Universal Properties of the Universe in Modified Loop Quantum Cosmology

Jamal Saeed and Christian Brown
(with, Gerald Cleaver,
Rui Pan, Anzhong Wang)
GCAP-CASPER
Physics Department
Baylor University

Loops'24 International Conference on Quantum Gravity
FT Lauderdale, Florida
May 6– 10, 2024

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1. Introduction

- Loop quantum gravity (LQG) is a canonical quantization of Einstein's theory, based on Hamiltonian formalism & canonical quantization of Ashtekar's variables [A. Ashtekar, PRL57 (1986) 2244]: ***holonomies of connections & fluxes of triads.***

- A very successful application of LQG is loop quantum cosmology (LQC) [M. Bojowald, PRL86 (2001) 5227; A. Ashtekar, T. Pawlowski, P. Singh, PRD74 (2006) 084003].

- LQC is constructed by applying LQG techniques to cosmological models within the supermini-space approach [A. Ashtekar, P. Singh, CQG 28 (2011) 213001].

- LQC resolves the big-bang singularity generically because of its fundamental result [T. Thiemann, Modern Canonical Quantum General Relativity, (Cambridge University Press, 2007)]:

  ``quantum gravity effects always lead the area operator to have a non-zero minimal area gap."

2. Modified Loop Quantum Cosmology (LQC)

- In the framework of LQC, two fundamental gradients were adopted:

  - **The supermini-space assumption:** that is, first assume that the spacetime is homogeneous and isotropic, and then write down the corresponding classical Hamiltonian, before quantization.

  - **Using the classical relation between the Euclidean & Lorentzian terms**,

$$\mathcal{H}^{(L)}_{\text{grav}} = \gamma^{-2}\mathcal{H}^{(E)}_{\text{grav}}$$

so the total Hamiltonian is written only in terms of the Euclidean term,

$$\mathcal{H}_{\text{grav}} = -\gamma^{-2}\mathcal{H}^{(E)}_{\text{grav}}$$

Then, one needs to quantize **only** the Euclidean term $\mathcal{H}^{(E)}_{\text{grav}}$.

2. Modified LQC (Cont.)

- However, none of them is in the spirit of LQG:

  - In LQG, the quantization of the **Euclidean** & **Lorentzian** terms usually follow different processes [T. Thiemann, Modern Canonical Quantum General Relativity, (Cambridge University Press, 2007)]. In the flat FLRW universe, the two terms are proportional classically. However, it is expected that in the quantum level they should be different.

  - In LQG, the operations of symmetry reduction and quantization do not communicate. As a result, first imposing symmetry and then quantizing the reduced Hamiltonian in general is different from first quantizing the Hamiltonian of the whole system and then imposing the symmetry.

2. Modified LQC (Cont.)

- To answer the first question, Yang, Ding & Ma (YDM) quantized these two terms differently by closely following the processes of LQG and obtained the following effective Hamiltonian [PLB682 (2009) 1]:

$$\mathcal{H}_{\mathrm{mLQC\text{-}I}} = \frac{3}{8\pi G}\left\{\frac{p^{1/2}\sin^2(\bar{\mu}c)}{\bar{\mu}^2} - \frac{p^{1/2}(\gamma^2+1)\sin^2(2\bar{\mu}c)}{4\bar{\mu}^2\gamma^2}\right\} + \mathcal{H}_M,$$

$\mathcal{H}_M$: the matter Hamiltonian.

- A systematical derivation of the above Hamiltonian was provided later by M. Assanioussi *et al.*, [PRL121 (2018) 081303].

- The above effective Hamiltonian can be also obtained from the top-down approach [A. Dapor and K. Liegener,, PLB 785 (2018) 506; M. Han, H.G. Liu, PRD104 (2021) 024011].

2. Modified LQC (Cont.)

- The modified Friedmann of mLQC-I takes the form [B.-F. Li, P. Singh, A. Wang, PRD97 *(2018) 084029*],

$$H^2 = \begin{cases} \frac{8\pi\alpha G\rho_\Lambda}{3}\left(1 - \frac{\rho}{\rho_c^{\mathrm{I}}}\right)\mathcal{F}_-\left(\rho, \rho_c^{\mathrm{I}}\right), & (t < t_B), \\ \frac{8\pi G\rho}{3}\left(1 - \frac{\rho}{\rho_c^{\mathrm{I}}}\right)\mathcal{F}_+\left(\rho, \rho_c^{\mathrm{I}}\right), & (t > t_B), \end{cases}$$

$$\mathcal{F}_-(\rho_c^{\mathrm{I}}) = \mathcal{F}_+(\rho_c^{\mathrm{I}})$$

$$\rho_\Lambda \equiv \gamma^2\rho_c/[(1+\gamma^2)(1-5\gamma^2)]$$

$$\rho_c^{\mathrm{I}} \equiv \rho_c/[4(1+\gamma^2)]$$

$\mathcal{F}_\pm$: finite and non-zero; $t = t_B$: the time of the bounce

- Then, asymptotically, we find

$$H^2 \approx \begin{cases} \frac{8\pi\alpha G}{3}\rho_\Lambda, & (t \ll t_B), \\ \frac{8\pi G}{3}\rho, & (t \gg t_B), \end{cases}$$

$$\alpha \equiv (1-5\gamma^2)/(\gamma^2+1) \simeq 0.68$$

- It is clear that now the spacetime is not symmetric with respect to the bounce, sharply in contrast to LQC [A. Ashtekar, P. Singh, CQG 28 (2011) 213001].

2. Modified LQC (Cont.)

- In particular, after the bounce ($t > t_B$), the modified Friedmann approaches the classical one.

$$H^2 \approx \begin{cases} \frac{8\pi\alpha G}{3}\rho_\Lambda, & (t \ll t_B), \\ \frac{8\pi G}{3}\rho, & (t \gg t_B), \end{cases}$$

- Before the bounce ($t < t_B$), a de Sitter spacetime with an effective cosmological constant $\rho_\Lambda \approx 0.03\rho_{pl}$ soon dominates the evolution of the Universe.

- The above model was systematically studied in the post-bounce phase by Li, Singh & Wang for a scalar field with various potentials [B.-F. Li, P. Singh, A. Wang, PRD97 *(2018) 084029; D98 (2018) 066016; D100 (2019) 063513*].

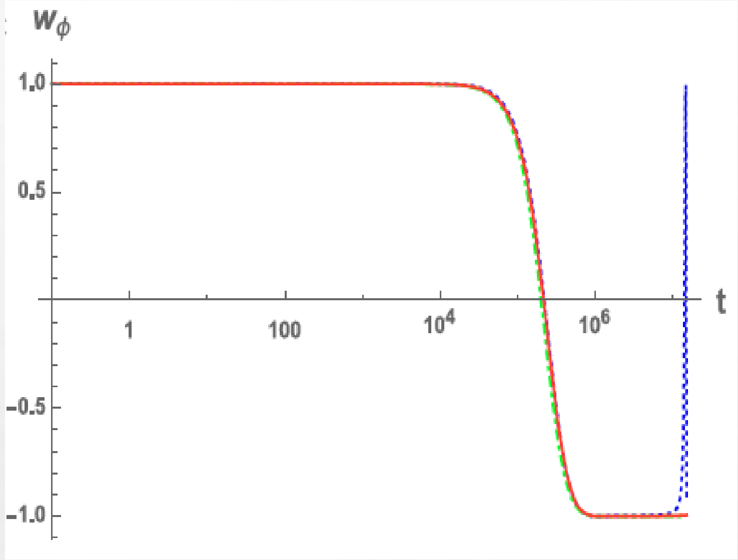
- Among various interesting properties, it was found that the evolution of the universe after the bounce can be divided into three different phases universally:

Bouncing, transition, inflationary phases!

2. Modified LQC (Cont.)

where

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} \simeq \begin{cases} +1, & \text{bouncing} \\ 0, & \text{transition} \\ -1, & \text{inflationary} \end{cases}$$



- It was found that this division is independent of the potentials, as long as the initial condition at the bounce is kinetic-energy dominated: $\dot{\phi}_B^2 \gg 2V(\phi_B)$
- During the bouncing phase, the solution is universal, given by

$$\begin{aligned} a(t) &= \left[1 + 24\pi G\rho_c^{\mathrm{I}} t^2\right]^{1/6}, \\ \phi(t) &= \phi_{\mathrm{B}} \pm \frac{m_{\mathrm{Pl}}}{2\sqrt{3\pi}} \sinh^{-1}\left(\sqrt{24\pi G\rho_c^{\mathrm{I}}}\, t\right) \end{aligned}$$

2. Modified LQC (Cont.)

- In addition, the slow-roll inflationary phase is attractive [B.-F. Li, P. Singh, A. Wang, PRD*100 (2019) 063513*]:

$$P^{\mathrm{I}}(\text{not realized}) \lesssim \frac{\int_{-5.158}^{0.917} d\omega^{\mathrm{I}}}{\int_{-\phi^{\mathrm{I}}_{\max}}^{\phi^{\mathrm{I}}_{\max}} d\omega^{\mathrm{I}}} \simeq 1.12 \times 10^{-5},$$

where $P^{\mathrm{I}}(\text{not realized})$ denotes the probability of inflationary phase that is not realized.

3. Evolution of the Universe in Pre-bounce Phase

- In the pre-bounce phase, the modified Friedmann equation reads [B.-F. Li, P. Singh, A. Wang, PRD97 *(2018) 084029*]:

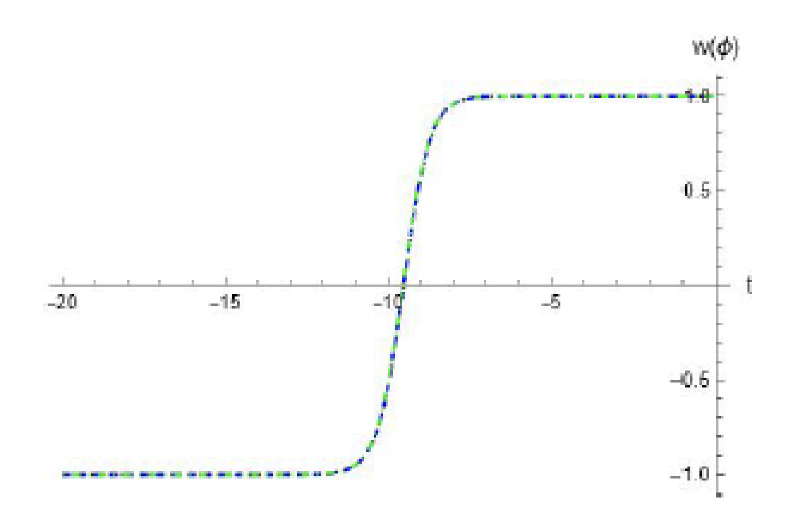
$$H^2 = \frac{8\pi\alpha G\rho_\Lambda}{3}\left(1-\frac{\rho}{\rho_c^{\mathrm{I}}}\right)\mathcal{F}_-\left(\rho,\rho_c^{\mathrm{I}}\right) \simeq \frac{8\pi\alpha G}{3}\rho_\Lambda$$

- Thus, no matter what initial conditions are imposed at the bounce, as $t \ll t_B$, the Universe will be dominated by this **Planck-size cosmological constant**, and the spacetime soon becomes de Sitter.

- We find that this is indeed the case. In particular, we consider the kinetic-energy dominated initial conditions at the bounce for various potentials, and find that the whole pre-bounce phase can be always divided into two subphases:

pre-bouncing & pre-de Sitter

3. Evolution of the Universe in Pre-bounce Phase (Cont.)



- For the chaotic potential $V(\phi) = \frac{1}{2}m^2\phi^2$ we find $w(\phi) \simeq 0$ at $t \simeq -9.8t_p$. Afterwards, the spacetime enters completely into the de Sitter phase.

- In fact, we find that at $t = t_d \simeq -2.0t_p$, we already have $\rho_\Lambda \gg \rho_\phi$, that is, the universe already enters the de Sitter phase.

- In the pre-bounce phase, the expansion factor can be well approximated as

$$a(t) = \begin{cases} (1 + d_0\pi\rho_c^I t^2)^{1/6}(1 + d_2t^2 + d_3t^3 + d_4t^4), & \text{pre-bouncing,} \\ a(t_d)\exp\left\{-\sqrt{\frac{8\pi\alpha G\rho_\Lambda}{3}}(t - t_d)\right\}, & \text{de Sitter,} \end{cases}$$

3. Evolution of the Universe in Pre-bounce Phase (Cont.)

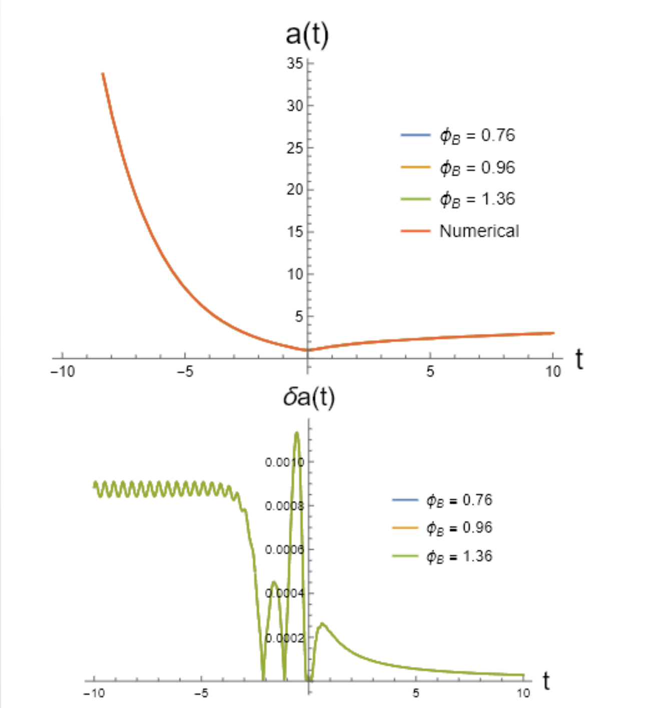
- For the scalar field, we find that the whole pre-bounce phase can be written in the form

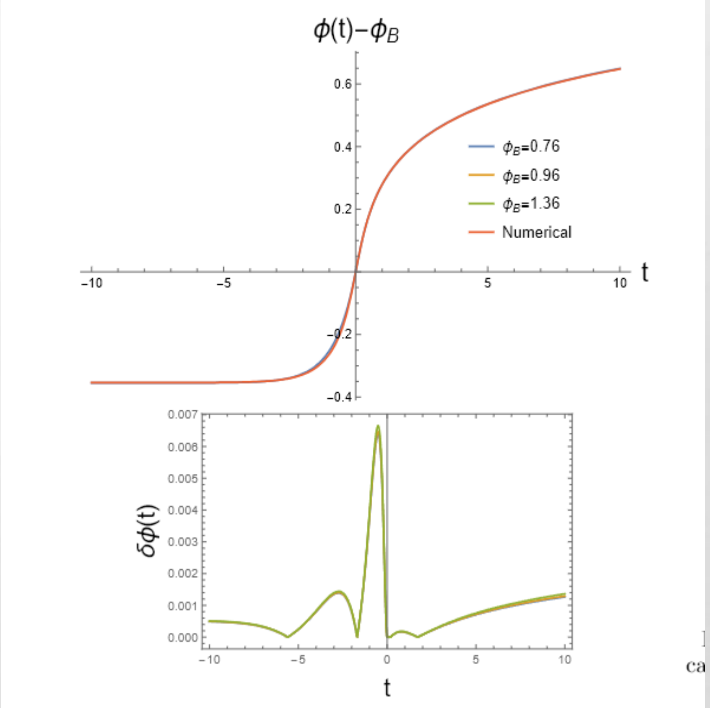
$$\phi = \phi_0 e^{e_1 t + e_2} + \phi_D,$$

- The numerical and analytical solutions of

$a(t)$ $\phi(t)$

$$\delta\mathcal{F} \equiv \left| \frac{\mathcal{F}_N - \mathcal{F}_A}{\mathcal{F}_N} \right|$$





3. Evolution of the Universe in Pre-bounce Phase (Cont.)

- We find similar behaviors for other potentials, including the Starobinsky, alpha-attractor, natural inflationary potentials.

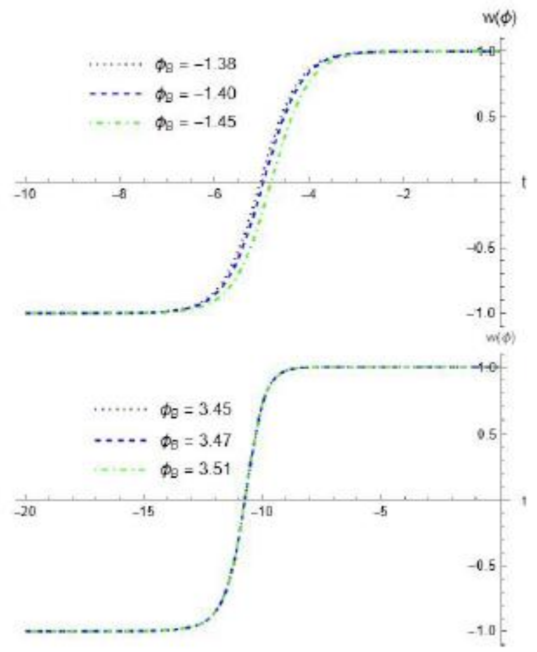
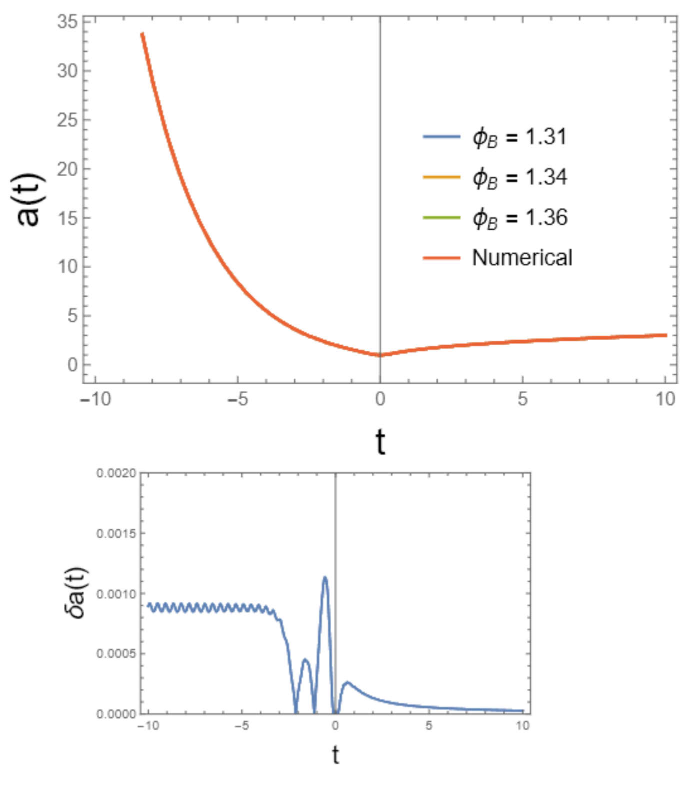
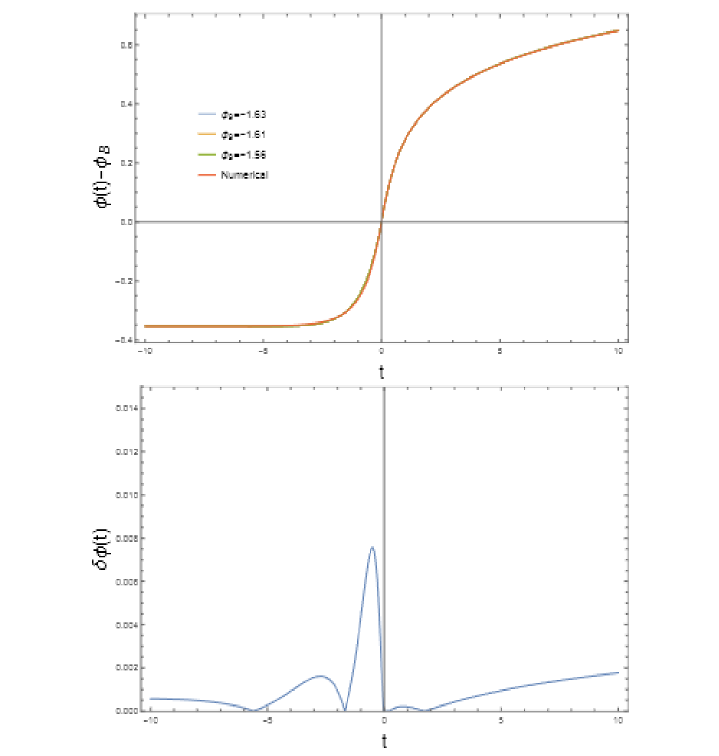


FIG. 14. The equation of state for the Starobinsky potential for different values of ϕ_B that result in greater than 50 eFolds for both cases $\dot{\phi}_B > 0$ and $\dot{\phi}_B < 0$





3. Evolution of the Universe in Pre-bounce Phase (Cont.)

- The equation of state for natural inflation:

$$V\left(\phi\right) = \Lambda^4 \left[1 + \cos\left(\phi/\mu\right)\right]$$

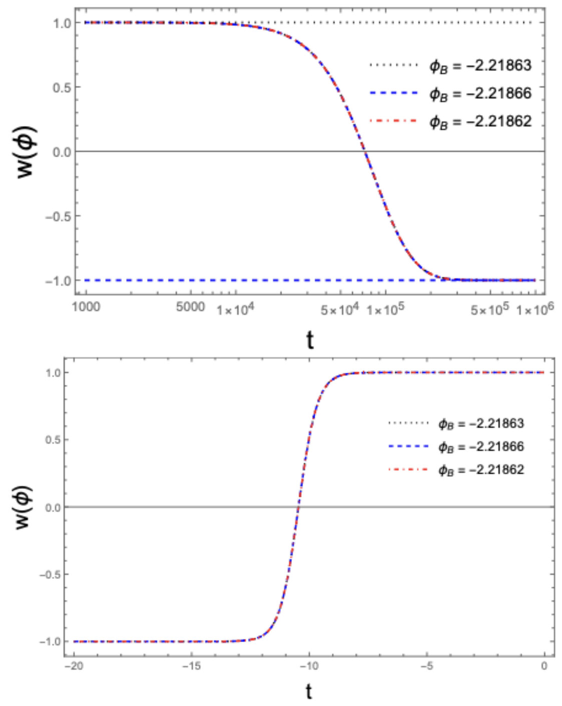


FIG. 23: The equation of state for the Natural potential for different values of ϕ_B that result in greater than 50 eFolds for both cases $\dot{\phi}_B > 0$ $\mu = 0.3$

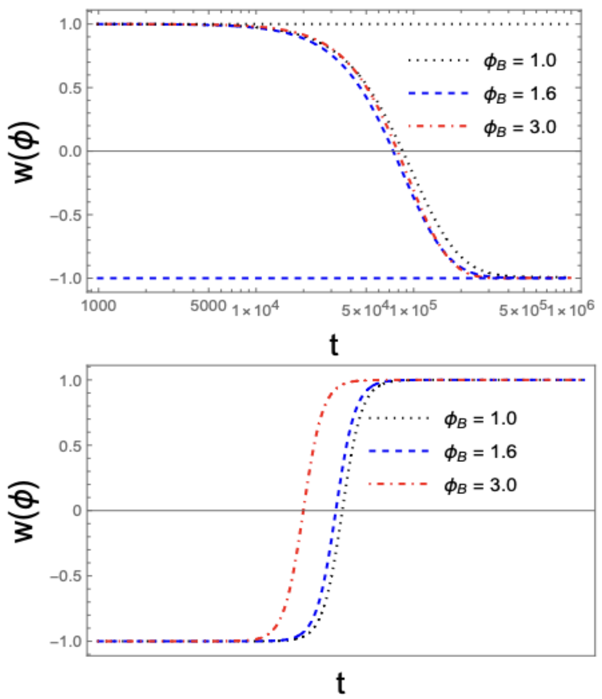


FIG. 33: The equation of state for the Natural potential for different values of ϕ_B that result in greater than 50 eFolds for both cases $\dot{\phi}_B < 0$ $\mu = 1$

4. Conclusions

- We study systematically the background evolution of the Universe in mLQC-I in the framework of the effective Hamiltonian dynamics, which contains the leading-order corrections of the quantum gravitational effects of LQG.

- Due to the domination of the effective cosmological constant in the pre-bounce phase, which has the size of the Planck scale, the evolution can be simply divided into two subphases, *the pre-bouncing & the pre-de Sitter phases*.

- We find the universal behavior of the evolution in these subphases, and the corresponding background solution of the expansion factor $a(t)$ and the scalar field $\phi(t)$ can be well approximated by analytical solutions.

- **With these, we are ready to study quantum cosmological perturbations.**