Motivating semiclassical gravity

An approximation for bipartite quantum systems

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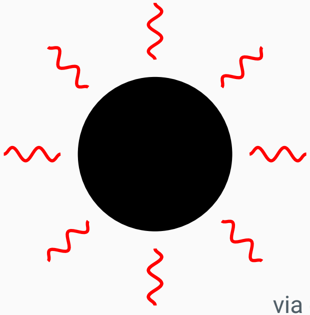
There is ***no consensus*** on a quantum theory of gravity to date.

. . . so we rely on approximations with ***classical*** gravity and ***quantum*** matter.

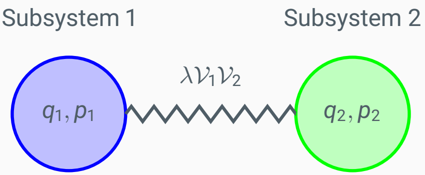


Quantized matter
carrying energy to infinity
from a classical black hole

Black hole mass is corrected
via energy conservation ($dM/dt \sim -1/M^2$)

We need to account for ***backreaction*** but do so in an ***ad hoc*** manner.

1. Are such ***classical-quantum*** approximations derivable from the fundamentals?
2. If so, how good are they, or what are their ***regimes of validity***?



A bipartite system (subsystem 1 + subsystem 2)

$$\mathcal{H}(q_1, p_1, q_2, p_2) = \mathcal{H}(q_1, p_1) + \mathcal{H}(q_2, p_2) + \lambda\mathcal{V}_1(q_1, p_1)\mathcal{V}_2(q_2, p_2) \text{ (\textbf{\textit{Classical-classical}})}$$

$$\hat{H} = \hat{H}_1 \otimes \hat{I}_2 + \hat{I}_1 \otimes \hat{H}_2 + \lambda\hat{V}_1 \otimes \hat{V}_2 \text{ (\textbf{\textit{Quantum-quantum}})}$$

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A classical-quantum approximation for this system

Classical eqs. with ***quantum*** expectations $\begin{cases} \partial_t q_1 = \partial_{p_1}\left(\mathcal{H}_1 + \lambda \langle\psi|\,\hat{V}_2\,|\psi\rangle\,\mathcal{V}_1\right) \\ \partial_t p_1 = -\partial_{q_1}\left(\mathcal{H}_1 + \lambda \langle\psi|\,\hat{V}_2\,|\psi\rangle\,\mathcal{V}_1\right) \end{cases}$

Quantum eq. with ***classical*** trajectory $\left\{\iota\partial_t\,|\psi\rangle = \left(\hat{H}_2 + \lambda\mathcal{V}_1(q_1,p_1)\hat{V}_2\right)|\psi\rangle\right.$ [1]

This is reminiscent of the ***semiclassical Einstein equation***: $G_{\alpha\beta} = M_{Pl}^{-2}\,\langle\psi|\,\hat{T}_{\alpha\beta}\,|\psi\rangle$.

[1] V. Husain, I. Javed, and S. Singh, Phys. Rev. Lett. 129, 111302 (2022).

Could the said classical-quantum (CQ) approximation be derived from the known correct quantum-quantum (QQ) dynamics, which is given by the following?

$$\iota\partial_t \left|\psi\right\rangle = \hat{H}\left|\psi\right\rangle$$
$$\hat{H} = \hat{H}_1 \otimes \hat{I}_2 + \hat{I}_1 \otimes \hat{H}_2 + \lambda \hat{V}_1 \otimes \hat{V}_2$$

Others too have attempted to derive this CQ approximation but ***without much success***.[2]

Our approach relies on ***somewhat different assumptions*** from theirs.

[2] T. Singh and T. Padmanabhan, Ann. Phys. (N.Y.) 196, 296 (1989)
C. Kiefer and T. P. Singh, Phys. Rev. D 44, 1067 (1991).

$$\imath\partial_t |\psi\rangle = \hat{H} |\psi\rangle \xrightarrow{\text{Approximation}} \begin{cases} \partial_t q_1 = \partial_{p_1} \left(\mathcal{H}_1 + \lambda \langle\psi| \hat{V}_2 |\psi\rangle \mathcal{V}_1 \right) \\ \partial_t p_1 = -\partial_{q_1} \left(\mathcal{H}_1 + \lambda \langle\psi| \hat{V}_2 |\psi\rangle \mathcal{V}_1 \right) \\ \imath\partial_t |\psi\rangle = \left(\hat{H}_2 + \lambda \mathcal{V}_1(q_1, p_1) \hat{V}_2 \right) |\psi\rangle \end{cases}$$

Assumptions allowing the approximation

We find that the CQ approximation is valid if the following hold.

1. Coupling parameter λ is small.
2. Entanglement between subsystems is small.
3. Quantum state of subsystem 1 is a ***semiclassical*** state.

We start by assuming that the system is in a nearly product state (entanglement is small):

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle + \mathcal{O}(\lambda).$$

We could then read off ***effective Hamiltonians*** from the equations of motion for the subsystems' reduced density matrices:

$$\iota\partial_t\rho_{1,2} = \left[H^{\text{eff}}_{1,2}, \rho_{1,2}\right] + \mathcal{O}\left(\lambda^2\right),$$

where $\hat{H}^{\text{eff}}_{1,2} = \hat{H}_{1,2} + \lambda \langle\psi_{2,1}| \hat{V}_{2,1} |\psi_{2,1}\rangle \hat{V}_{1,2}$.

Finally, we assume a sharply peaked semiclassical state for subsystem 1 such that

$$\partial_t q_1 \approx \partial_{p_1}\mathcal{H}_1 + \lambda \langle\psi_2| \hat{V}_2 |\psi_2\rangle \partial_{p_1}\mathcal{V}_1$$

and

$$\partial_t p_1 \approx -\partial_{q_1}\mathcal{H}_1 - \lambda \langle\psi_2| \hat{V}_2 |\psi_2\rangle \partial_{q_1}\mathcal{V}_1,$$

where $q_1 = \langle\psi_1| \hat{q}_1 |\psi_1\rangle$, $p_1 = \langle\psi_1| \hat{p}_1 |\psi_1\rangle$, and $\langle\psi_1| \hat{V}_1 |\psi_1\rangle \approx \mathcal{V}_1(q_1, p_1)$.

Being an approximation after all, the CQ scheme holds for a ***finite amount of time***.

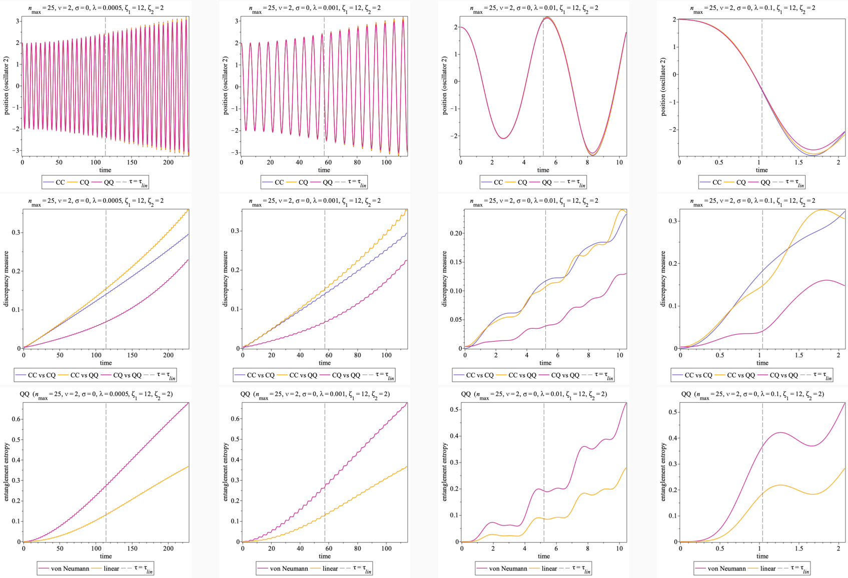
Failure could be determined by two different time scales.

1. ***Ehrenfest time:*** characteristic time for the spread of subsystem 1 quantum state
2. ***Scrambling time:*** characteristic time for the growth of entanglement between subsystem 1 and subsystem 2

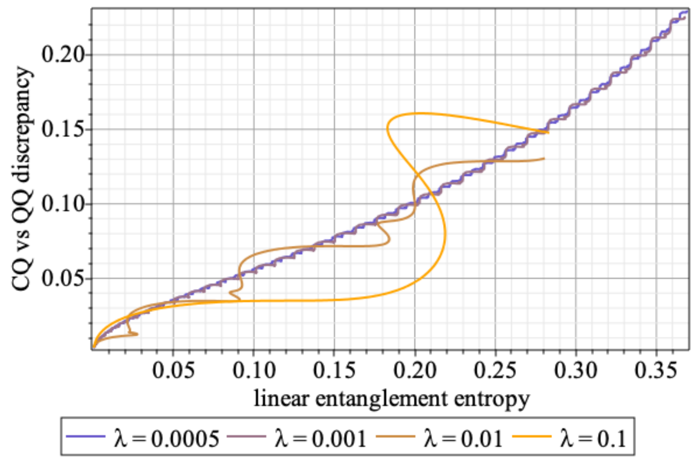
Ehrenfest time is well understood: it becomes longer as the energy of subsystem 1 gets higher.

Scrambling time is calculated using linear perturbation theory as timescale for linear entanglement entropy to grow from 0 to $\mathcal{O}(1)$.

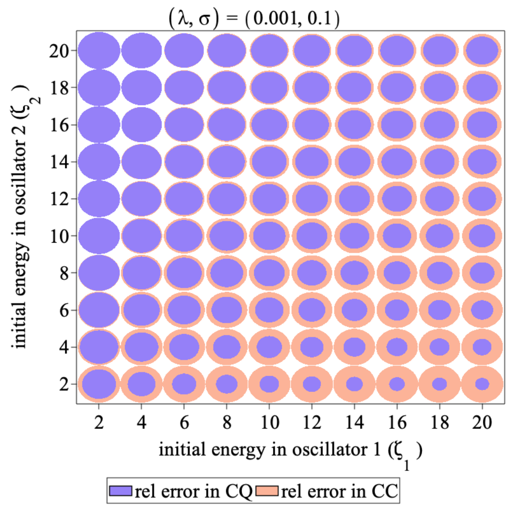
Results and discussion CQ vs. QQ and CC



Results and discussion Error and entropy



Results and discussion Ehrenfest verification



Key messages

1. ***Derivation*** of a classical-quantum approximation like $G_{\alpha\beta} = M_{Pl}^{-2} \langle\psi| \hat{T}_{\alpha\beta} |\psi\rangle$
2. Approximation failure after a (***calculable***) finite amount of time

Future directions

1. Explicit ***generalization*** to gravity remains to be seen (e.g., parametric resonance).
2. ***Long-term behavior*** of entropy ($S \sim 2/3 \ln(E)$) asks for further exploration.

Thank you!
Questions?