Shell-crossing singularities/shockwaves in effective Lemaître-Tolman-Bondi collapse

Francesco Fazzini

University of New Brunswick

In collaboration with Lorenzo Cipriani, Viqar Husain & Edward Wilson-Ewing

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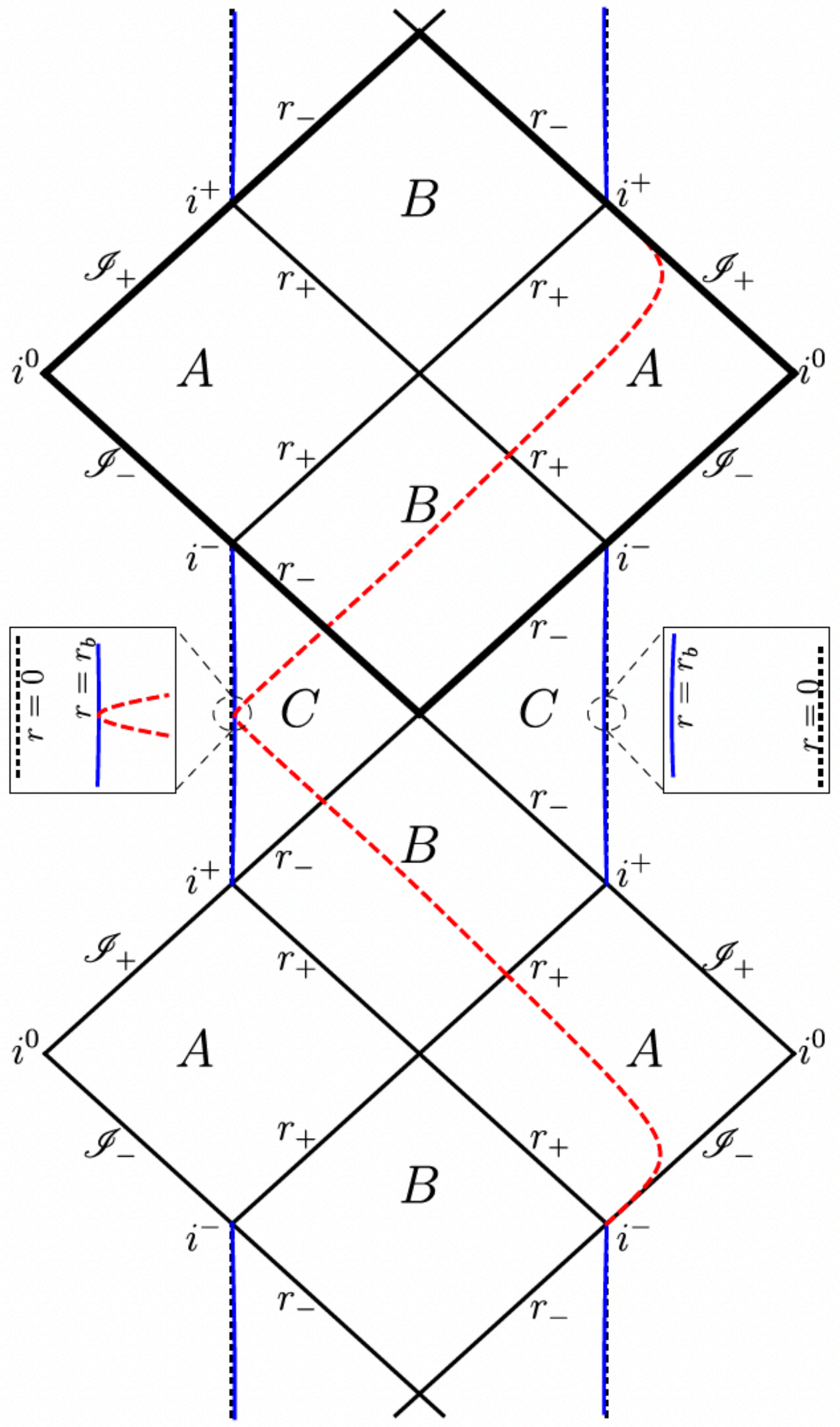
Cipriani, FF, Wilson Ewing, arXiv:2404.04192

Introduction

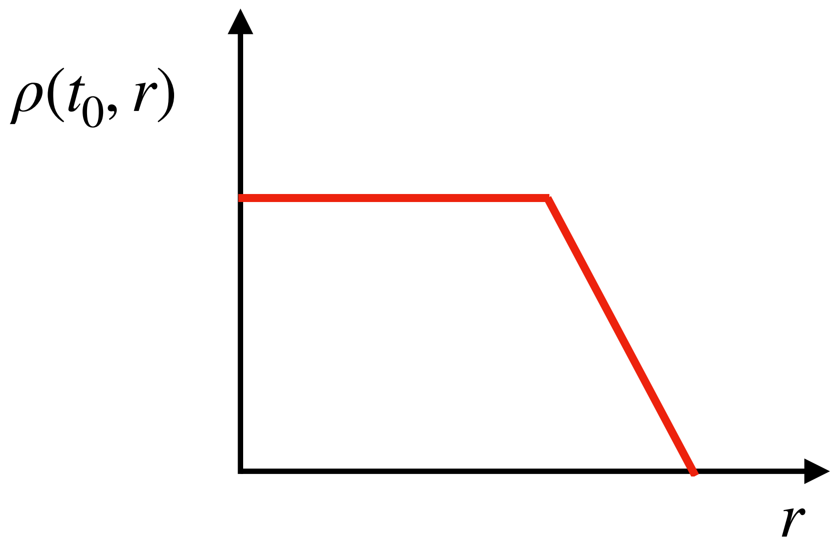
Spherically symmetric effective dust collapse with an infinitely sharp boundary (quantum OS model) predicts a symmetric dynamics around the bounce point $\left(\rho_{bounce} = \dfrac{3}{8\pi G\gamma^2\Delta} \equiv \rho_c\right)$.

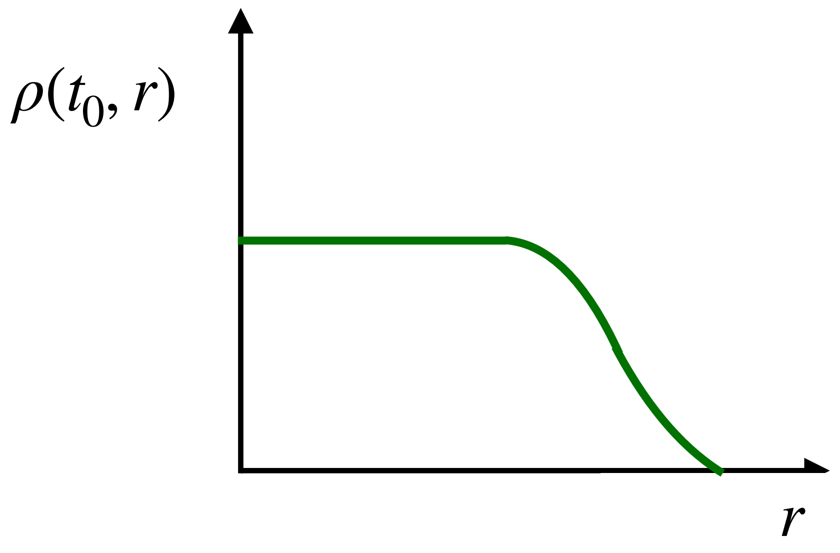
[Lewandowski, Ma, Yang, Zhang, 2023; Giesel, Liu, Singh, Weigl, 2023; FF, Rovelli, Soltani, 2023]

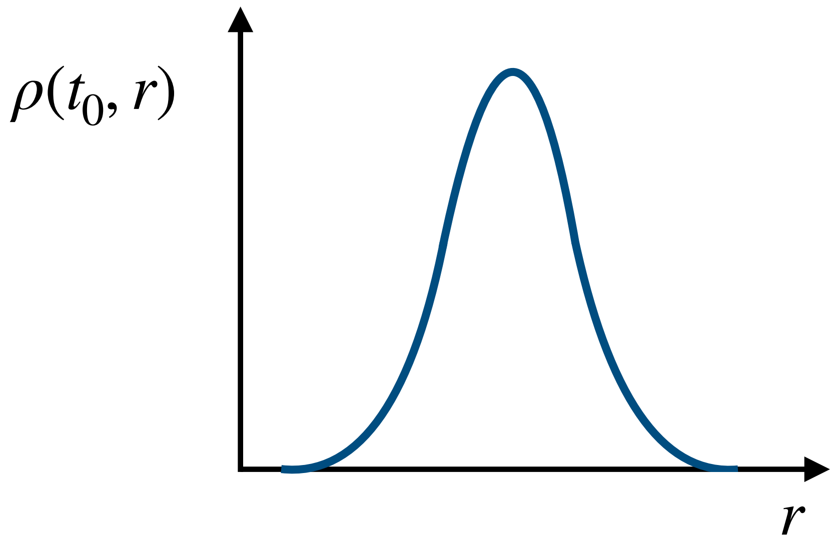


[Lewandowski, Ma, Yang, Zhang, 2023]

- Is the OS model a good prototype?
- How does the picture change if we consider continuous initial energy density profiles?





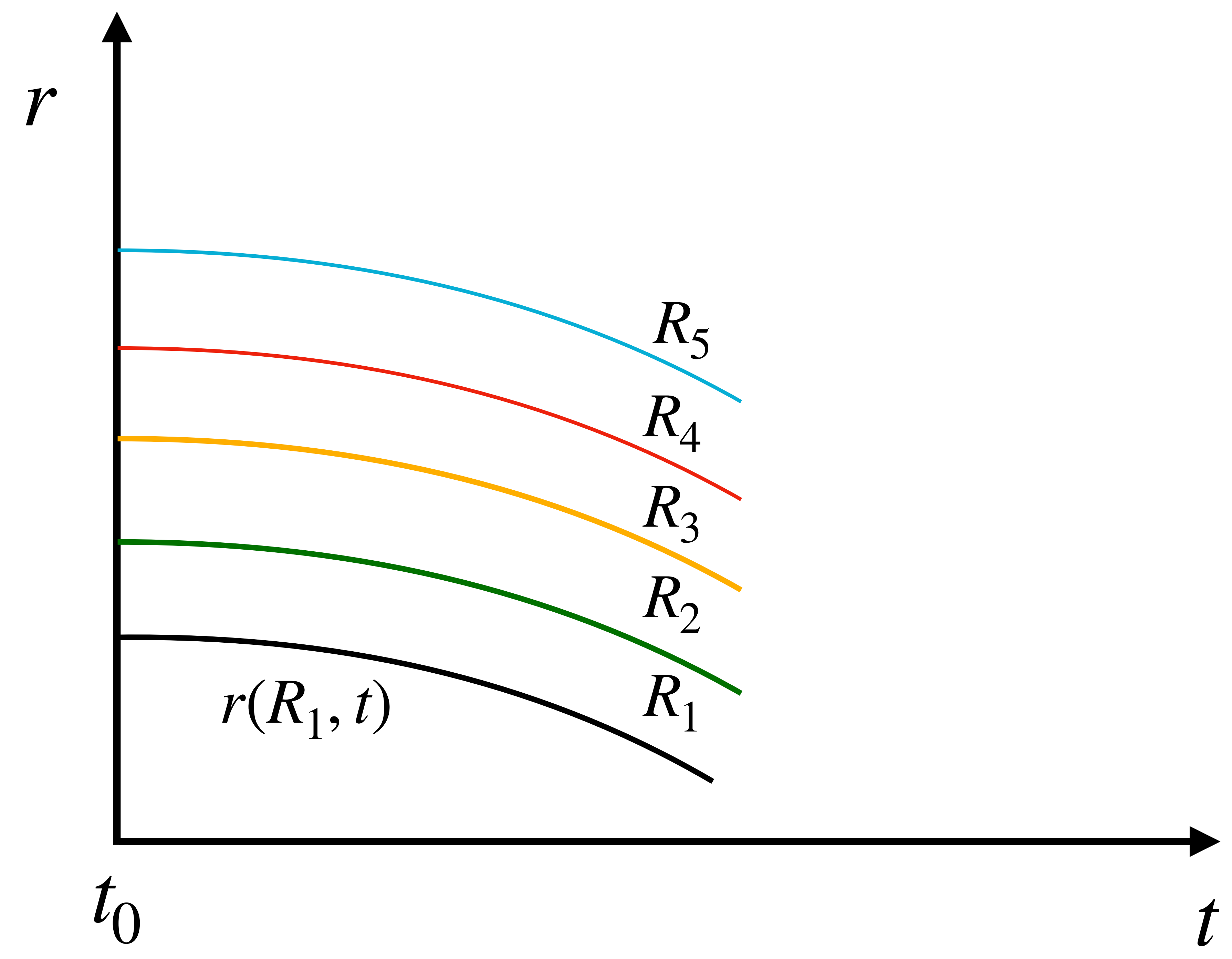


Lemaître-Tolman-Bondi metric

The effective metric describing star collapse in LTB coordinates in the marginally bound case reads:

$$ds^2 = -dt^2 + \left[\partial_R r(R,t)\right]^2 dR^2 + r(R,t)^2 d\Omega^2$$

This metric describes both the matter and vacuum region of the space-time.



Interpretation: we can imagine to divide the spatial part of the manifold in spherical shells parametrized by the radial coordinate R. The solution $r(R,t)$ of the EOMs is the areal radius r of the shell R at time t.

Effective dynamics in LTB coordinates

The LTB effective equations for spherically symmetric dust collapse (marginally bound case):

$$\left(\frac{\dot{r}}{r}\right)^2 = \frac{2Gm}{r^3}\left(1 - \frac{2\Delta Gm}{r^3}\right),$$

[Giesel, Liu, Singh, Weigl, 2023]

where Δ is the area gap in LQG: $\Delta \sim l_P^2$ and $m(R)$ is the mass function and is fixed by the initial energy density profile.

The general solution of the EOM is:

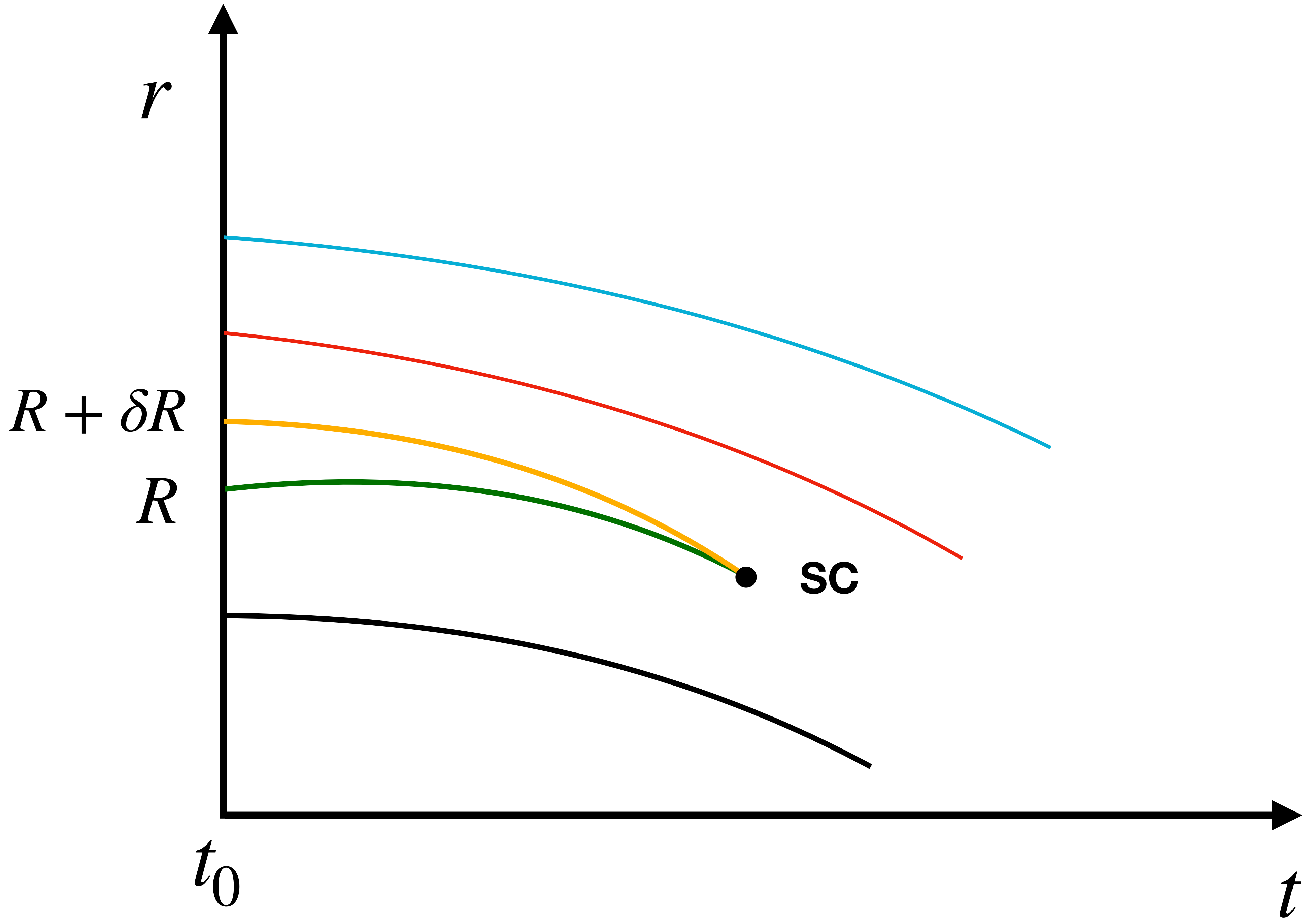
$$r(R,t) = (2Gm(R))^{1/3}\left[\frac{9}{4}\left(t - \alpha(R)\right)^2 + \Delta\right]^{1/3}.$$

$\Longrightarrow$ Each shell R should bounce at: $t = \alpha(R)$.

But: LTB equations break down when the solution develops *shell crossing singularities*.

Shell-crossing singularities in LTB space-times

The dust energy density is given by: $$\rho(R,t) = \frac{\partial_R m(R)}{4\pi r^2 \partial_R r(R,t)}\,.$$



- If $\partial_R r(R,t) = 0$ for some shell R at some time t a shell-crossing (SC) arises.

- Moreover, if for the same R: $\partial_R m(R) \neq 0$ (matter region)

$$\Rightarrow \quad \rho(R,t) = +\infty, \quad R_{\mu\nu} g^{\mu\nu} = +\infty.$$

A shell-crossing singularity (SCS) forms: it is a physical *weak* singularity.

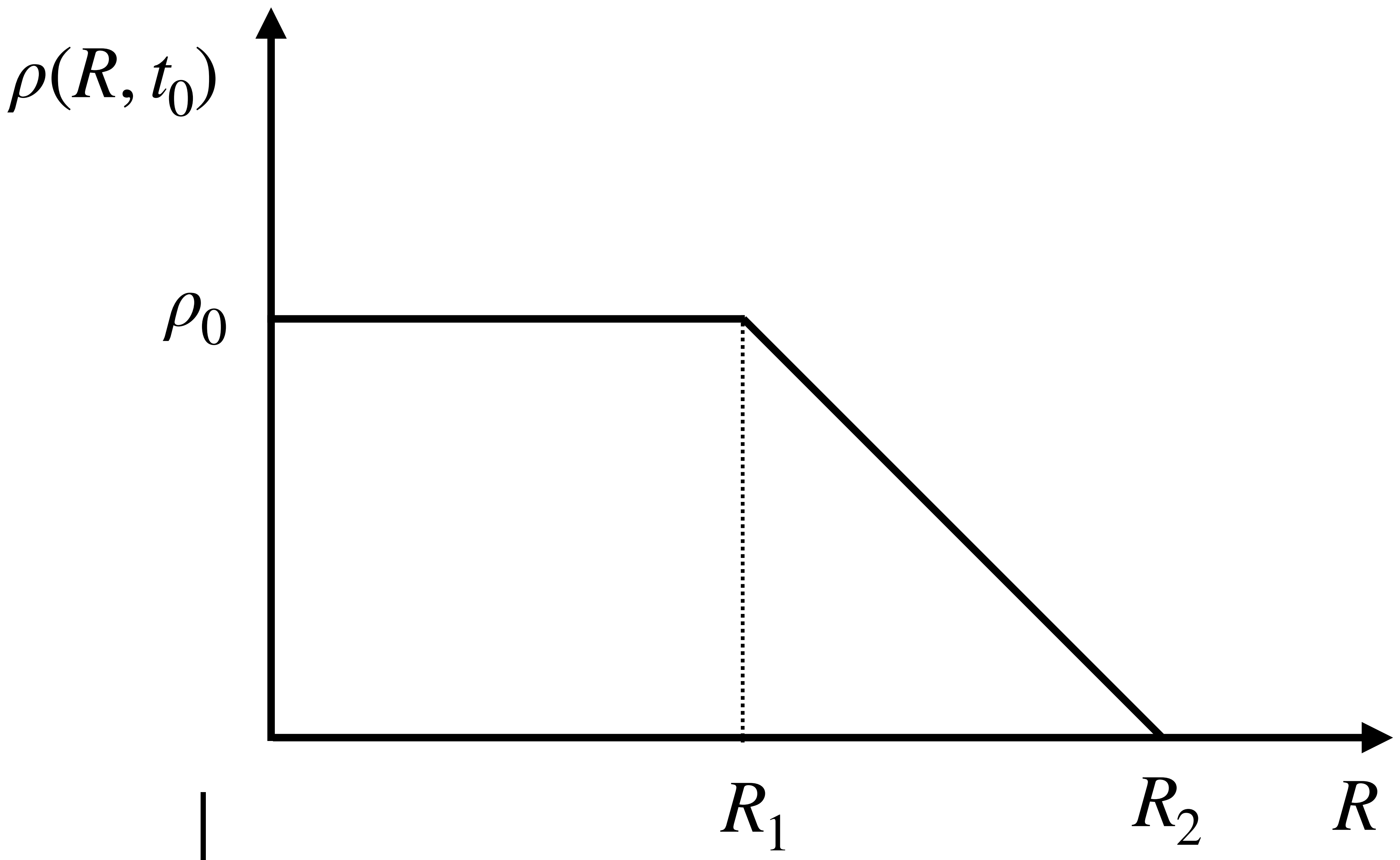
In classical GR, many initial configurations develop SCS, but one can choose suitable initial profiles that don't develop such singularities *[Hellaby, Lake, 1984]*; the same holds in LQC *[Singh, 2009]*.

Can SCS be avoided also in the effective star collapse dynamics?

Shell-crossing singularities for a simple initial profile

Let's consider the initial energy-density profile:

$$\rho(R, t_0) = \begin{cases} \rho_0, & R < R_1 \\ \rho_0 \cdot \dfrac{R_2 - R}{R_2 - R_1}, & R_1 < R < R_2 \\ 0, & R < R_2 \end{cases}$$



In the intermediate region $R_1 < R < R_2$:

- $\partial_R m(R) \neq 0$
- $\partial_R r(R, t) = 0$ implies:

$$\frac{1}{\sqrt{\dfrac{R^3}{2Gm} - \Delta}} \left| \frac{R_2 - R_1}{R_2 - R} - \frac{R_2 - R_1}{R_2 - \frac{3}{4}R - \frac{R_1^4}{4R^3}} \right| \geq \sqrt{\frac{\rho_0}{\rho_c}}$$

Independently from the precise value of ρ_0, R_1, R_2, for R (smaller than but) sufficiently close to R_2 $\longrightarrow$ SCS

$\Longrightarrow$ Also for profiles arbitrarily close to OS ($R_2 - R_1 \ll 1$), SCS necessarily form and the dynamics changes significantly.

Shell-crossing singularity theorem

Theorem: "*for the marginally bound case, a shell-crossing singularity will necessary form at some R if the initial energy density profile is non-negative, continuous, of compact support and for which $m(R)$ is not everywhere zero*."

Additionally, if $\partial_R \rho(R, t_0) \leq 0$, the time at which shell-crossing singularity forms will be:

$$t_{bounce}(R) < t_{SCS}(R) < t_{bounce}(R) + \frac{2}{3}\sqrt{\Delta}$$

White hole shockwave/SCS identification

LTB coordinates cannot be used to study the dynamics when shell crossing singularities form.

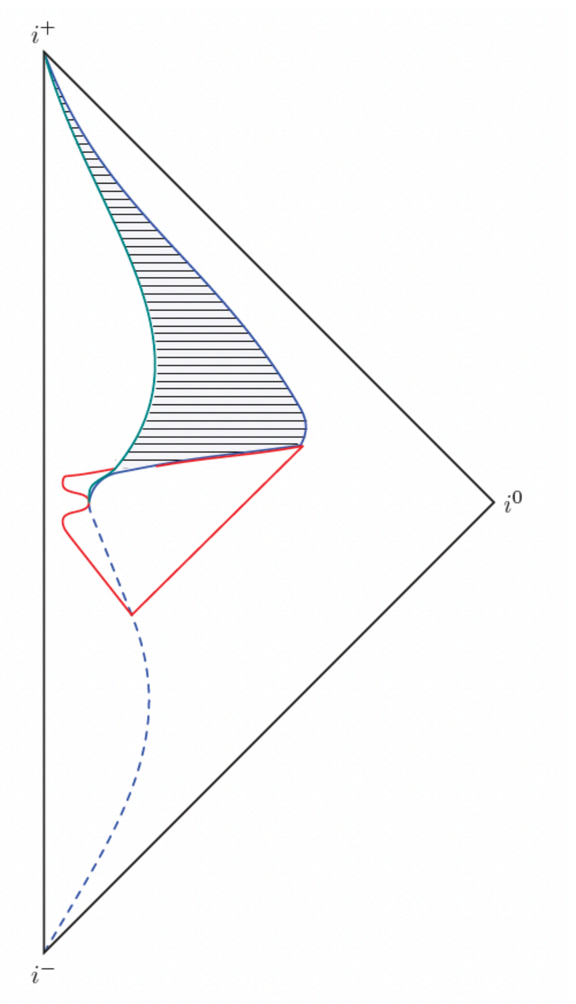
However:

LTB Coordinates		**PG Coordinates**
Shell-crossing singularity in decoupled ODEs	=	Characteristic crossing in the PDE/discontinuity in the gravitational field

The dynamics beyond characteristic crossing in a PDE can be studied by using the integral form of the equations (this is commonly done to study shockwaves in fluid dynamics for example, but also for SCS in classical GR *[Nolan, 2003]*).

↓

Outgoing propagating shockwave of matter driven by a shock in the gravitational field during the post-bounce dynamics.
[Husain, Kelly, Santacruz, Wilson-Ewing, 2022]



Beyond marginally bound configurations

The LTB equations in the most general case ($\varepsilon(R) \neq 0$): $$\left(\frac{\dot{r}}{r}\right)^2 = \left(\frac{2Gm}{r^3} + \frac{\varepsilon}{r^2}\right)\left[1 - \Delta\left(\frac{2Gm}{r^3} + \frac{\varepsilon}{r^2}\right)\right],$$

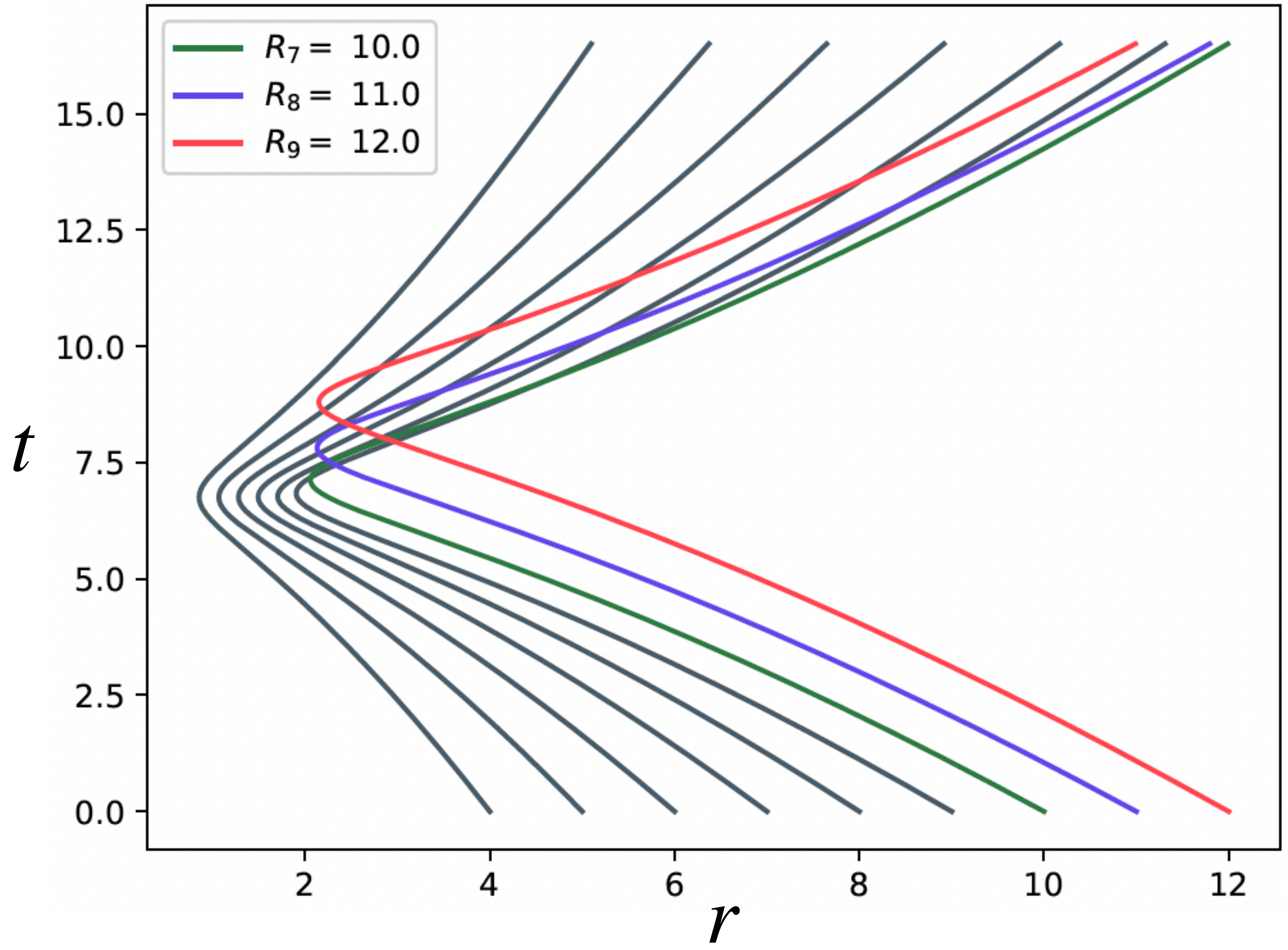
For the initial profile: $$\rho(R, t_0) = C\left(1 - \tanh\frac{R - R_0}{\sigma}\right),$$

$$\varepsilon(R, t_0) = \begin{cases} -\alpha\frac{R^2}{R_0^2}, & \text{for } R < R_0 \\ -\alpha, & \text{for } R \geq R_0 \end{cases}$$

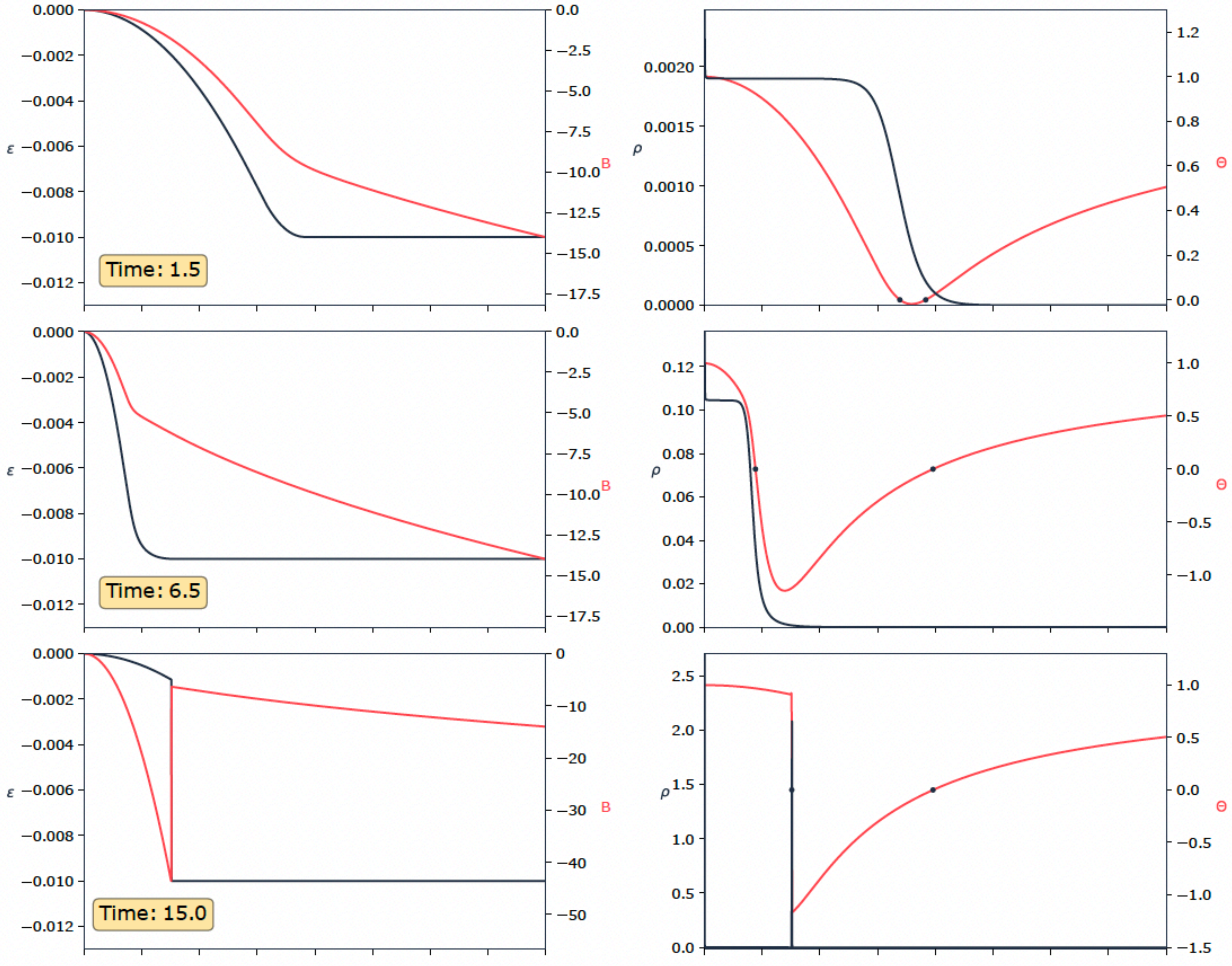
And parameters:

$$C \propto m_{tot} = 5, \quad R_0 = 10, \quad \sigma = 1.1, \quad \alpha = 0.01$$

[Cipriani, FF, Wilson-Ewing, 2024]



Beyond marginally bound configurations: numerical simulation



→ **Collapse phase**

→ **Bounce phase**

→ **Shockwave phase**

Conclusions

- Shell-crossing singularities arise in effective dust collapse for a wide class of initial energy density profiles, and in particular for each continuous non-negative profile with compact support, including profiles arbitrarily close to OS.

  $\Longrightarrow$ Quantum Oppenheimer-Snyder model is not a good prototype to describe effective star collapse.

- Even if the effective equations are decoupled in LTB coordinates, such equations break down after SCS. One has to change coordinates (for example PG) and study the resulting PDEs in their integral form.

- Weak solutions of the dynamics develop shocks in the gravitational field in correspondence of the shell-crossing singularity (shockwave of matter), propagating outward together with the shockwave.

Thank you for your attention!