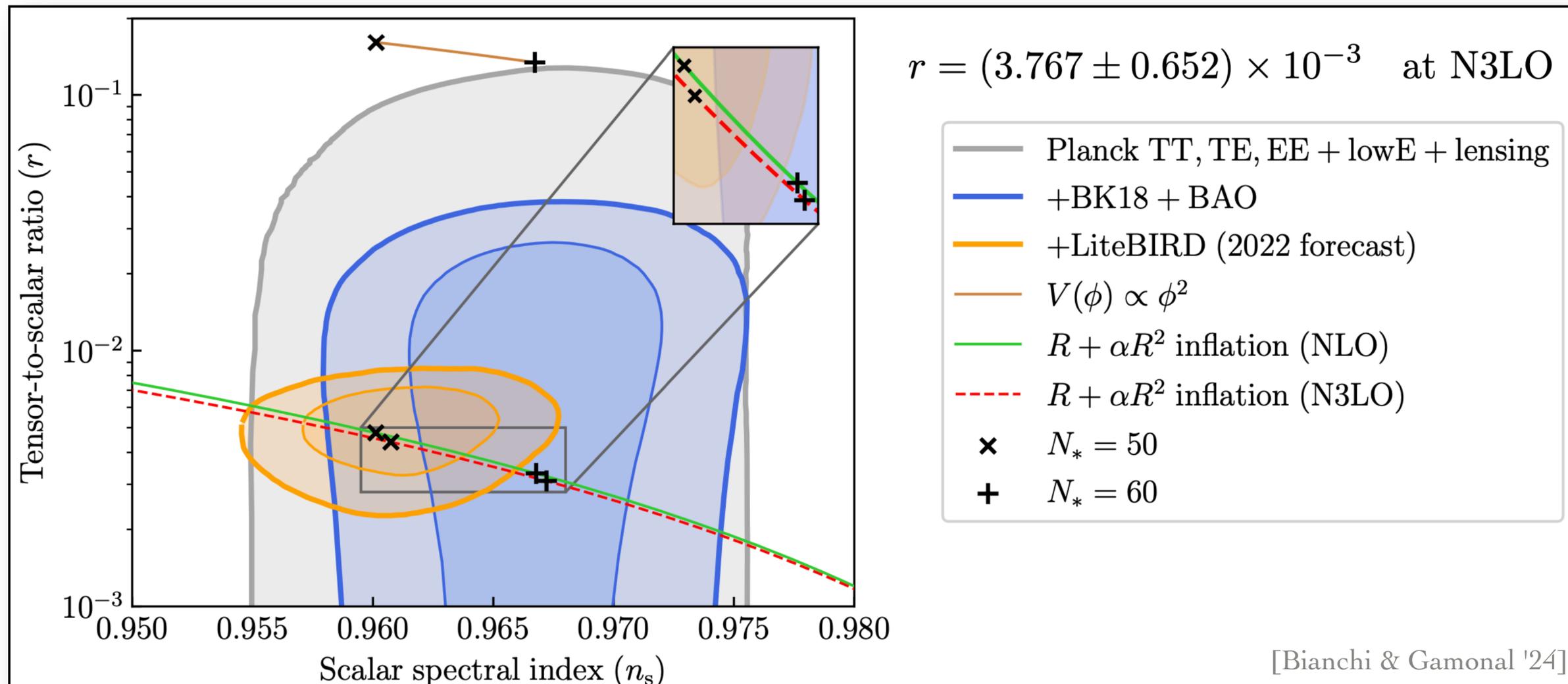
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7% decrease in tensor-to-scalar ratio:

$$r \equiv \frac{\mathcal{A}_t}{\mathcal{A}_s} = \frac{12}{N_*^2} + \frac{2 \ln(2N_*)}{N_*^3} - (2.36\dots) \frac{24}{N_*^3} + \mathcal{O}(N_*^{-4})$$

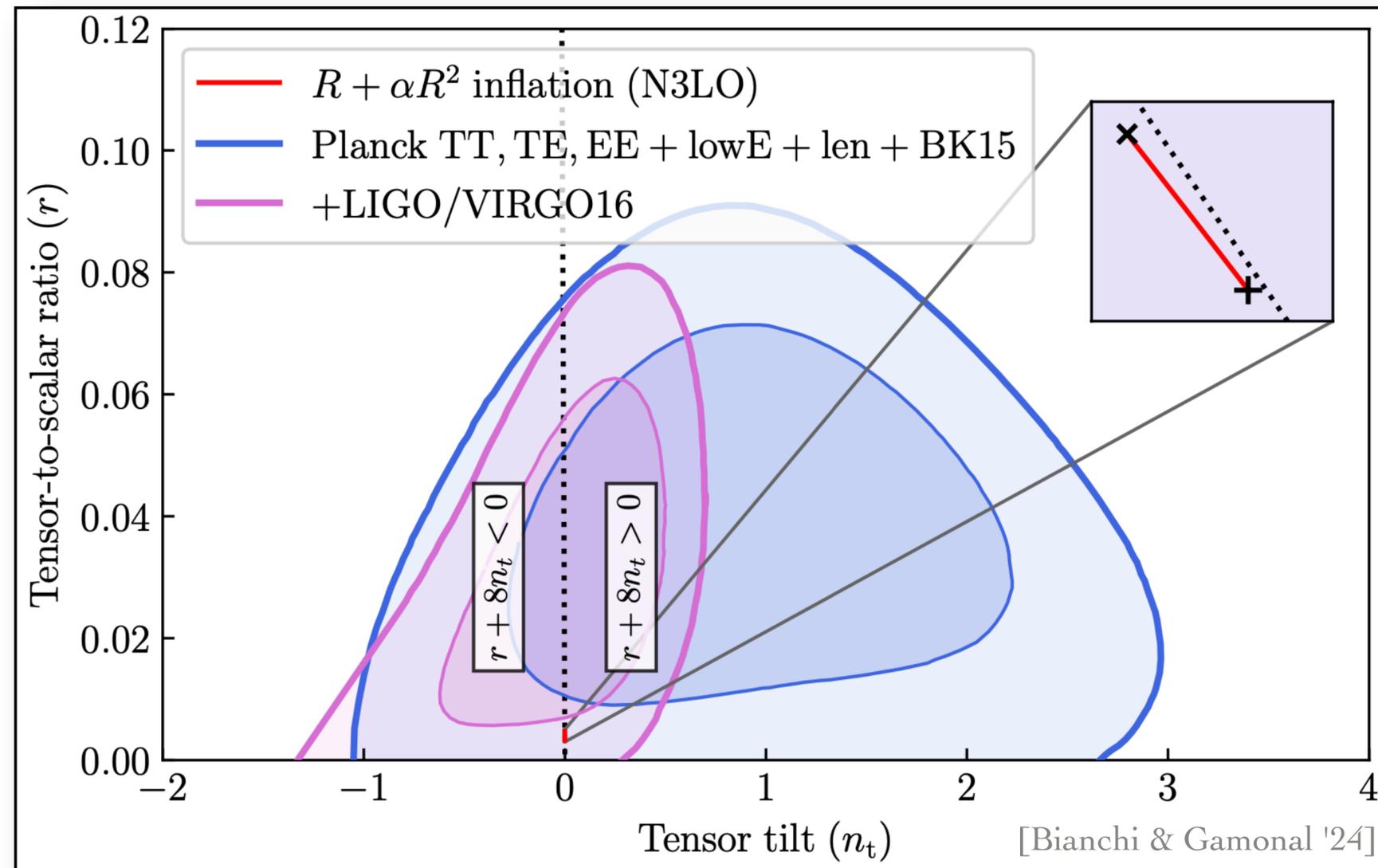
$$= 1 + 2 - \ln(2) - \gamma_E - \left(-\frac{1}{2} + \frac{\ln(2)}{24} + \frac{\ln(3)}{12} - \frac{\ln(20)}{24} - \frac{19 \tan^{-1}\left(\frac{1}{\sqrt{39}}\right)}{12\sqrt{39}} - \frac{19 \cot^{-1}\left(\frac{3\sqrt{39}}{7}\right)}{12\sqrt{39}} \right)$$



THE IMPLEMENTATION: STAROBINSKY INFLATION IN THE GEOMETRIC FRAME

Precise value for deviation from exact consistency condition:

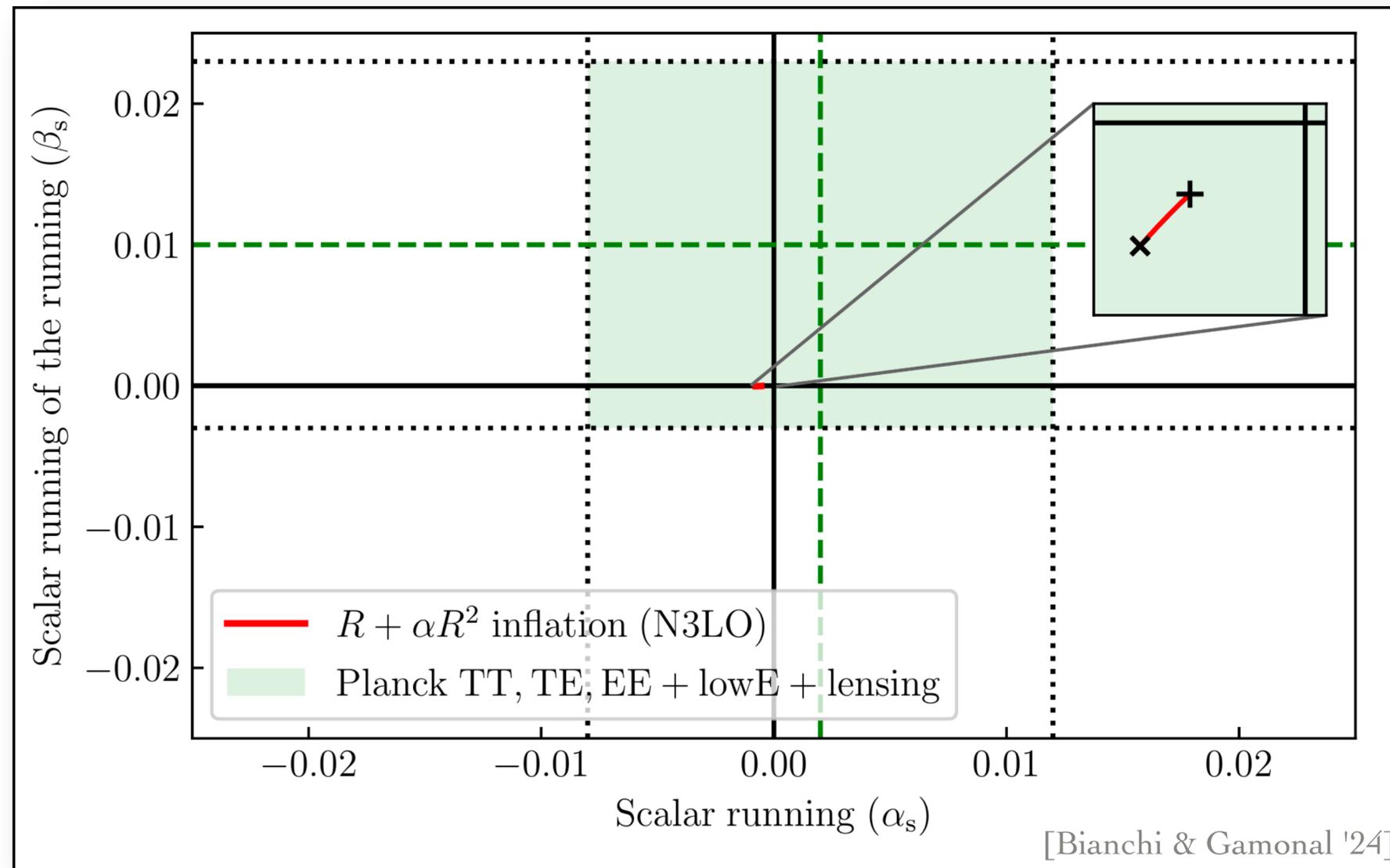
$$r + 8n_t = -\frac{48}{N_*^3} + \mathcal{O}(N_*^{-4}) = (-3.031 \pm 0.809) \times 10^{-4} \quad \text{for } N_* = 55 \pm 5 \quad \text{at N3LO}$$



THE IMPLEMENTATION: STAROBINSKY INFLATION IN THE GEOMETRIC FRAME

Running and running-of-the-running for scalar modes:

$$\alpha_s = (-6.626 \pm 1.180) \times 10^{-4} \quad \text{at N3LO} \quad \text{for } N_* = 55 \pm 5$$
$$\beta_s = (-2.526 \pm 0.674) \times 10^{-5} \quad \text{at N3LO}$$



CONCLUSIONS AND FUTURE WORK

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- **Possible additional considerations (from RHS):**

1. Consider other vacuum choices: $\mathcal{A}_s = \frac{G\hbar H_*^2}{\pi\epsilon_{1H_*}} |\alpha_k - \beta_k|^2 (1 + \dots)$

[e.g. Danielsson '02; Broy '16]

2. Consider closed cosmologies: $\mathcal{A}_s = \frac{G\hbar H_*^2}{\pi(\epsilon_{1H_*} + \frac{K}{\mathcal{H}})} (1 + \dots)$

[e.g. Bonga, Gupt & Yokomizo '16; Kiefer & Vardanyan '22]

3. Consider parity-violating models: $v''(y) + \left(1 - \frac{2}{y^2} \pm \gamma \frac{\sigma}{y}\right) v(y) = 0$

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- **Long term goal (from LHS):** Explore the emergence of states from LQG/Spinfoams in Quantum Cosmology and build a bridge towards RHS and phenomenology.

