Quantization of a black hole interior model in Loop Quantum Cosmology

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CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Introduction

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Objective

Use LQC to explore the quantum aspects of the **simplest** black hole scenario (Schwarzschild). In doing so, we contemplate:

- ✦ A recent **effective** proposal made by Ashtekar, Olmedo, and Singh (AOS).
- ✦ A study of the black hole **interior** geometry.
- ✦ A complete quantum description.

Motivation for the AOS model

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Effective formulation

- EoM lack Hamiltonian form without an **extended formulation**.
- Reducing the model alters the symplectic structure, making it too complex for quantization. The extended version seems more manageable.

Framework of the extended AOS model

The metric in the interior region takes the **Kantowski-Sachs** (KS) form:

$$ds^2 = -N(\tau)^2 d\tau^2 + \frac{p_b^2(\tau)}{L_o^2 |p_c(\tau)|} dx^2 + |p_c(\tau)|(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

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The extended phase space has **4 canonical pairs** of degrees of freedom

$$\{b, p_b\} = \gamma, \quad \{c, p_c\} = 2\gamma, \quad \{\delta_b, p_{\delta_b}\} = 1, \quad \{\delta_c, p_{\delta_c}\} = 1. \tag{2}$$

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The dynamics are subject to three constraints

$$\tilde{H}^{\text{eff}}[\tilde{N}], \quad \Psi_b[\lambda_b], \quad \Psi_c[\lambda_c]. \quad (3)$$

For a densitized lapse $\tilde{N}$, the effective Hamiltonian is defined as

$$\tilde{H}^{\text{eff}}[\tilde{N}] = -\tilde{N}L_o\left[\Omega_b^2 + \frac{p_b^2}{L_o^2} + 2\Omega_b\Omega_c\right], \quad \Omega_j = \frac{p_j\sin(\delta_j j)}{\gamma L_o\delta_j} \text{ for } j = b \text{ or } c. \quad (4)$$

Dynamics of the extended AOS model

To define the other two constraints, one must first define the quantities

$$O_b = -\frac{1}{2\Omega_b}\left[\Omega_b^2 + \frac{p_b^2}{L_o^2}\right], \qquad O_c = \Omega_c. \tag{5}$$

(O_b, O_c) are **constants of motion** if (δ_b, δ_c) are also constants of motion. They coincide with the **black hole mass** m **on-shell**.

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The constraints $\Psi_j[\lambda_j] = \lambda_j[K_j(O_b, O_c) - \delta_j]$ follow the **AOS prescription**

$$K_b(m,m) \xrightarrow{m\gg 1} \left(\frac{\sqrt{\Delta}}{\sqrt{2\pi}\gamma^2 m}\right)^{1/3}, \quad K_c(m,m) \xrightarrow{m\gg 1} \frac{1}{2L_o}\left(\frac{\gamma\Delta^2}{4\pi^2 m}\right)^{1/3}. \tag{6}$$

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The dynamics of the AOS model emerge through $\tilde{H}^{\text{eff}}[\tilde{N}]$ on the **constraint surface** when the inverse of $\tilde{N}$ is fixed to $2\Omega_b$.

Loop quantization

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Quantum representation

- For the geometry, we employ the **triad representation**.
- For the δ-parameters, we use the standard (Schrödinger) representation.
- The kinematic Hilbert space is obtained by taking the tensor product.
- A basis of (generalized) eigenstates is represented by $\{|\tilde{\mu}_b, \tilde{\mu}_c, \delta_b, \delta_c\rangle\}$.

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Quamntum operators [e.g. for the angular sector]

$$\hat{\tilde{p}}_c|\tilde{\mu}_b, \tilde{\mu}_b, \delta_c, \delta_c\rangle = \gamma\tilde{\mu}_c|\tilde{\mu}_b, \tilde{\mu}_c, \delta_b, \delta_c\rangle, \quad \hat{\delta}_c|\tilde{\mu}_b, \tilde{\mu}_c, \delta_b, \delta_c\rangle = \delta_c|\tilde{\mu}_b, \tilde{\mu}_c, \delta_b, \delta_c\rangle,$$

$$2i\widehat{\sin(\delta_c c)}|\tilde{\mu}_b, \tilde{\mu}_c, \delta_b, \delta_c\rangle = |\tilde{\mu}_b, \tilde{\mu}_c + 2, \delta_b, \delta_c\rangle - |\tilde{\mu}_b, \tilde{\mu}_c - 2, \delta_b, \delta_c\rangle.$$

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The Hamiltonian constraint operator becomes

$$\hat{H}^{\text{eff}} = -L_o \left[\hat{\Omega}_b^2 + \hat{\delta}_b^2 \frac{\hat{\tilde{p}}_b^2}{L_o^2} + 2\hat{\Omega}_b \hat{\Omega}_c \right], \tag{7}$$

where we use the **MMO** prescription to define

$$\hat{\Omega}_j = \frac{1}{2\gamma L_o} |\hat{\tilde{p}}_j|^{1/2} \left[\widehat{\sin(\delta_j j)} \widehat{\text{sign}(\tilde{p}_j)} + \widehat{\text{sign}(\tilde{p}_j)} \widehat{\sin(\delta_j j)} \right] |\hat{\tilde{p}}_j|^{1/2}. \tag{8}$$

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The eigenvalues of $\hat{O}_c = \hat{\Omega}_c$ represent the mass of the black hole.

Mass operators

$\hat{\Omega}_j^2$ operator

4-unit step difference operator, essentially self-adjoint, positively defined, with a positive, continuous, and nondegenerate spectrum. It leaves invariant

$$^{(4)}\mathcal{H}^{\pm}_{\tilde{\varepsilon}_j} = \overline{\text{span}\left\{|\tilde{\mu}_j\rangle \, : \, \tilde{\mu}_j \in {}^{(4)}\mathcal{L}^{\pm}_{\tilde{\varepsilon}_j}\right\}}, \qquad {}^{(4)}\mathcal{L}^{\pm}_{\tilde{\varepsilon}_j} = \{\pm(\tilde{\varepsilon}_j + 4n) \, : \, n \in \mathbb{N}\}. \tag{9}$$

for $\varepsilon_j \in (0, 4]$. Its eigenstates, $\hat{\Omega}_j^2 |e^{\tilde{\varepsilon}_j}_{m_j^2}\rangle = m_j^2 |e^{\tilde{\varepsilon}_j}_{m_j^2}\rangle$, depend on **one** initial data.

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$\hat{\Omega}_j$ operator

2-unit step difference operator, essentially self-adjoint, with real, continuous, and nondegenerate spectrum. It leaves invariant

$$ {}^{(2)}\mathcal{H}^{\pm}_{\tilde{\epsilon}_j} = {}^{(4)}\mathcal{H}^{\pm}_{\tilde{\epsilon}_j} \otimes {}^{(4)}\mathcal{H}^{\pm}_{\tilde{\epsilon}_j+2}, \qquad |e^{\tilde{\epsilon}_j}_{m_j}\rangle = |m_j|^{1/2}[|e^{\tilde{\epsilon}_j}_{m_j^2}\rangle \otimes i\text{sign}(-m_j)|e^{\tilde{\epsilon}_j+2}_{m_j^2}\rangle]. \tag{10} $$

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Hamiltonian constraint

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In a generalized eigenspace of the $\hat{\delta}_b$ and $\hat{\Omega}_c$ operators, the constraint can be reexpressed on the radial sector as

$$\hat{\mathcal{Q}}_b\,(m_c)\,|\psi^{\tilde{\epsilon}_b}_{\delta_b}\rangle = \left[(\hat{\Omega}_b + m_c)^2 + \delta_b^2 \frac{\hat{\tilde{p}}_b^2}{L_o^2}\right] |\psi^{\tilde{\epsilon}_b}_{\delta_b}\rangle = m_c^2 |\psi^{\tilde{\epsilon}_b}_{\delta_b}\rangle. \tag{11}$$

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- Continuous solutions, for any black hole mass $\mathbf{m} = \mathbf{m_c}$, appear if we instead search for them in a much larger set, namely the **algebraic dual** of the linear span of the eigenstates of $\hat{\tilde{p}}_b$.

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Our choice

Solutions depend on where you look for them. We opt to proceed with the second case to favor a **continuous classical limit**.

Construction of the states

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The δ-contributions amount to impose Dirac deltas on the wave function of the physical state such that

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To ensure that the radial solution depends on one inital data, we impose on the closest two points to the origin of our superselected sector

$$\langle \tilde{\epsilon}_b | \left(\hat{\Omega}_b + m + \sqrt{m^2 - \delta_b^2 \hat{\tilde{p}}_b^2 / L_o^2} \right) |\psi_{\delta_b}^{\tilde{\epsilon}_b}\rangle = 0. \tag{13}$$

This expression represents the classical behavior of Ω_b when $\delta_b \tilde{p}_b$ is small.

Properties of the states

Formally, the physical states satisfy: $\hat{H}^{\text{eff}}|\xi_p\rangle = 0$, $\hat{\Psi}_b|\xi_p\rangle = 0$, and $\hat{\Psi}_c|\xi_p\rangle = 0$.

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Physical states

$$|\xi_p\rangle = \int_{\mathbb{R}} \mathrm{d}m \sum_{\tilde{\mu}_b,\tilde{\mu}_c} \xi(m)\psi^{\tilde{\epsilon}_b}_{\delta_b}(\tilde{\mu}_b)|_{\delta_b=\tilde{K}_b(m)} e^{\tilde{\epsilon}_c}_m(\tilde{\mu}_c)\,|\tilde{\mu}_b,\tilde{\mu}_c,\delta_b=\tilde{K}_b(m),\delta_c=\tilde{K}_c(m)\rangle. \quad (14)$$

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- We can construct states peaked on **large masses**.
- We can caracterize a physical Hilbert space by defining an adequate inner product for these states.

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Further researches

- ✦ Inclusion of matter fields.
- ✦ Consideration of perturbations.
- ✦ Extension of the results to the exterior.