Loops '24

De Sitter horizon entropy

from a simplicial Lorentzian path integral

- 2403.02119

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This talk:

- **Partition function computing the quantum gravity Hilbert space dimension of a (D-1)-ball of space from a Lorentzian path integral:**

$\Lambda > 0$

$$Z \sim e^{S_{\mathrm{dS}}}$$

How can this be reproduced from an oscillatory, Lorentzian integral?!

- **Simplicial formulation of the <u>3-dimensional</u> computation**

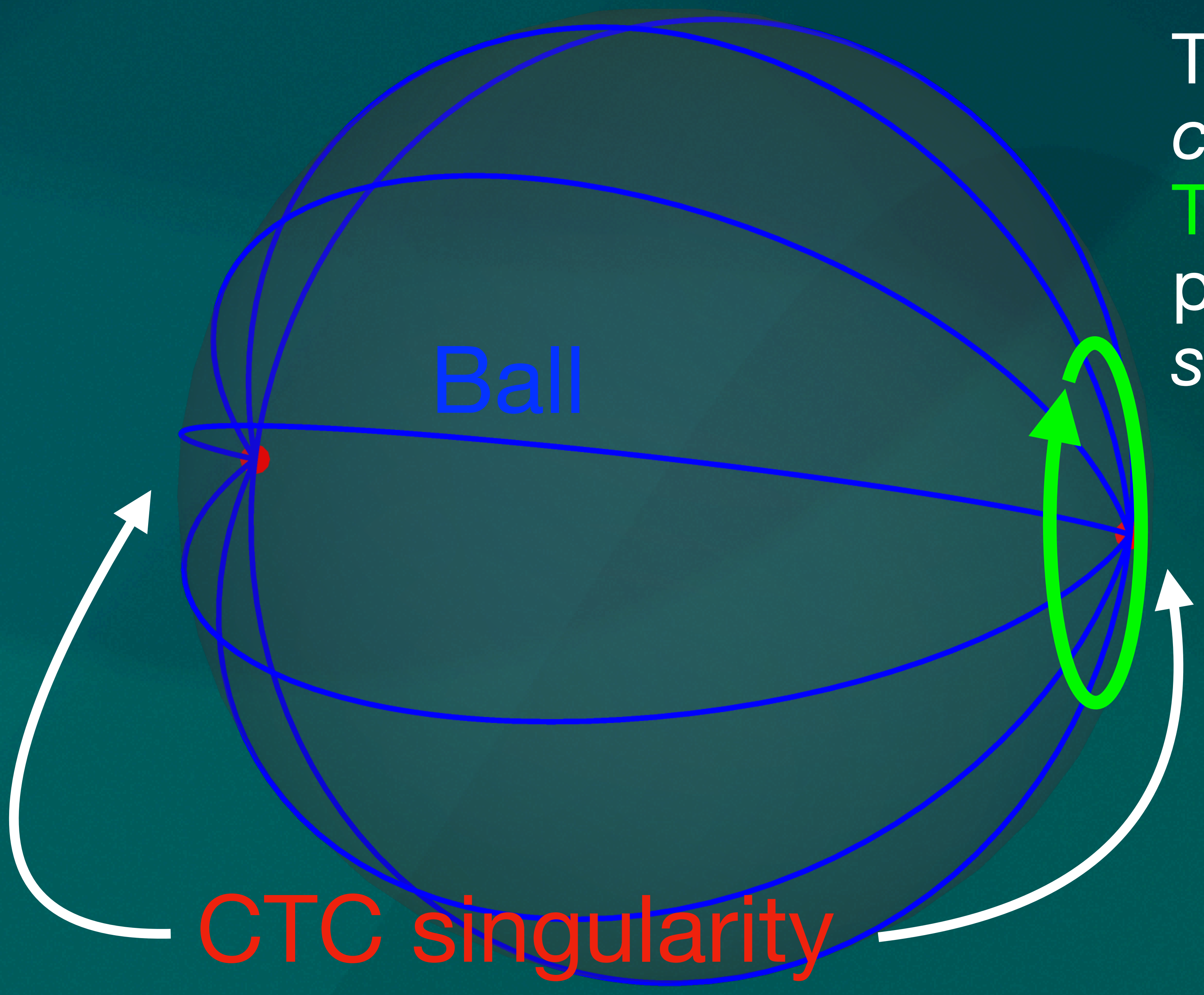
Along the way: Off-shell causality violations, complex metrics and actions, convergence of vacuum fluctuations…

Full GR computation:

$$Z_{\mathcal{H}} = \mathrm{Tr}\mathbb{1}_{\mathcal{H}} = \int \mathrm{D}p_{\mathrm{phys}}\mathrm{D}q_{\mathrm{phys}} e^{\imath \oint p_{\mathrm{phys}} \dot{q}_{\mathrm{phys}} \mathrm{d}t} \sim \int \mathrm{D}p\mathrm{D}q\,\delta(C_i) e^{\imath \oint p\dot{q}\mathrm{d}t} = \int \mathrm{D}p\mathrm{D}q\mathrm{D}N^i e^{\imath \oint p\dot{q}\mathrm{d}t - \imath N^i C_i}$$

‘Lorentzian’ path integral

2D example:



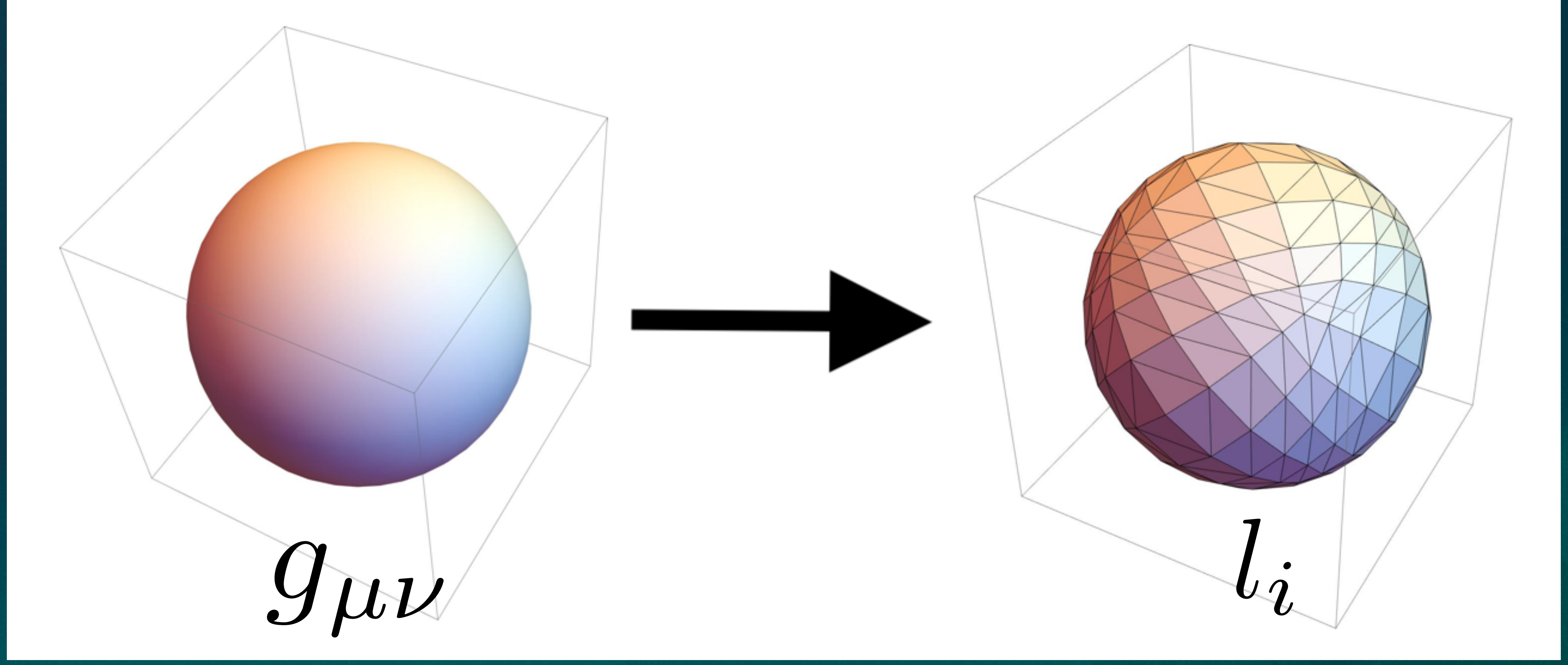
This foliation forces upon us a *chronotopology*: There are Closed Timelike Curves (CTCs) contractible to points of a 0-sphere… There is a *CTC singularity*

CTC → $\mathrm{Im}\, S_{\mathrm{EH}} = A/4G$

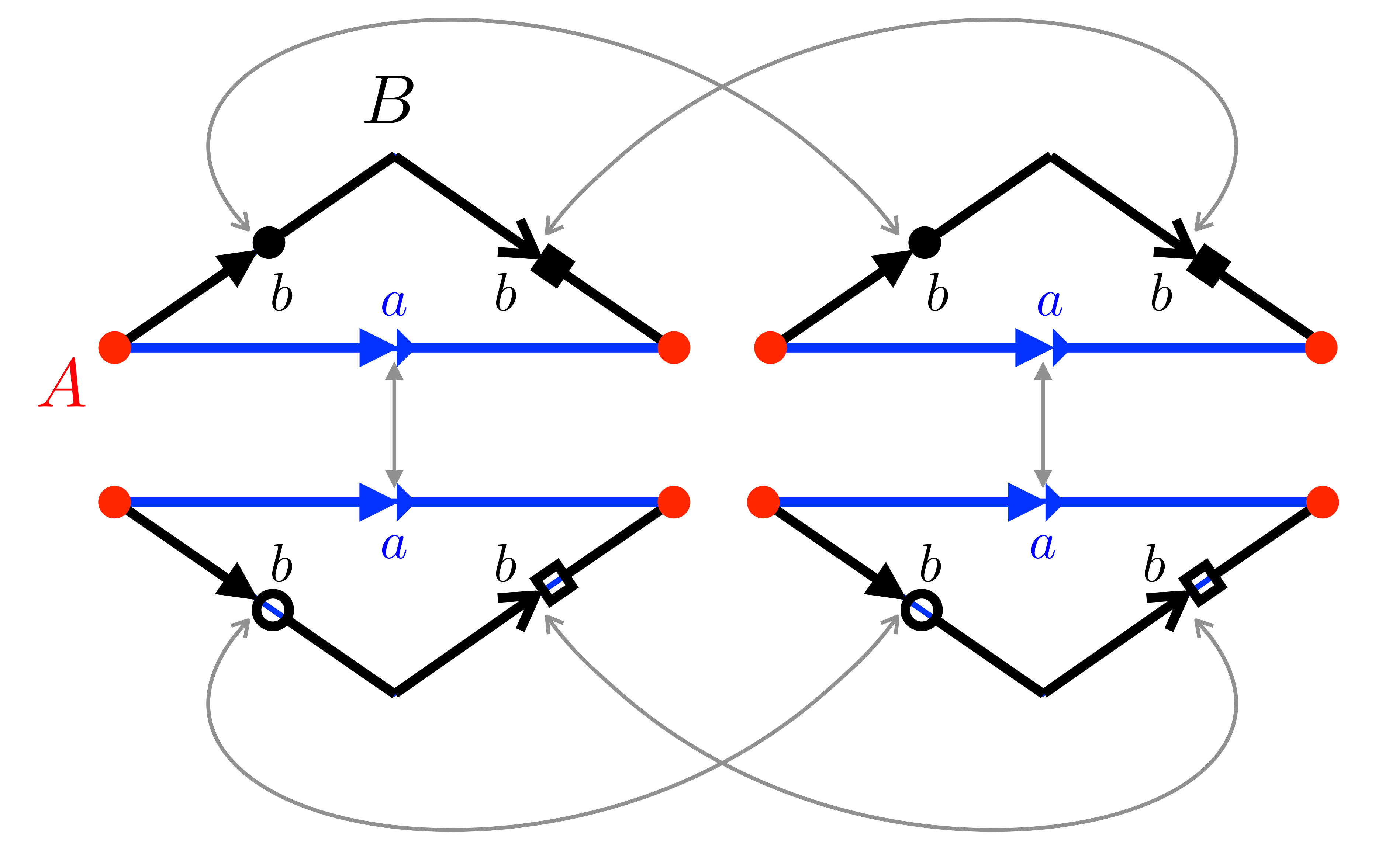
[Jacobson & Banihashemi ‘22]

Regge calculus setup

2D analogues of our computation



Discretization: $g_{\mu\nu} \to (s_a, s_b) \equiv (s_a, s_h)$



Discretization

$$Z_{\mathrm{EH}} = \int \mathrm{D}g\, e^{W} \longrightarrow \int \prod_e \mathrm{d}l_e \mu(\{l_e\})\ e^{W}.$$

Strategy: Compute the `complex Regge’ exponent W ($s_h \to r_h e^{\imath\phi}$)

and explore deformations of the Lorentzian contour

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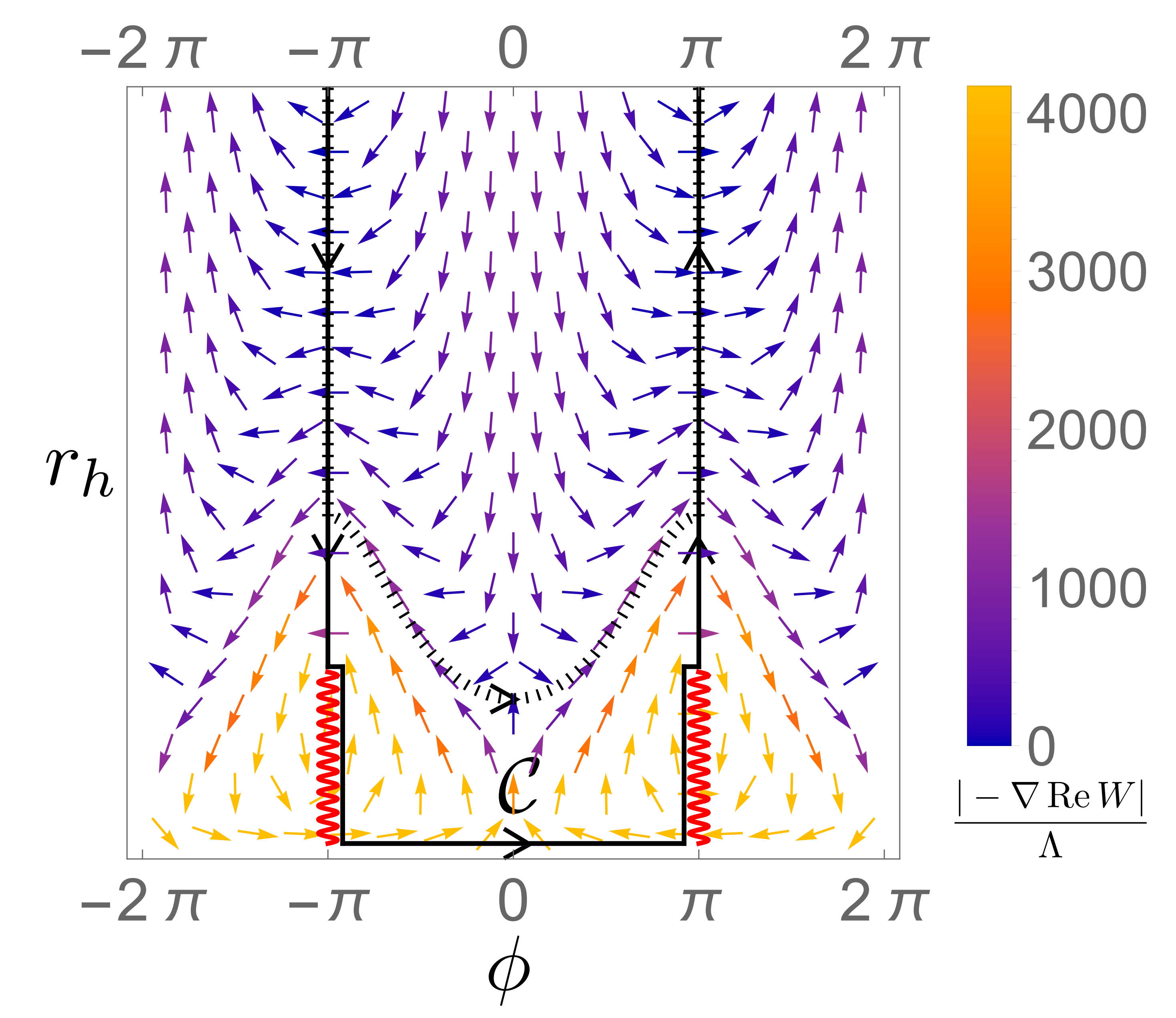
$$Z = \lim_{\varepsilon\to 0}\int_0^\infty \mathrm{d}s_a \mu_a(s_a)\int_{\mathcal{C}} \mathrm{d}s_h \mu_h(s_h) e^{W}$$

Fixed-area path integral

Euclidean regime $s_a \lesssim 1/\Lambda$



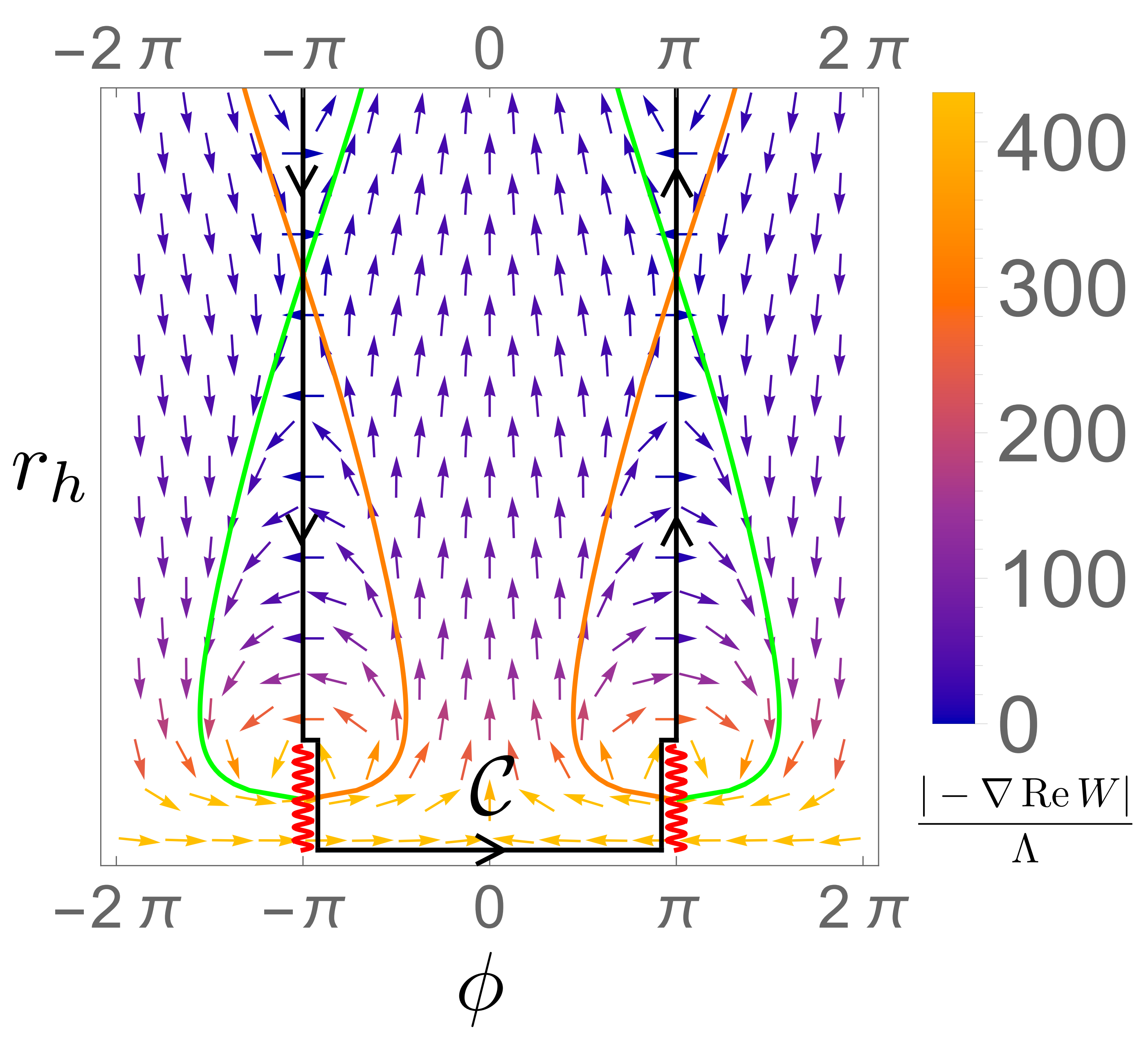
Steepest descent flow shows that the integral is indeed approximated by a *Euclidean* saddle point evaluation! So the continuum expectation is recovered!

$$Z \sim e^{S_{\mathrm{dS}}}$$

Lorentzian regime $s_a \gtrsim 1/\Lambda$

The exponential enhancement due to the causal irregularities makes the *right* Euclidean saddle 'accessible'… But here it would seem to backfire

[Marolf '22]



For the class of measures $\mu_h = s_h^{\alpha}, \quad \alpha < -\frac{1}{2}$, the fixed-length integrals in this regime vanishes! Because the integral of the arc at infinity from $-\pi$ to π vanishes.

Conclusions

Our discrete formulation shows that:

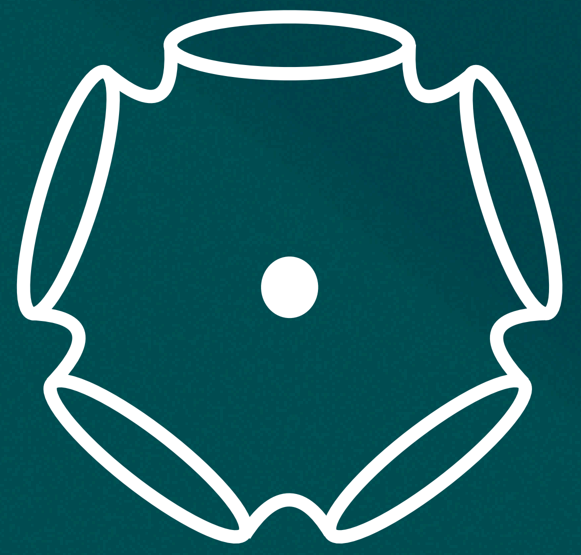
1. Euclidean saddles dominate in the semiclassical limit (if CTC singularities are included).

2. Very large CTC singularities do not contribute at all! The enhancement of CTC singularities can be ‘tamed’.

We do not expect our conclusions to change for $D > 3$

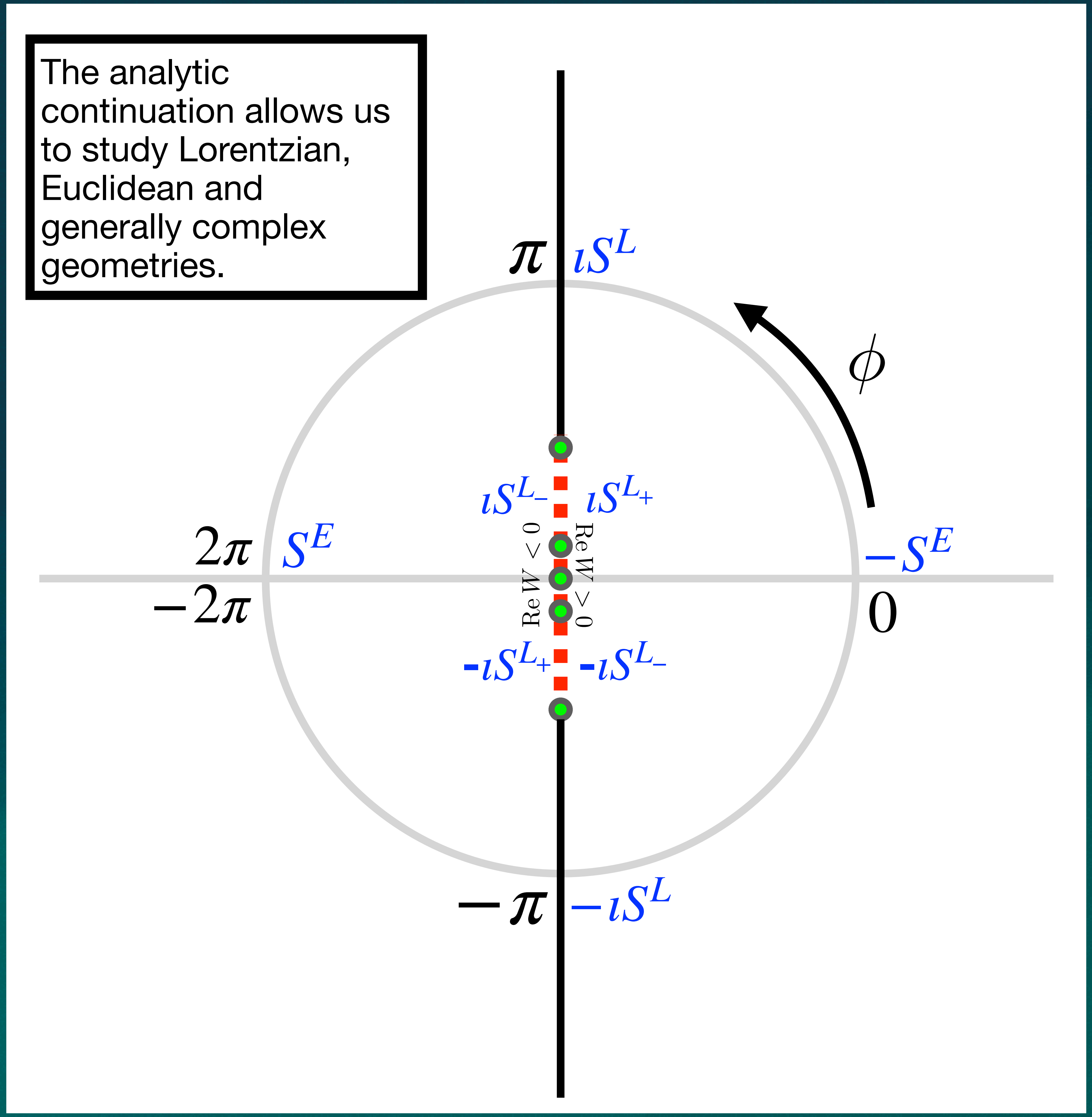
Outlook

- Application of quantum Regge calculus to other thermodynamic situations (*e.g.* Black hole evaporation process in the real-time replica paradigm)
- Study of the contributions of causal irregularities in the EPRL spin foam model
- Comparison with no-boundary proposal essential singularity
- Exploration of off-shell fluctuation convergence criterion using Regge calculus



[Dittrich, Asante, JPA ’21]

Regge action analytic structure



$$W = 6\sqrt{s_a}\left[\pi + 2\imath \log\left(\frac{e^{-\frac{\imath\phi}{2}}\sqrt{s_a}+\imath\sqrt{12 r_h}}{\sqrt{e^{-\imath\phi}s_a+12r_h}}\right)\right] + 4e^{\frac{\imath\phi}{2}}\sqrt{9r_h+3e^{-\imath\phi}s_a}\left[\pi+\imath\log\left(\frac{-s_a+6e^{\imath\phi}\left(r_h+\imath\sqrt{r_h\left(3r_h+e^{-\imath\phi}s_a\right)}\right)}{s_a+12e^{\imath\phi}r_h}\right)\right] - \Lambda e^{\frac{\imath\phi}{2}} s_a\sqrt{r_h/3}$$

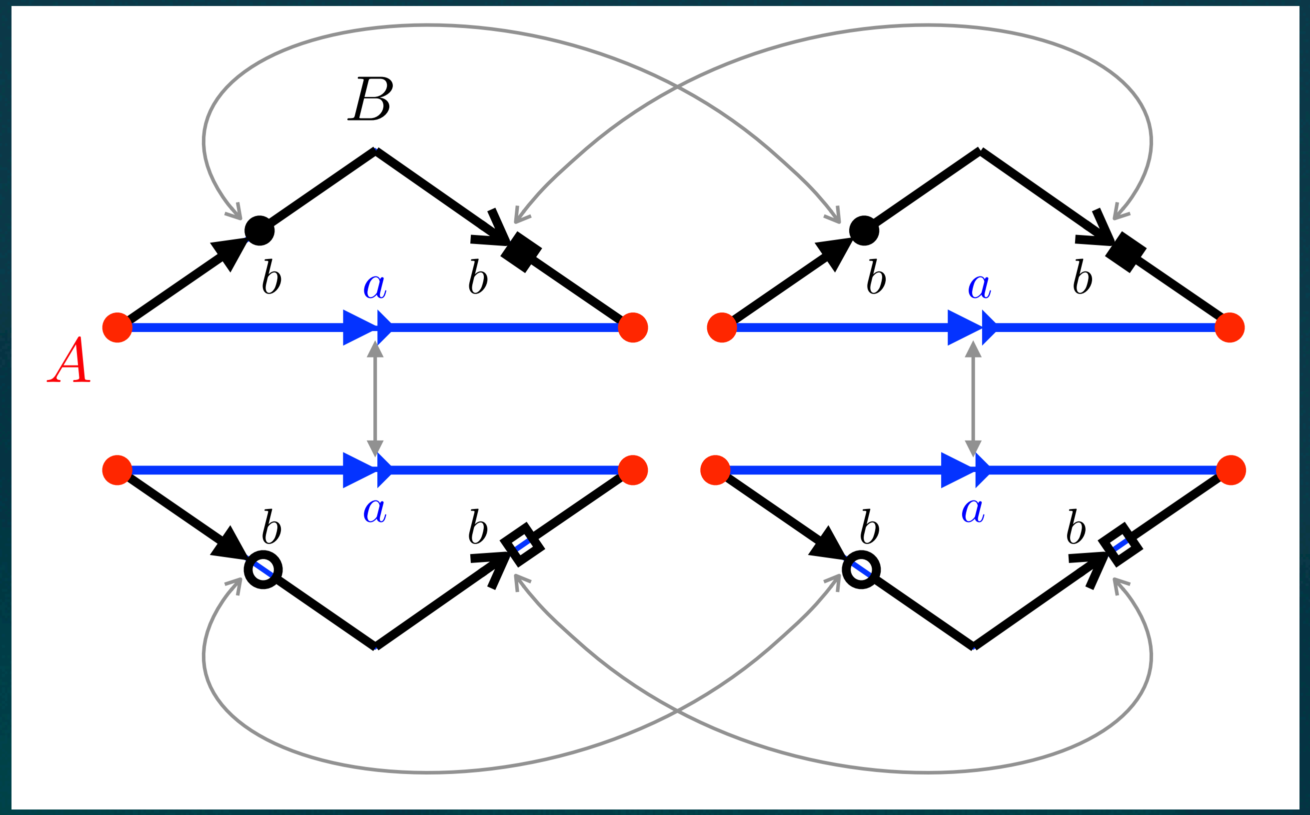
The Lorentzian Regge action has a branch-cut when the light cone structure is irregular… **Which side should one choose?**

$$Z = \lim_{\varepsilon\to 0}\int_0^\infty \mathrm{d}s_a \mu_a(s_a)\int_{\mathcal{C}} \mathrm{d}s_h \mu_h(s_h) e^W$$

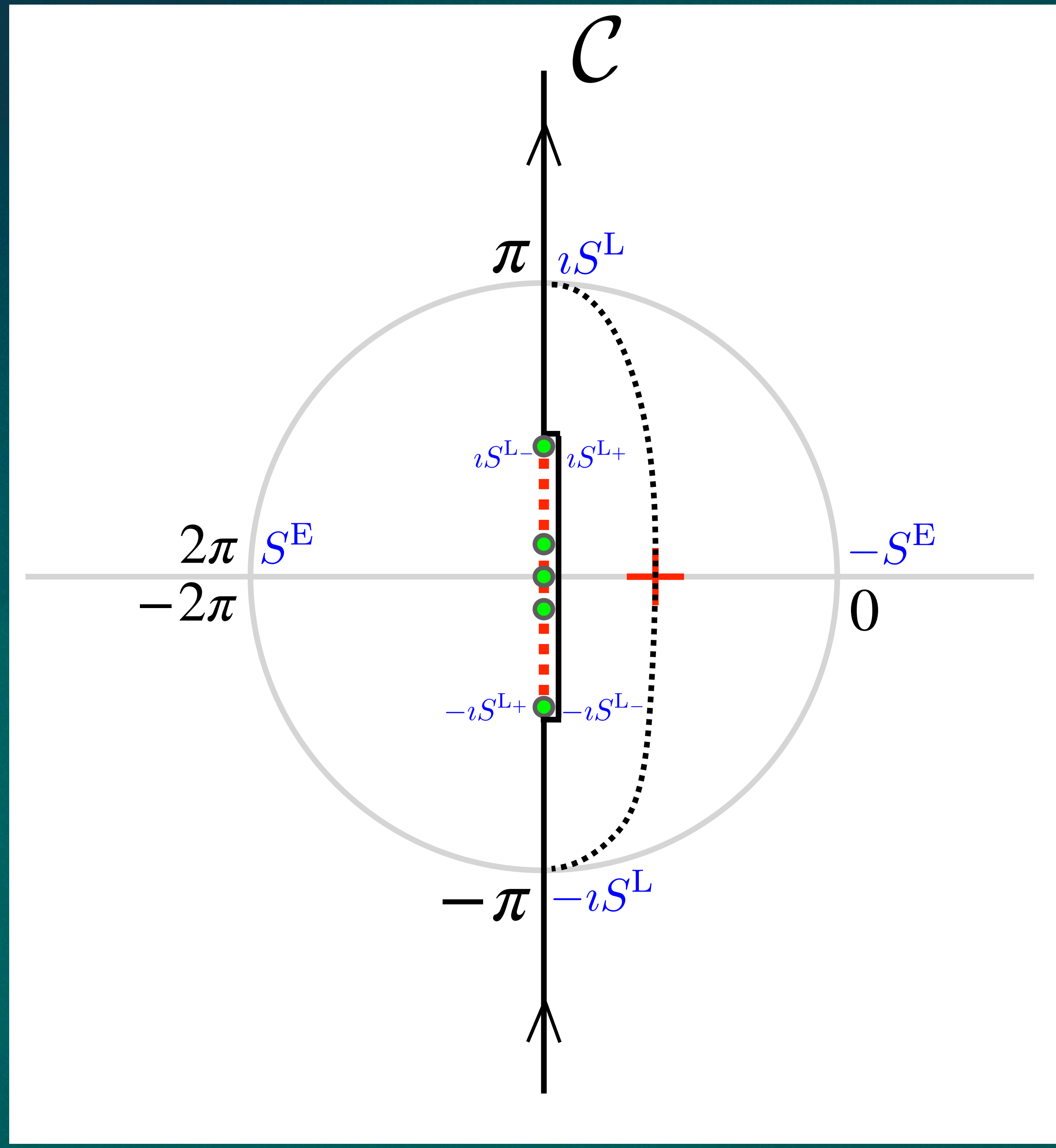
Parallels with the real-time no-boundary proposal

The fixed-length path integrals are similar to microsuperspace no-boundary (NB) transition amplitudes.

In fact, the saddle point threshold behavior is mimicked.



Are Lorentzian saddles discrete avatars of the continuum complex saddles?



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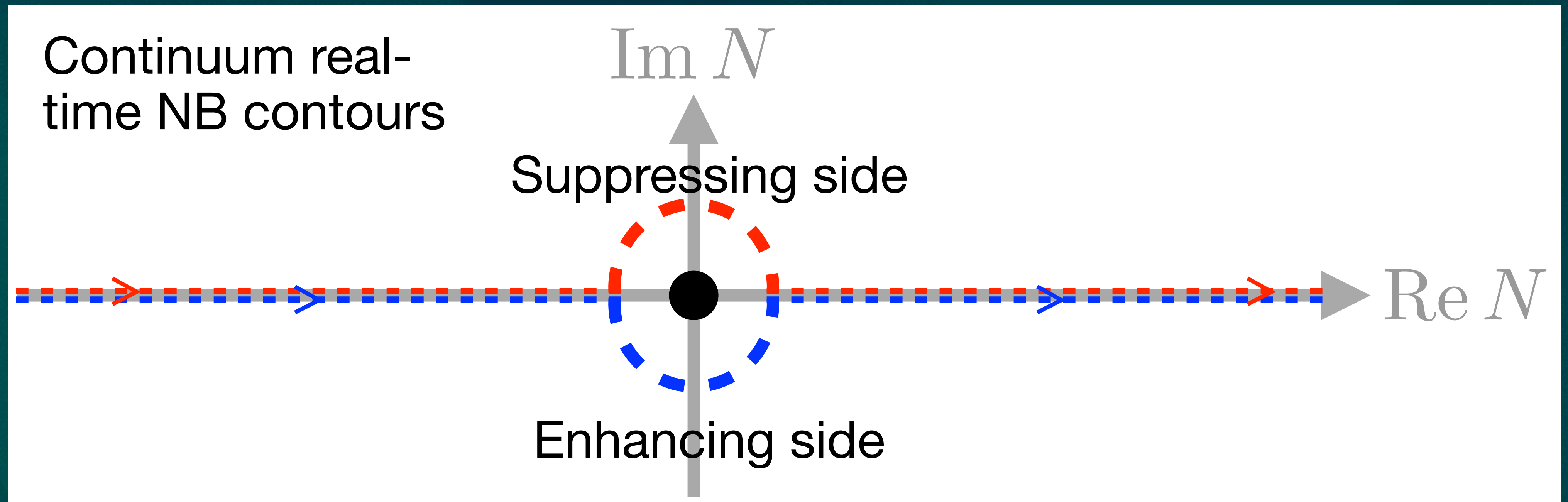
Are branch-cuts blown-ups of continuum essential singularity?



[Feldbrugge, Lehners , Turok ’17 …]

[Diaz Dorronsoro, Hartle, et al. ’17]