Entanglement in QFT: Lessons from Minkowski and deSitter space

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Work in collaboration with B. Bonga, P. Calizaya-Cabrera, B. Elizaga-Navascués, E. Martin-Martinez, S. Nadal-Gisvert, P. Ribes-Metidieri, K. Yamaguchi.

Goal:

Understand/quantify the entanglement content of QFT's, its spatial distribution, and its relation to curvature.

Interesting applications:

de Sitter (cosmology)

Connection with quantum gravity \bigcirc

The approach

Entanglement is all around in QFT

Simplest example: Free scalar field, Minkowski st, Minkowski vacuum.

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But region A hosts infinitely many field degrees of freedom.

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von Neumann entropy a region diverges. Cut-off makes it finite. But then, unclear interpretation.

A complementary approach:

Study entanglement between an *a priori* **specified set of finitely many field d.o.f.**

(For similar lines of thought see e.g. Bianchi-Satz 2019)

Some relevant concepts

Free massless scalar field in 3+1 dim

Single-mode subsystem: \blacksquare

Consider a complex solution of the Klein-Gordon eqn. $f(x)$ such that $\langle f | f \rangle \neq 0$ $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 1 \end{bmatrix}$ **F** $\begin{bmatrix} 0 & 1 \end{bmatrix}$ der a complex solution of the Klein-Gordon *R*

Define the operator: \hat{C} = 0 indicates the \hat{C} = \hat{C} e the operator: $O_f = \langle f | \Phi \rangle$

$$
\hat{O}_f=\langle f|\hat{\Phi}\rangle
$$

In this section, we apply the formalism presented above

a pure field and a pure field and a pure momentum operator. We evaluate the set of the set of the set of the s

will also analyze the LN of bipartitions of \mathbb{R}^n of \mathbb{R}^n of \mathbb{R}^n of \mathbb{R}^n versus of \mathbb{R}^n

$$
[\hat{O}_f, \hat{O}_f^{\dagger}] = \langle f | f \rangle \neq 0
$$

 $\textbf{Single-mode subsystem} = \textbf{sub-aldgebra generated from } \hat{O}_f \textbf{ and } \hat{O}_f^\dagger$ to a simple, yet illustrative, situation: two modes sup- \log bingle-mode subsystem \equiv sub-abigebrow \hat{A} is a function computation computed in \hat{A} enerated from O_f and O_f of inverse energy (mutual information and energy $\frac{J}{\sqrt{J}}$

Notation: ${f}$ = Single-mode subsystem 1 derive $\iint_J f^{(0)}$ expansions in any number of space $\iint_J f^{(0)}$ t ransformation restricted to one subsystem, and these subsystem, and these subsystem, and these subsystem, and these subsystem, and the set of α dus near in variant under such "local" transforma-

 $\{\cdot\}$ indicates $g = \alpha f + \beta f^*$ with $|\alpha|^2 - |\beta|^2 = 1$ defines the same single-mode subsyst. sates $g = \alpha J + \beta J$ with $|\alpha| - |\beta|$ field and pure momentum operators, respectively. The defines the same single-mode subsyst.

If the field is prepared in a quasi-free state $|0\rangle$ **(Gaussian)** $\qquad \qquad \qquad \hat{\rho}_f^{\text{red}}$ can be mixed

Simple way of computing $\hat{\rho}_f^{\text{red}}$: take advantage it is a Gaussian state

$$
\sigma_f = \begin{pmatrix} \langle \{\hat{O}_f, \hat{O}_f\} \rangle & \langle \{\hat{O}_f, \hat{O}_f^{\dagger}\} \rangle \\ \langle \{\hat{O}_f, \hat{O}_f^{\dagger}\} \rangle & \langle \{\hat{O}_f^{\dagger}, \hat{O}_f^{\dagger}\} \rangle \end{pmatrix} \qquad \qquad \Omega_f = \begin{pmatrix} \Omega(f, f) & \Omega(f, f^*) \\ \Omega(f, f^*) & \Omega(f^*, f^*) \end{pmatrix}
$$

state independent

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Example: entropy and purity

 $\pm i\nu$ = eigenvalues of $\sigma_f \cdot \Omega_f$

$$
S[\hat{\rho}_f^{\text{red}}] = \left(\frac{\nu+1}{2}\right) \log\left(\frac{\nu+1}{2}\right) - \left(\frac{\nu-1}{2}\right) \log\left(\frac{\nu-1}{2}\right) \quad \text{entropy of} \quad \hat{\rho}_f^{\text{red}}
$$

 $\hat{\rho}_f^{\text{red}}$ is pure iff the eigenvalues of $\sigma_f \cdot \Omega_f$ are equal to $\;\pm i\;$

If ${f}$ mixed ${f}$ must be entangled with other field modes

⇢ **region A**

It turns out that one cat It turns out that one can lind a It turns out that one can find a single-mode subsystem $\{\bar{f}\}$ encoding all entangled with $\{f\}$

FIG. 2. Illustration of two spacelike separated balls of ra-

dius *R* in a *t*=constant Cauchy hypersurface in *D* + 1 dimensional $\lceil \sqrt{P} \rceil = p$ partly supported in region $\{3 \}$ defined in region $\{3 \}$ defined in region $\{3 \}$ (ˆ *^A*(*B*)*,* ⇧ˆ *^A*(*B*)), as shown in Eq. (15). **= Partner of**

In this section, we apply the formalism presented above **to a situation in the function:** *[Hotta, Schützhold, Unruh 2015; Trevison, Yamaguchi, Hotta 2019; H* gion *A*, and *c* is an arbitrary constant with dimensions **[Hotta, Schützhold, Unruh 2015; Trevison, Yamaguchi, Hotta 2019; Hackl, Johnson 2019]**

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derive analytical expressions in any number of space di-Partner from the complex structure J: quantities are invariant under such "local" transforma-

 $\frac{1}{\sqrt{2}}$ on the introduce a mass to avoid infrareduce a mass to avoid infrare α and β and β and β mattin-Martinez, N ing operators defining the modes of interest as (ˆ *^A,* ⇧ˆ *^A*), [Agullo, Martin-Martinez, Nadal-Gisvert, Yamaguchi, to appear]

$$
\bar{f} = \Pi_f^\top (J\,f)
$$

Where

 \blacksquare T complex at $\overline{ }$ $\overline{\text{a}}$ are $\overline{\text{a}}$ f $\textbf{complex structure of } |0\rangle$

 \bullet $\Pi^+ - 1 - \Pi$, \mathbf{H} \mathbf{H} \mathbf{H} \mathbf{H} \mathbf{H} where ~*xⁱ* is the center of the ball *i*, and ⇥(*x*) is the Heav- $\mathbf{I} = (f \langle f, \cdot \rangle - f^* \langle f^*, \cdot \rangle)$: p pactly supported within a ball of radius *R* centered at **: projector orthogonal to subsystem** *{f}*

Patterner of
$$
\{f\}
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For f compactly supported, $\bar{f}(x)$ falls off at spatial infinity at a rate determined by the **two-point function** $\langle \hat{\Phi}(x) \hat{\Phi}(x') \rangle$

[Ribes-Metidieri, Agullo]

Minkoswki spacetime

$\langle \hat{\Phi}(x) \hat{\Phi}(x') \rangle \sim \frac{1}{r^2}$ **Two-point function:**

B ⇢ **region A**

Example:
$$
f(\vec{x}) = N \left(1 - \left(\frac{r}{R}\right)^2\right)^2 \times \Theta\left(1 - \frac{r}{R}\right)
$$

[Agullo, Martin-Martinez, Nadal-Gisvert, Yamaguchi, to appear]

The spatial support of the partner serves to quantify the spatial distribution of entanglement

de Sitter spacetime (Cosmological patch) **Massive scalar field, with small mass:** $\mu^2 \equiv \frac{m^2}{3H^2} \ll 1$

Bunch-Davies vacuum: $|0\rangle$

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A few results: [Agullo, Bonga, Ribes-Metidieri]

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Entropy of $\{f\}$ is larger in dSitter than in Minkwowski **BD** vacuum in dS space is more *{f}* \bigcirc

entangled than Minkowski vac.!

(Compatible with many previous results for the entropy of regions; see e.g. Maldacena 2013)

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But entanglement distributed very differently. Spread across much larger distances in dS. \bigcirc

Consequences:

 \mathcal{L} will most \mathcal{L} is the LN in situation in which it was in which it will in which it was in which Given two modes of compact support in dS

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Individually, they are more mixed \bigcirc ividually, they are more mixed

where $\overline{}$ is the center of the ball $\overline{}$ is the Heave $\overline{}$

~*xi*; *A* is a normalization constant determined below and

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Individually, they are more mixed

Consider two *D* dimensional balls, *A* and *B*, with ra-*R* ind they are less entangled with each other than they would be in Min We find they are less entangled with each other than they would be in Minkoswki st ! \bigcirc let ⇢ be the distance between their centers in units of *R*.

more entanglement with the partner is detrimental for within region *A* and *B*, respectively, and defined as fol-(Intuition: more entanglement with the partner is detrimental for entanglement with other modes) **Interesting consequences for cosmology**

Does Inflation generates entanglement??

(Long debate)

My answer: No if we only have access to local observables