Effective LQG-BH: A Covariant, Improved-Dynamics Scheme

Idrus Husin Belfaqih

Joint work with Suddhasattwa Brahma, Martin Bojowald, and Erick Evan Duque

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Introduction

Motivation:

- Recently, a consistent model of effective loop quantum gravity successfully constructed beyond homogeneous spacetime [Bardaji, Brizuela'21; Bojowald, Duque'23; Bardaji, Brizuela'24].
- Demand for covariance solution for $\bar{\mu}$-scheme for BH solution [Kelly, Santa Cruz, Ewing'20; Gambini, Olmedo, Pullin'20].

> "...the mechanism that resolves the singularity can also trigger conceptually undesirable features that can be subtle and are often uncovered only after a detailed examination. Therefore, the quantisation scheme has to be chosen rather astutely." [Ashtekar, Olmedo, Singh'23].

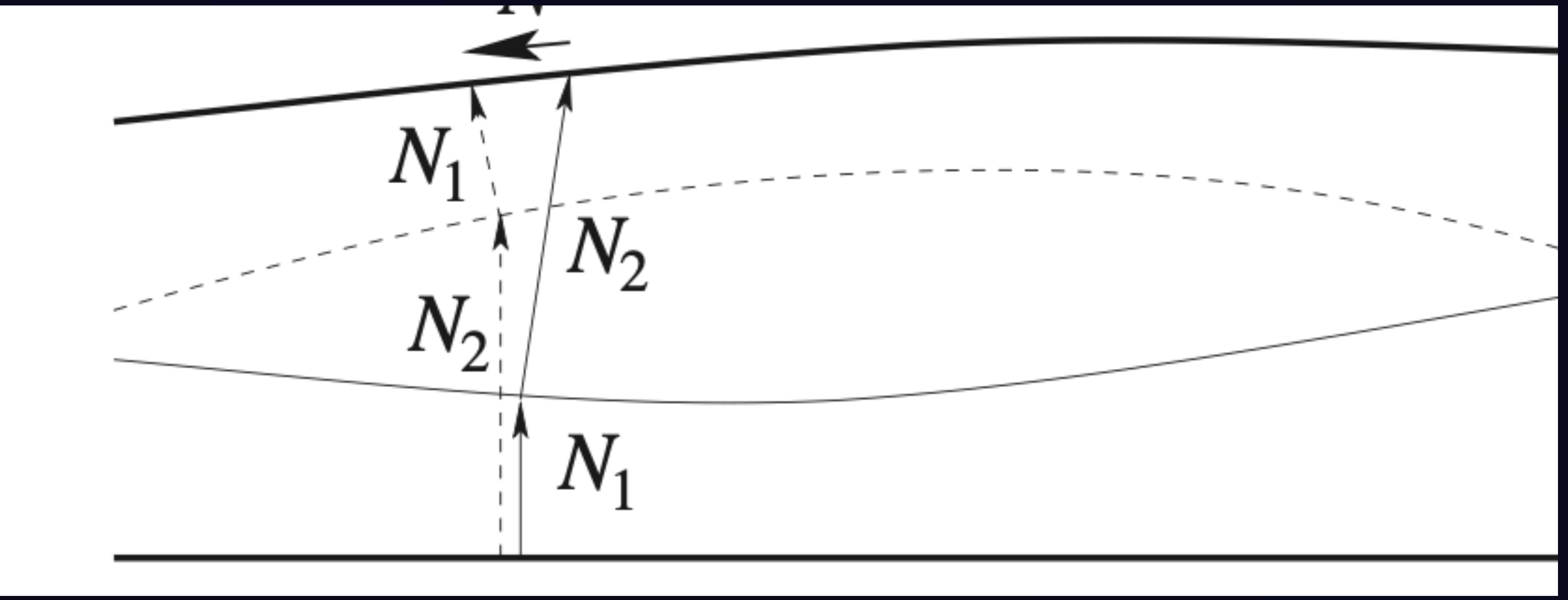
Framework:

- The canonical formulation of GR, with the phase space, is described by triads and extrinsic curvature $(E^I, ; K_I) = (E^x, E^\varphi; K_x, K_\varphi)$.
- Emergent modified gravity (EMG) [Bojowald, Duque'23]: Line-element constructed from the deformation algebra.

Kinematical and Dynamical Structure

Off-Shell

- The constraints algebra corresponds to hypersurface deformation algebra
  - $\{H_x[N^x], H_x[M^x]\} = H_x[NM' - MN']$
  - $\{\tilde{H}[N], H_x[N^x]\} = -\tilde{H}[N^x N']$
  - $\{\tilde{H}[N], \tilde{H}[M]\} = -H_x\left[\tilde{q}^{xx}(NM' - MN')\right]$



- The surface $(K_\varphi = \pm\pi/2\tilde{\mu}, K_x = 0)$ is a reflection-symmetry surface in the phase space which manifests as a reflection surface on-shell.

On-Shell

- The covariance condition required

$$\{\tilde{g}_{\mu\nu}(E^I; K_I), H[\epsilon] + H_x[\epsilon^x]\}\Big|_{O.S} = \mathcal{L}_\zeta \tilde{g}_{\mu\nu}\Big|_{O.S}$$

- The emergent line-element

$$ds^2 = -N^2 dt^2 + \tilde{q}_{xx}\left(dx + N^x dt\right)^2 + \tilde{q}_{\theta\theta} d\Omega^2$$

- The reflection surface symmetry on-shell corresponds to the solution of

$$1 + \tilde{\mu}^2(E^x)\left(1 - \frac{2M}{\sqrt{E^x}} - \frac{\Lambda}{3}E^x\right) = 0$$

Stationary Gauges

Schwarzschild Gauge ($N^x = 0$ and $E^x = x^2$)

$$ds^2 = -\left(1 - J(x)\right)\frac{dt^2}{\alpha^2\chi^2} + \frac{dx^2}{\chi^2\left(1 - J(x)\right)\left(1 + \mu^2(x)\left(1 - J(x)\right)\right)} + x^2 d\Omega^2$$

with:

$$J(x) = \frac{2M}{x} + \frac{\Lambda x^2}{3}$$

Horizon structure

$$x_H \approx 2M$$

$$x_\Lambda \approx \sqrt{\frac{3}{\Lambda}}$$

Reflection surface

$$1 + \tilde{\mu}^2(x)\left(1 - \frac{2M}{x} - \frac{\Lambda}{3}x^2\right) = 0$$

Related by a coordinate transformation

GP- Gauge ($N = 1$ and $E^x = x^2$)

$$\mathrm{ds}^2_{\mathrm{GP}} = -\,\mathrm{dt}^2_{\mathrm{GP}} + \frac{\chi^{-4}}{\alpha^2\varepsilon^2}\left(1 + \mu^2\left(1 - \mathrm{J(x)}\right)\right)^{-1}\left(\mathrm{dx} + s\chi\sqrt{\mathrm{J(x)} - \mathrm{J(x_0)}}\sqrt{1 + \mu^2\left(1 - \mathrm{J(x)}\right)}\,\mathrm{dt}_{\mathrm{GP}}\right)^2 + \mathrm{x}^2\mathrm{d}\Omega^2 .$$

Asymptotically flat

$$\mathrm{ds}^2_{\mathrm{GP}} = -\,\mathrm{dt}^2_{\mathrm{GP}} + \frac{\chi(\infty)^2}{\chi^4}\left(1 + \mu^2\left(1 - \frac{2\mathrm{M}}{\mathrm{x}}\right)\right)^{-1}\left(\mathrm{dx} + s\chi\sqrt{\frac{2\mathrm{M}}{\mathrm{x}}}\sqrt{1 + \mu^2\left(1 - \frac{2\mathrm{M}}{\mathrm{x}}\right)}\,\mathrm{dt}_{\mathrm{GP}}\right)^2 + \mathrm{x}^2\mathrm{d}$$

Homogeneous Gauge

Homogeneous Patch ($N^x = 0,\ N' = 0$)

○ Schwarzschild Gauge

Ricci scalar finite for any $\mu(x)$ as long as $x_\mu^{(-)} \neq 0$ and $x_\mu^{(+)} \neq \infty$

$$\mathscr{R}|_{x=x_\mu^{(i)}} = \frac{2}{(x_\mu^{(i)})^2} + \frac{2\chi^2}{(x_\mu^{(i)})^2}\mu^2 \left(\frac{M}{x_\mu^{(i)}} - \frac{\Lambda (x_\mu^{(i)})^2}{3}\right)\left(\frac{3M}{x_\mu^{(i)}} + \Lambda(x_\mu^{(i)})^2 - 2\right) - \frac{2\chi^2}{x_\mu^{(i)}}\left(\frac{3M}{x_\mu^{(i)}} + \Lambda (x_\mu^{(i)})^2 - 2\right)(\ln\mu)'|_{x=x_\mu^{(i)}}$$

At large scales ($\Lambda x^2 \gg 1$ and $M/x \ll 1$), the Ricci scalar

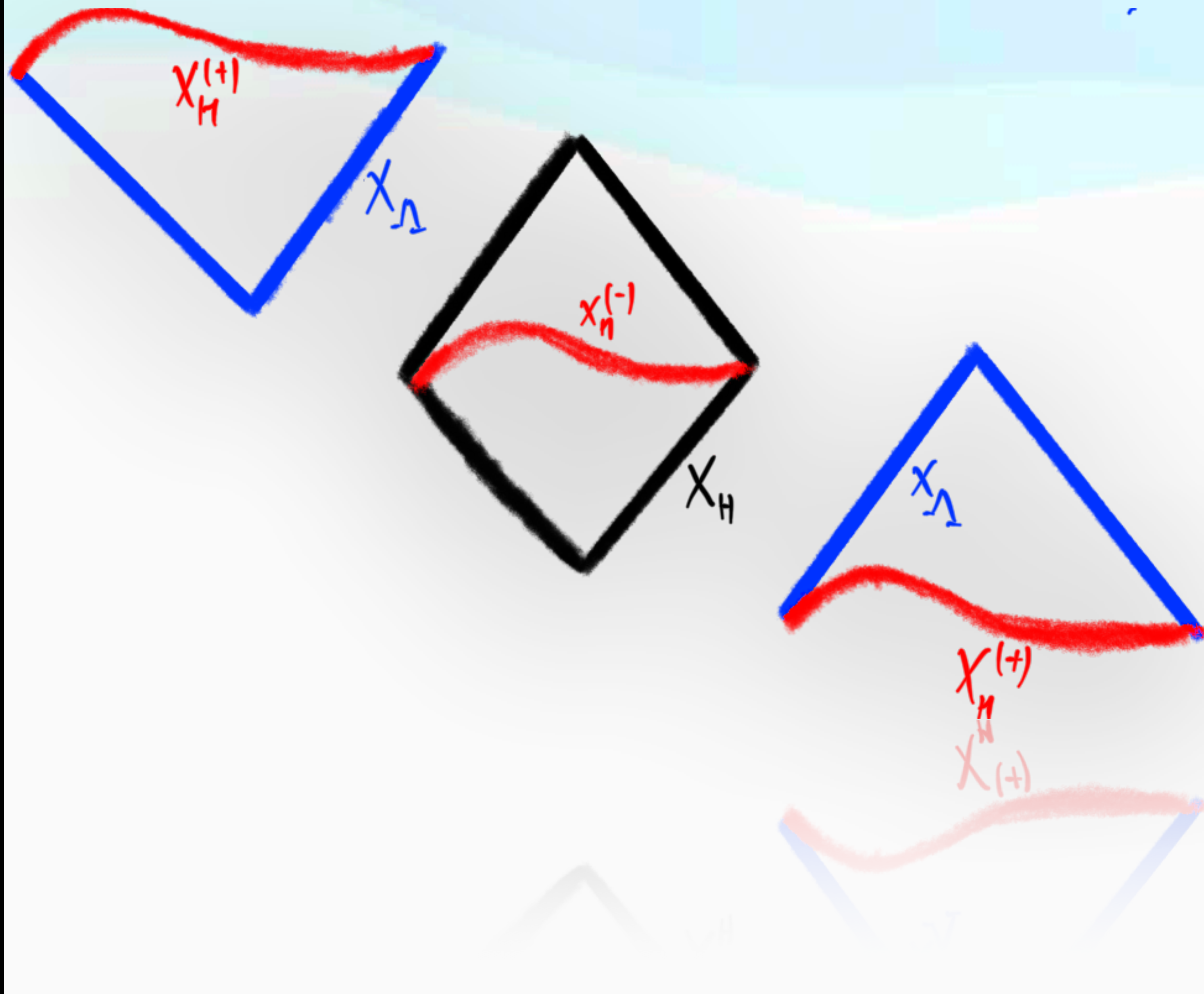
$$\mathscr{R} \approx 4\chi^2\Lambda + \frac{2}{x^2}\left(1-\chi^2+\chi^2\mu^2\left(1+x\left(\ln\mu^2\right)'\right)\right) + 4\chi^2\Lambda\mu^2\left(\frac{3}{2}+\frac{5x}{12}(\ln\mu^2)'\right) - 2\chi^2\mu^2\Lambda^2x^2\left(1+\frac{x}{6}(\ln\mu^2)'\right)$$

Finite at $x \to \infty$ if $\mu(x)$ falls off at least as $1/x$.

○ Internal Time Gauge

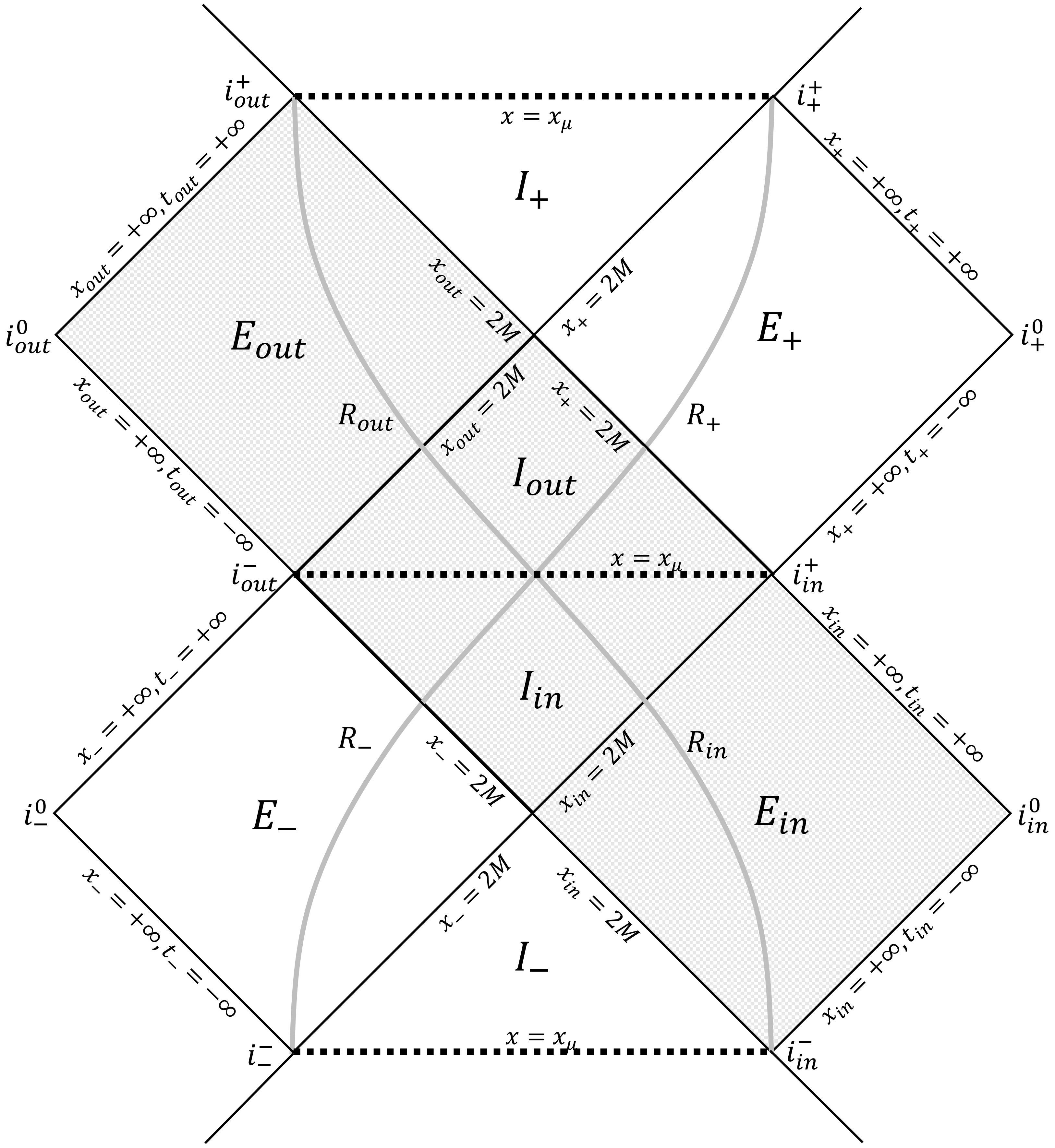
Line element

$$ds^2 = -4E^x\frac{\tilde\mu^2}{\mu^2}\left|1-\Lambda E^x + \left(1-2\frac{\partial\ln\mu^2}{\partial\ln E^x}\right)\frac{\tilde\mu^2}{\mu^2}\frac{\sin^2\left(\tilde\mu t_\varphi\right)}{\tilde\mu^2}\right|^{-2}\frac{dt_\varphi^2}{\chi^2} + \frac{\tilde\mu^2}{\mu^2}\frac{\sin^2\left(\tilde\mu t_\varphi\right)}{\tilde\mu^2}\frac{dx_h^2}{\alpha^2\chi^2} + E^x d\Omega^2$$



Global Structure

Vacuum Schwarszchild Solution



1. Asymptotic limit ($x \to \infty$):

$$ds^2 \approx -\frac{dt^2}{\alpha^2\chi(\infty)^2} + \left(1+\mu(\infty)^2\right)^{-1}\frac{dx^2}{\chi(\infty)^2} + x^2 d\Omega^2 .$$

Requiring asymptotic flatness implies $\alpha = \chi(\infty)^{-1}$ *and* $\chi(\infty) = 1/\sqrt{1+\mu(\infty)^2}$.

2. Zero mass limit ($M \to 0$):

$$ds^2 \approx -\frac{dt^2}{\alpha^2\chi^2} + \left(1+\mu^2\right)^{-1}\frac{dx^2}{\chi^2} + x^2 d\Omega^2 .$$

- *Recovering flat space attained by* $\chi = 1/\sqrt{1+\mu^2}$

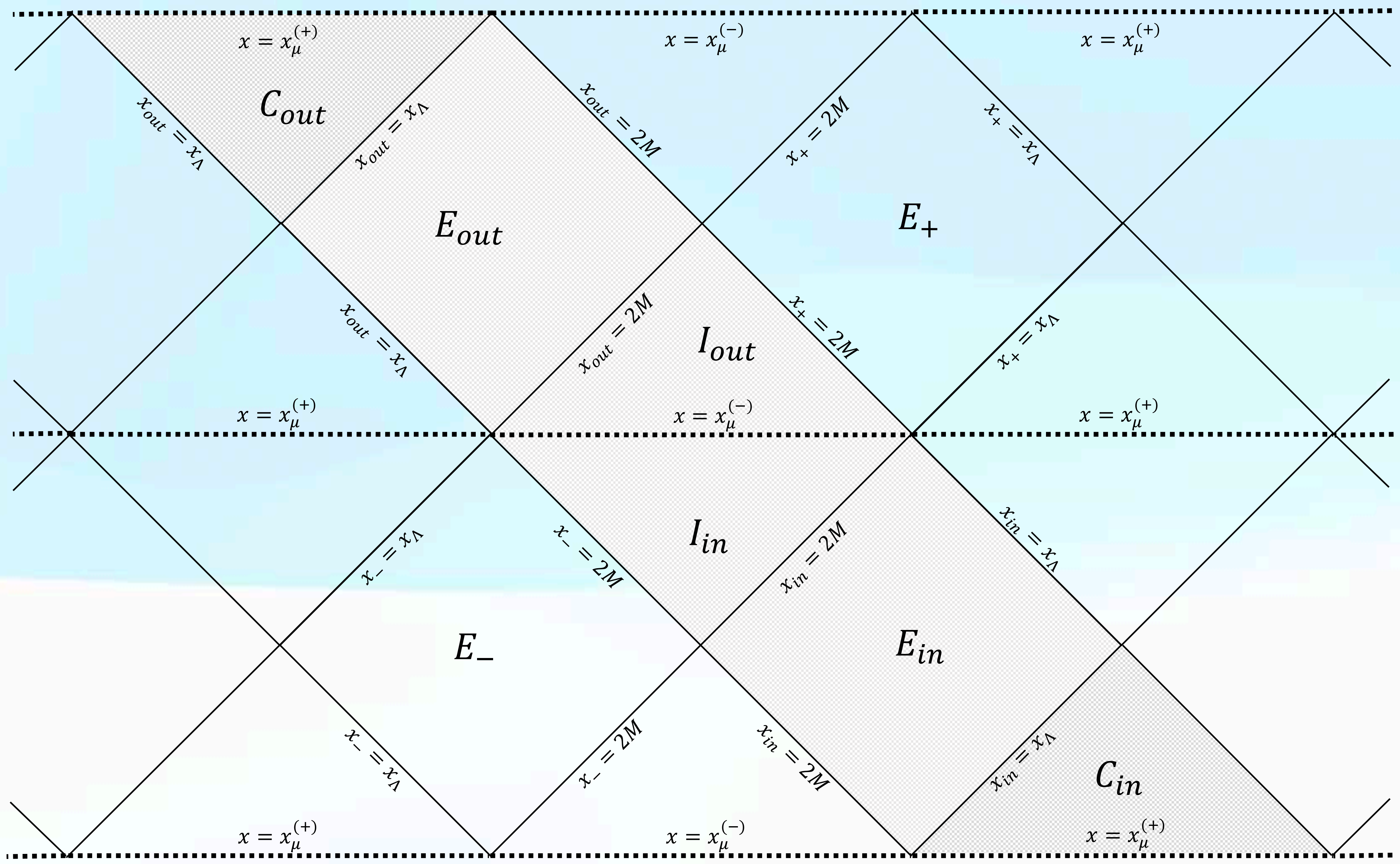
$$ds^2 = -\frac{1+\mu^2}{\alpha^2}dt^2 + ds^2_{Euclidean}$$

- *Recovering flat time attained by* $\chi^2 = \alpha^{-2} \equiv \chi_0^2 = \left(1+\mu_\infty^2\right)$

$$ds^2 = -dt^2 + \frac{1}{\chi_0^2\left(1+\mu^2(x)\right)}dx^2 + x^2 d\Omega^2$$

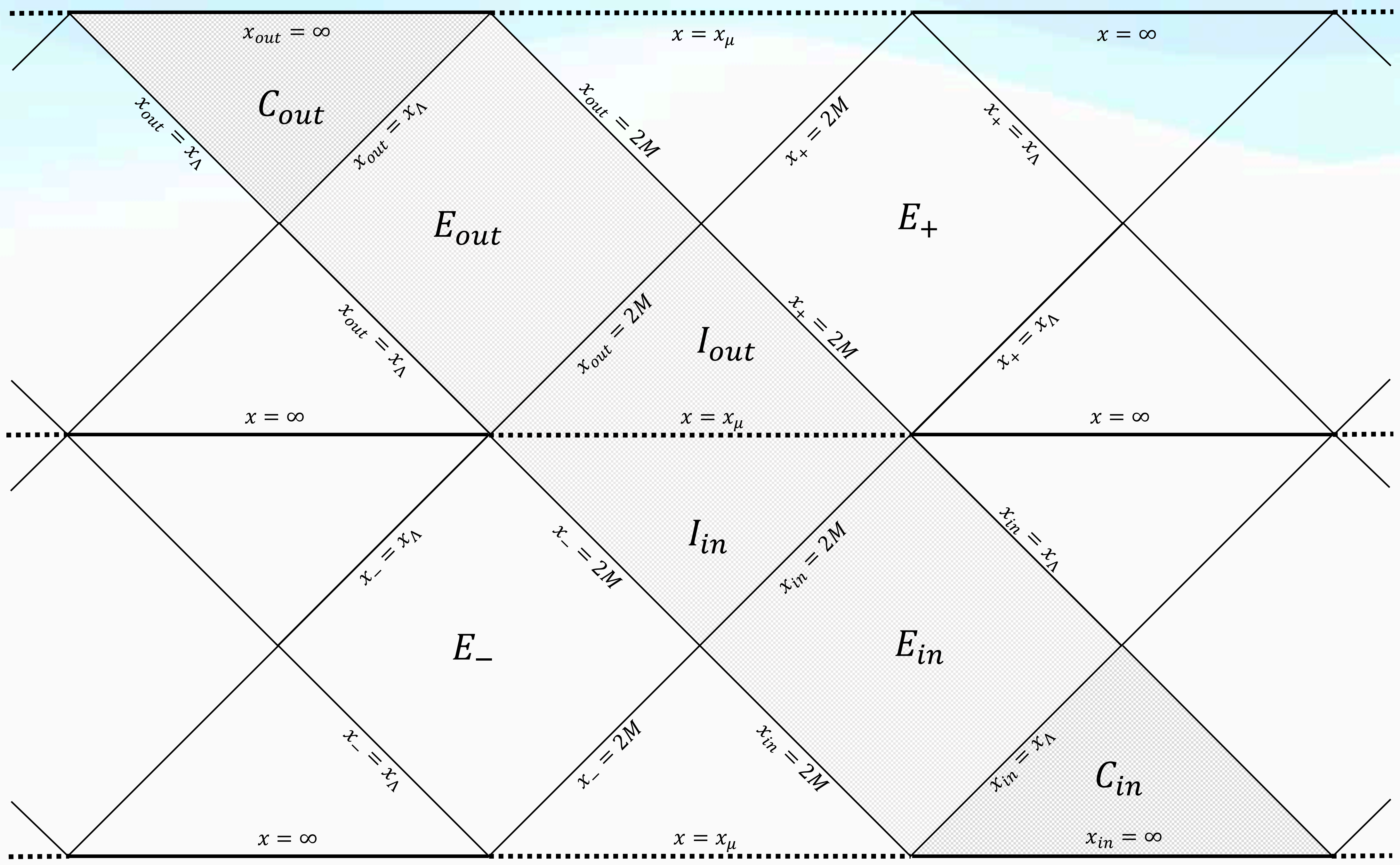
Global Structure

dS-Schwarzschild vacuum in μ_0-scheme



Large x effect

dS-Schwarzschild vacuum in $\bar{\mu}$-scheme



No IR effect

Spacetime Contrast

Exterior Physics

μ_0–Scheme

- Red-shift → No correction
- Deflection angle:

$$\Delta\phi \approx \pi\left(\left(1-\frac{\mu_0^2}{2}\right)\sqrt{1+\mu_0^2}-1\right)+\frac{4M}{x}\frac{1+3\mu_0^2/2}{1+\mu_0^2}$$

- Black-hole entropy (local)

(At the horizon) $S(2M)=S_{BH}\left(1+\frac{\mu_0^4}{48}+\mathcal{O}\left(\mu_0^6\right)\right)$

(Asymptotically) $S_\infty = S_{BH}\frac{1+2\mu_0^2}{1+\mu_0^2} > S_{BH}$

- $\mathbf{M\to 0}$ Limit → Minkowski spacetime

$\bar{\mu}$–Scheme

- Red-shift → No correction
- Deflection angle:

$$\Delta\phi \approx -\frac{\pi}{4}\frac{\tilde{\Delta}}{x^2}+\frac{4M}{x}\left(1-\frac{3}{8}\left(\frac{\tilde{\Delta}}{x^2}\right)^2+O\left(\frac{\tilde{\Delta}^3}{x^6}\right)\right)+O\left(M^2/x^2\right)$$

- Black-hole entropy (local)

(At the horizon) $S(2M)=S_{BH}\left(1+2\pi\frac{\tilde{\Delta}}{A_H}+\mathcal{O}\left(\frac{\tilde{\Delta}}{A_H}\right)^2\right)$

(Asymptotically) $S_\infty\to S_{BH}$

- $\mathbf{M\to 0}$ Limit → Non-smooth geometry at the Planckian scale

$$ds^2=-dt^2+\left(1+\frac{\tilde{\Delta}}{x^2}\right)^{-1}dx^2+x^2d\Omega^2$$

BH-Entropy

Brown-York quasi-local energy

$$E_{BY}(x) = x\chi_0 \left(\sqrt{1+\mu^2} - \sqrt{1-\frac{2M}{x}} \sqrt{1+\mu^2\left(1-\frac{2M}{x}\right)} \right)$$

An infinitesimal change δM will change the Brown-York quasi-local energy which can be associated with thermodynamics relation $\delta E_{BY} = T\delta S$, which provides:

$$S(x) = \chi_0 \frac{8\pi\bar{x}^2}{15\mu^4} \left[\sqrt{1+\mu^2\left(1-\frac{2M}{\bar{x}}\right)} \left(3+\mu^2\left(1+\frac{3M}{\bar{x}}\right) - 2\mu^4\left(1+\frac{M}{\bar{x}}-\frac{6M^2}{\bar{x}^2}\right)\right) - \sqrt{1+\mu^2}\left(3+\mu^2-2\mu^4\right)\right]$$

Important points:

➡ Correct classical limit: $S(x) \xrightarrow[\mu\to 0]{\chi_0\to 1} \pi(2M)^2 = \frac{A_{\rm H}}{4} = S_{\rm BH}$

➡ Asymptotic limit: $S(\infty) = \left(1+\mu_\infty^2 + O\left(\mu_\infty^4\right)\right) S_{\rm BH}$

➡ Horizon: $S(2M) = \left(1 + \frac{\mu_{\rm H}^2 - \mu_\infty^2}{2} + O\left(\mu_{\rm H}^2\mu_\infty^2, \mu_{\rm H}^4, \mu_\infty^4\right)\right) S_{\rm BH}$

Spacetime Contrast

Interior Physics

μ_0–Scheme

- Minimum and maximum radius

$$x^{(-)}_{\mu_0} \approx \frac{2M\mu_0^2}{1+\mu_0^2} \text{ and } x^{(+)}_{\mu_0} \approx \sqrt{\frac{3}{\Lambda}\frac{1+\mu_0^2}{\mu_0^2}}$$

UV/IR mixing

- Geometric condition:
  - Violation of NGC → $R_{\alpha\beta}v^{\alpha}_{(i)}v^{\beta}_{(i)} = -\frac{x^{(-)}_{\mu 0}}{x^3} \le 0$
  - Violation of TGC → $R_{\alpha\beta}u^{\alpha}_{(i)}u^{\beta}_{(i)} = -\frac{3Mx^{(-)}_{\mu 0}}{2x^4} \le 0$
- Geodesic completeness

$$\tau_{\text{cross}} = \pi\left(2M + x_{\mu_0}\right)$$

congruences will diverge near $x^{(-)}_{\mu_0}$

$\bar{\mu}$–Scheme

- Minimum radius

$$x_{\Delta} = (\tilde{\Delta}M)^{1/3}\frac{\left(1+\sqrt{1+\tilde{\Delta}/(27M^2)}\right)^{2/3} - \left(\tilde{\Delta}/(27M^2)\right)^{1/3}}{\left(1+\sqrt{1+\tilde{\Delta}/(27M^2)}\right)^{1/3}}$$

- Geometric condition
  - Violation of NGC → $R_{\mu\nu}v^{\mu}_{(i)}v^{\nu}_{(i)} = \frac{2}{x^2}\frac{\tilde{\Delta}}{x^2}\left(1-\frac{3M}{x}\right) \le 0$
  - Violation of TGC → $R_{\mu\nu}u^{\mu}_{(i)}u^{\nu}_{(i)} = \frac{3M}{x^3}\frac{\tilde{\Delta}}{x^2}\left(1-\frac{3M}{x}\right) \le 0$
- Geodesic completeness

Conclusion

Summary

- Consistent (anomaly-free and general covariance) solution for arbitrary holonomy parameter μ.
- An eternal structure, transitioning from a regular black hole to a white hole, remains valid for any holonomy parameter, with the sole condition that its decay is no less than $1/x$.
- The $\bar{\mu}$ resolves the pathologies appear from μ_0-scheme
  * The large deflection angle of a light ray in the zero mass limit.
  * Monotonically increasing black hole entropy
  * Large-scale correction (UV/IR mixing).
- The non-smooth geometry at the Planckian scale achieves uniquely for $\mu \neq \mu_0$.

This isnt over yet...

Outlook

- *Gravitational collapse* ⟶ *A way to see the actual trajectory of ingoing matter (physical description for BH to WH)*
- *Hawking evaporation…possible BH remnant?*
- *Quasinormal modes (BH stability).*

Teaser

There is additional freedom which has a rich impact:

- Dilation potential
- Coupled oddly to K_φ
- Serve as a U(1)-gauge field under wick rotation
- Time-dependent dark energy
- Euclidean transition (No-boundary proposal)

Thank You!