QUANTUM GRAVITY AT THE NULL ASYMPTOTE

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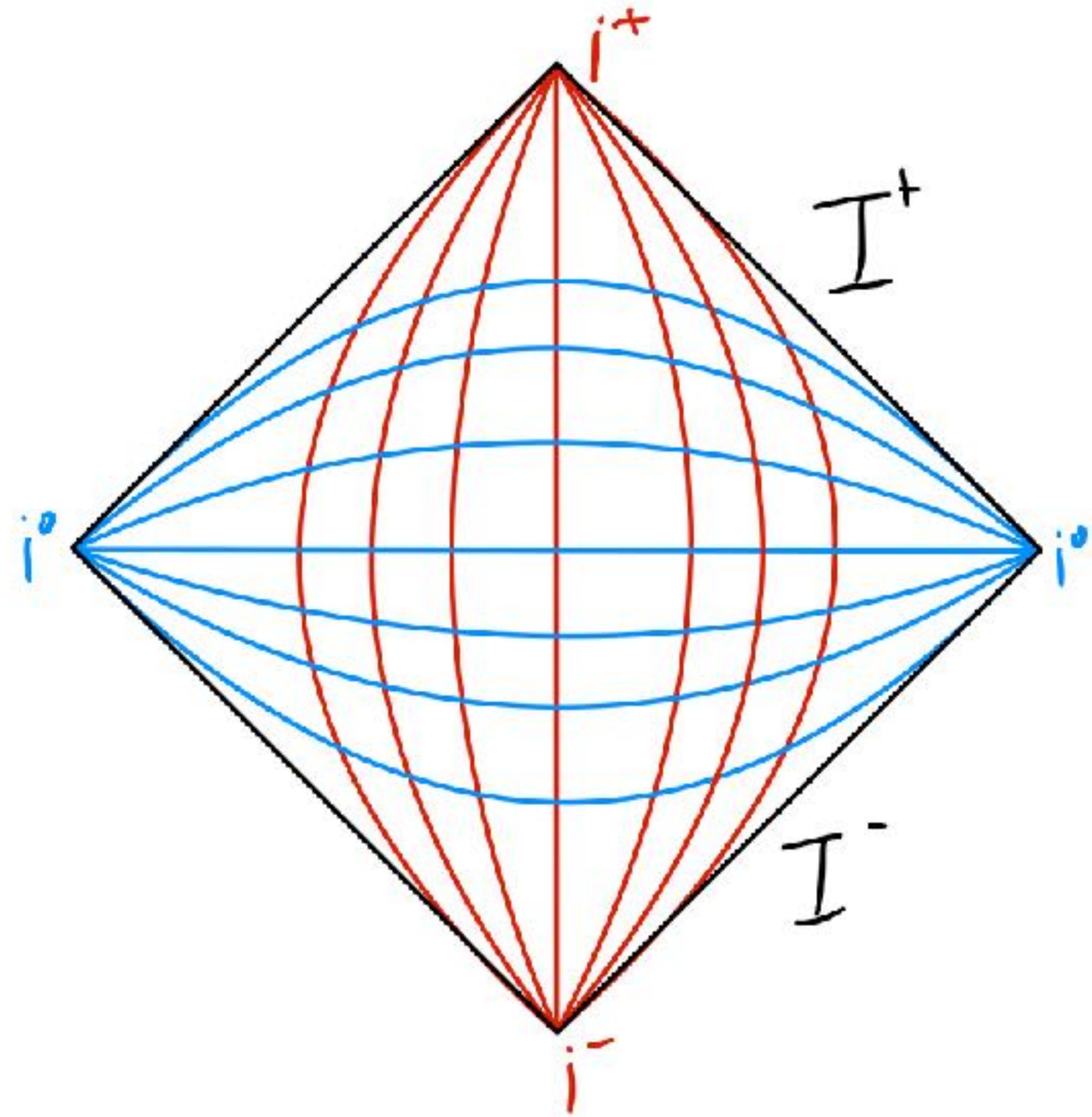
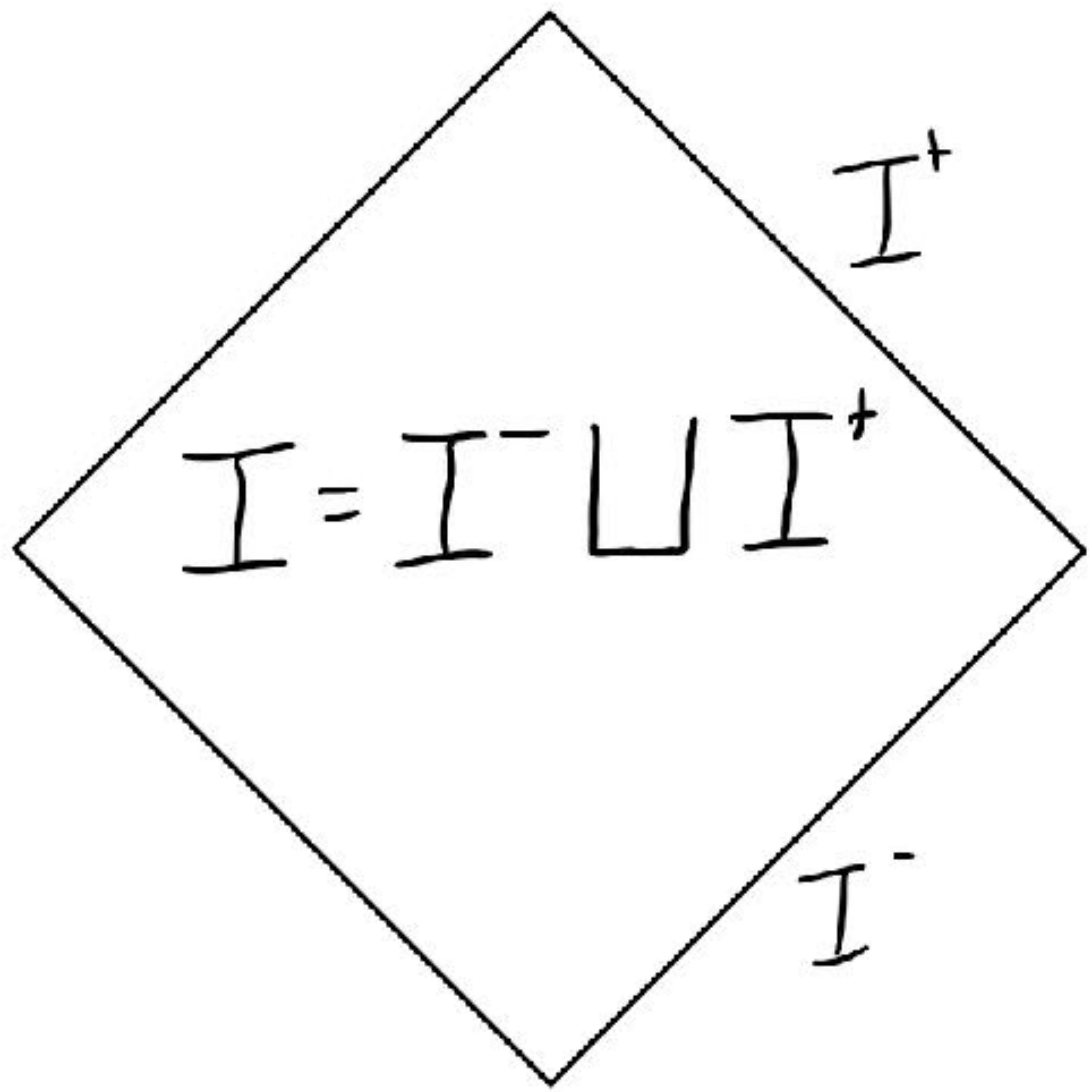
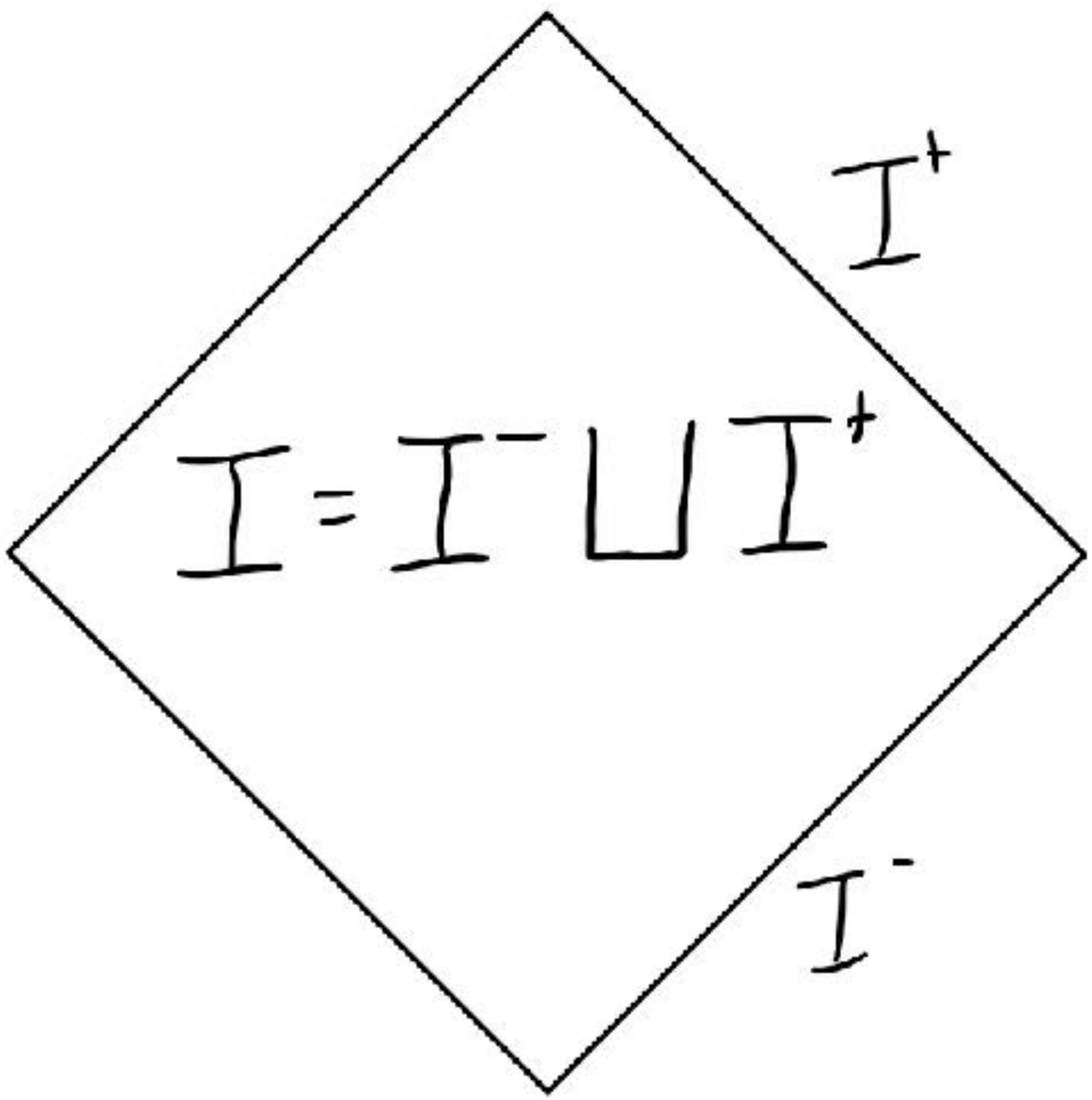
THE (ALMOST) PLAYGROUND



THE (ACTUAL) PLAYGROUND



RADIATIVE MODES



- Asymptotic Symmetries described by the BMS group

$$\mathcal{B} = \mathcal{S} \ltimes \mathcal{L}$$

- Space of connections forms an affine space and parametrized by a Sym + TT 3x3 matrix

$$\sigma_{ab} : \{D\} - \{D'\}$$

- 9 - 3 - 3 - 1 = 2 d.o.f of GR
- Hence, we can interpret σ_{ab} as our radiative modes
- N_{ab} carries gauge invariant curvature information

SYMPLECTIC STRUCTURE

- Symplectic Form

$$\Omega|_{\{D\}}(\sigma, \sigma') := \frac{1}{8\pi G} \int_{\mathcal{I}} \left[\sigma_{ab} \mathcal{L}_n \sigma'_{cd} - \sigma'_{ab} \mathcal{L}_n \sigma_{cd}\right] q^{ac} q^{bd} \epsilon_{mnp} dS^{mnp}$$

- Hamiltonian

$$H_\xi(\{D\}) = \frac{1}{16\pi G} \int_{\mathcal{I}} [N_{ab}(\mathcal{L}_\xi D_c - D_c \mathcal{L}_\xi) l_d + 2N_{ab} l_c D_d \alpha] q^{ac} q^{bd} \epsilon_{mnp} dS^{mnp}$$

- Bondi News Smearing - our observables

$$[N[f]](\{D\}) := \int_{\mathcal{I}} N_{ab} f_{cd} q^{ac} q^{bd} \epsilon_{mnp} dS^{mnp}$$

- Poisson Bracket

$$\{N[f], N[f']\} = \int_{\mathcal{I}} [f_{ab} \mathcal{L}_n f'_{cd} - f'_{ab} \mathcal{L}_n f_{cd}] q^{ac} q^{bd} \epsilon_{rst} dS^{rst} = \Omega(f, f')$$

OBSERVABLE ALGEBRA

- Abstractly defined generators are R-linear and Hermitian

$$\hat{N}[af + bf'] = a\hat{N}[f] + b\hat{N}[f'] \qquad \hat{N}^{\star}[f] = \hat{N}[f]$$

- Commutation relation

$$[\hat{N}[f], \hat{N}[f']] = i\hbar\Omega(f, f')\mathbb{I}$$

- Convenient to use the Weyl algebra as it is closed

$$\hat{W}[f] = \exp\{(i/\hbar)\hat{N}[f]\}$$

- One can use the GNS construction to find a representation of Weyl algebra on a Hilbert space.

CONSTRUCTION OF THE HILBERT SPACE I

- Decompose f into + and - frequency parts to induce a complex structure J

$$J f_{ab} = i f_{ab}^{+} - i f_{ab}^{-}$$

$$f_{ab}^{+}(u, \theta, \phi) = \int_0^\infty \tilde{f}_{ab}(\omega, \theta, \phi) e^{i\omega u} d\omega \qquad f_{ab}^{-} = (f_{ab}^{+})^*$$

- Because J is independent of conformal frame and is compatible with the symplectic structure, we have a Kähler space

- Use following (Hermitian) inner product to Cauchy complete space of smearings: h

$$\langle f | f' \rangle := \frac{1}{2}(\Omega(f, J f') + i\Omega(f, f'))$$

CONSTRUCTION OF THE HILBERT SPACE II

- Define VEV from which GNS construction can be carried out

$$\langle 0|\hat{W}[f]|0\rangle := \exp\{(-1/2\hbar)\Omega(f, Jf)\}$$

- The representation space is isomorphic to a Fock space

$$\mathcal{H} = \oplus_{n=0}^{\infty} h_n^{\text{sym}}$$

- The normalized vector in $h_0^{\text{sym}} = \mathbb{C}$ defines the vacuum state and elements of h_n^{sym} are obtained via the creation operator

$$\hat{a}^{\dagger}[f] = \frac{1}{2}(\hat{N}[f] + i\hat{N}[Jf])$$

- This gives us our asymptotic Fock space

SPACE OF STATES

- $|0\rangle$ is a coherent state peaked at a chosen vacuum $\{D^\circ\}_0$
- Our space of states Γ_0 is all $\{D^\circ\}$ with $\langle\sigma|\sigma\rangle < \infty$
- Γ_0 and h are isomorphic via $\sigma_{ab} \to f_{ab} = \sigma_{ab}$
- Γ_0 contains no other classical vacuum states, hence $\mathcal{H}$ contains no states corresponding to vacuum states apart from $|0\rangle$

ASYMPTOTIC GRAVITONS

- The action of a Poincaré subgroup $\mathcal{P}$ of BMS can be split into two irreducible components $\mathcal{H}_R \oplus \mathcal{H}_L$
- Right/Left - handed sectors have positive/negative helicity if the positive frequency part is self-dual/anti self-dual

$$\epsilon^{mnp} l_p q_{nb} f^+_{am} = \pm i f^+_{ab}$$

- The mass Casimir has eigenvalue of 0 for both sectors while the helicity Casimir for $\mathcal{H}_R/\mathcal{H}_L$ is ±2
- These excitations can be interpreted as gravitons! Cool!
- Asymptotic particle states of the <u>exact</u> theory rather than bulk particle states of the <u>linearized</u> theory

INFRARED ISSUES AND INADEQUACY OF THE FOCK REPRESENTATION

- Requiring states to be finite implies a vanishing IR charge

$$\langle\sigma|\sigma\rangle < \infty \implies \int_{-\infty}^{\infty} du N_{ab}(u,\theta,\phi) = 0$$

- On the level of radiative modes, this implies that $\{D^+\} = \{D^-\}$
  - But this implies our symmetry group reduces from $\mathcal{B}$ to $\mathcal{P}$
  - BORING!!
- This implies that the Fock representation for gravitational radiation is inadequate

OPEN QUESTIONS

- How do we go beyond the Fock representation?
- Can we find a loop representation of the null asymptote?
- What if we had a cosmological constant?
- What are the perturbative dynamics of our theory?
- Can we construct a non-perturbative S-matrix?
- Is there a non-perturbative infrared triangle?
- and more!

BIBLIOGRAPHY

Ashtekar, A. (2015). "Geometry and physics of null infinity." Surveys in Differential Geometry, 20(1), 99–122. https://doi.org/10.4310/sdg.2015.v20.n1.a5

Ashtekar, A., Campiglia, M., & Laddha, A. (2018). "Null Infinity, the BMS Group and Infrared issues." General Relativity and Gravitation, 50(11). https://doi.org/10.1007/s10714-018-2464-3

Ashtekar, A. (1981). "Asymptotic quantization of the gravitational field." Physical Review Letters, 46(9), 573–576. https://doi.org/10.1103/physrevlett.46.573