Dynamical frames, relational subsystems and gauge-invariant entanglement entropy

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QISS THE QUANTUM INFORMATION STRUCTURE OF SPACETIME

Based on: quant-ph/2308.09131 with I. Kotecha, P. A. Höhn

& ongoing work with S. Carrozza, P. A. Höhn, J. Kirklin

LOOPs'24 Fort Lauderdale FLORIDA

INTRODUCTION AND MOTIVATIONS

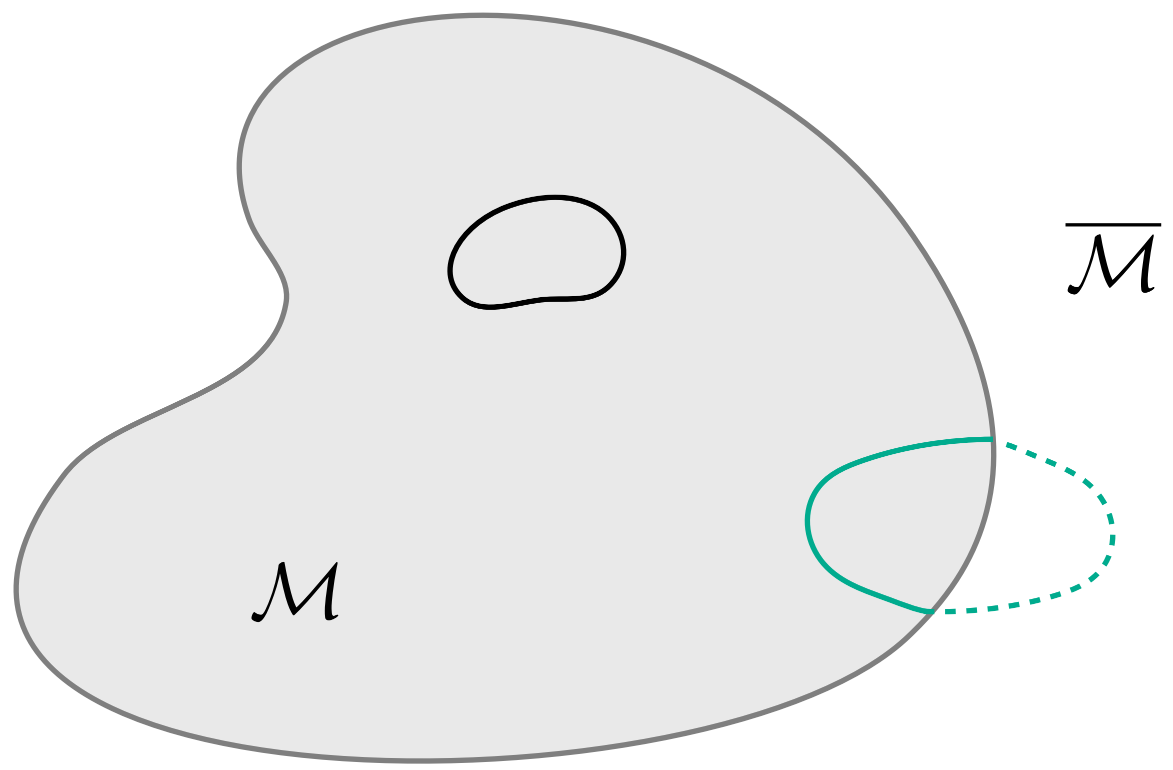
Operationally, subsystems are distinguished by subalgebras of physically accessible observables [Zanardi '01; Zanardi, Lidar, Lloyd '03]

often relative to external frame, e.g. the Lab, or to notion of locality of a background spacetime (external to the fields of interest).

What if no external relatum is available and/or there is tension between locality and gauge-invariance?

In constrained/gauge systems and gravity:

- Kinematical notion of subsystems generically **not** inherited at gauge-inv. level;
- Non-local gauge-invariant observables;
- Partitioning vs. cross-boundary observables. (edge modes) [Donnelly, Freidel, Francois, Geiller, Gomes, Pranzetti, Riello, Speranza, Speziale, Wieland, Carrozza, Eccles, Höhn,…]



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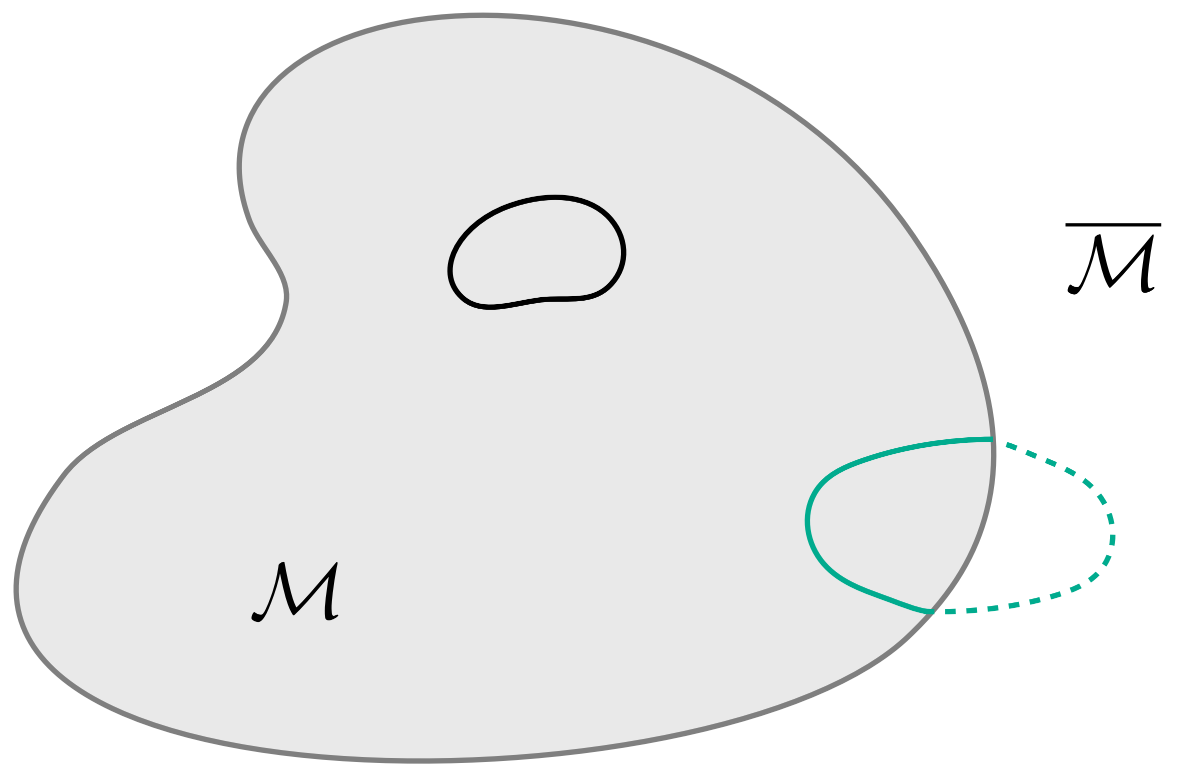


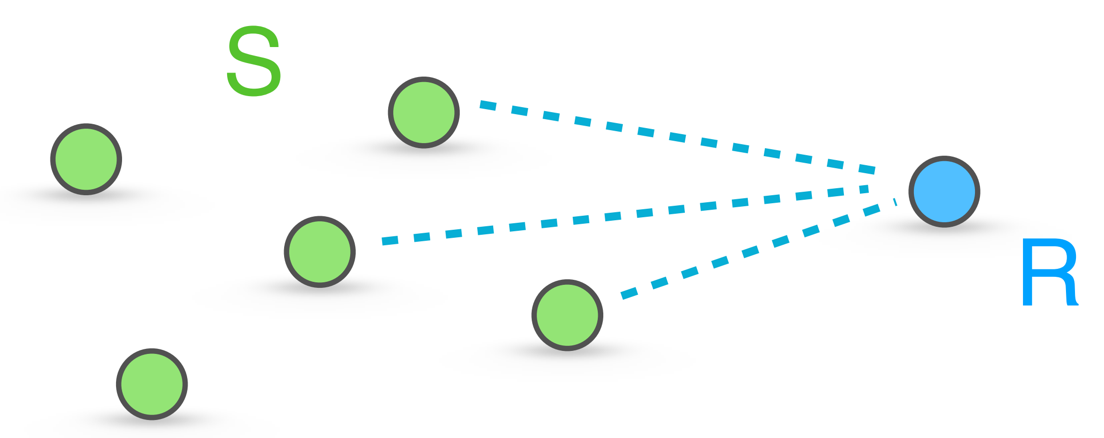
SUBSYSTEM RELATIVITY

Idea: **Use internal reference frames & relational observables** ← MULTIPLE CHOICES → **Gauge-inv. subsystems depend on the relational observables accessible in the chosen internal frame**

[Ahmad Ali, Galley, Höhn, Lock, Smith '21; de la Hamette, Galley, Höhn, Loveridge, Müller '22; Höhn, Kotecha, **FMM** '23; Carrozza, Höhn, Kirklin, **FMM** to appear]

Consequences: frame-dependent gauge-inv. properties of subsystems + alternative proposal for entanglement entropy

PLAN OF THE TALK

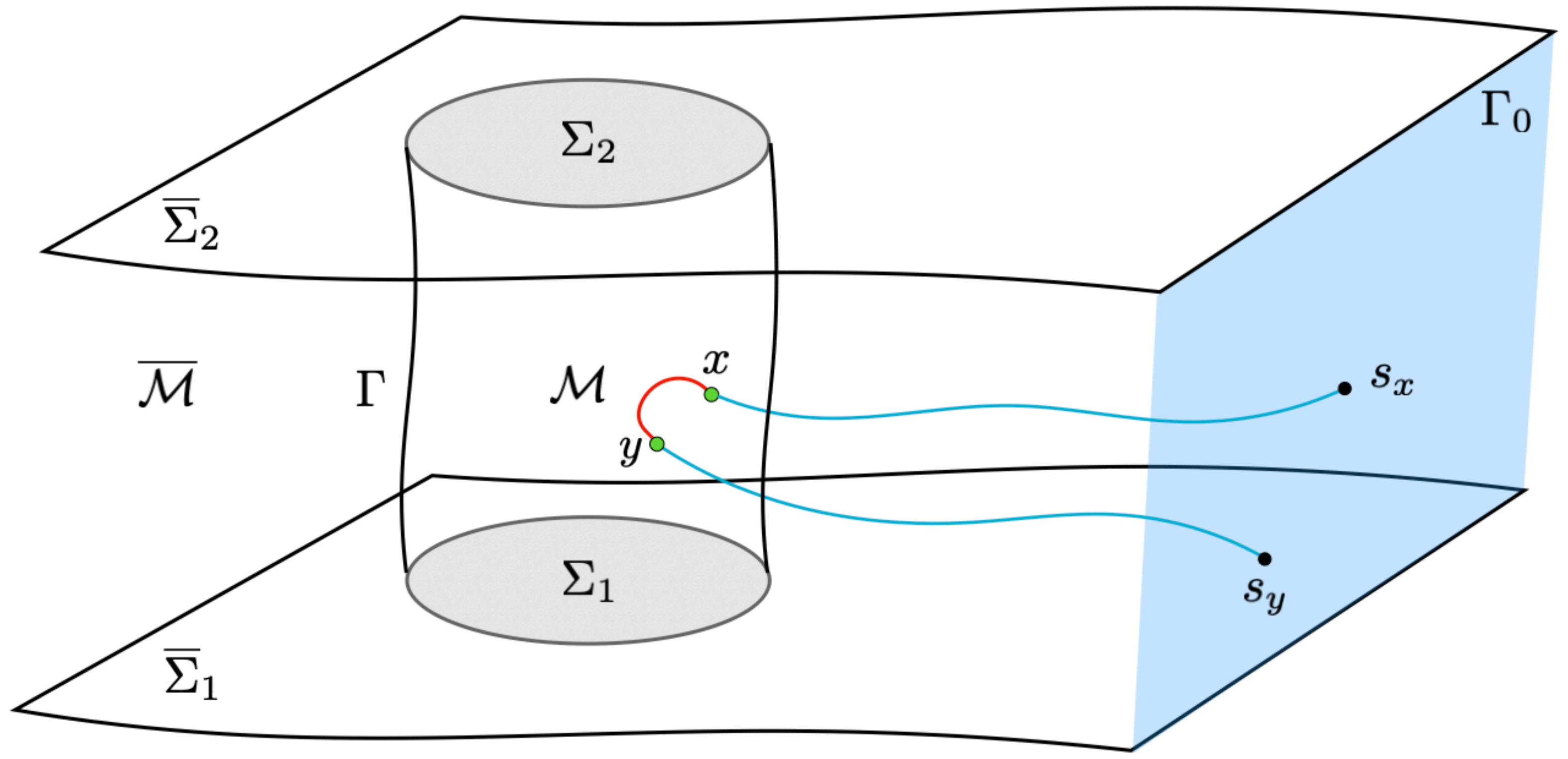


- Illustration of subsystem relativity in a simple mechanical toy model example

    Split kinematical DoFs into system of interest S + frame R

    Subsystem gauge-invariantly defined via relational observables relative to chosen frame

    Different frames identify **distinct** subalgebras of relational observables



- Comparison between subsystem entropy with constraints via center construction

    Different assignment of gauge invariant subalgebras → Different notion of entropy (proper entanglement entropy?)

- Generic finite dimensional constrained/gauge systems and gauge field theory

EXAMPLES OF SUBSYSTEM RELATIVITY

- Special relativity with tetrads frames [de la Hamette, Galley, Höhn, Loveridge, Müller '21; Höhn, Kotecha, **FMM** '23] ⇒ Relativity of simultaneity from relativity of subsystems [Höhn, Kotecha, **FMM** '23]

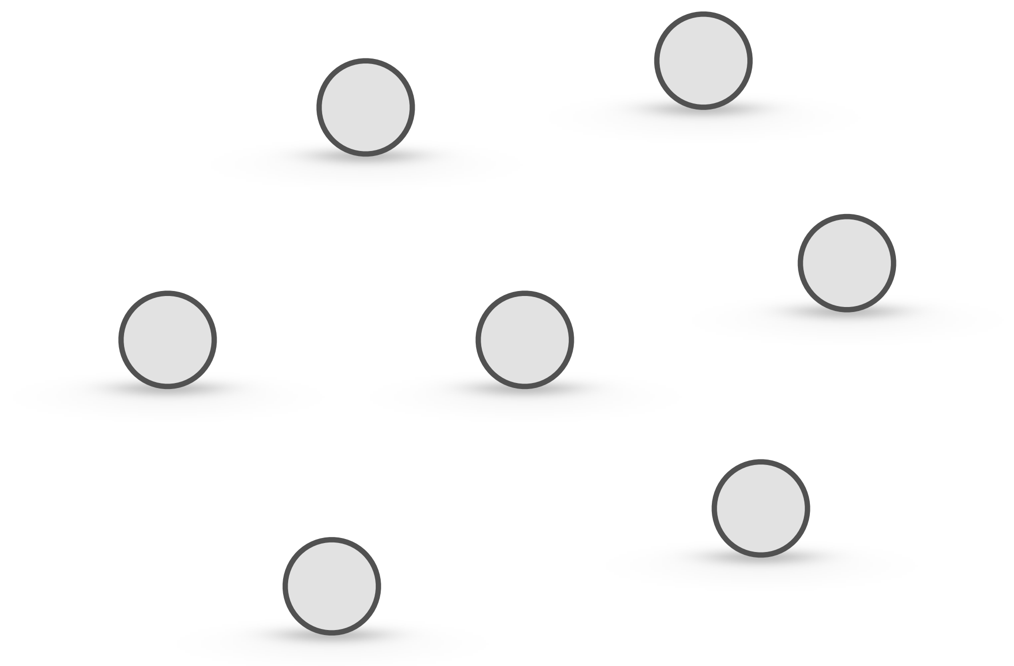
- Finite-dim. quantum systems: external-frame independent description of DoF of interest relative to the remaining DoFs (used as internal frame) ⇒ Relativity of correlations, subsystem dynamics, equilibrium and non-equilibrium themodynamics [Höhn, Kotecha, **FMM** '23]

- Subregions in gauge theories and gravity: edge modes frames [Carrozza, Höhn '21; Carrozza, Eccles, Höhn '22] ⇒ Frame-dependent subregional gauge-inv. algebras [Carrozza, Höhn, Kirklin, **FMM** to appear]

- Regulatisation of gravitational entropy via introduction of observer (from Type III to Type II algebras) [Chandrasekaran, Longo, Pennington, Witten '22; Kudler-Flam, Leutheusser, Satishchandran '23; Jensen, Sorce, Speranza '23; Freidel, Gesteau to appear] ⇒ Observer dependence of gravitational entropy [De Vuyst, Eccles, Höhn, Kirklin '24]

MAIN MESSAGE IN A NUTSHELL

(Total) system of particles with translation invariance:



Kinematical factorisation into subsystems (e.g. absolute positions/relative to external frame)

$\nparallel$

Physical/external-frame-indep. factorisation into subsystems (e.g. translation invariant relative distances)

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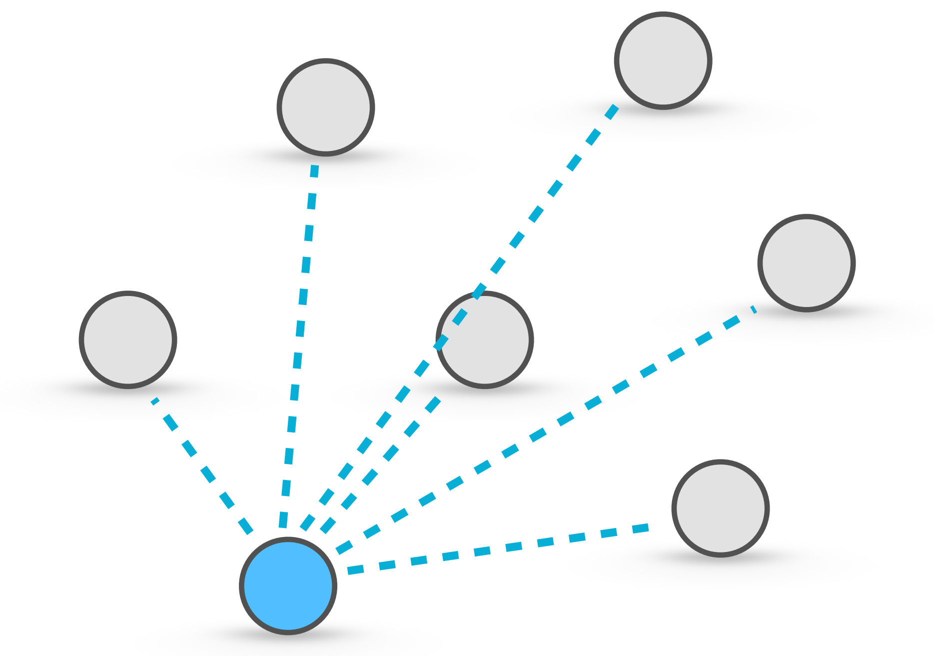

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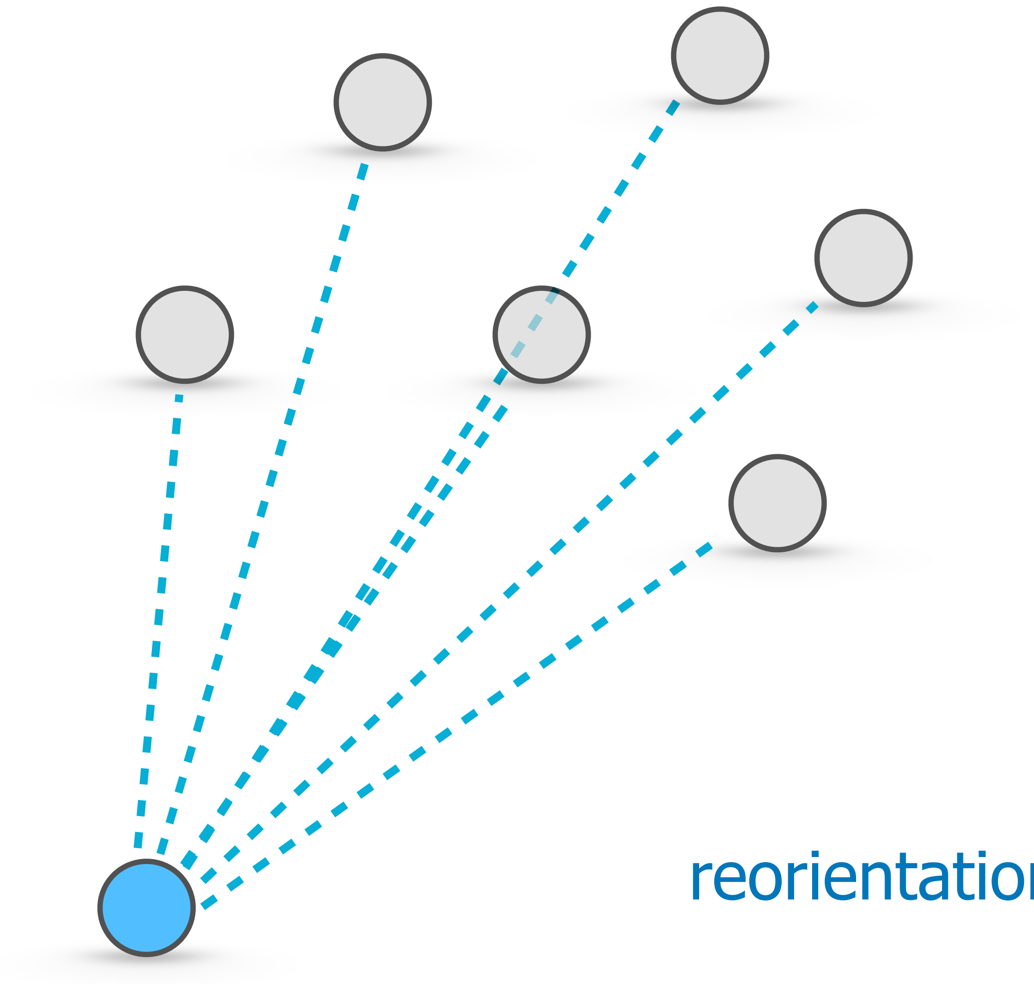

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Physical/external-frame-indep. factorisation into subsystems (e.g. translation invariant relative distances)

reorientation (change of state) of internal frame →

- description of remaining particles relative to external frame unchanged
  (absolute positions of the other particles unchanged)
- description relative to internal frames changed
  (relations between the frame and the other particles are changed)

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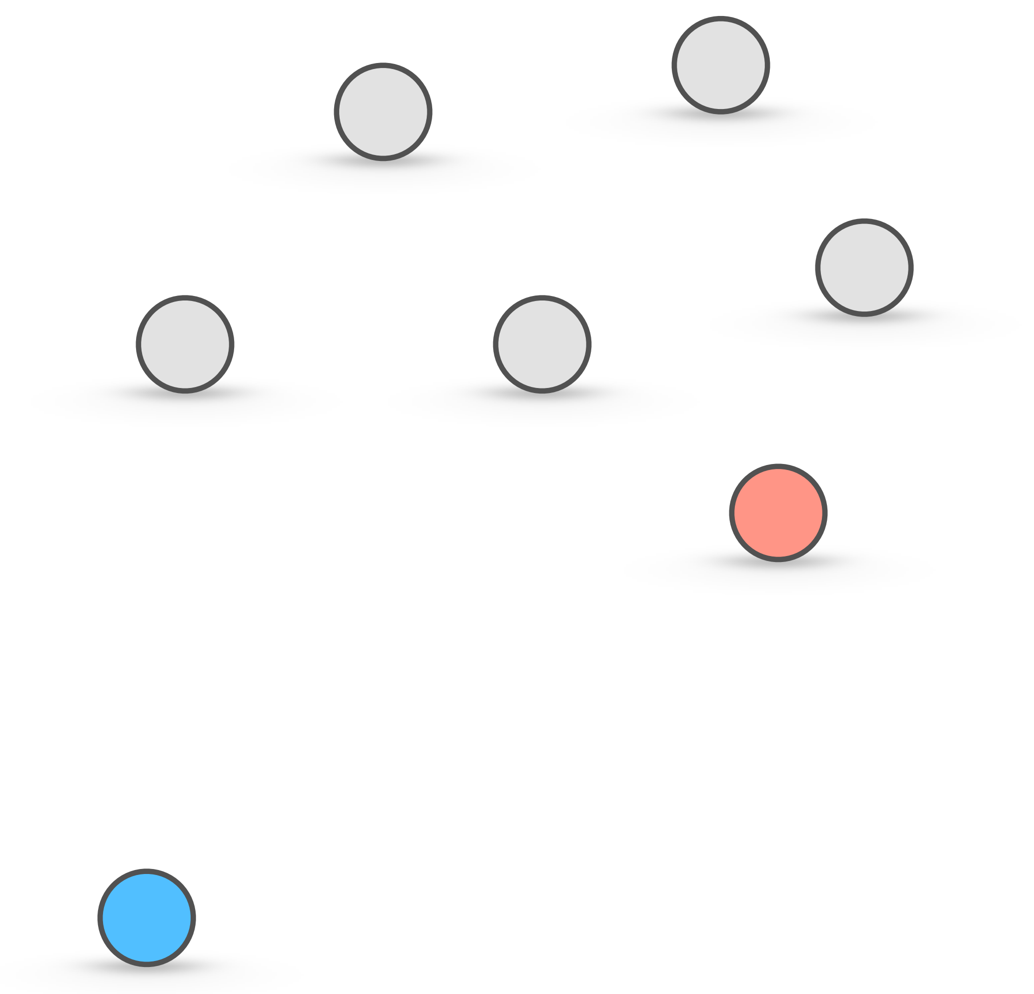

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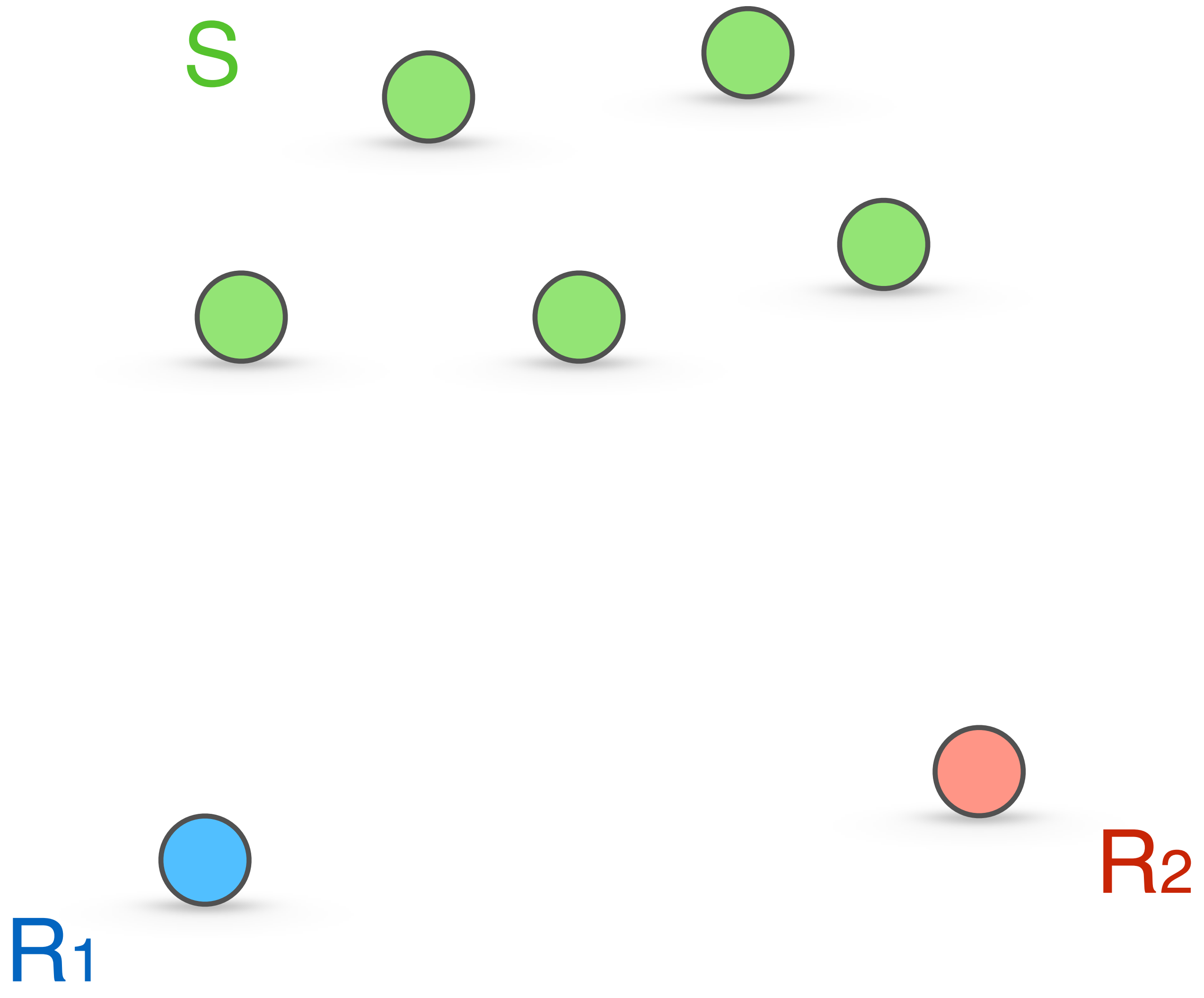

Kinematical factorisation into subsystems (e.g. absolute positions/relative to external frame)

$$\nparallel$$

Physical/external-frame-indep. factorisation into subsystems (e.g. translation invariant relative distances)

Description of S relative to R_1 invariant under reorientations of R_2, but relative to R_2 it changes since relations between S and R_2 change

$$\mathcal{A}^{\rm phys}_{S|R_1} \neq \mathcal{A}^{\rm phys}_{S|R_2}$$

Different internal frames identify distinct relational notions of subsystems

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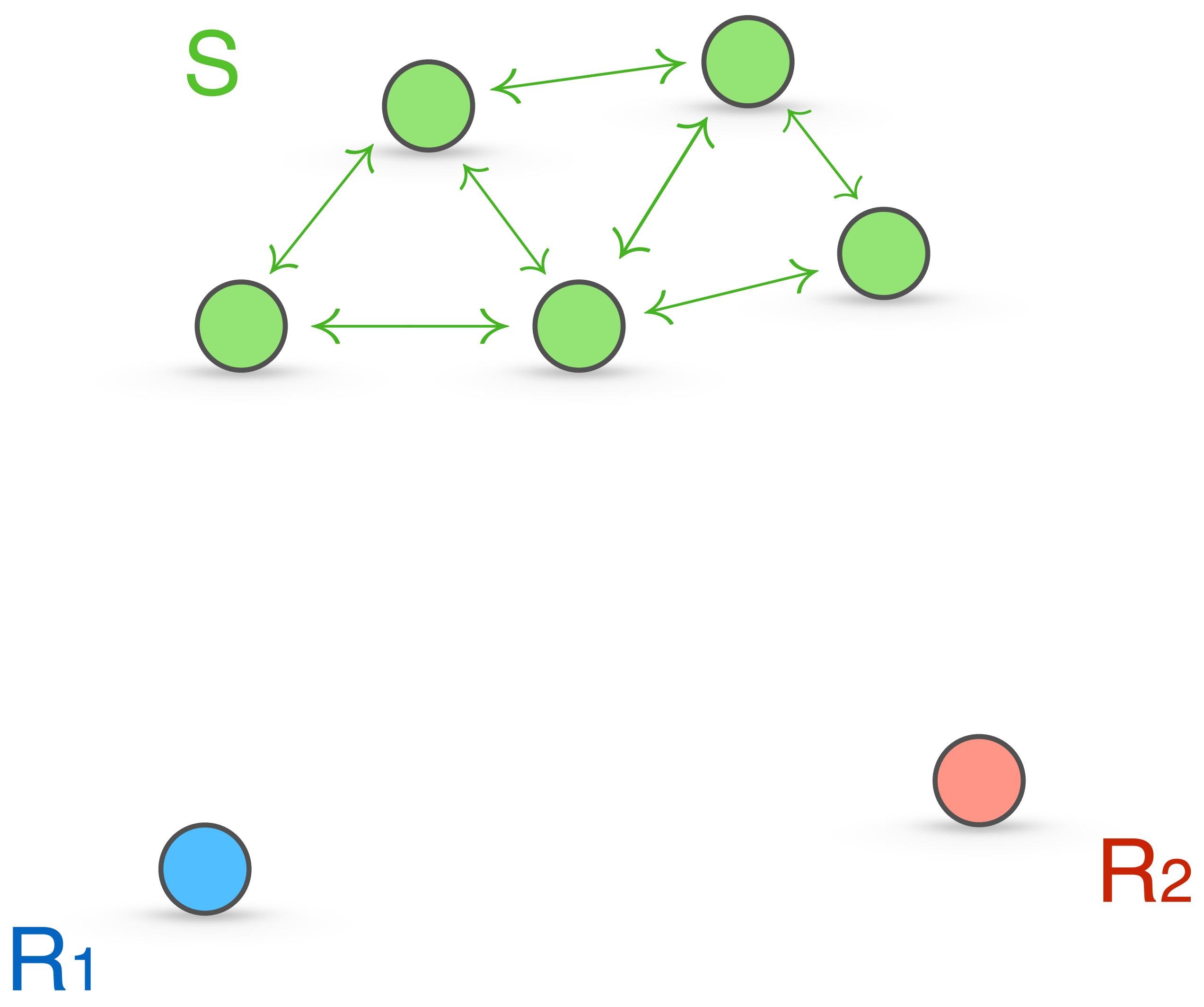

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Different internal frames identify distinct relational notions of subsystems

Internal relations to S are invariant under reorientations of both frames (frame-independent relational observables)

$$\mathcal{A}^{\text{phys}}_{S|R_1} \cap \mathcal{A}^{\text{phys}}_{S|R_2} \neq \emptyset$$

[Höhn, Kotecha, **FMM** '23]

Gauge-inv. properties of subsystems such as correlations, entropies, dynamics (open vs. closed), equilibrium and non-eq. thermodynamics contingent on the internal frame

RELATIONAL vs CENTER CONSTRUCTION

N+M particles in 1D with translation invariance $G = (\mathbb{R}, +)$ → $P_{\text{tot}}|\psi_{\text{phys}}\rangle = 0$ → $\mathcal{A}_{\text{phys}} = \{q_2 - q_1, \ldots, q_{N+M} - q_1, p_2, \ldots, p_{N+M}\}$

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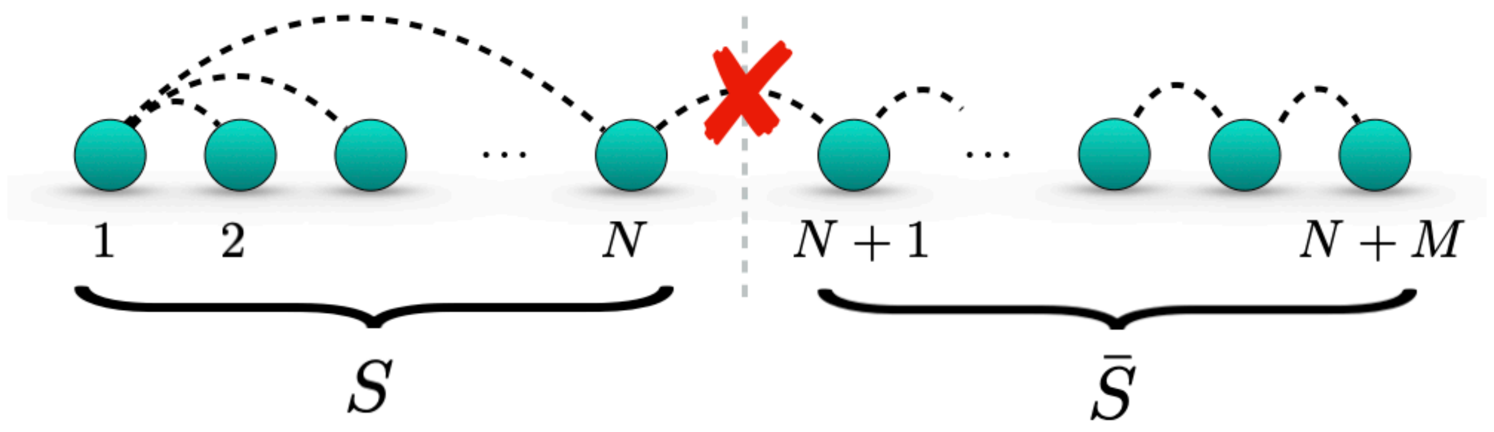
Center construction



[Casini, Huerta, Rosabal '13; Donnelly '11, '14; Van Acoleyen, Bultinck, Haegeman, Marien, Scholz, Verstraete '15,…]

Assign "regional"/internal gauge-inv. algebras to kinematical complements

$$\mathcal{A}_S = \{q_2 - q_1, \ldots, q_N - q_1, p_1, \ldots, p_N\}$$

$$\mathcal{A}_{\bar{S}} = (\mathcal{A}_S)' = \{q_{N+2} - q_{N+1}, \ldots, q_{N+M} - q_{N+M-1}, p_{N+1}, \ldots, p_{N+M}\}$$

Non-trivial center

$$\mathcal{Z}_S = \mathcal{A}_S \cap (\mathcal{A}_S)' \neq \mathbb{C}\mathbf{1}$$

Algebra generated by $\mathcal{A}_S$ and its combatant is a strict subalgebra of $\mathcal{A}_{\text{phys}}$

$$\mathcal{A}_S \vee \mathcal{A}_{\bar{S}} = \bigoplus_z \mathcal{A}_S^z \otimes \mathcal{A}_{\bar{S}}^z \subset \mathcal{A}_{\text{phys}}$$

Entropy associated to local subalgebra (not entanglement entropy)

$$\mathcal{S}_{\text{vN}}(S) = \sum_z p_z \mathcal{S}_{\text{vN}}(\rho_S^z) + H(\{p_z\})$$

← classical Shannon

RELATIONAL vs CENTER CONSTRUCTION

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$P_{\rm tot}|\psi_{\rm phys}\rangle = 0$

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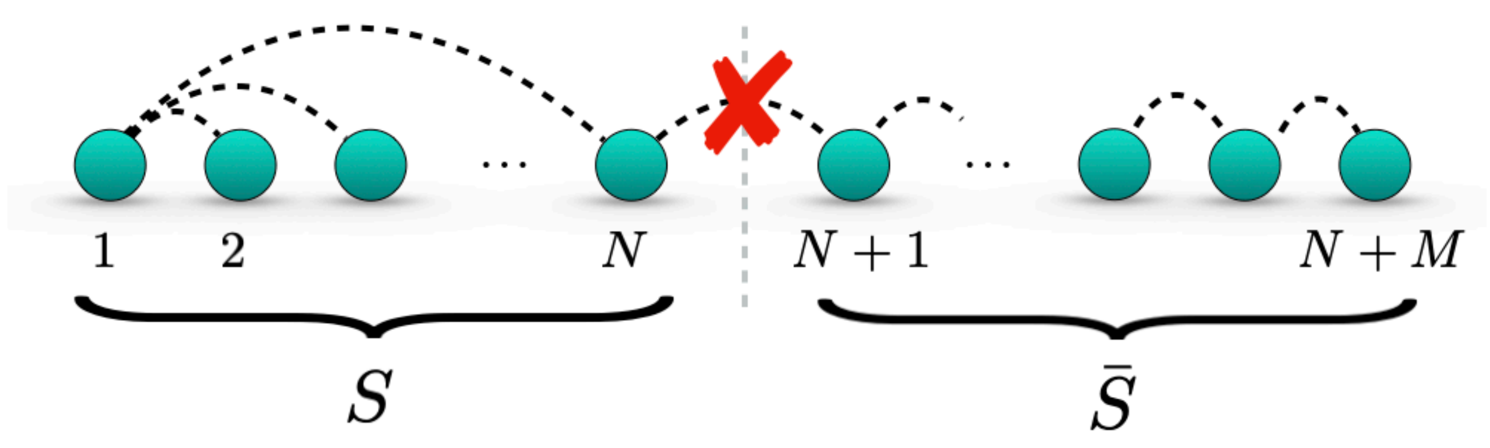

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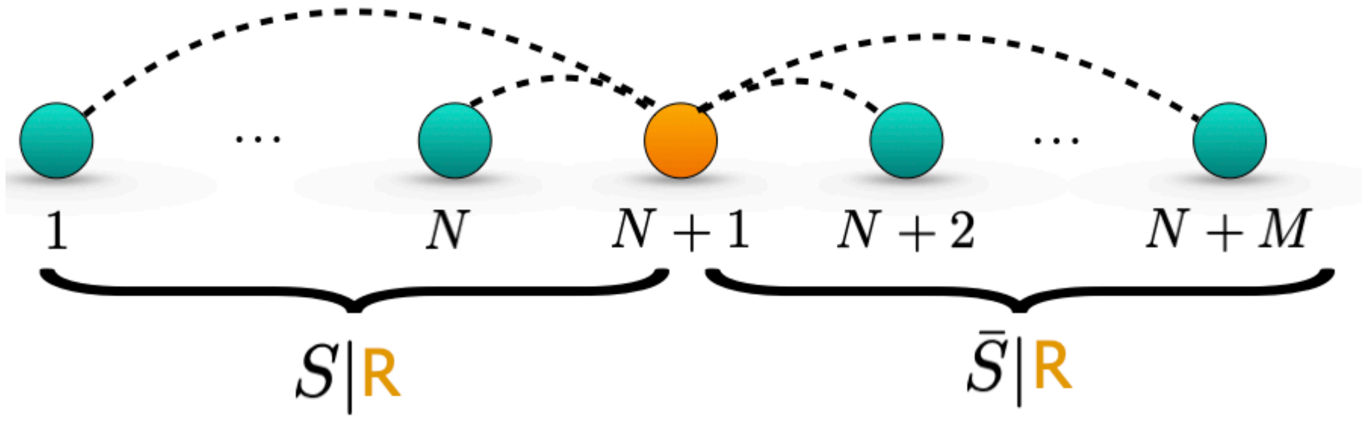
Relational construction



Assign gauge-inv. algebras via complements relative to R

$$\mathcal{A}^{\rm phys}_{S|R} = \{q_1 - q_R, \ldots, q_N - q_R, p_1, \ldots, p_N\}$$

$$\mathcal{A}^{\rm phys}_{\bar S|R} = \{q_{N+2} - q_R, \ldots, q_{N+M} - q_R, p_{N+2}, \ldots, p_{N+M}\}$$

No gauge-invariant data missing $\mathcal{A}^{\rm phys}_{S|R} \vee \mathcal{A}^{\rm phys}_{\bar S|R} = \mathcal{A}_{\rm phys}$

Proper entanglement entropy $\mathcal{S}_{\rm vN}(\rho_{S|1}) = -{\rm Tr}(\rho_{S|1} \log \rho_{S|1})$

generically frame dependent

Locality defined relationally (non-local combinations of $S\bar S$ kinematical DoFs)

Bigger subalgebras associated with subsystems

"regional"/internal algebras appear as the frame-indep. data $\mathcal{A}^{\rm phys}_{S|R_1} \cap \mathcal{A}^{\rm phys}_{S|R_2} = \mathcal{A}_S$

INTERNAL DYNAMICAL REFERENCE FRAMES

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Total system + action of some (unimodular) gauge group G

Split the total DoFs **into** a **(SUB)SYSTEM OF INTEREST** (subgroup of particle, subregion,…) **and FRAME** DoFs (constructed from the complement)

Dynamical frame associated with (gauge) symmetry group G

⟶ frame configurations associated to group elements

G-frame: subsystem “as non-invariant as possible” under G-action
to be used to parametrise the orbits of G

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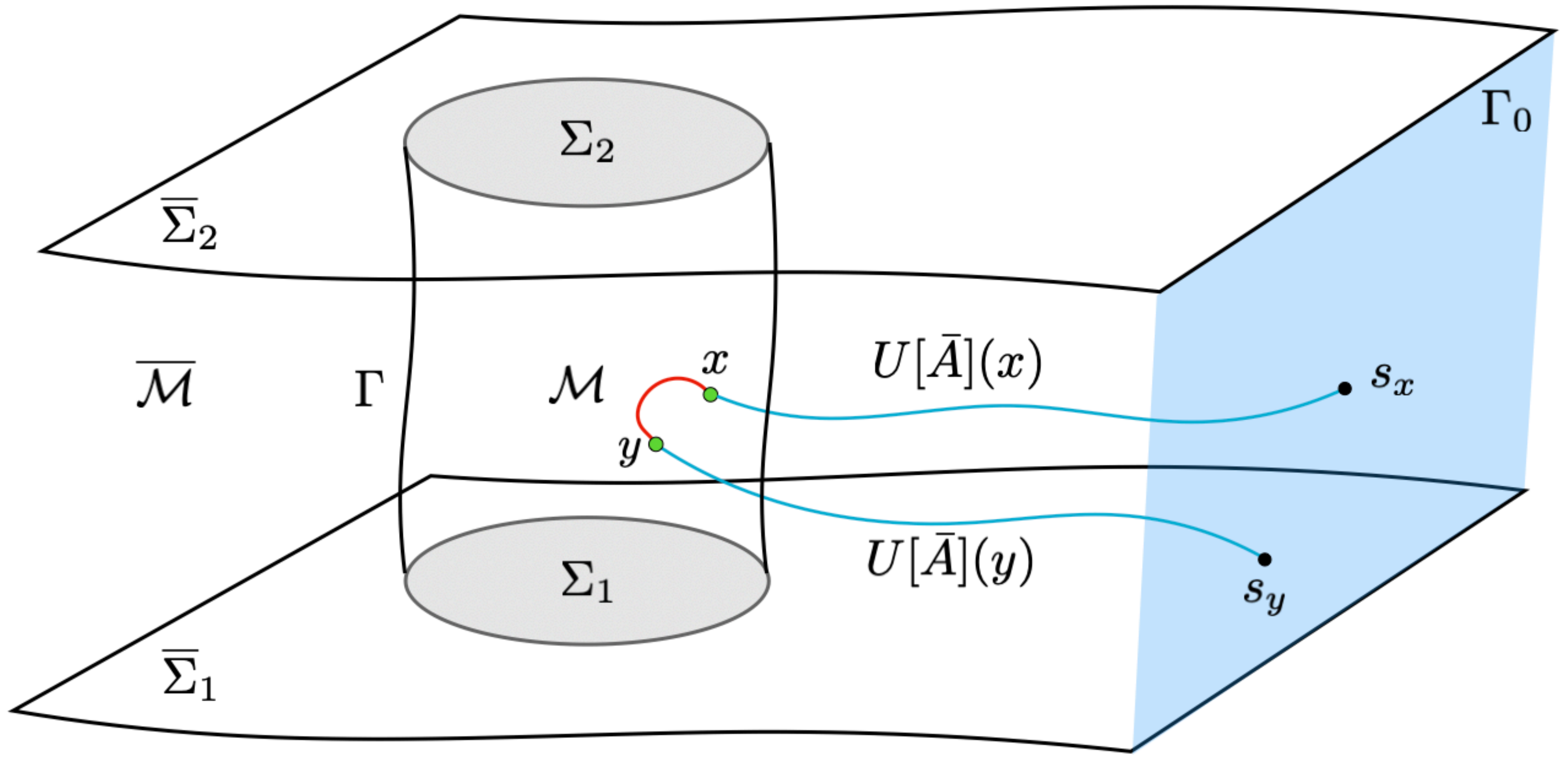
- Quantum reference frames: generalised coherent states $|\varphi(g)\rangle_R \to U_R(g')|\varphi(g)\rangle_R = |\varphi(g'g)\rangle_R$, $\int_G \mathrm{d}g\, |\varphi(g)\rangle\langle\varphi(g)|_R = \mathbf{1}_R$

  left G-action

  not necessarily orthogonal/perfectly distinguishable orientations

- Edge modes as group-valued "internalised" external frames
  (via e.g. Wilson lines originating in the complement)



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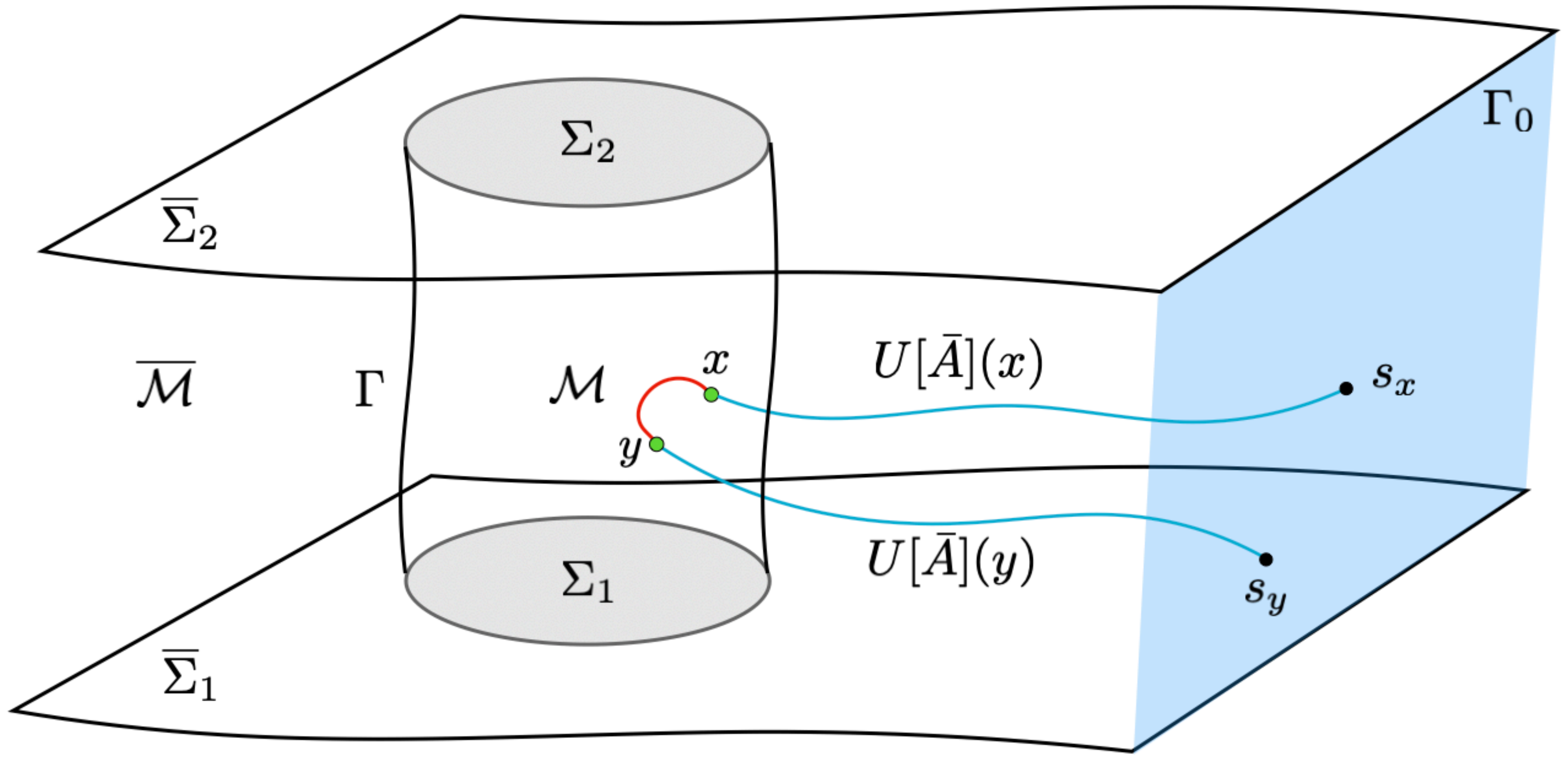


Gauge transf. acts from left on both system and frame DoFs

Symmetries = frame reorientations by right action (with g^{-1})

Relational frame-dressed observables: $O_{f|U}(g) = (Ug^{-1})^{-1} \triangleright f$

$$O_{f_S,R}(g) = \int_G \mathrm{d}g'\, U_{RS}(g') \Big(|\varphi(g)\rangle\langle\varphi(g)|_R \otimes f_S \Big) U^\dagger_{RS}(g')$$

value of f when frame is in orientation g (relational obs. as in Rovelli, Dittrich, Thiemann,…)

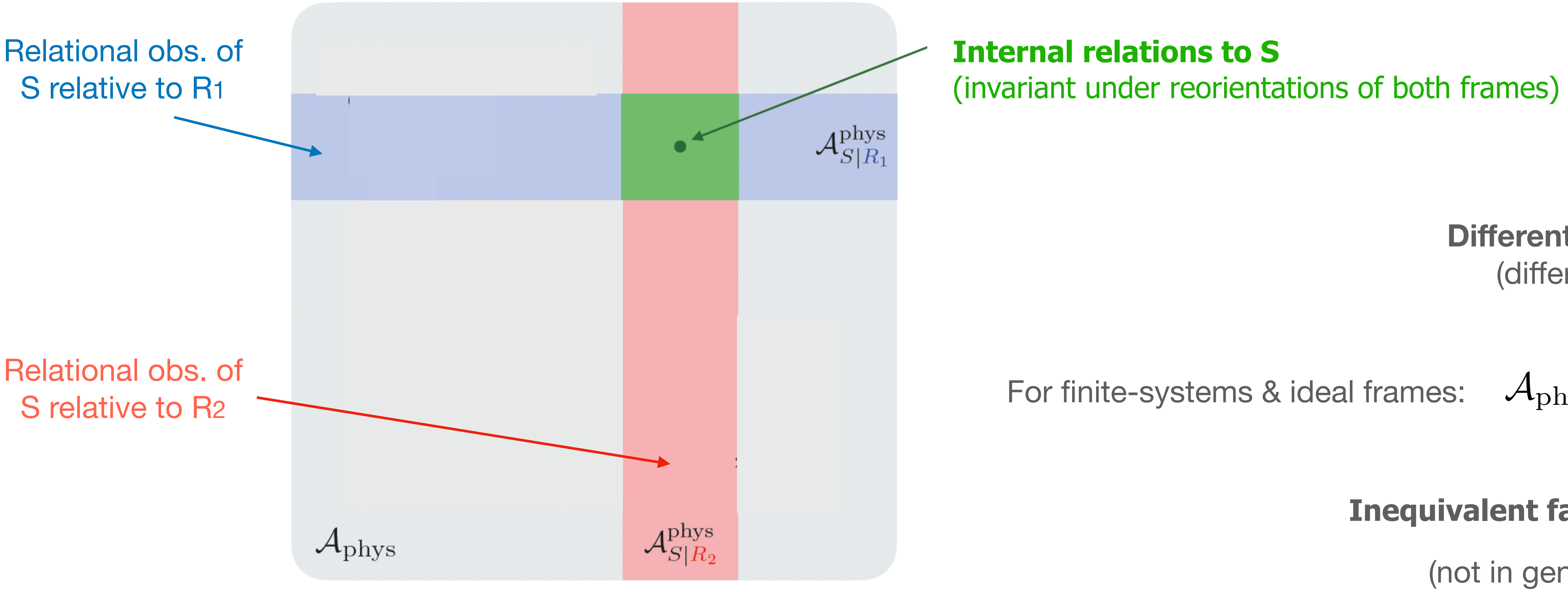
RELATIVITY OF SUBSYSTEMS

[Höhn, Kotecha, **FMM** '23]
[Carrozza, Höhn, Kirklin, **FMM** to appear]

- Observables describing S relative to R_1 invariant under reorientations of R_2, but relative to R_2 it changes since relations between S and R_2 change (and viceversa)

- Different frames identify distinct relational observable subalgebras inside total gauge inv. algebra $\mathcal{A}^{\text{phys}}_{S|R_1} \neq \mathcal{A}^{\text{phys}}_{S|R_2}$

- Non-trivial overlap $\mathcal{A}^{\text{phys}}_{S|R_1} \cap \mathcal{A}^{\text{phys}}_{S|R_2} \neq \emptyset$



Different relational ways to refer to a kinematical subsystem
(different frames identify different gauge inv. subsystems)

For finite-systems & ideal frames: $\mathcal{A}_{\text{phys}} \simeq \mathcal{A}^{\text{phys}}_{R_2|R_1} \otimes \mathcal{A}^{\text{phys}}_{S|R_1} \simeq \mathcal{A}^{\text{phys}}_{R_1|R_2} \otimes \mathcal{A}^{\text{phys}}_{S|R_2}$

Inequivalent factorisations of total algebra relative to the two frames

(not in general the same as factorisations across kinematical DoFs)

CONCLUSION

- We discussed internal dynamical frames in finite-dim. (quantum) systems and (classical) gauge field theories (edge modes)
- Gauge-invariant/relational notion of subsystems depend on the frame → frame-relativity of (gauge-invariant) properties of subsystems
- Alternative proposal for entanglement entropy in constrained systems and gauge theories ?

OUTLOOK

- Subsystem relativity in gauge theory (quantum)
- Gravitational subregions

} [work in progress with S. Carrozza, P. A. Höhn, and J. Kirklin + work in progress with P. A. Höhn, L. Marchetti, J. De Vuyst]

→ minisuperspaces,… [with F. Sartini and P. A. Höhn]

- Quantum gravity:
  - → Relational subsystems in spin networks, entanglement & quant. therm. in LQG
  - → Diffeo-inv. & relationalism in full QG and gravitational entropy [ongoing work with M. Bruno, E. Colafranceschi, and C. Rovelli]

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THANK YOU FOR YOUR ATTENTION !