Area metric actions and the Barbero-Immirzi parameter

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Area metric

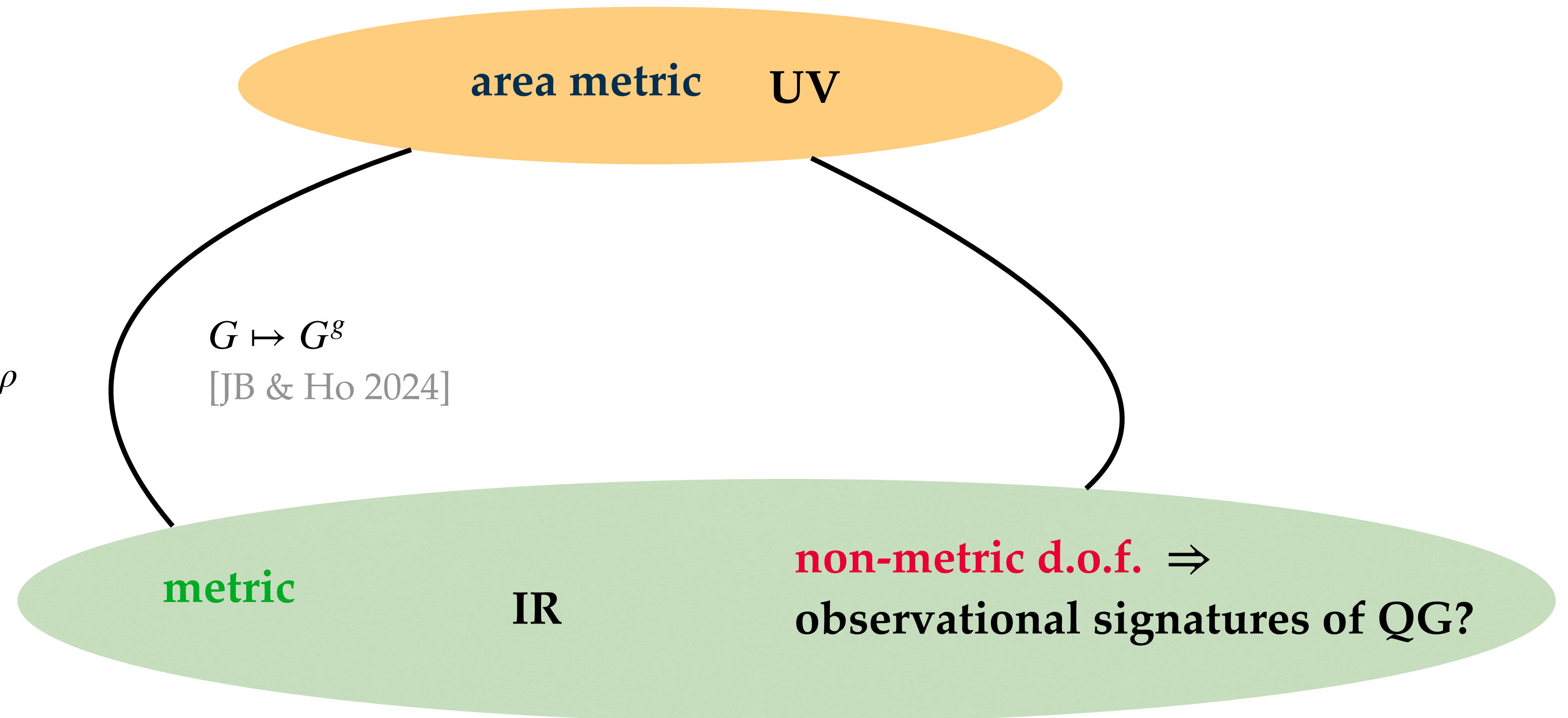[Schuller & Wohlfahrt 2005, Punzi et al 2006]

$$d = 4 : 21 - 1 = 10 + 10 \text{ d.o.f.}$$

$$G_{\mu\nu\rho\sigma} = G_{\rho\sigma\mu\nu} = -G_{\nu\mu\rho\sigma}$$

$$G_{\mu[\nu\rho\sigma]} = 0$$

induced area metric

$$G^g_{\mu\nu\rho\sigma}(g) = g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}$$

$$G \mapsto G^g$$

[JB & Ho 2024]

area metric **UV**

metric **IR**

non-metric d.o.f. $\Rightarrow$
observational signatures of QG?

1

Area (metric) variables in QG and beyond

LQG
[Rovelli & Smolin 1994, Ashtekar & Lewandowski 1996]
$$A_j = \gamma l_{pl}^2 \sqrt{j(j+1)}$$

(black-hole) entropy
[Bekenstein 1972-3, Hawking 1974]
$$S_{\text{BH}} = \frac{A}{4l_{pl}^2}$$
[Ryu & Takayanagi 2008]
"area law" for holographic entanglement entropy

fundamental action(s)

(Q)ED, YM: $S \supset G_{\mu\nu\rho\sigma}^g F^{\mu\nu} F^{\rho\sigma}$
Nambu-Goto: $S = \text{Area}(\Sigma)$

$G^g \mapsto G$ generalized background:
birefringence [Punzi et al 2009]
string worldsheet interactions [JB & Ho 2024]

modified non-chiral Plebanski theories
[Plebanski 1997, Reisenberger 1995, Pietri & Freidel 1999, Krasnov 2006-9, Alexandrov & Krasnov 2008, Freidel 2008, Speziale 2010]
$B \leftrightarrow g_\pm +$ 10 d.o.f.

[JB & Dittrich 2022]
G via $g_+ \overset{!}{=} g_- \equiv g$:
simplicity constraints: 10 sharp + 10 "weak"
$$\mathcal{L}_{eff}(h) = \mathcal{L}_{EH}(h) - \frac{1}{2}C_{\mu\nu\rho\sigma}\left(\frac{1}{\Box + m_+^2} + \frac{1}{\Box + m_-^2}\right)C^{\mu\nu\rho\sigma}$$
low-energy spin-foam effective action

semiclassical regime of (effective) spin foams
[Dittrich 2021, Dittrich & Kogios 2022]
Area-Regge continuum limit: $S[A_t] \to S[G]$
$G \leftrightarrow g +$ massive d.o.f.
$$\mathcal{L}_{eff}(h) = \mathcal{L}_{EH}(h) + \text{Weyl}^2 + \mathcal{O}(\lambda^4)$$

twisted geometries
[Dittrich, Padua-Argüelles 2023]
coherent $\sigma \leftrightarrow$ microscopic G_σ

Area metrics constitute a **unified framework for QG**.
They may provide a route towards establishing **connections between distinct QG theories** and understanding **universal features of QG**.

Generally covariant second-order quadratic Lagrangian for area metric perturbations

1. **expansion** $G = G^\delta +$ fluctuations $a \leftrightarrow (h_{\mu\nu}, \chi^+_{\mu\nu}, \chi^-_{\mu\nu})$

2. **most general ansatz** $\mathscr{L}(a) = \mathscr{L}(h_{\mu\nu}, \chi^+_{\mu\nu}, \chi^-_{\mu\nu})$

3. **diffeomorphism invariance** $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$, $\chi^\pm_{\mu\nu} \to \chi^\pm_{\mu\nu}$

$$\mathscr{L}(a) = \mathscr{L}_{EH}(h) + \frac{1}{2}\sum_\pm \alpha_\pm h_{\mu\nu}\chi^{\pm\mu\nu}p^2 + \frac{1}{2}(\beta_\pm p^2 + m^2_\pm)\chi^\pm_{\mu\nu}\chi^{\pm\mu\nu} \quad (\mathscr{L} \text{ parity-invariant} \Leftrightarrow \text{couplings "+"} \stackrel{!}{=} \text{"−"})$$

Effective metric action and propagator $\quad \rho_\pm \equiv \dfrac{\alpha^2_\pm}{\beta_\pm}, \; M^2_\pm \equiv \dfrac{m^2_\pm}{\beta_\pm}$

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Area metric actions from modified non-chiral Plebanski theories [JB & Dittrich 2022]

$$\mathscr{L}(a) = \mathscr{L}_{EH}(h) + \frac{1}{2}\sum_\pm \gamma_\pm h_{\mu\nu}\chi^{\pm\mu\nu}p^2$$

$$+ \frac{1}{2}(\gamma_\pm p^2 + M^2_\pm)\chi^\pm_{\mu\nu}\chi^{\pm\mu\nu}, \quad \gamma_\pm = 1 \pm \frac{1}{\gamma}$$

low-energy spin-foam effective action

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$$\mathscr{L}_{eff}(h) = \mathscr{L}_{EH}(h) - \frac{1}{2}{}^{(1)}C_{\mu\nu\rho\sigma}(h)\left(\frac{\rho_+}{p^2 + M^2_+} + \frac{\rho_-}{p^2 + M^2_-}\right){}^{(1)}C^{\mu\nu\rho\sigma}(h)$$

GR + nonlocal Weyl^2

$$\begin{aligned} \alpha_\pm &\leftrightarrow \gamma_\pm \\ \beta_\pm &\leftrightarrow \gamma_\pm \end{aligned}$$

$M^2_+ \equiv M^2_- \equiv M^2, \rho_+ + \rho_- = 2$ (additional shift symmetry):

$$\left(\text{Prop}\right)^{\text{spin-2}} \propto \frac{1}{p^2} + \frac{1}{M^2} \text{ ghostfree}$$

3

Lorentzian signature

$\chi^{E\pm} \in \mathbb{R}: \quad \mathscr{L}^E \supset \left(\chi^{E+}\right)^2 + \left(\chi^{E-}\right)^2$ positive definite

$\chi^{L+} = \overline{\chi^{L-}} \in \mathbb{C}: \quad \mathscr{L}^L \supset \left(\mathfrak{R}\left[\chi^{L+}\right]\right)^2 - \left(\mathfrak{I}\left[\chi^{L+}\right]\right)^2$ **indefinite**

$\mathscr{L}^L$ **coupled system of positive-energy d.o.f. interacting with negative-energy d.o.f.** $\Rightarrow$ **instabilities?**

couplings: (degenerate case with additional shift symmetry $\alpha_+^2 + \alpha_-^2 = 2$ ($\beta_+ = \beta_- \equiv 1$ via field redefinition) & equal masses $M_+^2 = M_-^2 \equiv M^2$)

$\alpha_+^L = \overline{\alpha_-^L} \equiv \mathfrak{R}\left[\alpha_+^L\right] + \iota\mathfrak{I}\left[\alpha_+^L\right], \quad \left(\mathfrak{R}\left[\alpha_+^L\right]\right)^2 - \left(\mathfrak{I}\left[\alpha_+^L\right]\right)^2 = 1$

Hamiltonian analysis of linearized area metric gravity

$\mathscr{H}$ **indefinite** — but **linearized dynamics stable**

2 massless spin-2 modes (graviton) + **5 massive modes with negative energy** (decoupled)

graviton arising from shifted spatial metric $h_{ab} + \mathfrak{R}\left[\alpha_+^L\right]\mathfrak{R}\left[\chi^{L+}\right]_{ab} - \mathfrak{I}\left[\alpha_+^L\right]\mathfrak{I}\left[\chi^{L+}\right]_{ab} \equiv H_{ab} \supset H_{ab}^{TT} \equiv H_{ab}^+ + H_{ab}^\times$

mixing of + and × polarizations: $\lambda\left(\gamma, M^2\right) \propto \dfrac{\mathfrak{R}\left[\alpha_+^L\right]\mathfrak{I}\left[\alpha_+^L\right]}{M^2}$

Prospects

- dynamics at **higher orders — instabilities? ∃ fine-tuned stable interactions?** implications for **LQG Hamiltonian?**

- **RG flows** for area metric Lagrangians: **masses** $M_\pm^2$ and **BI parameter** γ [JB, Dittrich, Eichhorn & Schiffer w.i.p.]
- phenomenology from area metric gravity: **parity-breaking / γ-dependent measurable effects**? corrections to **Newton potential**?

- non-perturbative constant area metric Lagrangian — classification of **area metric scalar invariants**?
- connection to **volume of twisted simplex** and **area-length/shape-matching constraints**?
- towards **area metric geometry**: area metric compatible connections and curvature

- improved understanding of **IR limit and geometric background reduction** $G \mapsto G^g$ (continuum + discrete)? **physical implications** for Λ? [JB & Ho 2024 + to do]

- generalization of **holographic entanglement entropy** derivations to area metric backgrounds? **coupling of fermions** to area metric?
- **LQG derivation** of **BH entropy** in area metric picture? **physical reinterpretation of large- vs small-j limit?**

- area metric backgrounds in **string theory**: consequences of **area metric fluctuations** on **quantized worldsheet action?** interpretation of **singular vertex operators?** relation to stringy **space-time uncertainty principle** and **GUP?** [JB & Ho 2024 + to do]

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