Waleed Sherif, Hanno Sahlmann Department Physik Friedrich-Alexander-Universität Erlangen-Nürnberg

Towards canonical LQG with neural networks

From the basics to 3d gravity in Smolin's weak coupling limit

[arXiv: 2402.10622, 2405.00661]

Learned some deep things about QG from LQG

Singularity avoidance

…

One big open question (in my mind):

(Physical!) quantum state of (1Å)4 of spacetime in this room?

(—> A. Perez: Planck scale DOF)

?

Idea

Solve constraints of canonical LQG numerically, without symmetry assumption.

- Cutoff: Single graph ɣ?
- **EXECUTOFF:** $\lim_{x \to \infty}$?
- **Exponential growth of Hilbert space:**

 $\dim\mathcal{H}_{\text{kin}} \sim (2j_{\text{max}} + 1)^{2|\gamma|}$

Problems:

Picture: solid state physics!

In this talk

Proof of principle, using

U(1) BF-Theory [2402.10622]

Of course: Can be solved analytically (but…)

\triangleright 3d Euclidean gravity in Smolin small coupling limit $SU(2) \longrightarrow U(1)^3$ [2405.00661]

Exponential growth

EXAMPLE Need smart Ansatz for the state EXAM Need sophisticated software tools

dim H = (2 mmax + 1)15 U(1),cutoff: charges *m* ∈ *{-mmax,…,mmax}*

Neural network quantum states

In principle

Weights wij

(m1, m2, …, mN) Ψ(m1, m2, …, mN)

Neural network quantum states

In practice

Uther methods: Tensor hetwork states, ex. [Cunningham,Dittrich,Steinhaus] Other methods: Tensor network states, ex. [Cunningham,Dittrich,Steinhaus]

! $\frac{1}{2}$ min

 $\langle C \rangle_{\Psi_w}$

Prosaic interpretation: Variational Ansatz for physical state Romantic interpretation: Special kind of brain learns about gauge invariance etc.

Plan: eventually release software package. Dream: Network weights as essence

NetKet

- **B** Differentiable programming for gradient descent
- Markov-chain Monte Carlo to compute $\langle C \rangle_{\Psi_w}$
- **▶ No need to hold entire C in memory**

We did:

- **EXA Custom representation of graphs, holonomies, fluxes**
- **Spatial volume operator, constraint operators**
-

Quantum constraints The classical theory considered so far is subject to the curvature and the Gauß $\bigcap_{v \in \mathcal{P}} \mathsf{A}^{\mathsf{A}}$ erned by the constraints (4) shown previously and these three constraints shown above are classically equivalent. However, it is argued that upon \mathbf{r}_i is argued that upon \mathbf{r}_i WUM II UI II COI ISTIM IIS

Master constraints: Waster constraints: are construinter. However, it is an argued that upon Λ

Thiemann-regularized Hamilton constraint: (TRC) for a physical state . Just a physical
Thiomann roquilarized Hamilton constraint (TDC) . Thiomann: n **Indianus in Pagence in 199** and the state in the state of the state of the state which is the -measure of the -
The -measure of the -measur $\hat{H}_{\text{max}}(\hat{N}) = \frac{2}{\epsilon} \sum_{\ell} \hat{q}^j \epsilon^{kl} N(\eta) \text{tr}(\hat{h}_{\text{max}}(\hat{N}) \hat{h}_{\text{max}}(\hat{N})$ Thiemann-regularized Hamilton constraint: (TRC) [Thiemann: QSD IV] *H* \hat{H} $T(\gamma)$ $\left(N\right)$ $=$ $\frac{1}{\hbar2}$ ˆ au
-*H* .
∶i ⊃ 2 $\sqrt{ }$ $\epsilon^{ij}\epsilon^{kl}N(v)\,{\rm tr}(\hat{h}% _{j}^{\dag}\epsilon^{kl}-\hat{h}(\hat{h}_{j}))\epsilon_{l}(v) \label{eq:4.14}%$

$$
\hat{G}|_{\gamma} = \sum_{v \in V(\gamma)} \sum_{i=1}^{3} (\hat{E}_{S(v),i})^2
$$
 (GauB)

$$
\hat{F}_{\gamma} = \sum_{\alpha \in L(\gamma)} tr \left[\left(\hat{h}_{\alpha} - \mathbb{1} \right) \left(\hat{h}_{\alpha}^{\dagger} - \mathbb{1} \right) \right]
$$
 (Curvature)

$$
H_{T(\gamma)}(N) = \frac{1}{\hbar^2} \sum_{\Delta,\Delta' \in T,v} \epsilon^{ij} \epsilon^{\kappa \iota} N(v) \operatorname{tr} (h_{\alpha_{ij}(\Delta')} h_{s_k(\Delta)} [h_{s_k(\Delta)}^{-1}, \sqrt{m_k} h_{s_k(\Delta)}] \cdot \mathcal{H}
$$

$$
=\frac{2}{\hbar^2}\sum\epsilon^{ij}\epsilon^{kl}N(v)\operatorname{tr}(\hat{h}_{\alpha_{ij}(\Delta')}\hat{h}_{s_k(\Delta)}[\hat{h}_{s_k(\Delta)}^{-1},\sqrt{\hat{V}_v}]\hat{h}_{s_l(\Delta)}[\hat{h}_{s_l(\Delta)}^{-1},\sqrt{\hat{V}_v}])
$$

(Gauß)

(Curvature)

THE EXECT CONSTRUCE: (TRC) [Thiemann: QSD IV]

Cutoffs

 tr(*he)* not gauge invariant **EXA** Non-trivial theory

Representations:

 $U(1)^3 \longrightarrow U(1)_q^3$

charges $\in \{-m_{\max}, -m_{\max}+1, \ldots, m_{\max}\}$

Graph:

!
!

Sanity checks: U(1) BF

Figure 3. The exact diagonalisation result for min ^h*C*ˆⁱ is shown in red and the NNQS result $\approx m_{max} \rightarrow \infty$ iooks reasonable

need many iterations to reach a good accuracy. The simulations of the simulations of the simulations of the simulations of the simulation shown in the simulation shown in the simulation shown in the simulation shown in the figure concluded with an accuracy in $\mathcal{L}(\mathcal{A})$ Sanity checks: U(1) BF

$\#(weights) = 4 * 10⁴$ *mmax —> ∞* looks reasonable *mmax = 8: dim H =* 1.4 * 106

U(1)³ with master constraints combination of solutions with real coefficients (see Appendix C in Eq.). We can consider the coefficients (see
The coefficients (see Appendix C in Eq.). We can consider the coefficients (see Appendix C in Eq.). We can c a better architecture. The following table summarises the values for min h*C*

- **Example 20 Convergence!**
	- **▶ Same architecture works**
- Way beyond exact diagonalization \sim and a standard commercial 8-core Apple Silicon M1 chip, without multiplot mu

 m_{max} = 2: $dim H = 3 * 10^{10}$ $\#(\text{weights}) = 6 * 10^3$ $n(vv) = 0$ 10

U(1)3 with master constraints

 m_{max} = 4 entails:

But:

- 2 42 000 weights
- **▶ 30 Minutes as base level HPC job**

Volume [Thiemann: QSD IV] Volume Thiomonn: CCD IV

U(1)3: Operators As previously mentioned, LQG comes with well-defined quantum geometric observables. Therefore, in this quantised model one has for example a well-defined notion of quantum

$$
\hat{V}(B) := \sum_{v \in V(\gamma) \cap B} \hat{V}_v, \qquad \hat{V}_v := \sqrt{\sum_I \left(\sum_{e,e' \text{ at } v} s \right)}
$$

B Can deal with complicated operators \mathbb{R} \mathbb{R} a Have checked expected behavior and the implementation of the imple are at least a pair of edges incident at the vertex with linearly independent tangents. The vertex with linearly independent tangents. The vertex with linearly independent tangents. The vertex with linearly independent ta

Cautious remarks

U(1)3 : Thiemann-regulated constraint fluxes, and their Poisson brackets, all associated to a triangulation *T*. To turn this regu-J(T)": THEITRITI-regulated CONStraint (Thiemann holonomies and fluxes by their operator counterparts and takes a certain limit: $\frac{1}{\sqrt{1-\frac{1$ ˆ ˆ *^T*()(*N*)*f* := lim *T,*✏(*N*)*f* (28) putational hurdle as the tools in this work to the top the top the NNQS and the (yet) work reliably or easily for non-Hermitian operators. All together, this now leaves [Thiemann: QSD IV]

 $\int_{\mathcal{F}}^{ij} e^{kl} N(v) \operatorname{tr} (h_{\alpha_{ij}(\Delta')} h_{s_k(\Delta)} [h_{s_k(\Delta)}^{-1}, \sqrt{V_v}] h_{s_l(\Delta)} [h_{s_l(\Delta)}^{-1}, \sqrt{V_v}])$ $s_k(\Delta)$ $[h]$ \hat{h}^{-1} $\frac{-1}{s_k(\Delta)},$ $\overline{}$ *V* \hat{V} *^v*]*h* \hat{h} $s_l(\Delta)$ $[h]$ \hat{h}^{-1} $\frac{-1}{s_l(\Delta)},$ $\overline{}$ *V* \hat{V}

\sqrt{V} via Taylor expansion (Mostly) no diffeo constraint $\sqrt{ }$ *V* \hat{V}

$$
\hat{H}_{T(\gamma)}(N) = \frac{2}{\hbar^2} \sum_{\Delta,\Delta' \in T,v} \epsilon^{ij} \epsilon^{kl} N(v) \operatorname{tr}(\hat{h}_{\alpha_{ij}(\Delta')} \hat{h}_{s_k(\Delta)} [\hat{h}_{s_k(\Delta)}^{-1}, \sqrt{\hat{V}_v}] \hat{h}_{s_l(\Delta)} [\hat{h}_{s_l(\Delta)}^{-1}, \sqrt{\hat{V}_v}])
$$

$$
\hat{C}_{\text{TRC}} = \hat{H} + \hat{H}^{\dagger} + \hat{G}.
$$

Solving C, CTRC ever, due to the technical diculties of implementing *C*

C, C_{TRC} look very different. So, do they have anything in common?

Solving C, observing CTRC Figure 8: The result of a *mmax* = 1 simulation where the master constraint *C*

Solving CTRC, observing C is being Figure 10: A *mmax* = 1 simulation where *C* ˆ TRC is being solved, shown in green, and *C*

TRC ? If one considers the entire Hilbert space *H*, which contains states

with complex valued complex valued control to the number of the addressed exactly by random can be addressed to

not exactly orthogonal to one another either. As such, we now \mathbf{r}_i and \mathbf{r}_i the significance significance \mathbf{r}_i **the preliminary:**

same calculation shows that this would again yield a negligible probability

$\hat{\bigcirc}$ $\sum_{i=1}^{n} \frac{1}{1} \sum_{i=1}^{n} \frac{1}{1$

\overline{V} VOIC CONTROLLED CONTROLLED AND THE UNITED STATES OF THE UNITED STATES OF THE UNITED STATES OF THE UNITED STATES

 $|\psi|^2 \geq 0.0308$) $\sim 10^{-194953}$ **implementation of diffeo = average over** graph symmetries

> Than you any i Overlap gets slightly bigger

dimensional property of the second terms of

The solutions have a small overlap: $Thes col$ ˆ F^{\dagger} ² as well as calculating the angle between them. In doing so, ille se The solutions have a small overlap: The solutions have a small overlap: Figure 12, the same simulation is done for di↵erent values of " with a number of trials being 50. The results indicate that individual vectors on *S*

$$
|\langle\Psi_{\hat{C}_{\rm TRC}}\mid\Psi_{\hat{C}}\rangle|^2\approx 0.0308\quad,\quad{\rm arccos}\,|\langle\Psi_{\hat{C}_{\rm TRC}}\mid\Psi_{\hat{C}}\rangle|\approx 1.394 {\rm rad}
$$

However, this is not an accident. If they were the solution of **C** \overline{a} , this is not an accident. If they **v** However, this is not an accident. If they were **we are well** where N is dimensional this looks in this looks in this looks in this looks in the first sight, it should of the first sight, it s Given the exponential decay shown in the figure above, we can then address the random picking procedure on *H* ^R as were that is not an accident. In they were the real subspace,

$$
P_{N}^{\mathbb{C}}(|\langle \Psi_{\hat{C}_{\text{TRC}}} | \Psi_{\hat{C}} \rangle|^2 \ge 0.0308) \sim 10^{-194953}
$$
 Implementation of diffeo = average over
\n
$$
P_{N}^{\mathbb{C}}(|\langle \Psi_{\hat{C}_{\text{TRC}}} | \Psi_{\hat{C}} \rangle|^2 \ge 0.0308) \sim 10^{-10}
$$
 On Exponential On Exponential On Exponential For Ex

precise statement regarding the probability of two states being " similar therefore needs

that into take the analogous picking process restricted to only the analogo

Summary

EXECTE Neural networks can in principle parametrize solutions of constraints of constr

▶ NetKet:

- * Strong enough to represent LQG-type constraint operators and the constraint of
- * Powerful numerical capabilities
- \triangleright Exponential growth of dim H always
	- * Cutoffs too severe?
	- * Start from gauge invariant descript
- \geq Need to go to 4d and SU(2), need (relative)
- **▶ We live exciting times**

Waleed Sherif

Sanity checks: U(1) BF

Gauge invariant subspace is selected

Figure 5: The amplitudes of the 3125 basis states obtained using the neural network in