

# Conformal Freeze-in of Light Dark Matter

**Sungwoo Hong**

**KAIST**

**Light Dark World International Forum 2022**

Based on works with

Gowri Kurup and Maxim Perelstein (1910.10160, 2207.10093)

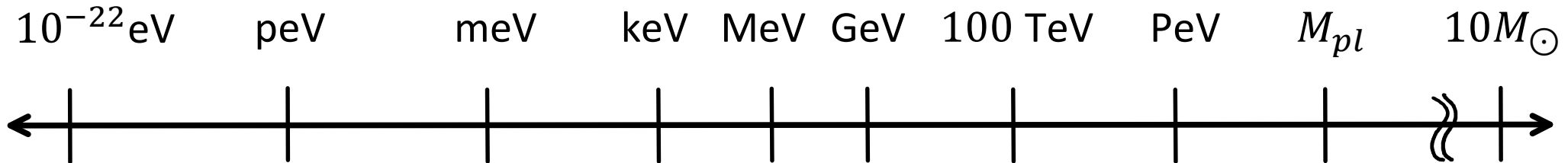
Wen Han Chiu and Liantao Wang (2209.10563)

# Dark Matter

**We are very excited and desperate!**

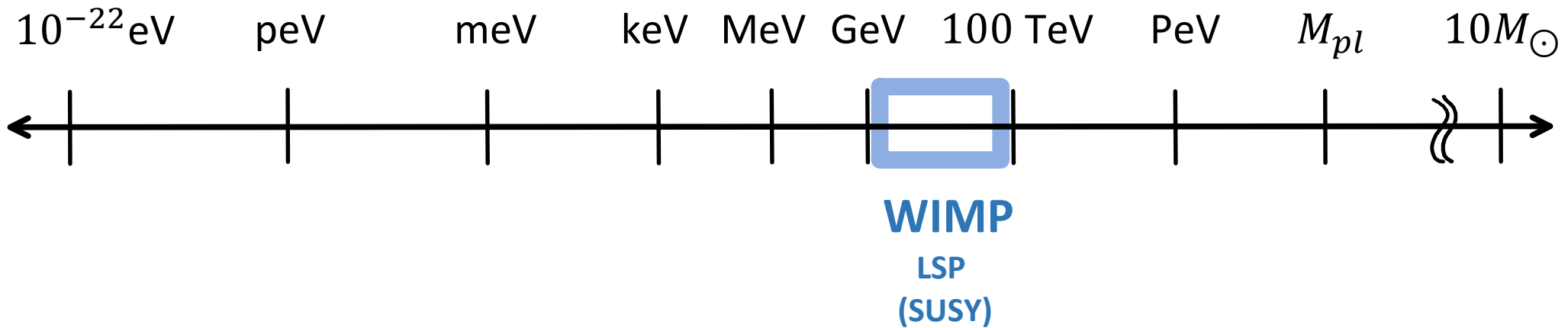
# Enormous Efforts to Uncover **Dark Matter**

☯ **Theory** : Majority of works assumes **DM** = **Particle** (-like)



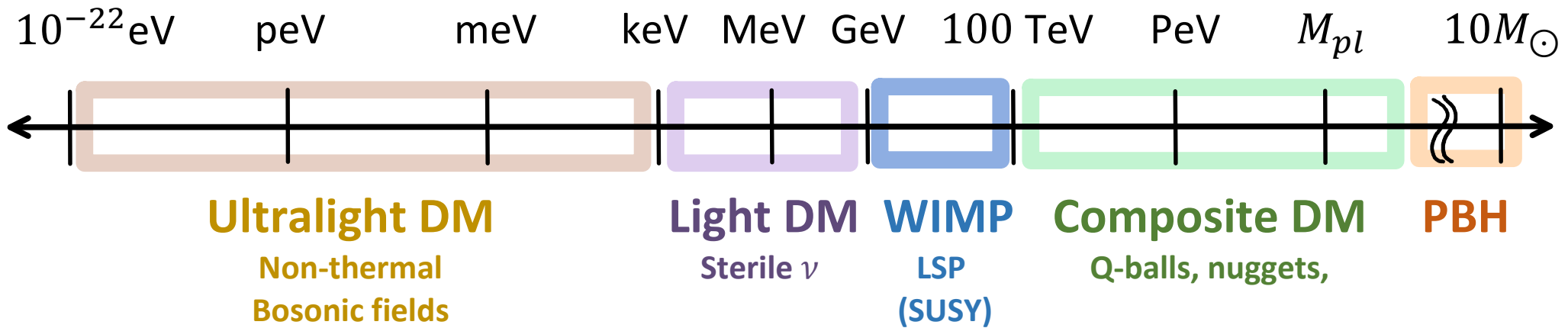
# Enormous Efforts to Uncover **Dark Matter**

☯ **Theory** : Majority of works assumes **DM** = **Particle** (-like)



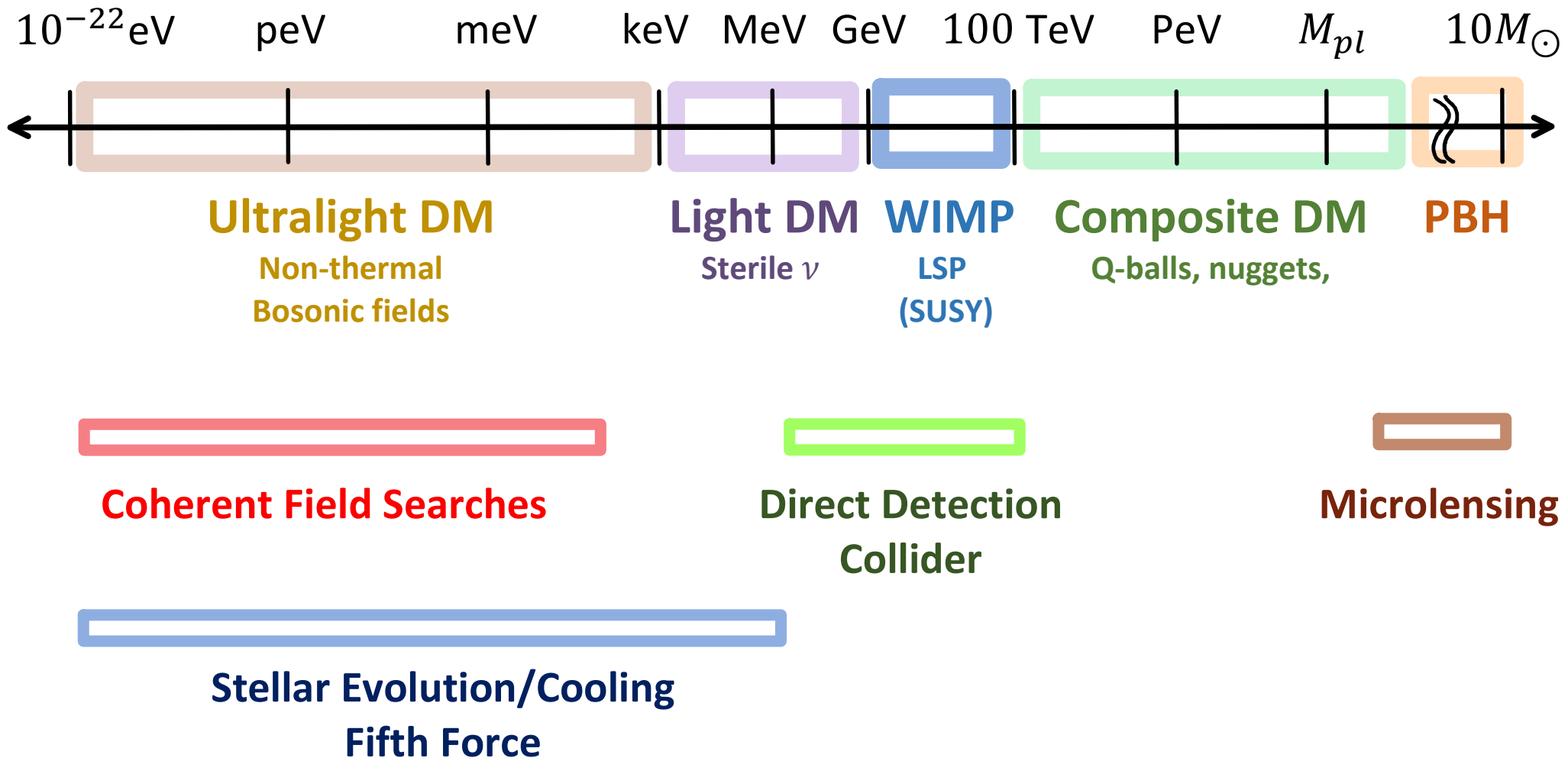
# Enormous Efforts to Uncover **Dark Matter**

☸ **Theory** : Majority of works assumes **DM** = **Particle** (-like)



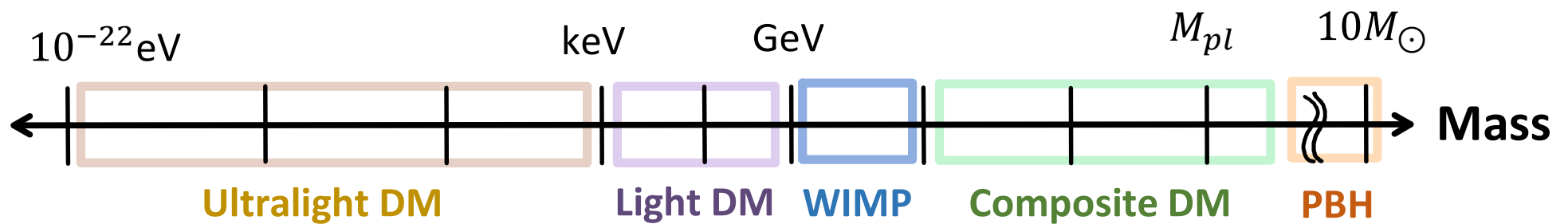
# Enormous Efforts to Uncover **Dark Matter**

🌀 **Experiments** : Diverse approaches, different scales, many of them



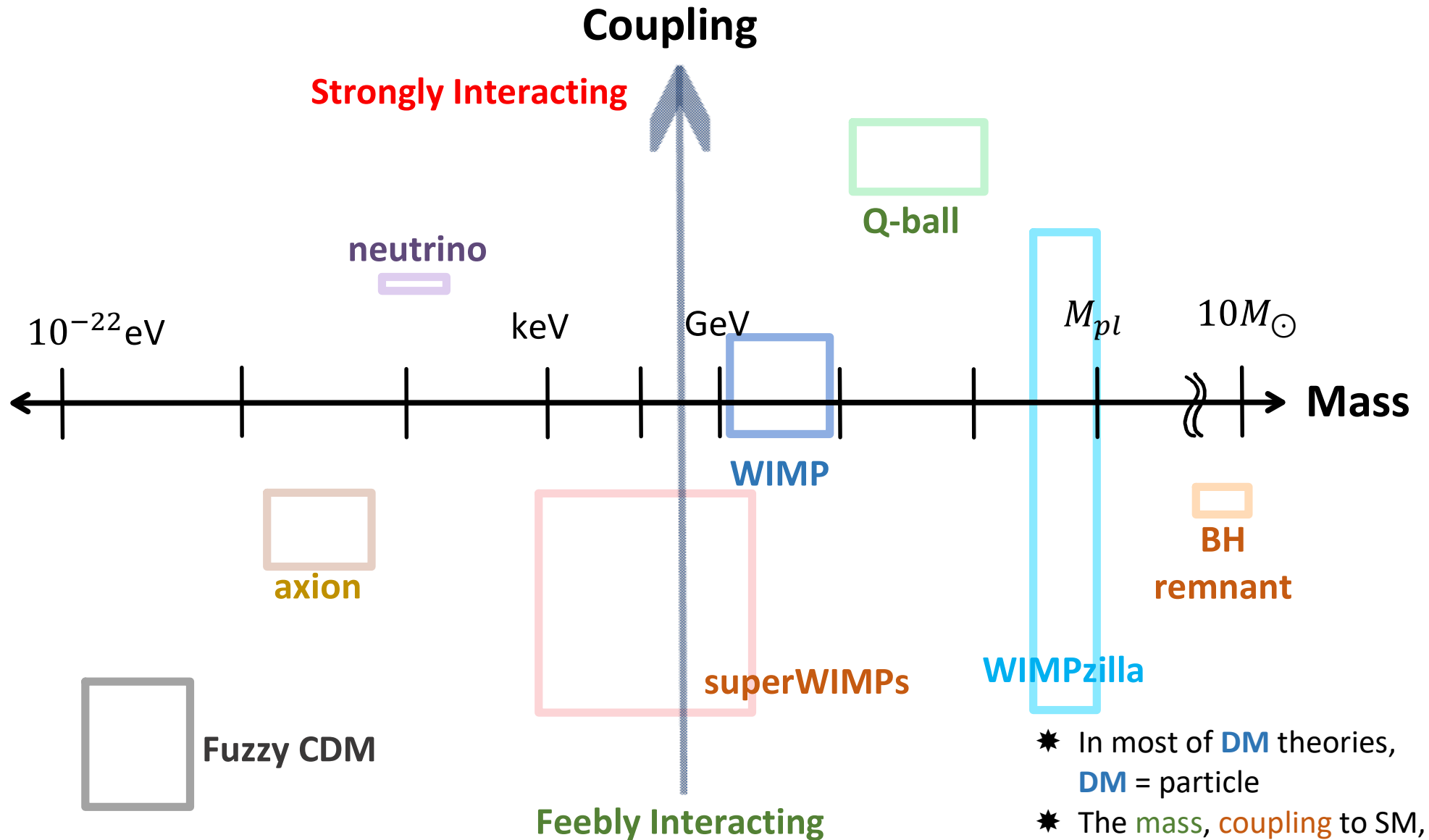
# Uncovering New Lampposts for DM

Let's be even more creative!



# Uncovering New Lampposts for DM

Let's be even more **creative**!

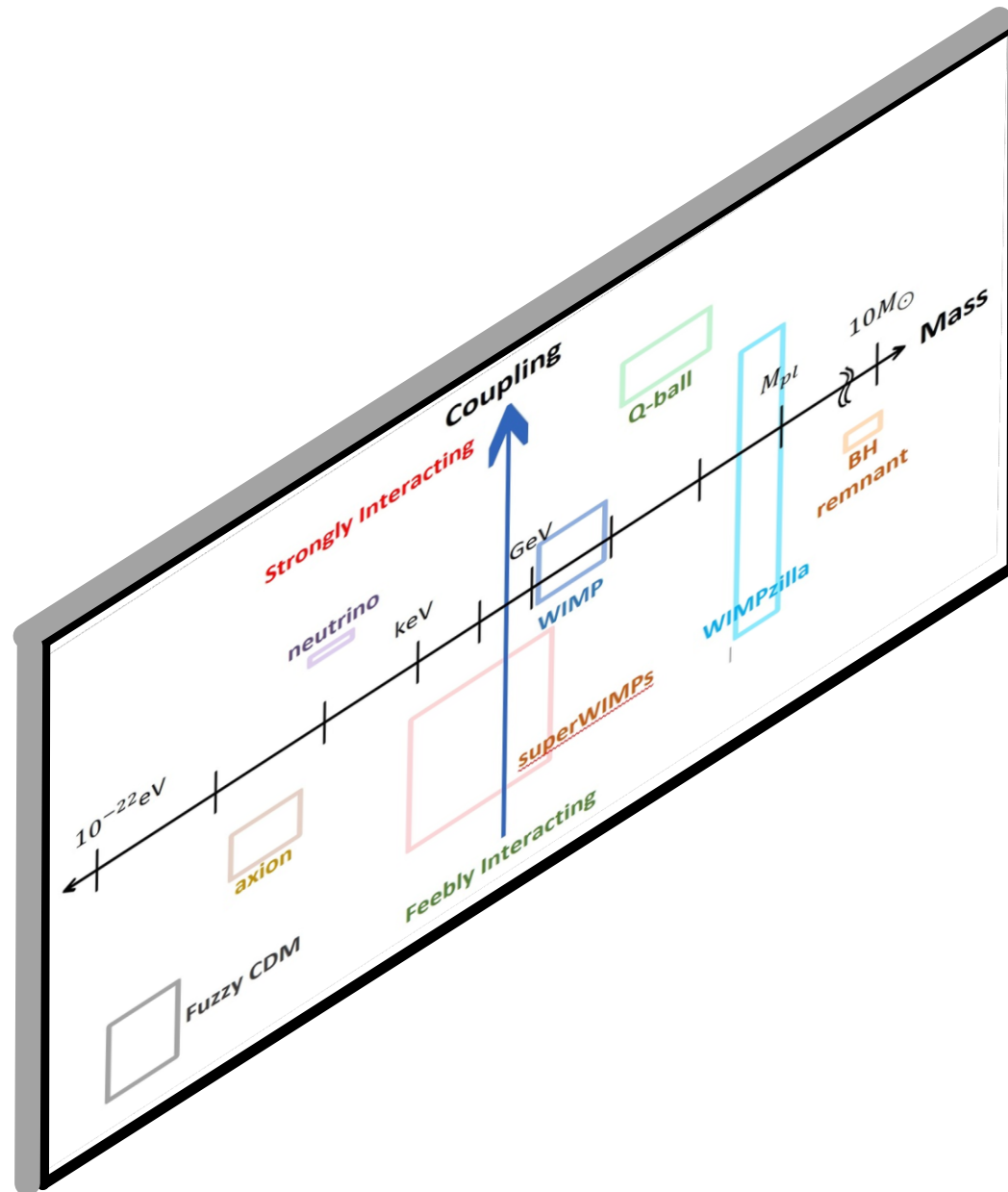


- ★ In most of **DM** theories, **DM** = particle
- ★ The **mass**, **coupling** to SM, **spin** etc are put by hand



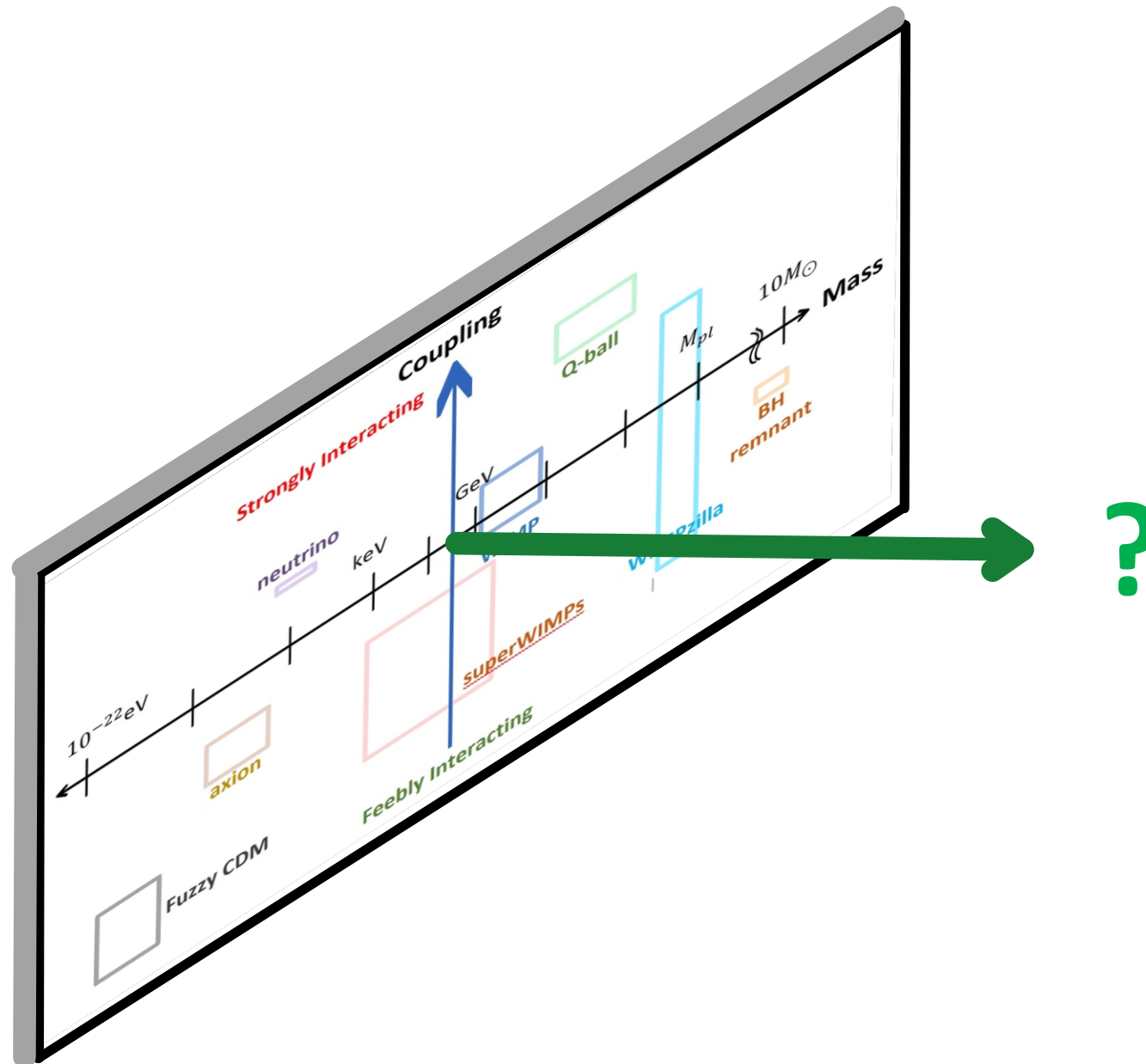
# Uncovering New Lampposts for DM

Let's be even more creative!



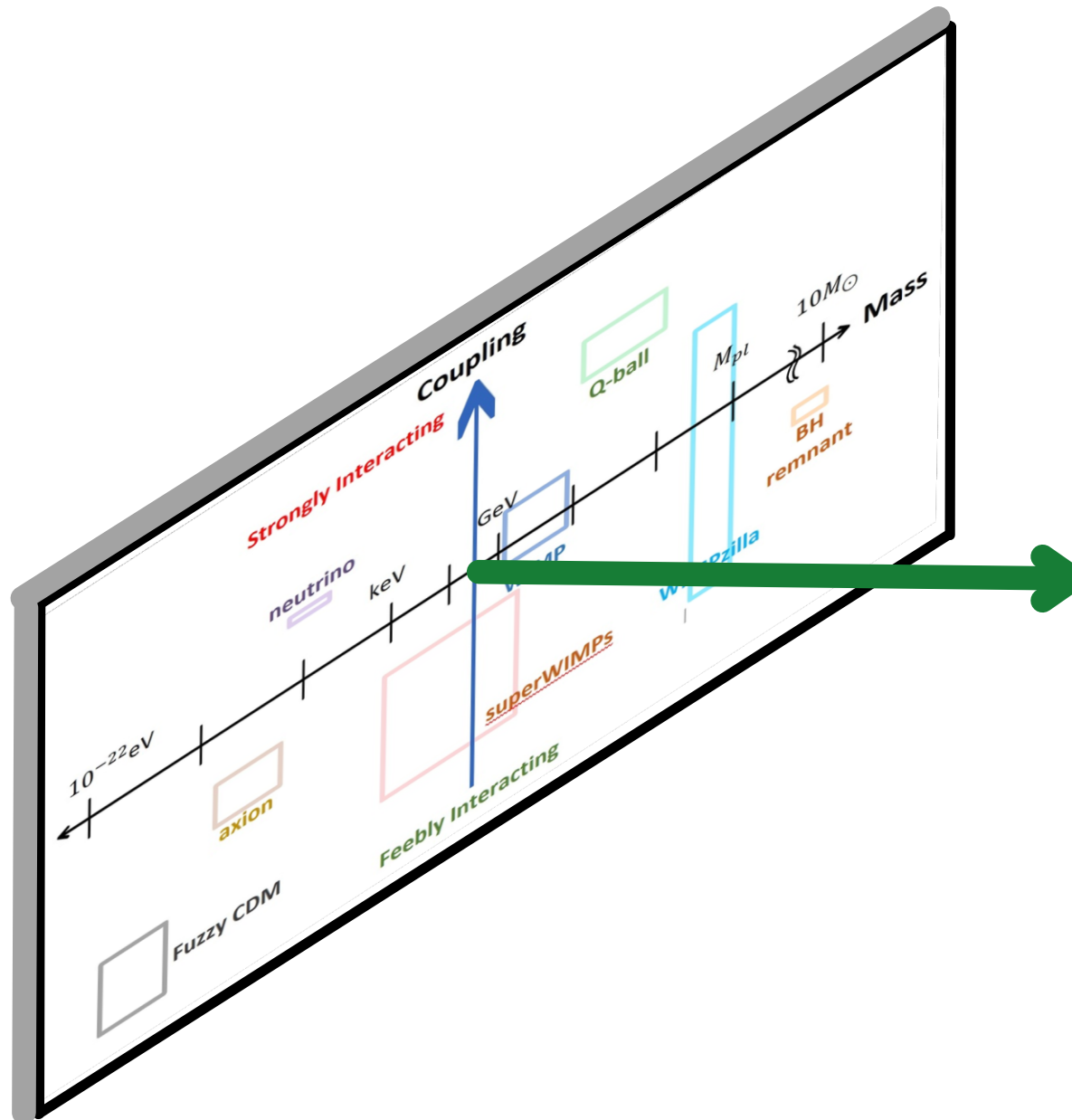
# Uncovering New Lampposts for DM

Let's be even more creative!



# Uncovering New Lampposts for DM

Let's be even more creative!



Conformal Freeze-In  
(COFI)

or

Continuum DM

or

?

# Outline

1. Introduction to Conformal Freeze-In
2. Robustness of COFI mechanism
3. COFI Phenomenology
  - DM Self-Interaction, WDM, Star Cooling
4. Conclusion and Outlook

# Outline

1. Introduction to Conformal Freeze-In

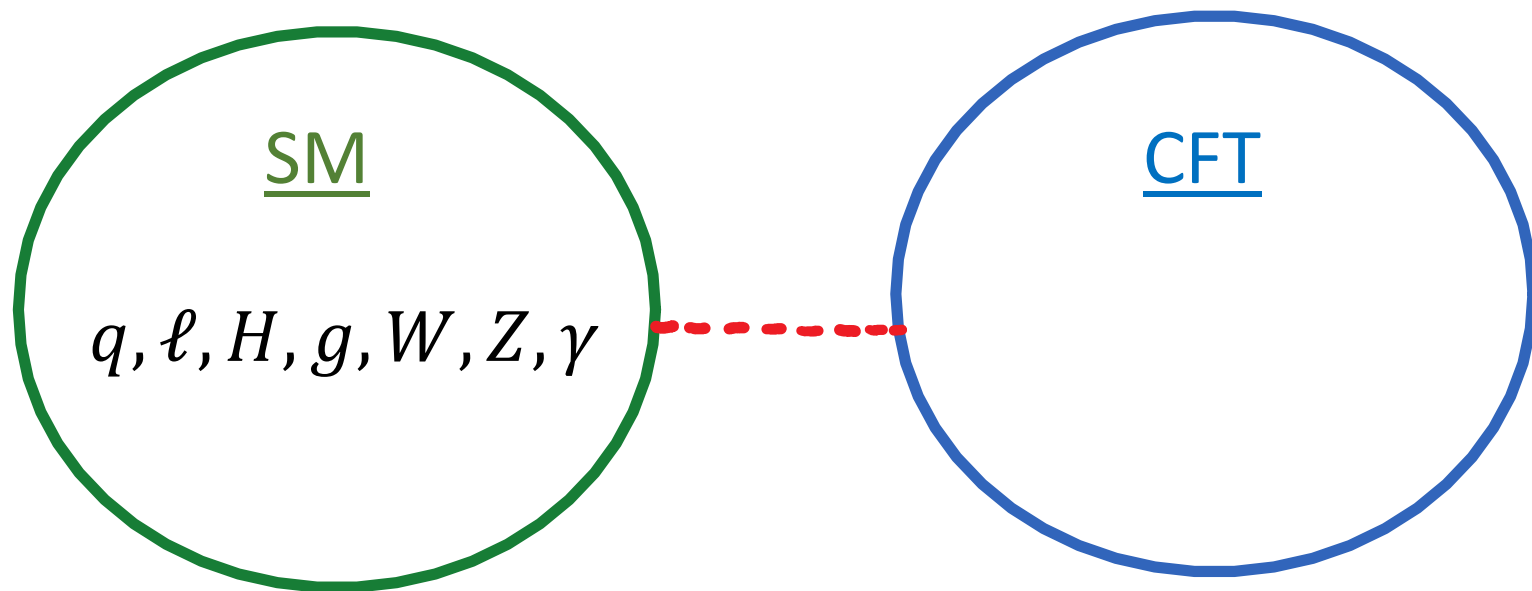
2. Robustness of COFI mechanism

3. COFI Phenomenology

- DM Self-Interaction, WDM, Star Cooling

4. Conclusion and Outlook

## Conformal Freeze-In (COFI)



$$\mathcal{L} \supset \lambda \mathcal{O}_{SM} \mathcal{O}_{CFT}$$

# Conformal Freeze-In (COFI)

(Q) Why Conformal Dark Sector?

(A1) CFT is mathematically well-defined QFT

UV CFT 

Not a particle

# Conformal Freeze-In (COFI)

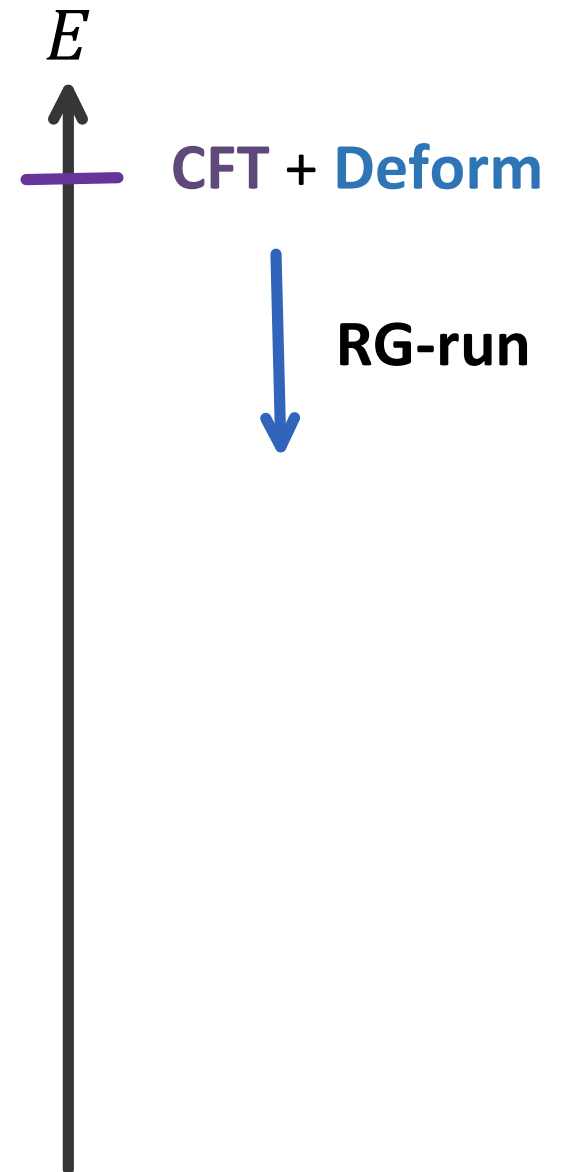
(Q) Why Conformal Dark Sector?

(A1) CFT is mathematically well-defined QFT

(A2) Every QFT is from CFT via deformation

UV CFT      QFT   particle  
Not a particle   

\*Deformation  
small breaking of  
conformal invariance



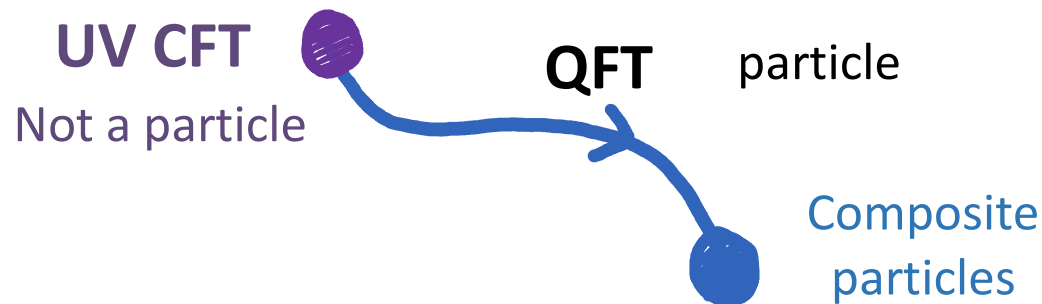


# Conformal Freeze-In (COFI)

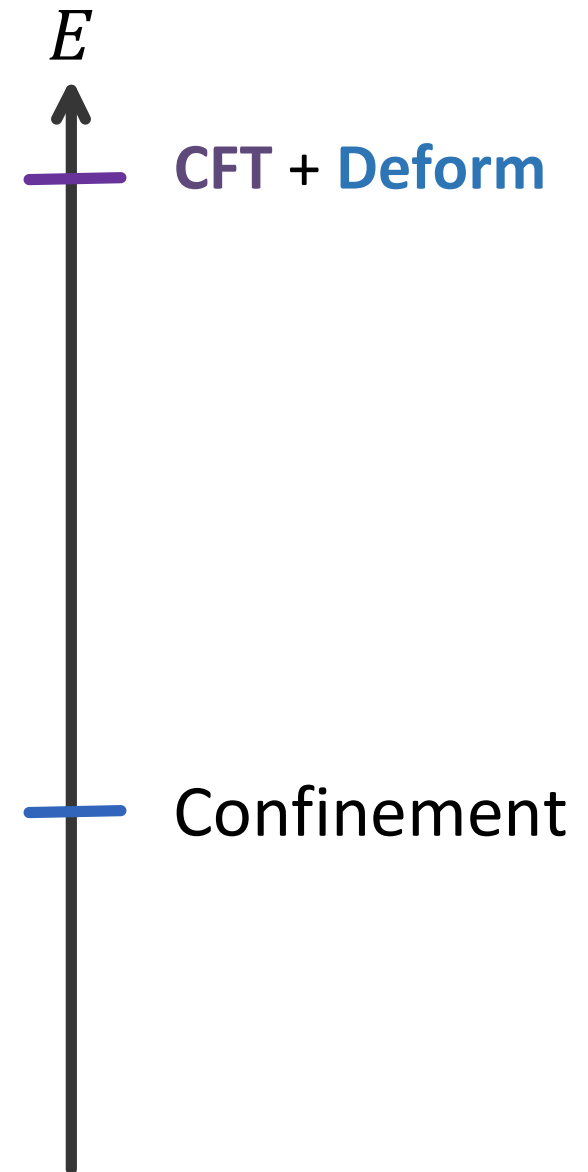
(Q) Why Conformal Dark Sector?

(A1) CFT is mathematically well-defined QFT

(A2) Every QFT is from CFT via deformation



\*Deformation  
small breaking of  
conformal invariance



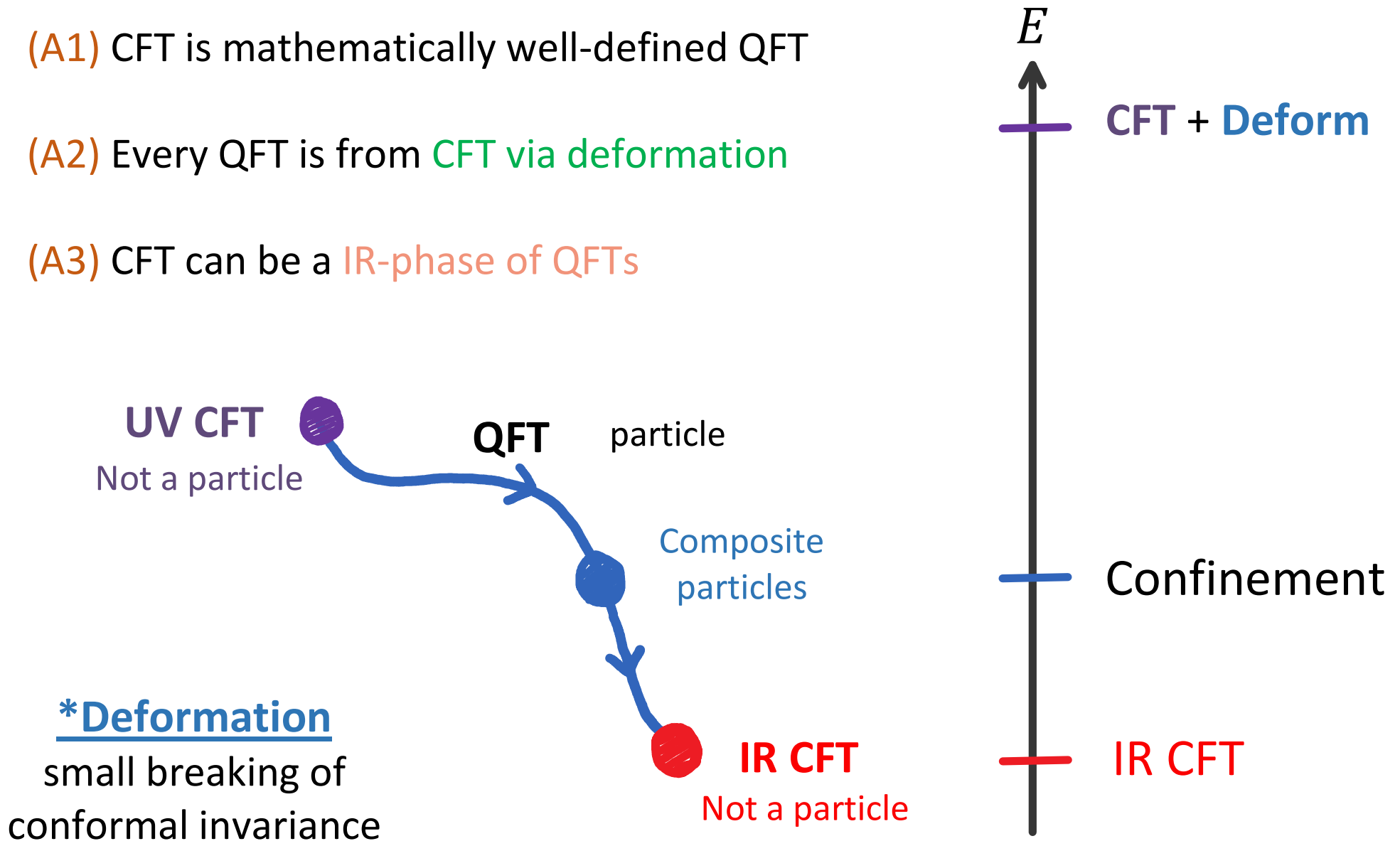
# Conformal Freeze-In (COFI)

(Q) Why Conformal Dark Sector?

(A1) CFT is mathematically well-defined QFT

(A2) Every QFT is from CFT via deformation

(A3) CFT can be a IR-phase of QFTs



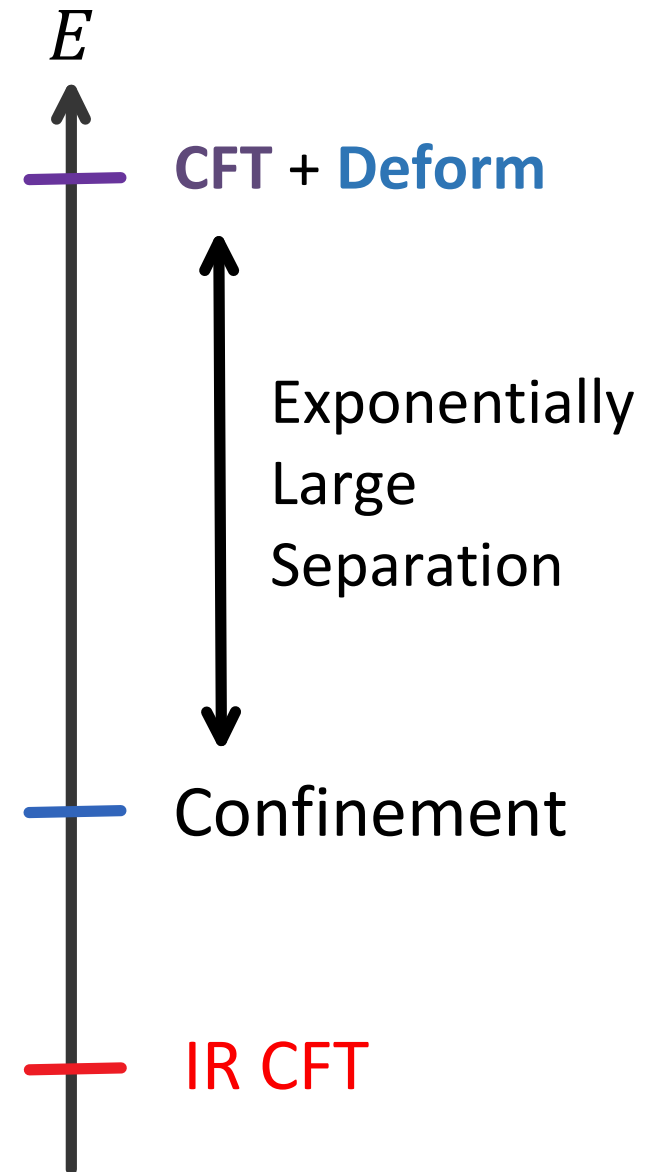
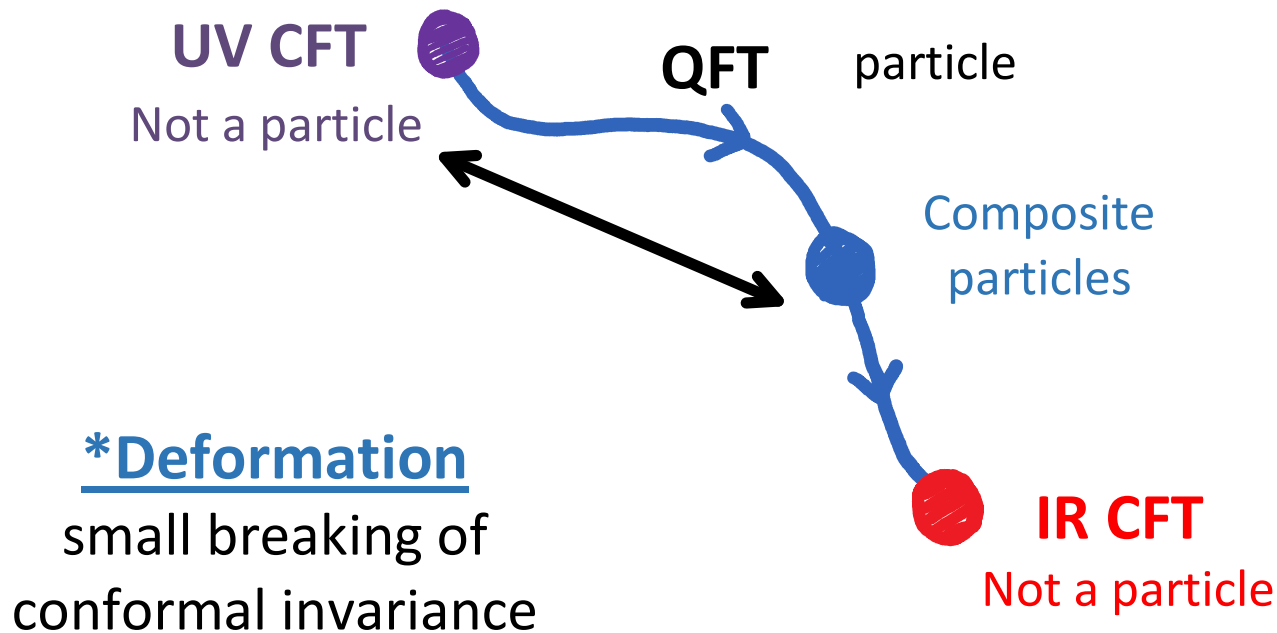
# Conformal Freeze-In (COFI)

(Q) Why Conformal Dark Sector?

(A1) CFT is mathematically well-defined QFT

(A2) Every QFT is from CFT via deformation

(A3) CFT can be a IR-phase of QFTs



# Conformal Freeze-In (COFI)

(Q) Why Conformal Dark Sector?

(A1) CFT is mathematically well-defined QFT

(A2) Every QFT is from CFT via deformation

(A3) CFT can be a IR-phase of QFTs

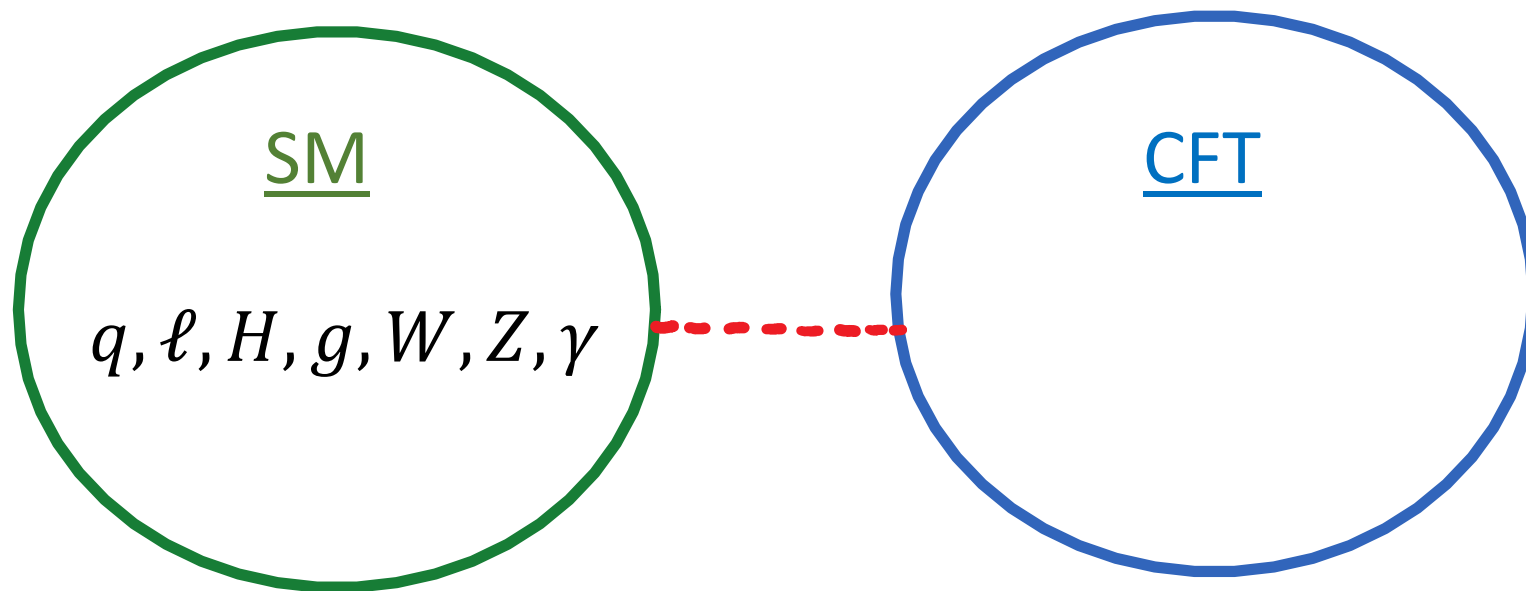
(A4) CFT features some nice universal properties thanks to large symmetry

$$\langle O(x)O(0) \rangle = \frac{c}{|x|^{2d}}$$

(A5) AdS/CFT-correspondence:  $CFT_D = AdS_{D+1}$

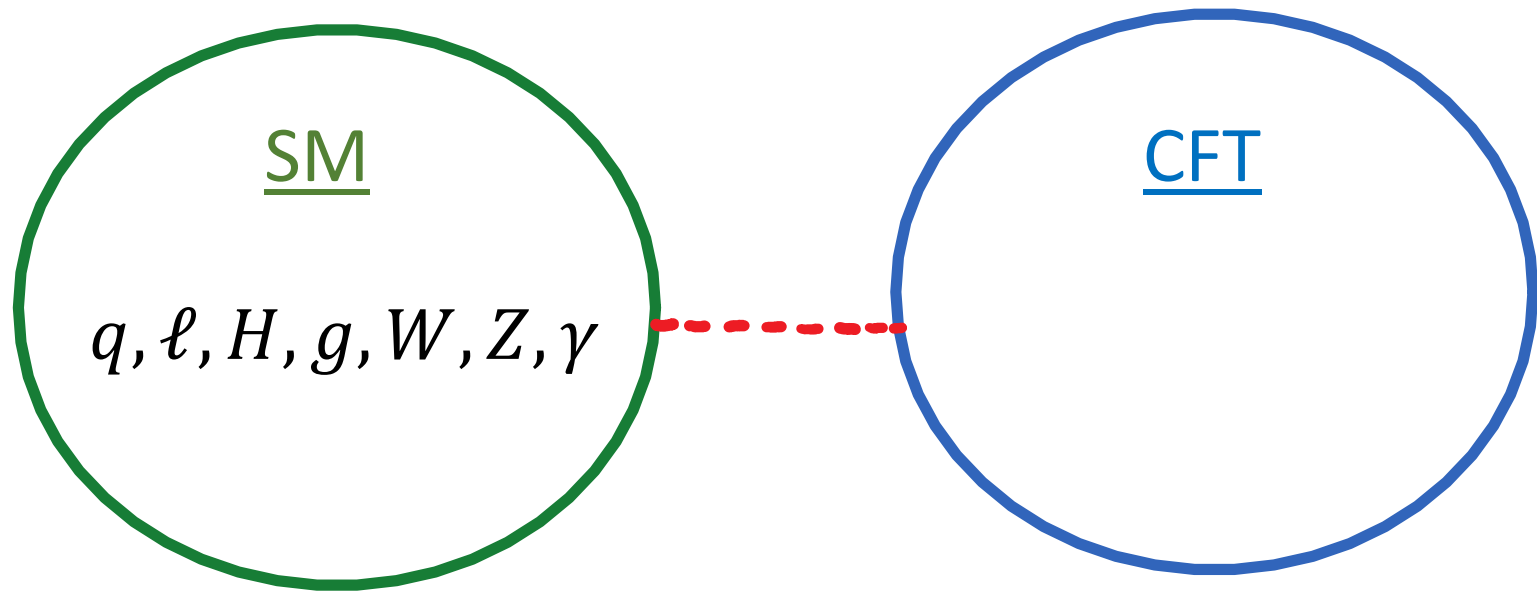
special class of CFTs admits a dual description in terms of gravitational theory in one higher dimension

## Conformal Freeze-In (COFI)



$$\mathcal{L} \supset \lambda \mathcal{O}_{SM} \mathcal{O}_{CFT}$$

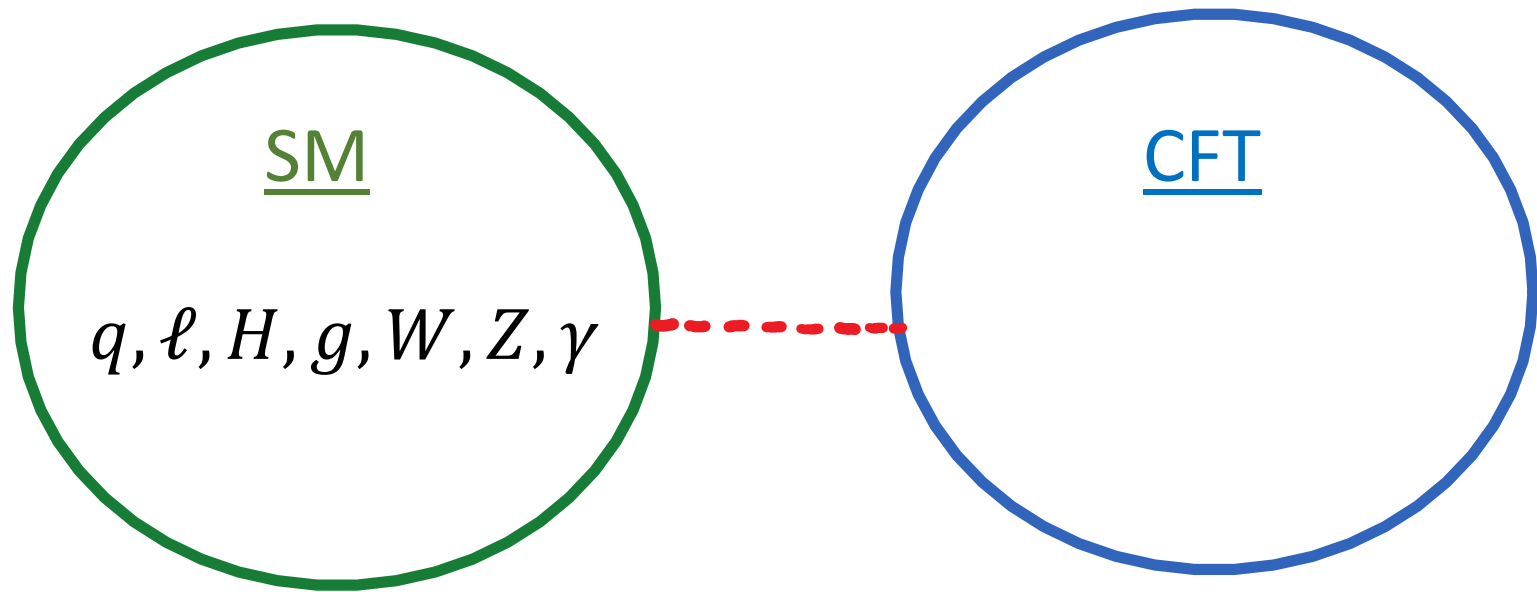
## Conformal Freeze-In (COFI)



$$\mathcal{L} \supset \lambda \mathcal{O}_{SM} \mathcal{O}_{CFT}$$

(Q) Under what conditions can this "conformal dark sector" turn into a "dark matter sector" ?

## Conformal Freeze-In (COFI)



$$\mathcal{L} \supset \lambda \mathcal{O}_{SM} \mathcal{O}_{CFT}$$

(Q) Under what conditions can this "conformal dark sector" turn into a "dark matter sector" ?

(A) Most likely\* answer: Conformal Freeze-In (COFI)

# Conformal Freeze-In (COFI)

We begin with

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^{D-4}} \mathcal{O}_{SM} \mathcal{O}_{CFT} \ , \quad D = d_{SM} + d_{CFT}$$



# Conformal Freeze-In (COFI)

We begin with

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^{D-4}} \mathcal{O}_{SM} \mathcal{O}_{CFT} , \quad D = d_{SM} + d_{CFT}$$

This simple interaction does MANY things:

- (i) It **breaks CFT**
- (ii) This breaking eventually leads to **generation of mass scale**  
CFT (radiation)  $\rightarrow$  Dark Matter  
 $m_g \ll m_{SM} \quad \rightarrow$  Light DM (keV - MeV)
- (iii) **Production of dark sector** via conformal freeze-in
- (vi) **Observations**

Consider  $\mathcal{O}_{SM} = H^\dagger H$

At  $v < E < \Lambda$

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^{d-2}} H^\dagger H \mathcal{O}$$

- Assume: this is the only CFT breaking term  
(in the limit  $\lambda \rightarrow 0$  we have a decoupled exact CFT + SM)

Consider  $\mathcal{O}_{SM} = H^\dagger H$

At  $v < E < \Lambda$

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^{d-2}} H^\dagger H \mathcal{O}$$

- Assume: this is the only CFT breaking term  
(in the limit  $\lambda \rightarrow 0$  we have a decoupled exact CFT + SM)

At  $E < v$  we get

$$\mathcal{L} \sim \frac{\lambda v^2}{\Lambda^{d-2}} \mathcal{O} \sim c \mathcal{O}$$

- A scalar deformation (to CFT) is induced by the EWPT.
- After this, CFT starts RG-running

Consider  $\mathcal{O}_{SM} = H^\dagger H$

At  $E < v$  we get

$$\mathcal{L} \sim \frac{\lambda v^2}{\Lambda^{d-2}} \mathcal{O}$$

- For  $d < 4$ , the effect of deformation grows in the IR
- Eventually, it becomes  $\mathcal{O}(1)$  effect at

$$m_g \sim \left( \frac{\lambda v^2}{\Lambda^{d-2}} \right)^{1/(4-d)}$$

Consider  $\mathcal{O}_{SM} = H^\dagger H$

At  $E < m_g$

- CFT goes through a **gap-creating** phase transition (PT)
- Possible IR phases ?

(i) **Confinement**

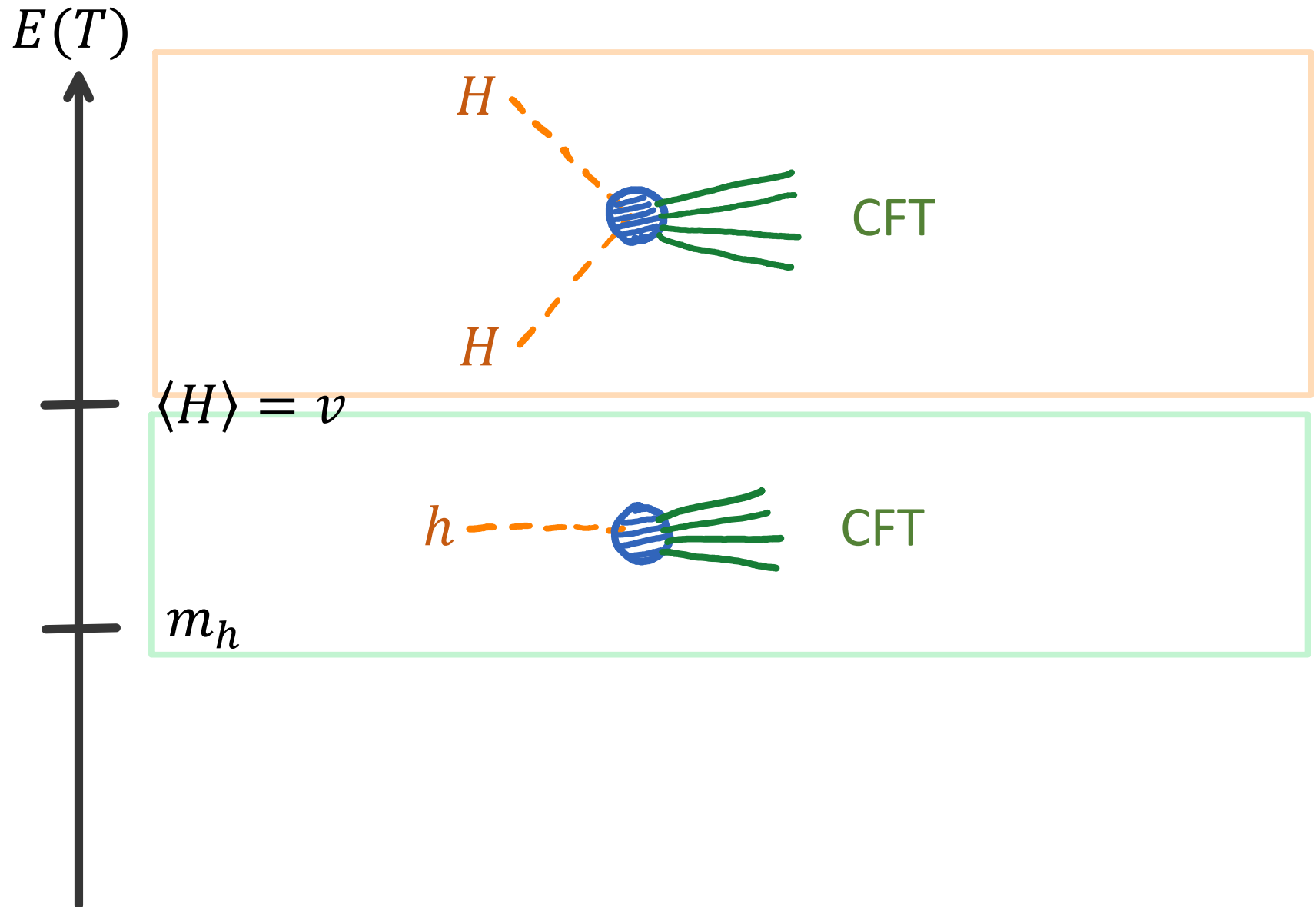
- most probable/standard
- spectrum : discretum of composite hadrons
- DM = light stable states, e.g. PNGB

(ii) **Gapped Continuum** (with C.Csaki, G.Kurup, S. Lee, M. Perelstein, W.Xue)

- more exotic possibility but many interesting features!
- UV-completion with non-thermal version of continuum DM ? (with G.Kurup, S.Lee, Y.Lee)

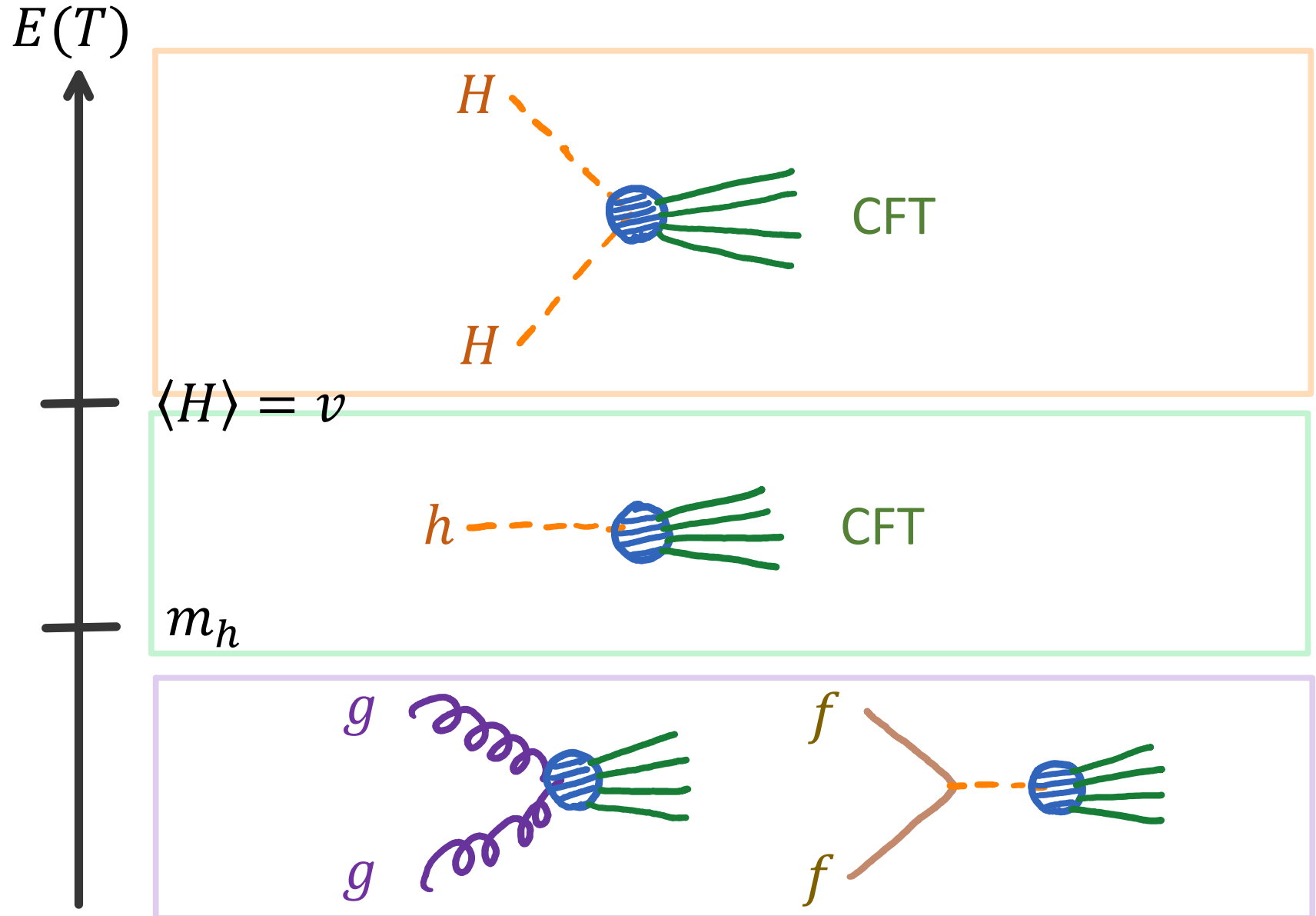
# COFI: Freeze-in Production

$$\mathcal{L} \supset \lambda H^\dagger H \mathcal{O}_{CFT}$$



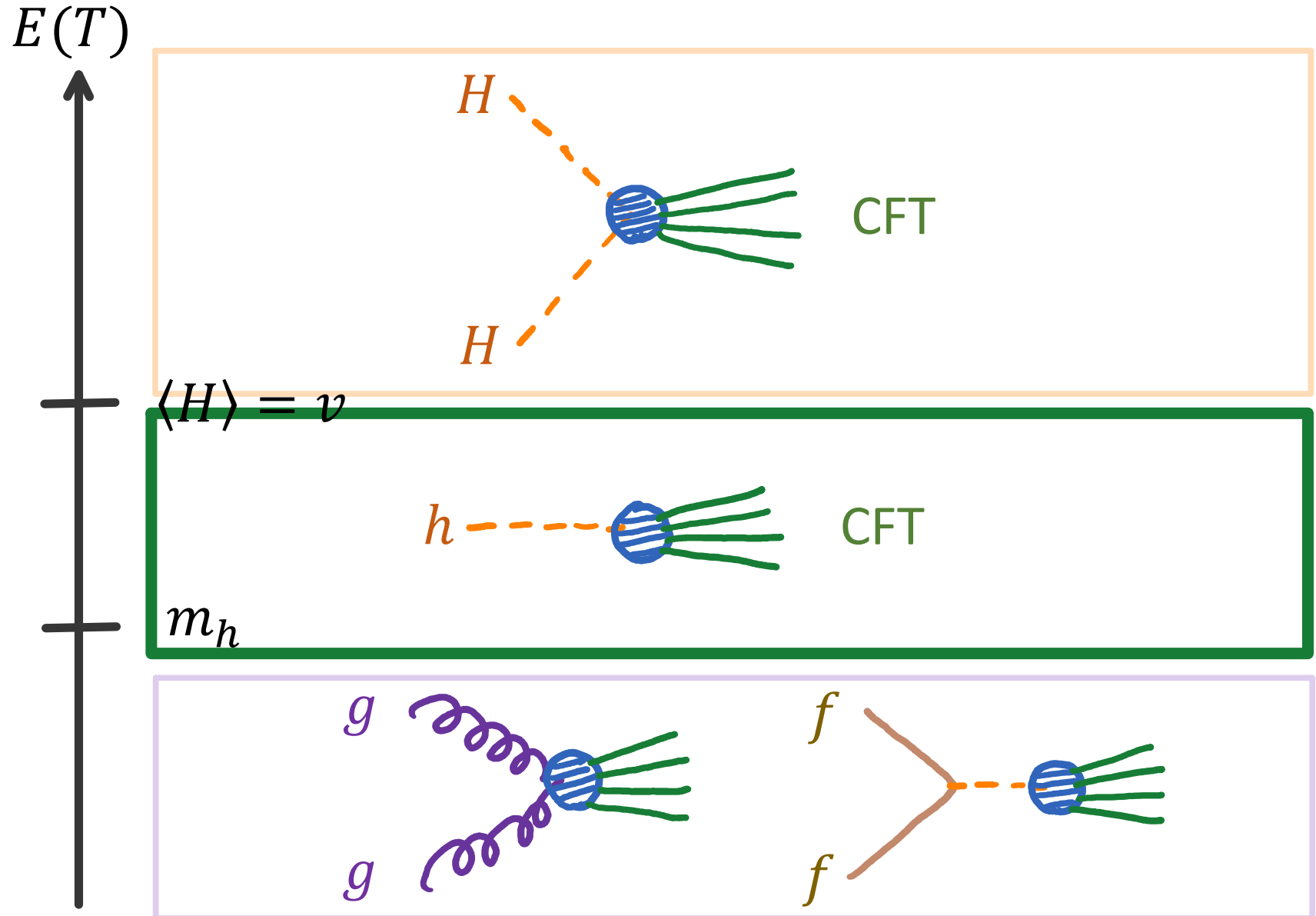
# COFI: Freeze-in Production

$$\mathcal{L} \supset \lambda H^\dagger H \mathcal{O}_{CFT}$$



# COFI: Freeze-in Production

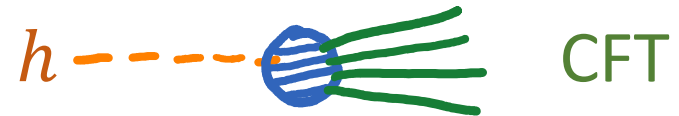
$$\mathcal{L} \supset \lambda H^\dagger H \mathcal{O}_{CFT}$$





# COFI: Computation

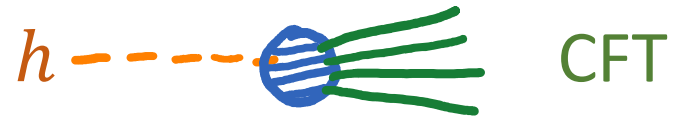
$$\mathcal{L} \supset \lambda H^\dagger H \mathcal{O}_{CFT}$$



- ★ **Conformal Freeze-In** completely **calculable theory** of **DM**
- ★ **Conformal Freeze-In** makes **precise predictions!**

# COFI: Computation

$$\mathcal{L} \supset \lambda H^\dagger H \mathcal{O}_{CFT}$$

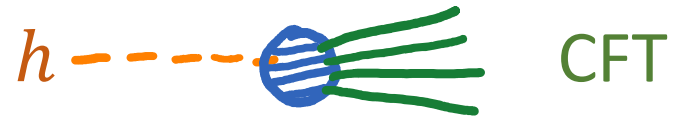


$$\frac{d\rho_{dark}}{dt} + 4H\rho_{dark} = \Gamma_E (SM \rightarrow CFT) \quad : \text{ Boltzmann Equation}$$

$$(\nabla_\mu T_\nu^\mu = \Gamma_E, \quad T_\mu^\mu = 0)$$

# COFI: Computation

$$\mathcal{L} \supset \lambda H^\dagger H \mathcal{O}_{CFT}$$

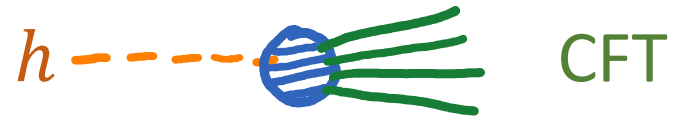


$$\frac{d\rho_{dark}}{dt} + 4H\rho_{dark} = \Gamma_E (SM \rightarrow CFT) \quad : \text{ Boltzmann Equation}$$

$$\Gamma_E (SM \rightarrow CFT) = \int \int d\Pi_h \frac{d^4 P_{CFT}}{(2\pi)^4} \langle \mathcal{O}^\dagger \mathcal{O} \rangle(P_{CFT}) f_h (2\pi)^4 \delta^4(p_h - P_{CFT}) |\widehat{M}|^2$$

# COFI: Computation

$$\mathcal{L} \supset \lambda H^\dagger H \mathcal{O}_{CFT}$$



$$\frac{d\rho_{dark}}{dt} + 4H\rho_{dark} = \Gamma_E (SM \rightarrow CFT) \quad : \text{ Boltzmann Equation}$$

$$\Gamma_E (SM \rightarrow CFT) = \int \int d\Pi_h \frac{d^4 P_{CFT}}{(2\pi)^4} \langle \mathcal{O}^\dagger \mathcal{O} \rangle (\mathbf{P}_{CFT}) f_h (2\pi)^4 \delta^4(p_h - P_{CFT}) |\widehat{M}|^2$$

$$\rho_{dark} (T) = \frac{\sqrt{5} M_{pl} f_d \lambda^2}{\pi^{3/2} \sqrt{g_*} v} m_h^{2d-4} T^4 \left( \frac{v^3}{T^3} - 1 \right)$$

$$f_d = \frac{\sqrt{\pi}}{(2\pi)^{2d}} \frac{\Gamma\left(d + \frac{1}{2}\right)}{\Gamma(d-1)\Gamma(2d)}$$

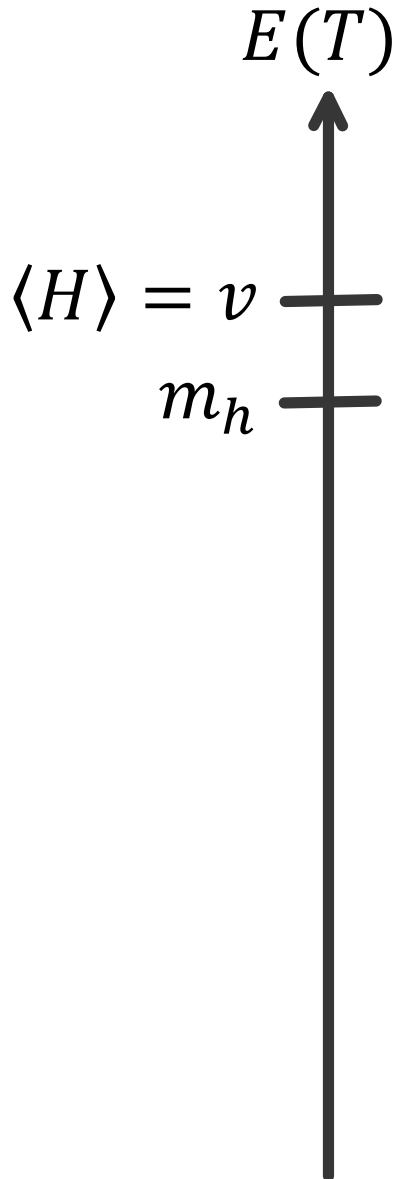
## COFI: Necessity of Mass Gap

$$\mathcal{L} \supset \lambda H^\dagger H \mathcal{O}_{CFT}$$



- ★ For **Dark Matter**,  $\rho \propto \frac{1}{a^3} = T^3$  vs  $\rho_{CFT} \propto \frac{1}{a^4} = T^4$
- ★ Conformal Dark Sector should be **gapped**, somehow!
- ★ As I showed, this occurs in **COFI** automatically and elegantly!

# COFI: Dynamical Gap Generation

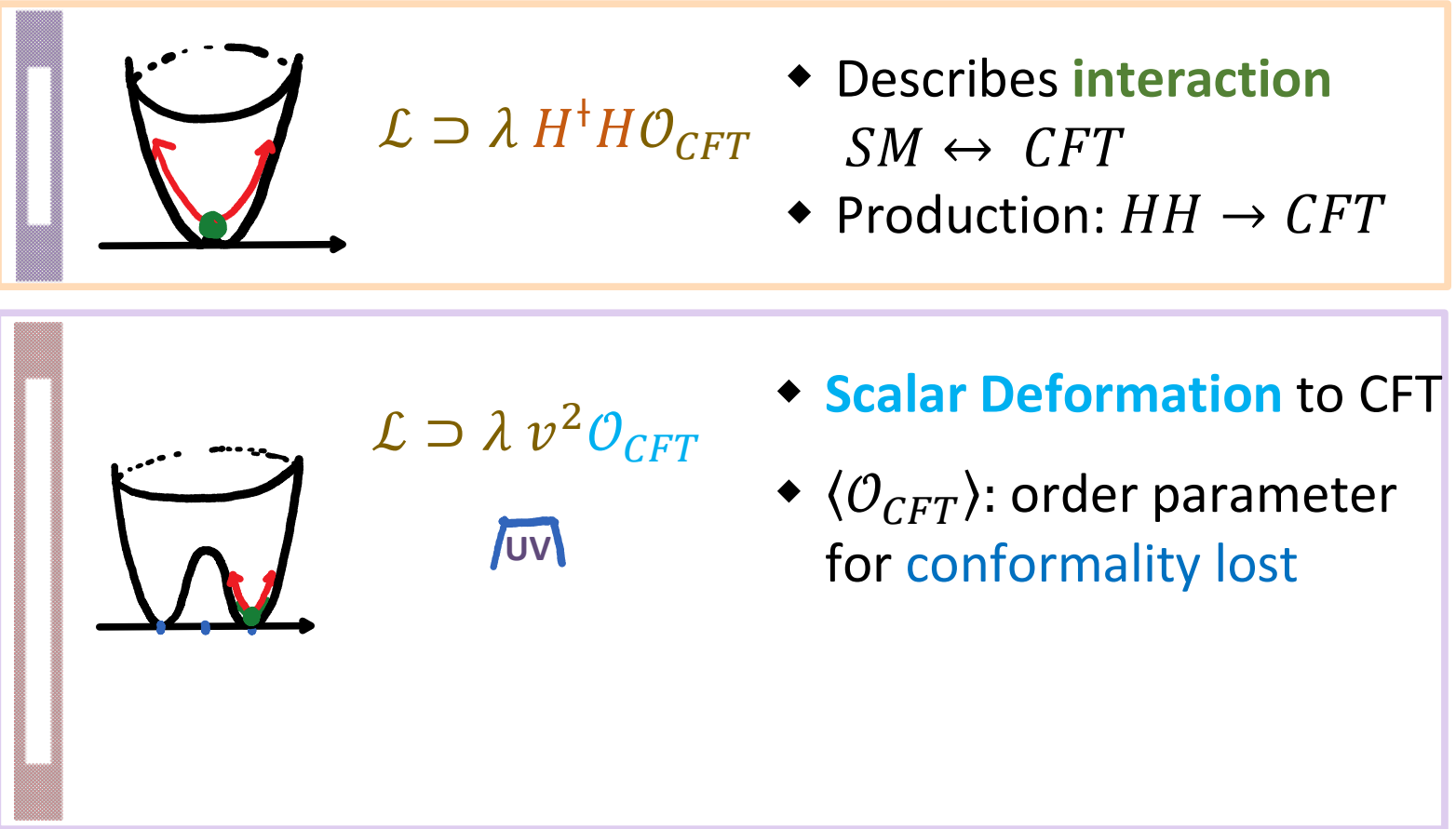
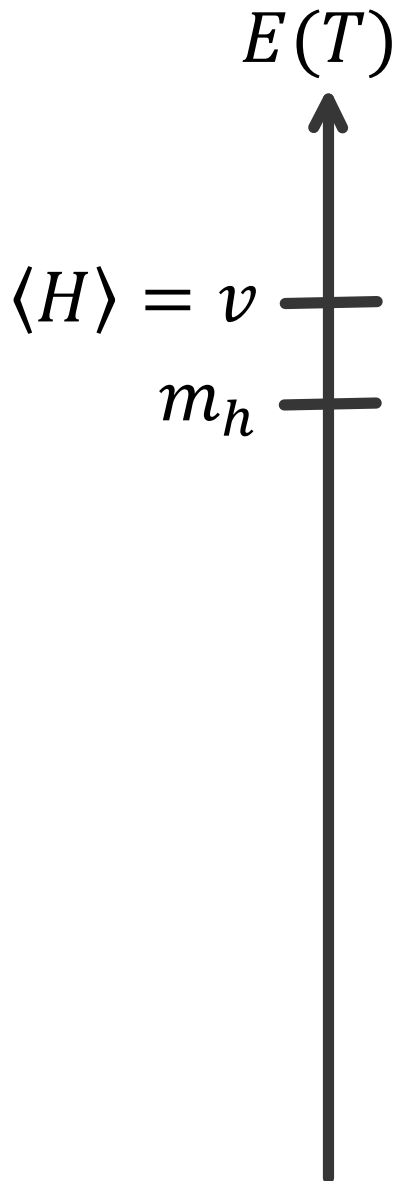


A diagram of a potential well. A green dot marks the minimum of the well. Two red arrows point upwards from the minimum to the walls of the well. A dashed line represents the top of the well.

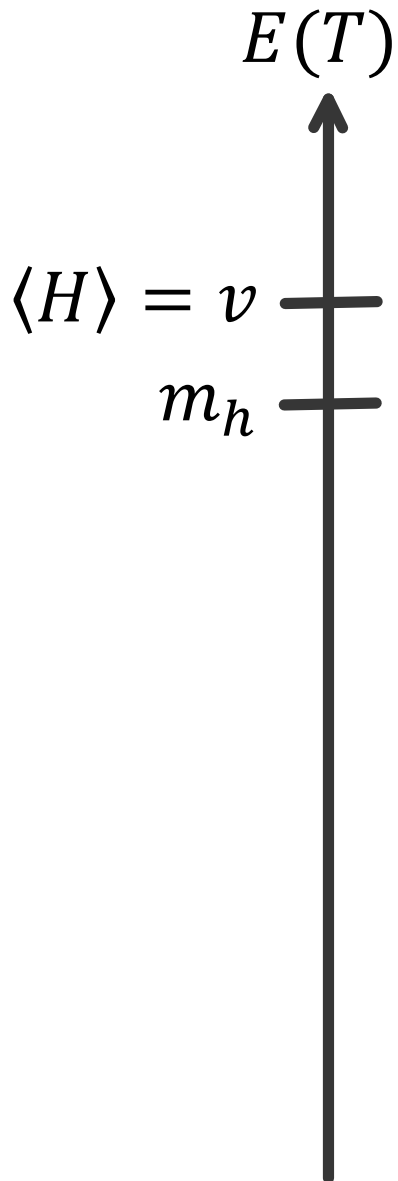
$$\mathcal{L} \supset \lambda H^\dagger H \mathcal{O}_{CFT}$$

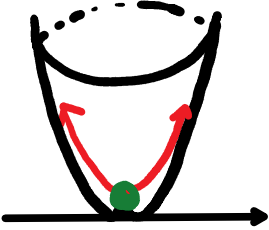
- ◆ Describes **interaction**  
 $SM \leftrightarrow CFT$
- ◆ Production:  $HH \rightarrow CFT$

# COFI: Dynamical Gap Generation



# COFI: Dynamical Gap Generation

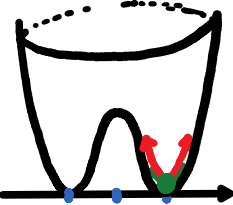




A potential well diagram showing a single minimum at the origin. A green dot marks the minimum, and two red arrows point upwards from it, indicating a transition or production process.

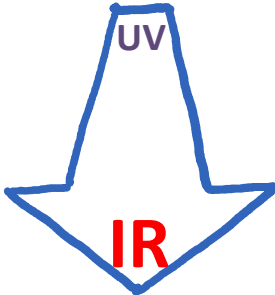
$\mathcal{L} \supset \lambda H^\dagger H \mathcal{O}_{CFT}$

- Describes **interaction**  $SM \leftrightarrow CFT$
- Production:  $HH \rightarrow CFT$



A potential well diagram showing a double-well structure. A green dot marks the minimum of the right well, and two red arrows point upwards from it. Three blue dots are marked on the x-axis at the minima of the wells.

$\mathcal{L} \supset \lambda v^2 \mathcal{O}_{CFT}$

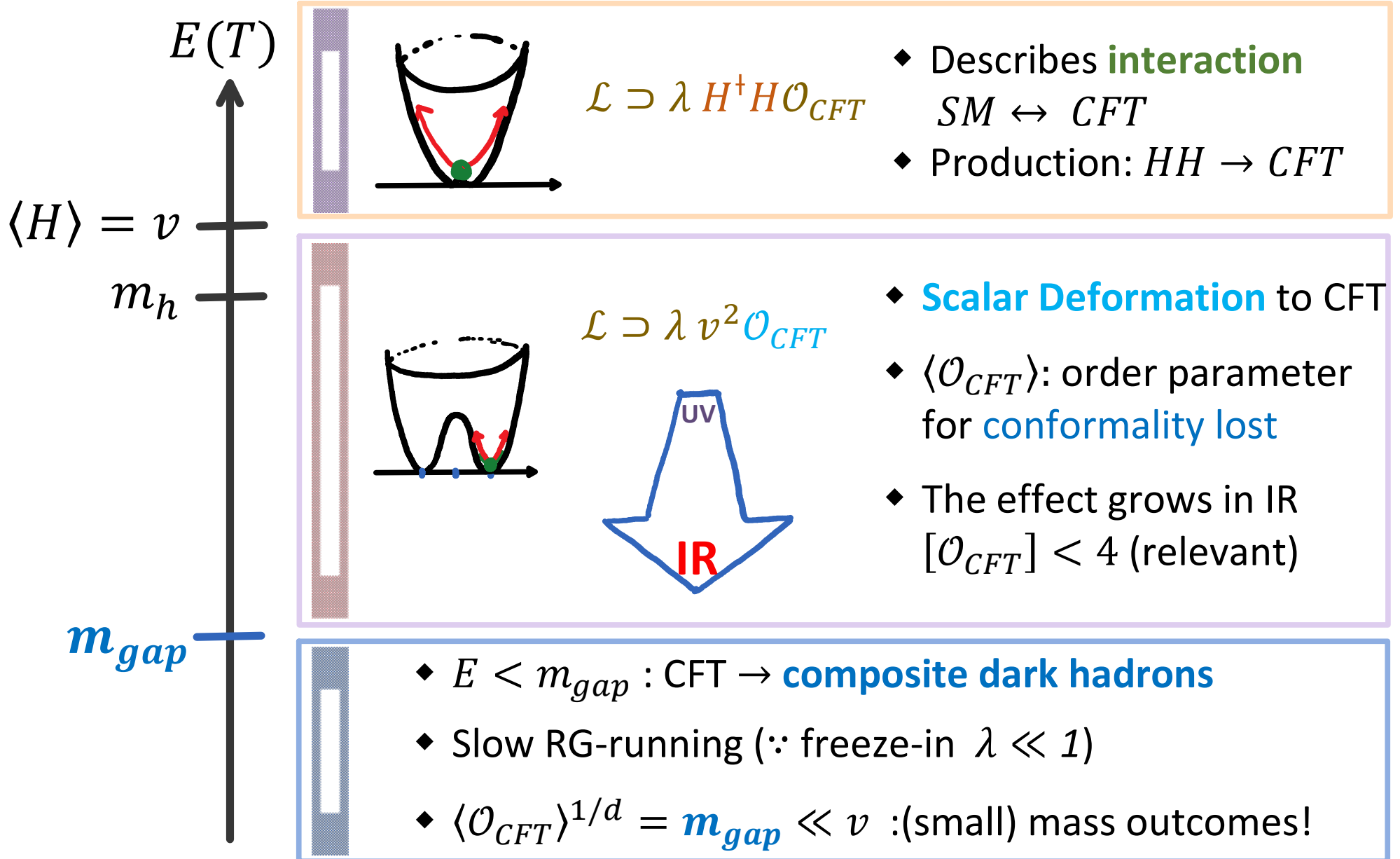


A blue arrow pointing downwards, labeled 'UV' at the top and 'IR' at the bottom, indicating the flow of the deformation from the ultraviolet to the infrared.

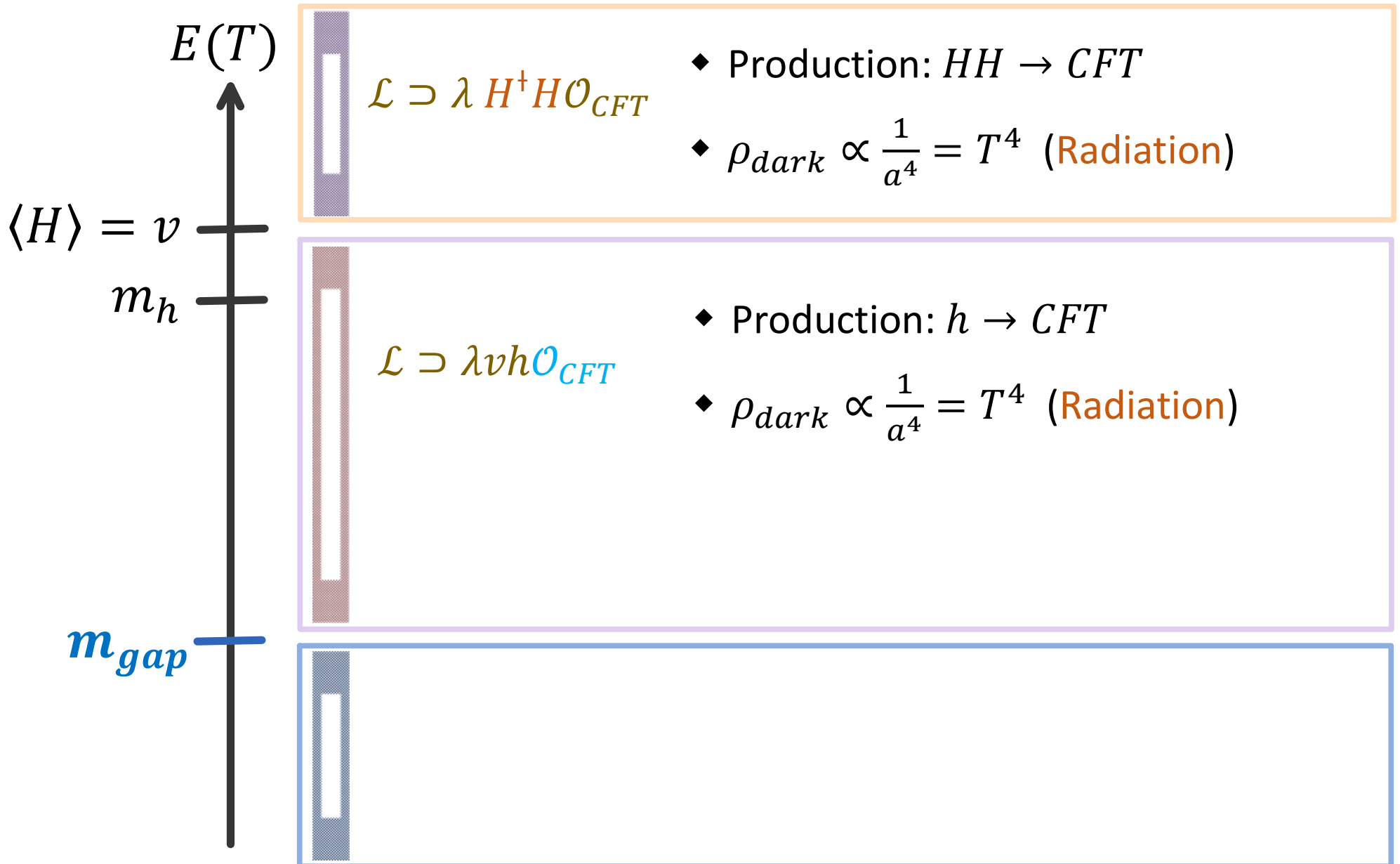
- Scalar Deformation** to CFT
- $\langle \mathcal{O}_{CFT} \rangle$ : order parameter for **conformality lost**
- The effect grows in IR  $[\mathcal{O}_{CFT}] < 4$  (relevant)



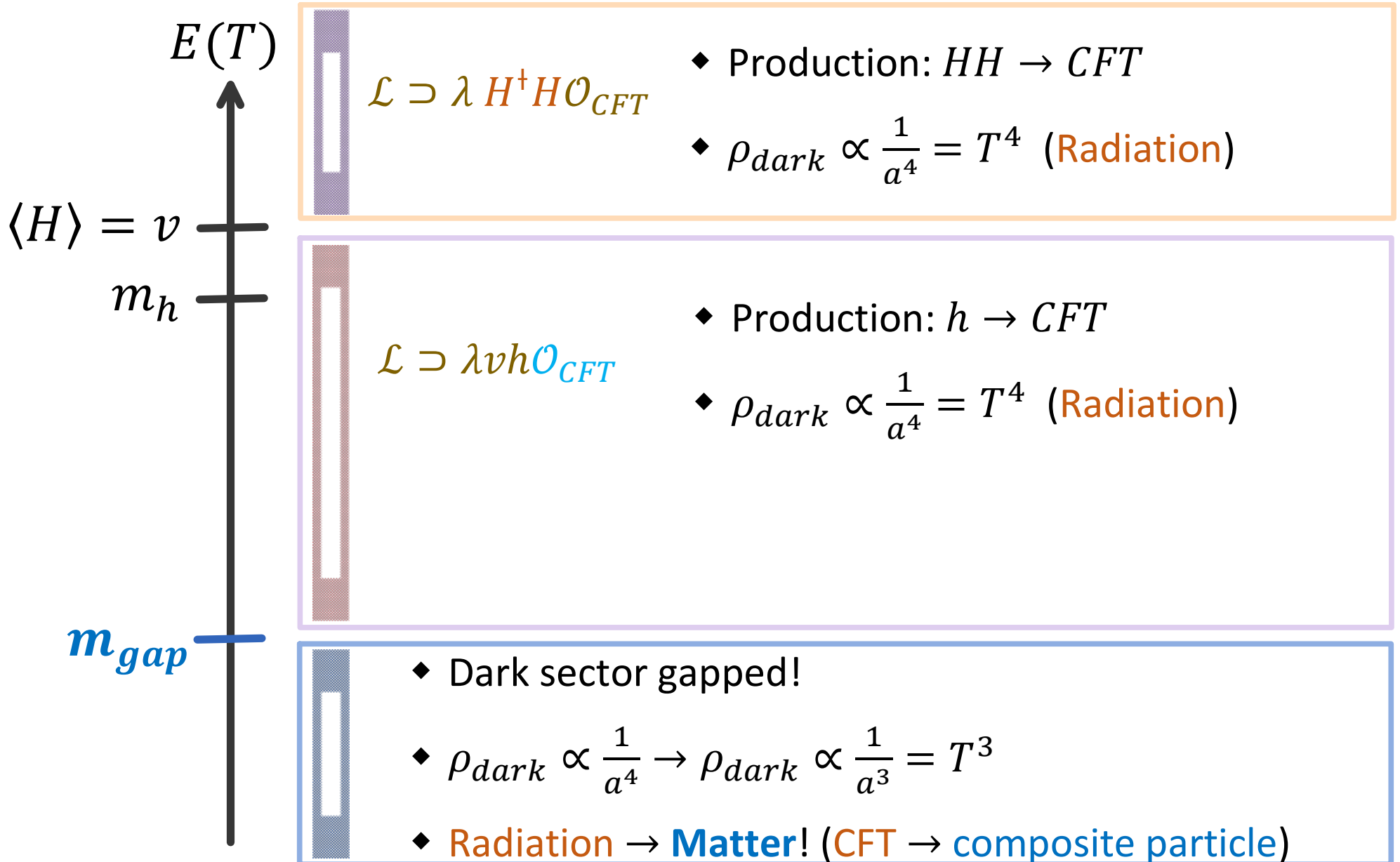
# COFI: Dynamical Gap Generation



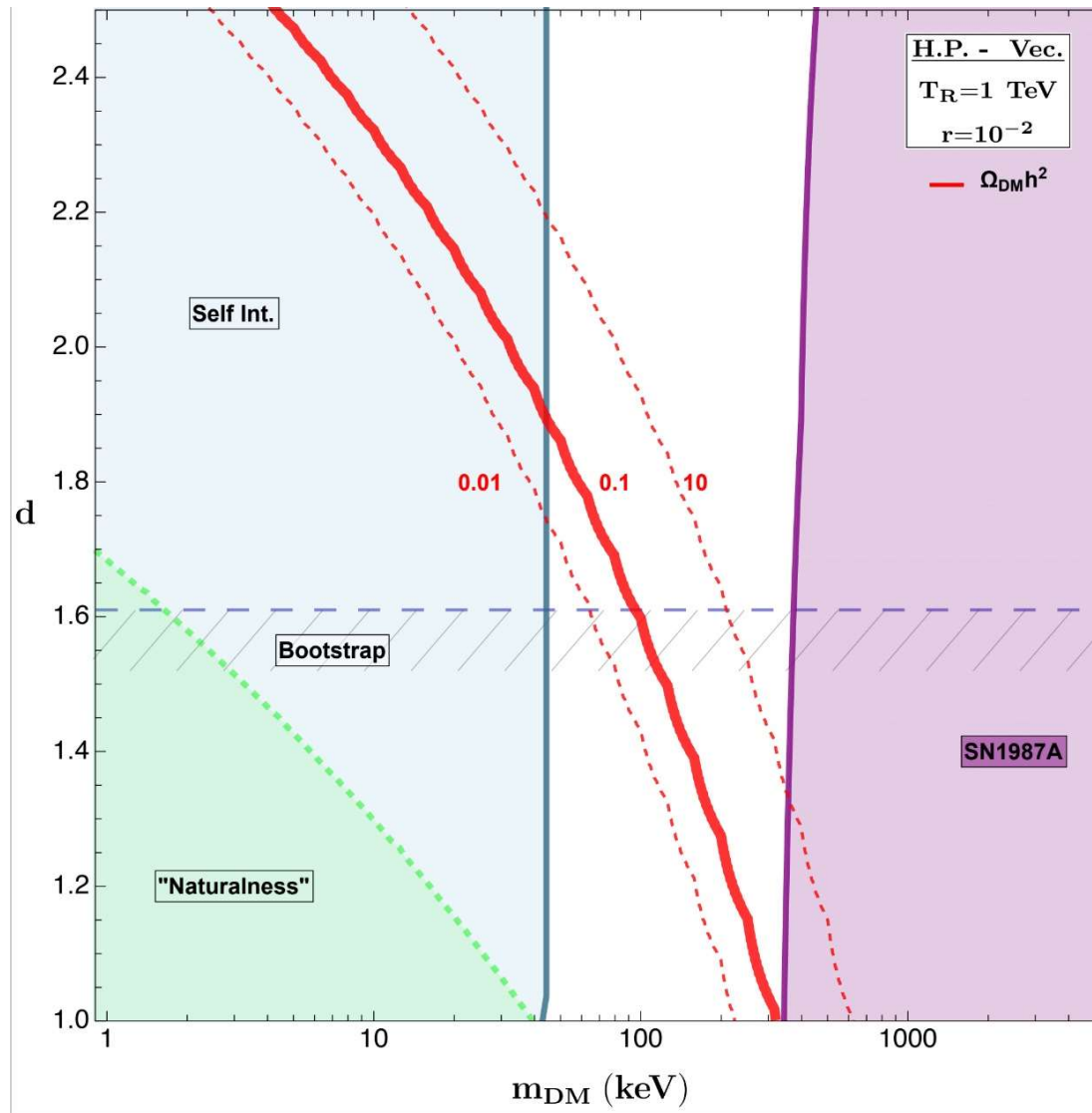
# COFI: DM from Conformal Dark Sector!



# COFI: DM from Conformal Dark Sector!

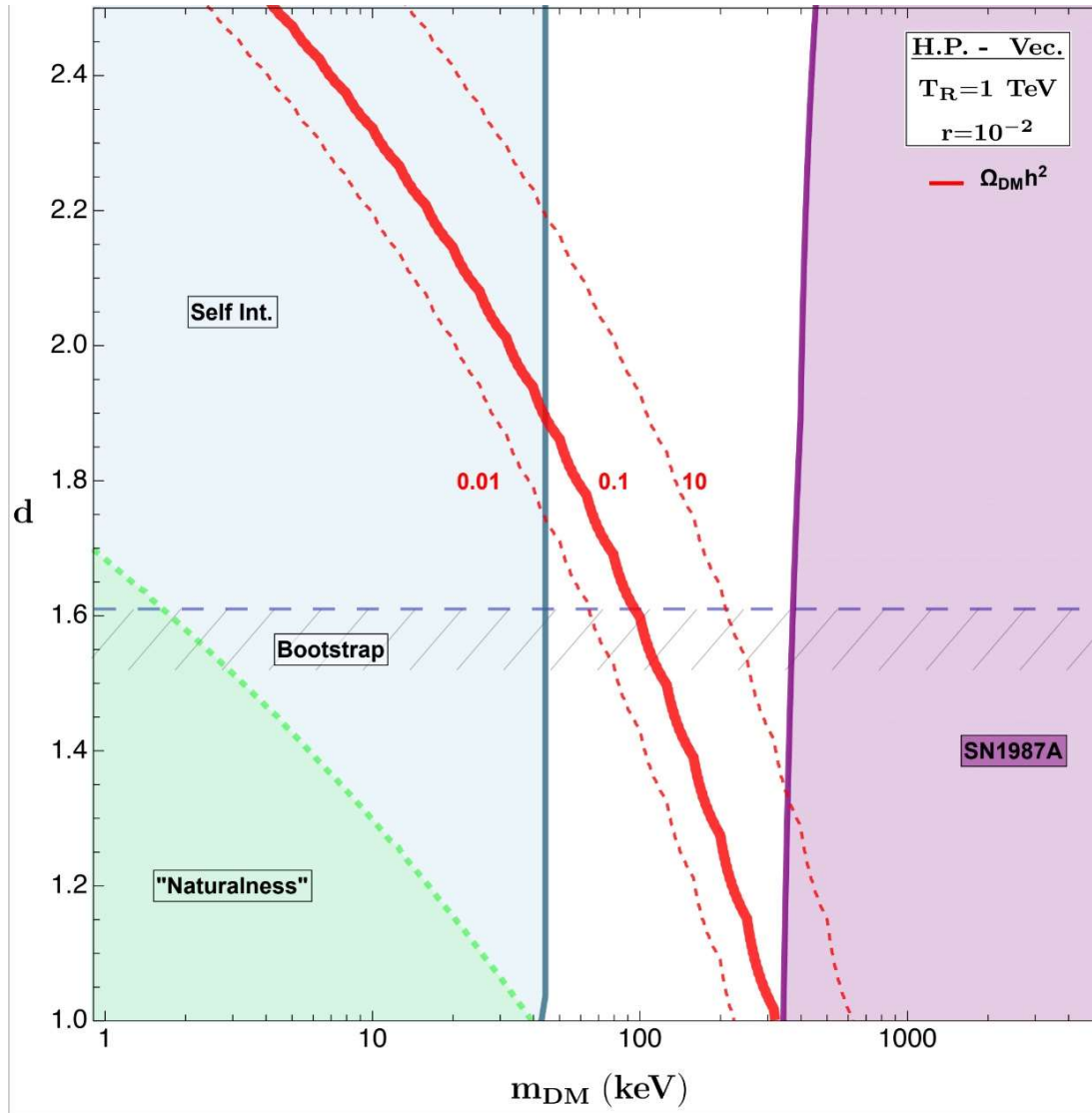


# COFI: DM from Conformal Dark Sector!



[Hong, Kurup, Perelstein '19,  
Hong, Kurup, Perelstein '22]

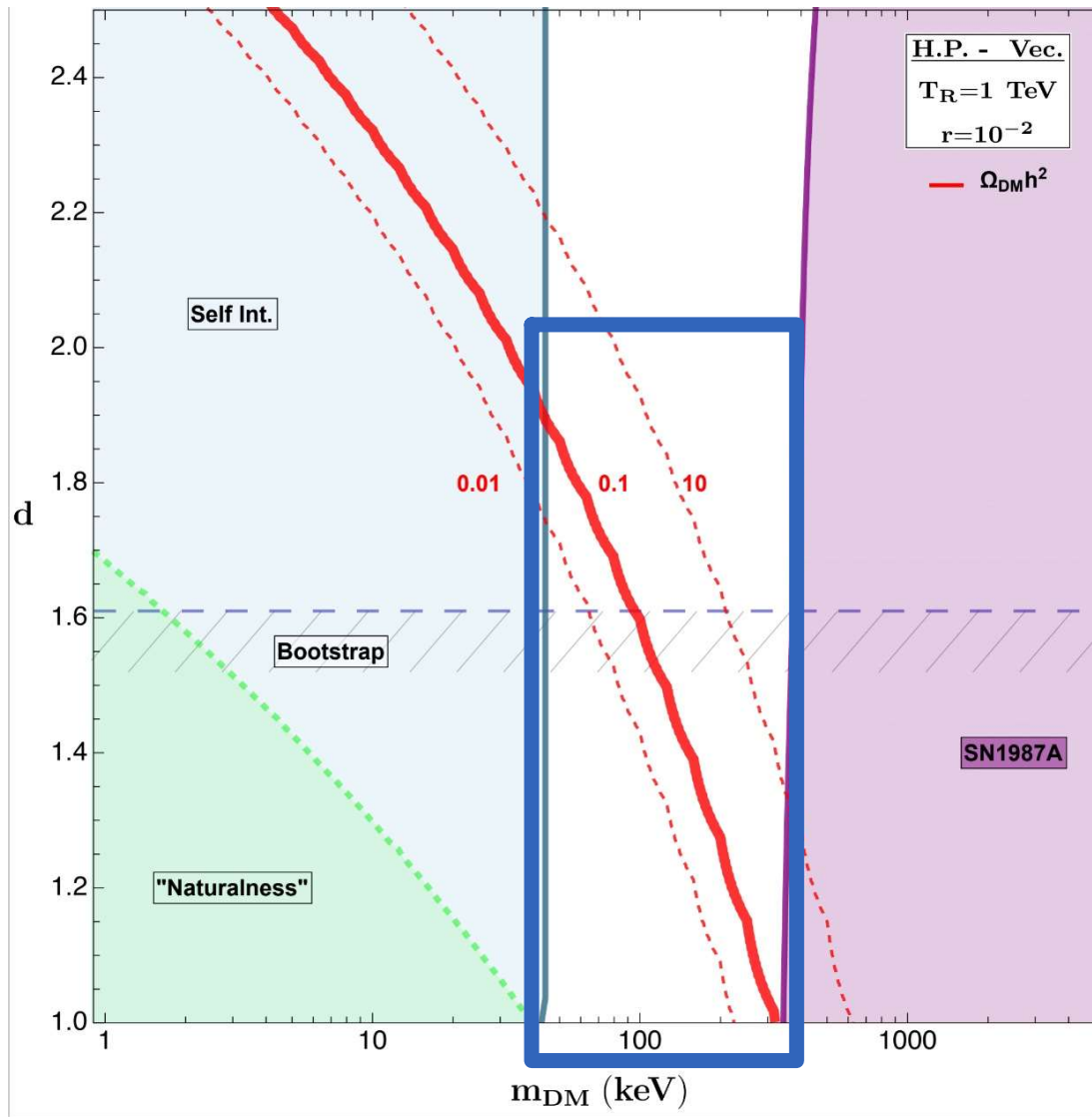
# COFI: DM from Conformal Dark Sector!



[Hong, Kurup, Perelstein '19,  
Hong, Kurup, Perelstein '22]

$$\mathcal{L} \supset \lambda \mathcal{O}_{SM} \mathcal{O}_{CFT}$$

# COFI: DM from Conformal Dark Sector!



[Hong, Kurup, Perelstein '19,  
Hong, Kurup, Perelstein '22]

$$\mathcal{L} \supset \lambda \mathcal{O}_{SM} \mathcal{O}_{CFT}$$

# Outline

1. Introduction to Conformal Freeze-In

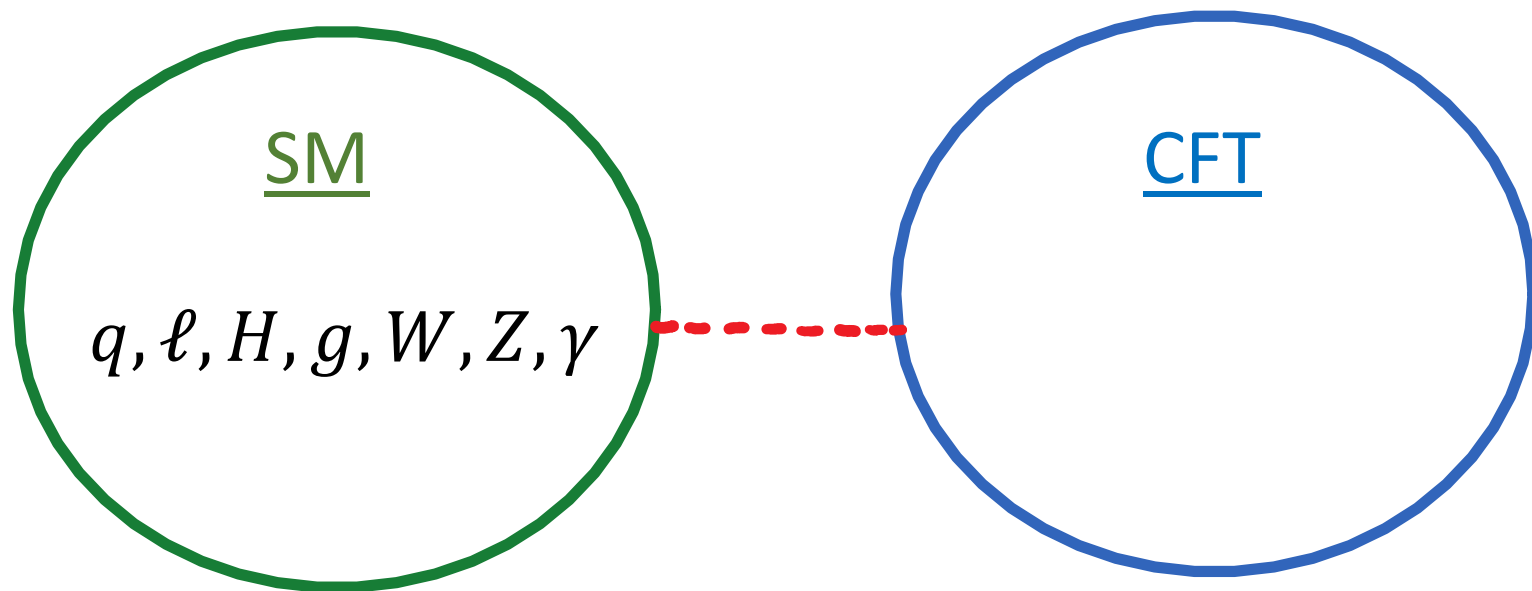
2. Robustness of **COFI mechanism**

3. COFI Phenomenology

- DM Self-Interaction, WDM, Star Cooling

4. Conclusion and Outlook

## Conformal Freeze-In (COFI)



$$\mathcal{L} \supset \lambda \mathcal{O}_{SM} \mathcal{O}_{CFT}$$



$\mathcal{O}_{SM}$  can be classified by ( $d_{SM} \leq 4$ )

I. scalar  $\mathcal{O}_{SM}$

(i)  $\langle \mathcal{O}_{SM} \rangle \neq 0$  :  $H^+ H$ ,  $H \bar{Q} q$ ,  $G_{\mu\nu} G^{\mu\nu}$

(ii)  $\langle \mathcal{O}_{SM} \rangle = 0$  :  $H \bar{L} \ell$ ,  $B_{\mu\nu} B^{\mu\nu}$ ,  $W_{\mu\nu} W^{\mu\nu}$

II. tensor  $\mathcal{O}_{SM}$  :  $B_{\mu\nu}$ ,  $HL$

$\mathcal{O}_{SM}$  can be classified by ( $d_{SM} \leq 4$ )

I. scalar  $\mathcal{O}_{SM}$

(i)  $\langle \mathcal{O}_{SM} \rangle \neq 0$  :  $H^+ H$ ,  $H \bar{Q} q$ ,  $G_{\mu\nu} G^{\mu\nu}$

(ii)  $\langle \mathcal{O}_{SM} \rangle = 0$  :  $H \bar{L} \ell$ ,  $B_{\mu\nu} B^{\mu\nu}$  "Operator Mixing Effects"

II. tensor  $\mathcal{O}_{SM}$  :  $B_{\mu\nu}$ ,  $HL$  "Operator Product Expansion"

Consider  $\mathcal{O}_{SM} = H \bar{L} \ell$

Recall this represents the case:  $\langle \mathcal{O}_{SM} \rangle = 0$

At  $v < E < \Lambda$  we have

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^d} H \bar{L} \ell \mathcal{O}$$

(Q) How does Conformality-lost ( $m_g$  generation) occur?

Consider  $\mathcal{O}_{SM} = H \bar{L} \ell$

Recall this represents the case:  $\langle \mathcal{O}_{SM} \rangle = 0$

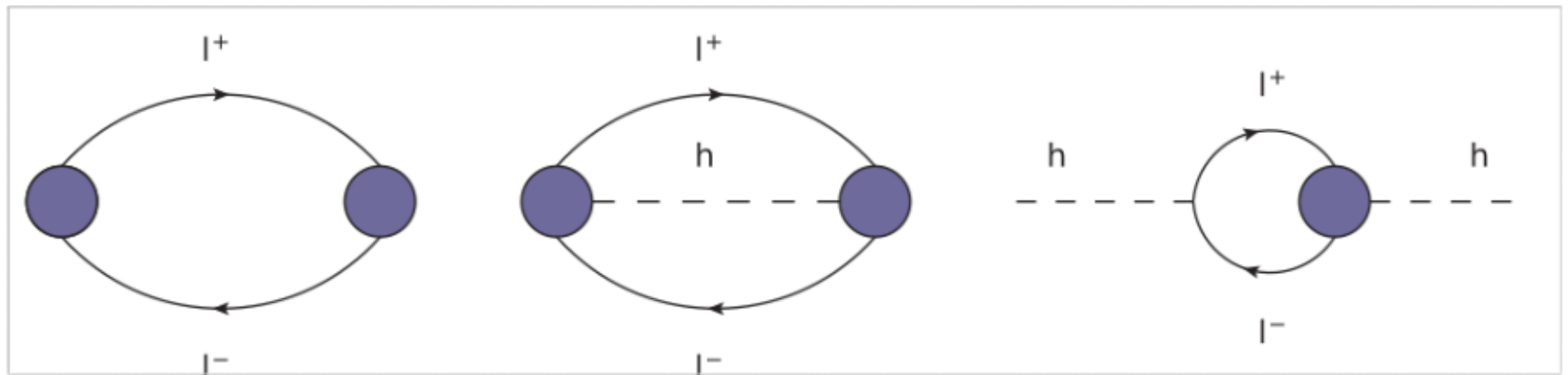
At  $v < E < \Lambda$  we have

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^d} H \bar{L} \ell \mathcal{O}$$

(Q) How does Conformality-lost ( $m_g$  generation) occur?

(A) "Operator Mixing Effect"

- Other operators are induced at tree or loop level



Consider  $\mathcal{O}_{SM} = H \bar{L} \ell$

Recall this represents the case:  $\langle \mathcal{O}_{SM} \rangle = 0$

At  $v < E < \Lambda$  we have

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^d} H \bar{L} \ell \mathcal{O}$$

"Operator Mixing Effect"

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^d} \frac{y_\ell \Lambda_{SM}^2}{16\pi^2} H^+ H \mathcal{O}$$

- This is just another kind of Higgs-portal  
(as far as **breaking of CFT** is concerned vs **production**)

Consider  $\mathcal{O}_{SM} = H \bar{L} \ell$

At  $E < v$  we have

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^d} \frac{y_\ell \Lambda_{SM}^2}{16\pi^2} H^+ H \mathcal{O} \rightarrow \frac{\lambda}{\Lambda^d} \frac{y_\ell \Lambda_{SM}^2}{16\pi^2} v^2 \mathcal{O}$$

- Again, EWPT induces a scalar deformation to CFT
- CFT starts RG-running

$$m_g \sim \left( \frac{\lambda}{\Lambda^d} \frac{y_\ell \Lambda_{SM}^2}{16\pi^2} v^2 \right)^{1/(4-d)}$$

$\mathcal{O}_{SM}$  can be classified by ( $d_{SM} \leq 4$ )

I. scalar  $\mathcal{O}_{SM}$

$$(i) \langle \mathcal{O}_{SM} \rangle \neq 0 : \quad H^+ H, \quad H \bar{Q} q, \quad G_{\mu\nu} G^{\mu\nu}$$

$$(ii) \langle \mathcal{O}_{SM} \rangle = 0 : \quad H \bar{L} \ell, \quad B_{\mu\nu} B^{\mu\nu}, \quad W_{\mu\nu} W^{\mu\nu}$$

II. tensor  $\mathcal{O}_{SM} : \quad B_{\mu\nu} , \quad HL$

$\mathcal{O}_{SM}$  can be classified by ( $d_{SM} \leq 4$ )

I. scalar  $\mathcal{O}_{SM}$

(i)  $\langle \mathcal{O}_{SM} \rangle \neq 0$  :  $H^+ H$ ,  $H \bar{Q} q$ ,  $G_{\mu\nu} G^{\mu\nu}$

(ii)  $\langle \mathcal{O}_{SM} \rangle = 0$  :  $H \bar{L} \ell$ ,  $B_{\mu\nu} B^{\mu\nu}$ ,  $W_{\mu\nu} W^{\mu\nu}$

II. tensor  $\mathcal{O}_{SM}$  :  $B_{\mu\nu}$ ,  $HL$

"tree-level"  $H^+ H \mathcal{O} \rightarrow v^2 \mathcal{O}$



$\mathcal{O}_{SM}$  can be classified by ( $d_{SM} \leq 4$ )

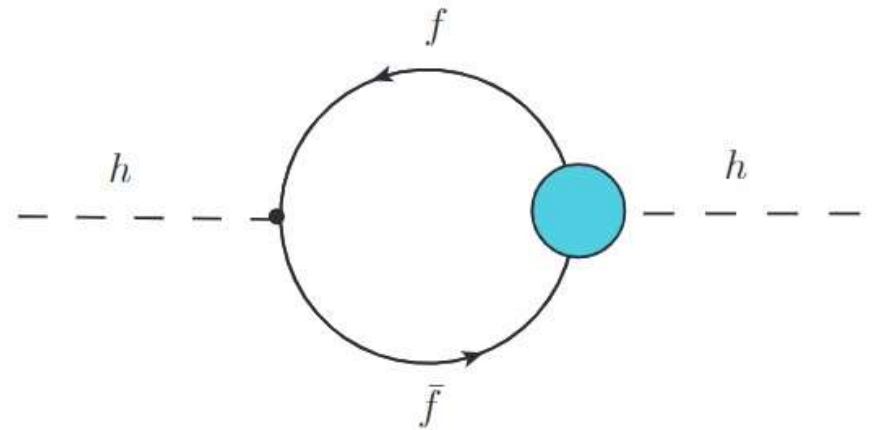
I. scalar  $\mathcal{O}_{SM}$

(i)  $\langle \mathcal{O}_{SM} \rangle \neq 0$  :  $H^+ H$ ,  $H \bar{Q} q$ ,  $G_{\mu\nu} G^{\mu\nu}$

(ii)  $\langle \mathcal{O}_{SM} \rangle = 0$  :  $H \bar{L} \ell$ ,  $B_{\mu\nu} B^{\mu\nu}$ ,  $W_{\mu\nu} W^{\mu\nu}$

II. tensor  $\mathcal{O}_{SM}$  :  $B_{\mu\nu}$ ,  $HL$

"radiative mixing"



$\mathcal{O}_{SM}$  can be classified by ( $d_{SM} \leq 4$ )

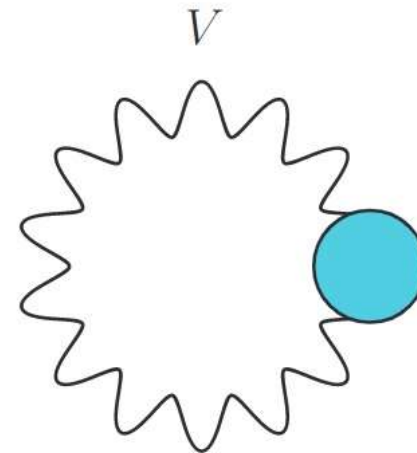
I. scalar  $\mathcal{O}_{SM}$

(i)  $\langle \mathcal{O}_{SM} \rangle \neq 0$  :  $H^+ H$ ,  $H \bar{Q} q$ ,  $G_{\mu\nu} G^{\mu\nu}$

(ii)  $\langle \mathcal{O}_{SM} \rangle = 0$  :  $H \bar{L} \ell$ ,  $B_{\mu\nu} B^{\mu\nu}$ ,  $W_{\mu\nu} W^{\mu\nu}$

II. tensor  $\mathcal{O}_{SM}$  :  $B_{\mu\nu}$ ,  $HL$

"radiative direct"



$\mathcal{O}_{SM}$  can be classified by ( $d_{SM} \leq 4$ )

I. scalar  $\mathcal{O}_{SM}$

(i)  $\langle \mathcal{O}_{SM} \rangle \neq 0$  :  $H^+ H$ ,  $H \bar{Q} q$ ,  $G_{\mu\nu} G^{\mu\nu}$

(ii)  $\langle \mathcal{O}_{SM} \rangle = 0$  :  $H \bar{L} \ell$ ,  $B_{\mu\nu} B^{\mu\nu}$ ,  $W_{\mu\nu} W^{\mu\nu}$

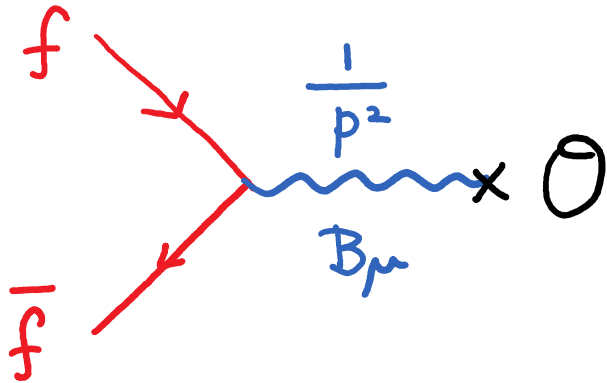
II. tensor  $\mathcal{O}_{SM}$  :  $B_{\mu\nu}$  ,  $HL$

$$\mathcal{O}_{SM} = B_{\mu\nu} \text{ (tensor operator)}$$

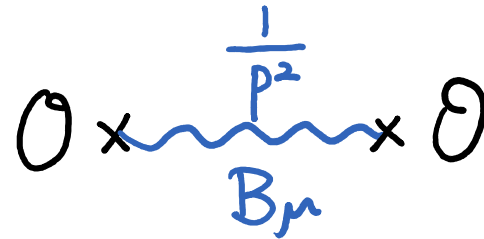
At  $v < E < \Lambda$  we have

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^{d-2}} B_{\mu\nu} \mathcal{O}^{\mu\nu}$$

- At this scale, there is **no local operator** induced by OME



$$\mathcal{L} \propto \bar{f} \sigma_{\mu\nu} f \mathcal{O}^{\mu\nu}$$



$$\mathcal{L} \propto \mathcal{O}_{\mu\nu} \mathcal{O}^{\mu\nu}$$

$$\mathcal{O}_{SM} = B_{\mu\nu} \text{ (tensor operator)}$$

At  $E < v$  we have

$$B_\mu = \cos \theta_w \gamma_\mu + \sin \theta_w Z_\mu$$

- Now, exchange of **massive Z boson** can generate a local operator by **OPE** for  $E \ll M_Z$  :

$$\mathcal{L} \sim \left( \frac{\lambda}{\Lambda^{d-2}} \right)^2 \frac{e_s \sin^2 \theta_w}{M_Z^{d_s-2d}} \mathcal{O}_s, \quad \mathcal{O}_{\mu\nu} \times \mathcal{O}^{\mu\nu} \sim e_s \mathcal{O}_s$$

- if  $d_s < 4$  this can result in a **mass scale generation**

$$m_g \sim \left( \left( \frac{\lambda}{\Lambda^{d-2}} \right)^2 \frac{e_s \sin^2 \theta_w}{M_Z^{d_s-2d}} \right)^{1/(4-d)}$$

$\mathcal{O}_{SM} = B_{\mu\nu}$  (tensor operator)

At  $E < v$  we have

$$B_\mu = \cos \theta_w \gamma_\mu + \sin \theta_w Z_\mu$$

- Now, exchange of **massive Z boson** can generate a local operator by **OPE** for  $E \ll M_Z$  :

$$\mathcal{L} \sim \left( \frac{\lambda}{\Lambda^{d-2}} \right)^2 \frac{e_s \sin^2 \theta_w}{M_Z^{d_s-2d}} \mathcal{O}_s, \quad \mathcal{O}_{\mu\nu} \times \mathcal{O}^{\mu\nu} \sim e_s \mathcal{O}_s$$

- if  $d_s < 4$  this can result in a **mass scale generation**
- currently, **no numerical CFT-bootstrap** bound exists.

$$\mathcal{O}_{SM} = B_{\mu\nu} \text{ (tensor operator)}$$

Emergent composite dark photon:

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^{d-2}} B_{\mu\nu} \mathcal{O}^{\mu\nu}$$

- At  $m_g$ , CFT confines and

$$\mathcal{O}^{\mu\nu} \sim \frac{m_g^{d-2}}{g_*} \rho^{\mu\nu}, \quad \rho^{\mu\nu} = \partial^\mu \rho^\nu - \partial^\nu \rho^\mu$$

$$\langle \theta \theta \rangle \sim \text{bubble diagram} \sim \frac{N}{16\pi^2} \sim \frac{1}{g_*^2}$$

$\mathcal{O}_{SM} = B_{\mu\nu}$  (tensor operator)

Emergent composite dark photon:

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^{d-2}} B_{\mu\nu} \mathcal{O}^{\mu\nu}$$

- At  $m_g$ , CFT confines and

$$\mathcal{O}^{\mu\nu} \sim \frac{m_g^{d-2}}{g_\star} \rho^{\mu\nu}, \quad \rho^{\mu\nu} \equiv \partial^\mu \rho^\nu - \partial^\nu \rho^\mu$$

- So, in the IR, we have

$$\mathcal{L} \sim \frac{\lambda}{g_\star} \left( \frac{m_g}{\Lambda} \right)^{d-2} B_{\mu\nu} \rho^{\mu\nu}, \quad \epsilon \sim \frac{\lambda}{g_\star} \left( \frac{m_g}{\Lambda} \right)^{d-2} \ll 1$$



$\mathcal{O}_{SM} = B_{\mu\nu}$  (tensor operator)

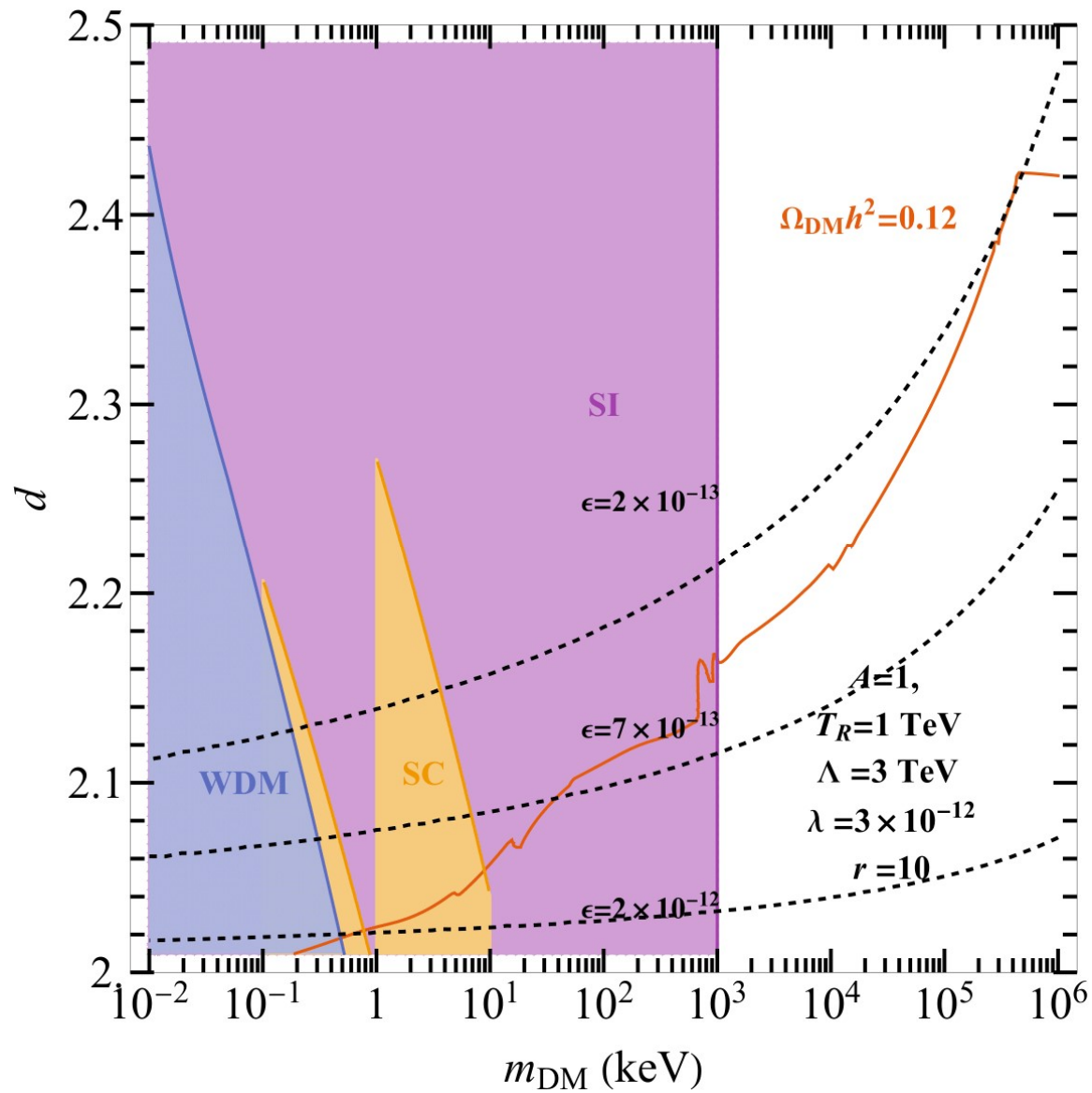
Emergent composite dark photon:

$$\mathcal{L} \sim \frac{\lambda}{\Lambda^{d-2}} B_{\mu\nu} \mathcal{O}^{\mu\nu} \rightarrow \epsilon B_{\mu\nu} \rho^{\mu\nu}, \quad \epsilon \sim \frac{\lambda}{g_\star} \left( \frac{m_g}{\Lambda} \right)^{d-2} \ll 1$$

- $\left( \frac{\lambda}{\Lambda^{d-2}} \right)$  controls COFI production
- It also partly determines  $m_g$  via OPE
- So up to  $g_\star \sim \mathcal{O}(1)$ , the kinetic mixing  $\epsilon$  fixed by othese data  
smaller  $\left( \frac{\lambda}{\Lambda^{d-2}} \right) \rightarrow$  smaller CFT breaking  $\rightarrow$  smaller  $m_g, \epsilon$

$\mathcal{O}_{SM} = B_{\mu\nu}$  (tensor operator)

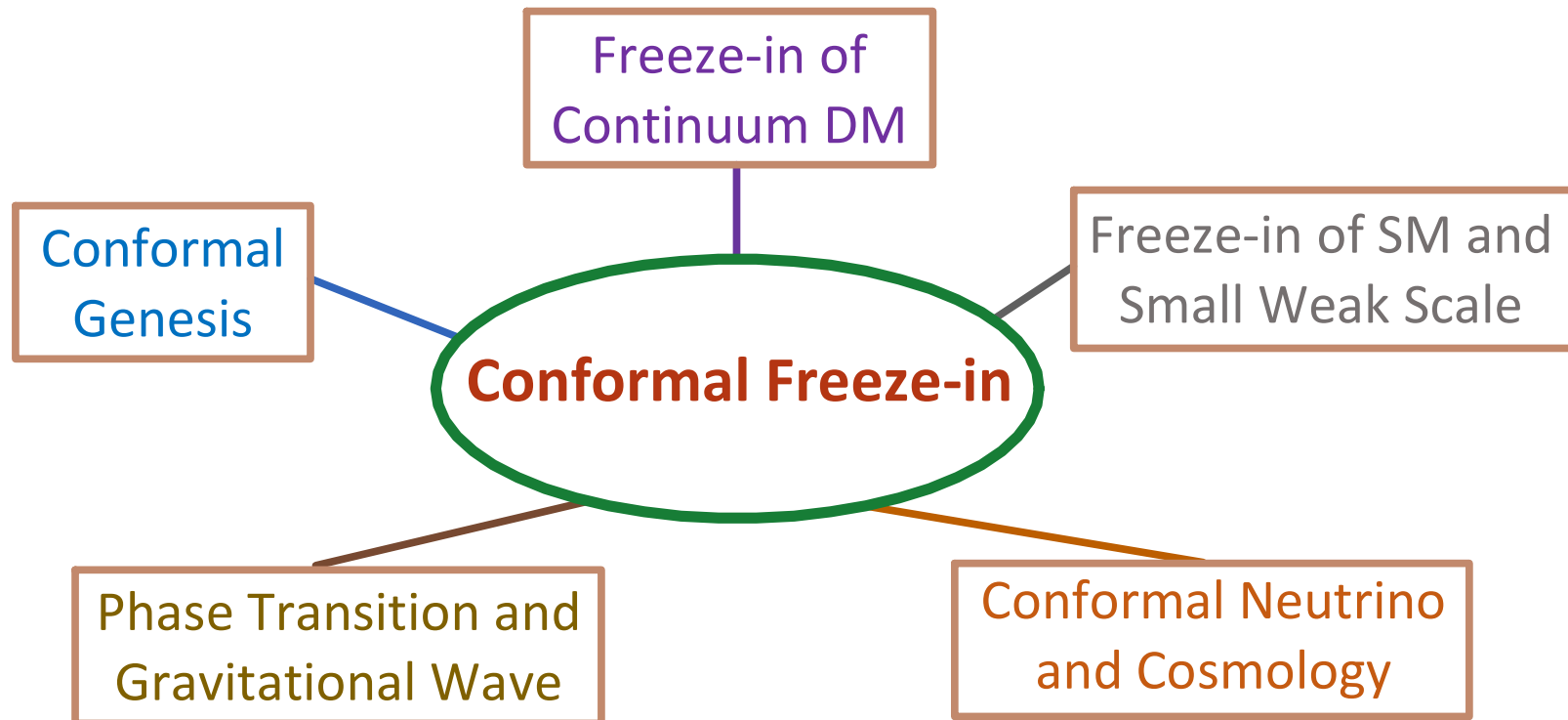
IR-dominant ( $d \leq 2.5$ )



# **Outline**

1. Introduction to Conformal Freeze-In
2. Robustness of COFI mechanism
3. COFI Phenomenology
  - DM Self-Interaction, WDM, Star Cooling
4. Conclusion and Outlook

# Conformal Freeze-In Physics!



# Outline

1. Introduction to Conformal Freeze-In
2. Robustness of COFI mechanism
3. **COFI** Phenomenology
  - DM Self-Interaction, WDM, Star Cooling
4. Conclusion and Outlook

# COFI Phenomenology

A general remark:

Freeze-in  $\rightarrow$  weak coupling

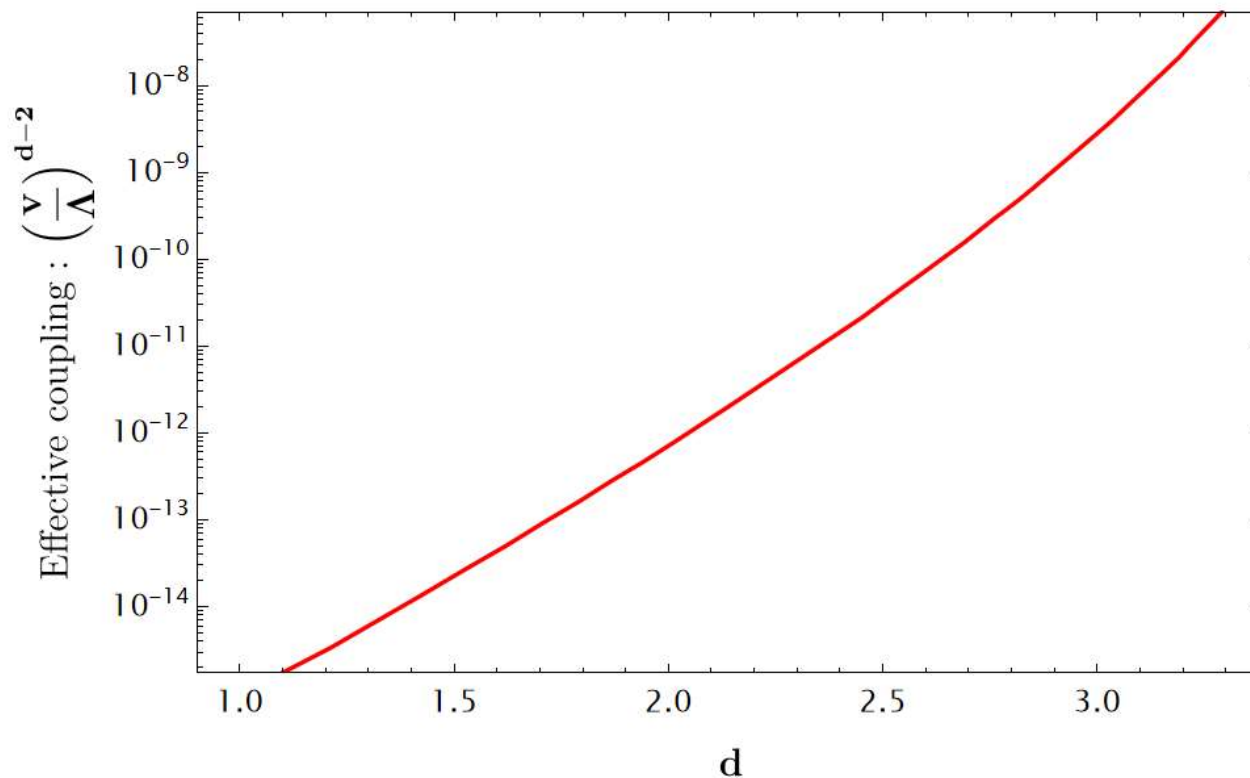
E.g.  $\mathcal{L} \sim \frac{\lambda}{\Lambda^{d-2}} H^+ H \mathcal{O} \Rightarrow g_{eff} \sim \left(\frac{\lambda v}{\Lambda}\right)^{d-2} \ll 1$

# COFI Phenomenology

A general remark:

Freeze-in  $\rightarrow$  weak coupling

*E.g.*  $\mathcal{L} \sim \frac{\lambda}{\Lambda^{d-2}} H^+ H \mathcal{O} \Rightarrow g_{eff} \sim \left(\frac{\lambda v}{\Lambda}\right)^{d-2} \ll 1$



# COFI Phenomenology

A general remark:

Freeze-in  $\rightarrow$  weak coupling

COFI is not very much visible at:

Colliders, Rare meson decays, CMB, BBN, ...

\*This is a general feature of most of freeze-in models

(Q) model building question: can there be COFI with light mediator (=kinematic enhancement) ?



# COFI Phenomenology

## (1) DM Self-interaction

- Observation of Bullet-cluster imposes

$$\frac{\sigma_{SI}}{m_{DM}} \leq 4500 \text{ GeV}^{-3} \sim \frac{1}{(100 \text{ MeV})^3}$$

# COFI Phenomenology

## (1) DM Self-interaction

- Observation of Bullet-cluster imposes

$$\frac{\sigma_{SI}}{m_{DM}} \leq 4500 \text{ GeV}^{-3} \sim \frac{1}{(100 \text{ MeV})^3}$$

- For a generic interacting CFT,  
we expect its hadronic IR phase is also interacting  $g_\star \sim \mathcal{O}(1)$
- If DM = genuine composite hadron

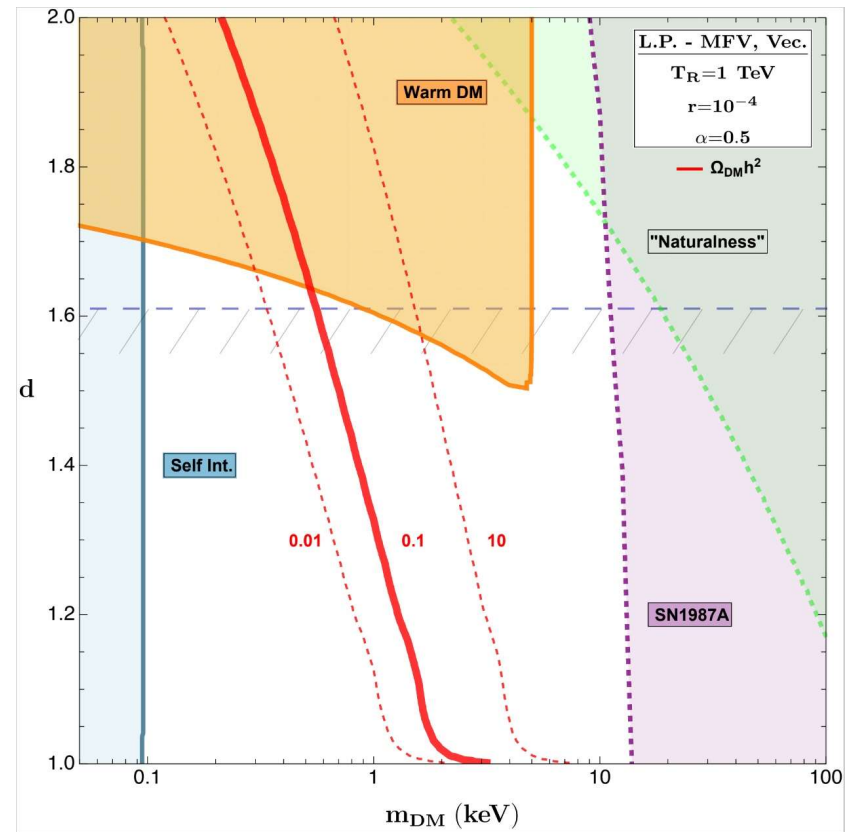
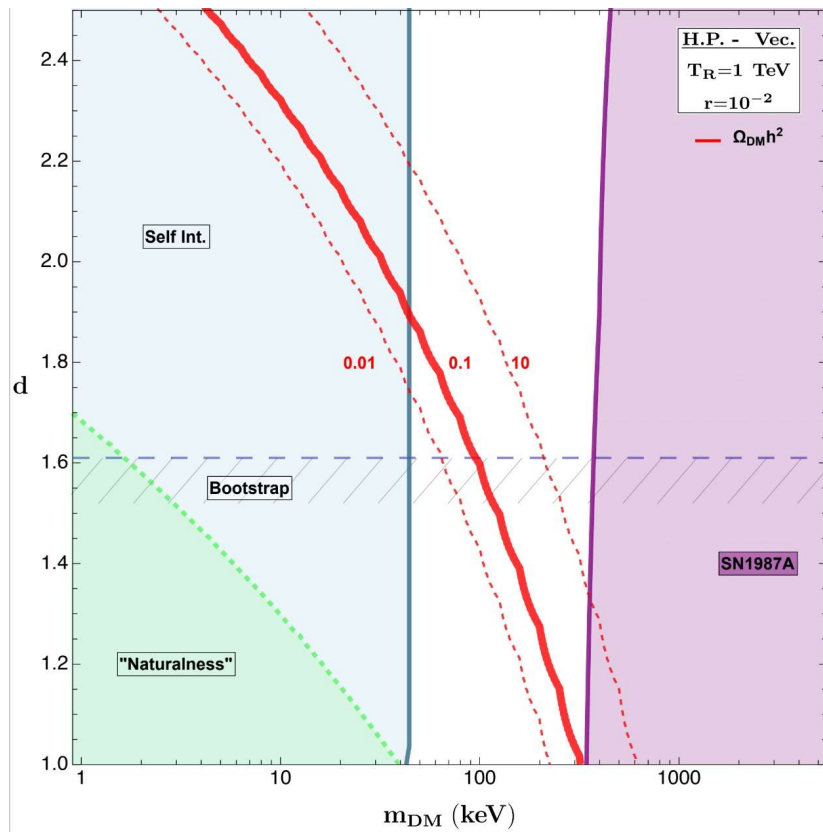
$$m_{DM} \approx m_g, \quad \sigma_{SI} \sim \frac{1}{8\pi m_g^2} \rightarrow \frac{\sigma_{SI}}{m_{DM}} \sim \frac{1}{8\pi m_g^3}$$

# COFI Phenomenology

## (1) DM Self-interaction

- In all but dark-photon-portal ( $B_{\mu\nu}\mathcal{O}^{\mu\nu}$ ),

DM relic density  $\Rightarrow 0.1 \text{ keV} \leq m_{DM} \leq \text{MeV}$



# COFI Phenomenology

## (1) DM Self-interaction

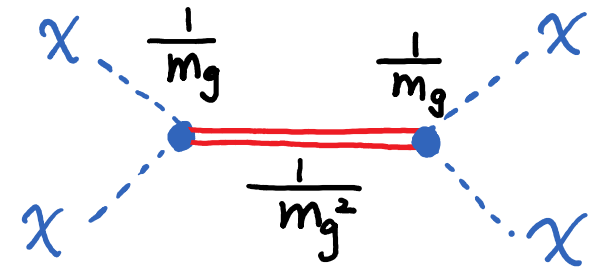
- Desired suppression of  $\sigma_{SI}$  possible

If **DM = PNGB** due to **derivative suppression**

(i) scalar mediator ( $\phi$ ) :  $\mathcal{O}_{CFT} \sim \frac{m_g^{d-1}}{g_\star} \phi$

$$\mathcal{L} \sim g_\star \frac{\phi}{m_g} (\partial\chi)^2$$

$$\rightarrow \sigma_{SI} \sim \frac{1}{8\pi} \frac{(m_{DM})^6}{m_g^8} = \frac{r^6}{8\pi m_g^2},$$



$$r = \frac{m_{DM}}{m_g} < 1$$

Typically we need :  $r \sim 0.1 - 0.01$

# COFI Phenomenology

## (1) DM Self-interaction

- Desired suppression of  $\sigma_{SI}$  possible

If **DM = PNGB** due to **derivative suppression**

(ii) vector mediator ( $\rho^\mu$ )

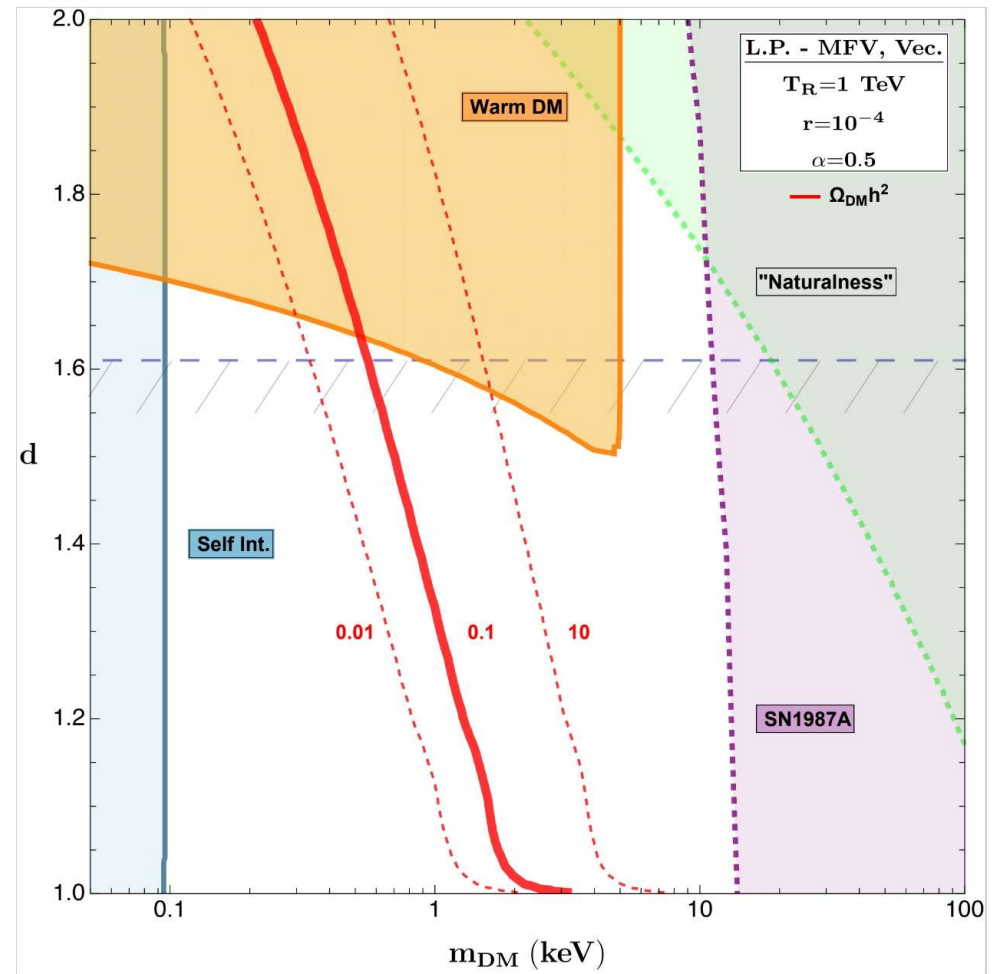
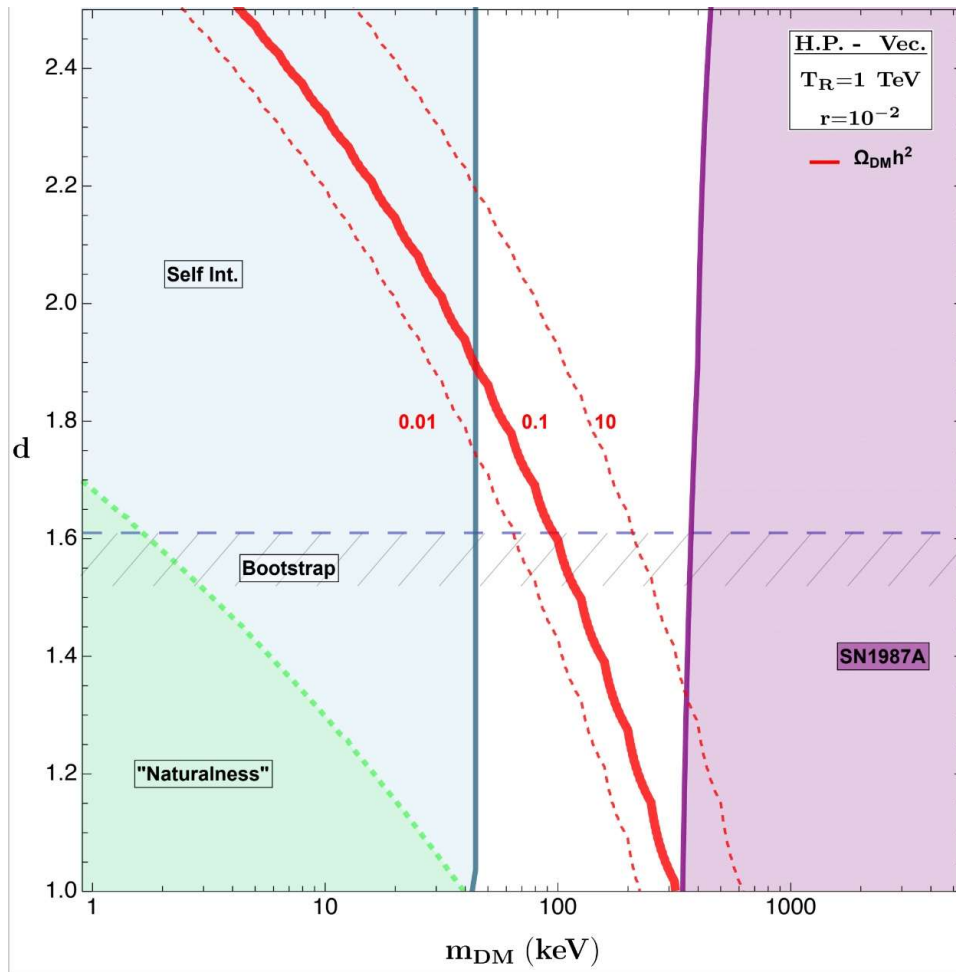
$$\mathcal{L} \sim g_\star \rho^\mu (\chi^\dagger \partial_\mu \chi + h.c.)$$

$$\rightarrow \sigma_{SI} \sim \frac{1}{8\pi} \frac{(m_{DM})^2}{m_g^4} = \frac{r^2}{8\pi m_g^2}, \quad r = \frac{m_{DM}}{m_g} < 1$$

This results in stronger bounds (but model-dependent)

# COFI Phenomenology

## (1) DM Self-interaction



# COFI Phenomenology

## (2) Warm DM bound

- If DM free-streams a comoving distance  $\lambda_{FS}$  it damps out structure below  $\lambda \leq \lambda_{FS}$
- Observation of DM halo of certain size places an upper bound

$$\lambda_{FS} < \lambda_{obs}$$

- Typically, this is stated in terms of

$$m_{DM} > (3.5 - 5.5) \text{ keV}$$

# COFI Phenomenology

## (2) Warm DM bound

- If  $T_d \neq T_{SM}$  the correct form of bound is

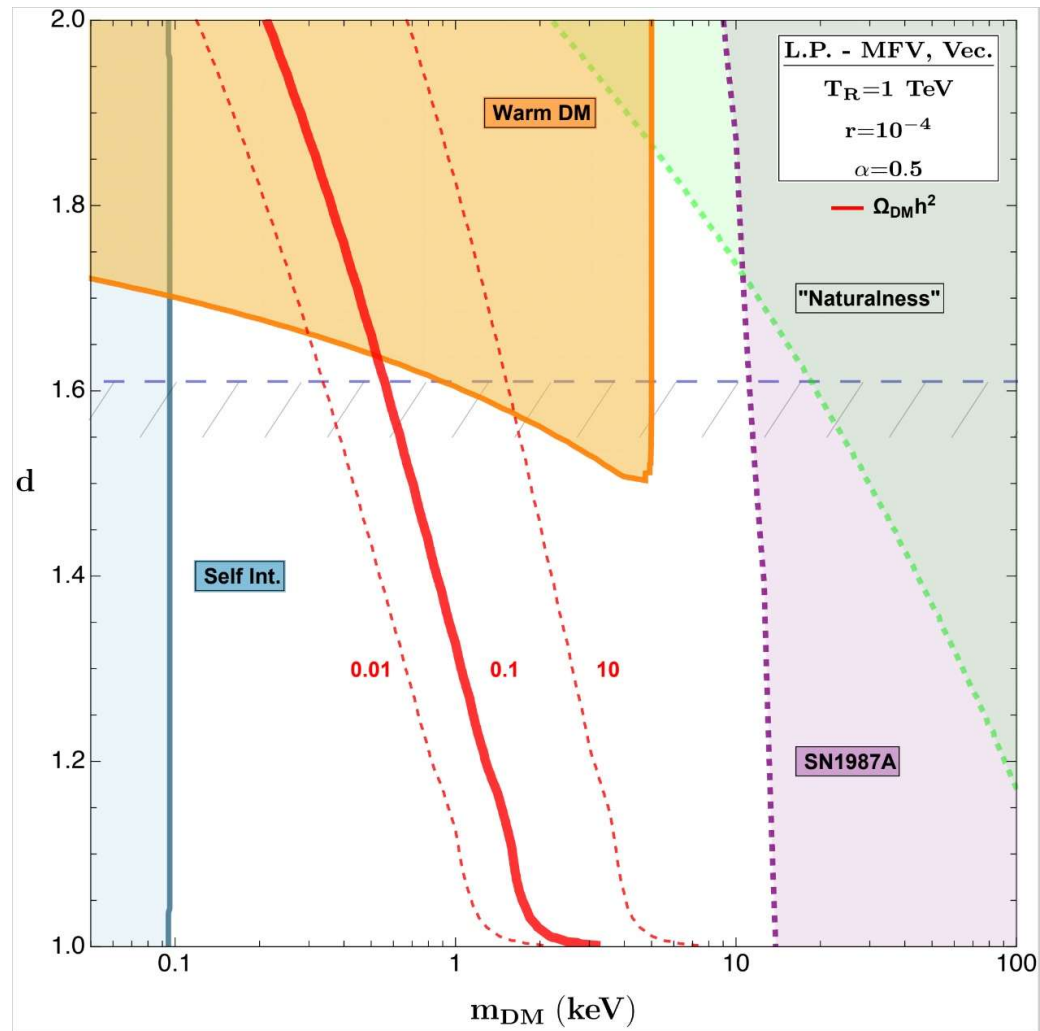
$$\lambda_{FS} \approx \frac{1}{m_{DM}} \left( \frac{T_d}{T_{SM}} \right) < \lambda_{obs}$$

- For us,  $T_d \ll T_{SM} \rightarrow$  bound relaxed linearly in  $\frac{T_d}{T_{SM}}$
- Moreover, since  $T_d(m_{DM}, d)$ , the bound can have non-trivial "shape"



# COFI Phenomenology

## (2) Warm DM bound



# COFI Phenomenology

## (3) Stellar evolution

- Recall: COFI = light and weakly interacting DM states
- Diverse stellar systems provide non-trivial constraints

### (i) Main Sequence (MS)

$$T \approx 1.3 \text{ keV}, \quad \epsilon \leq 0.2 \text{ erg g}^{-1} \text{s}^{-1}, \quad e^- \text{ not degenerate}$$

### (ii) Horizontal Branch (HB)

$$T \approx 10 \text{ keV}, \quad \epsilon \leq 10 \text{ erg g}^{-1} \text{s}^{-1}, \quad e^- \text{ not degenerate}$$

### (iii) Red Giant Branch (RGB)

$$T \approx 10 \text{ keV}, \quad \epsilon \leq 10 \text{ erg g}^{-1} \text{s}^{-1}, \quad e^- \text{ degenerate}$$

### (iv) Supernova (SN)

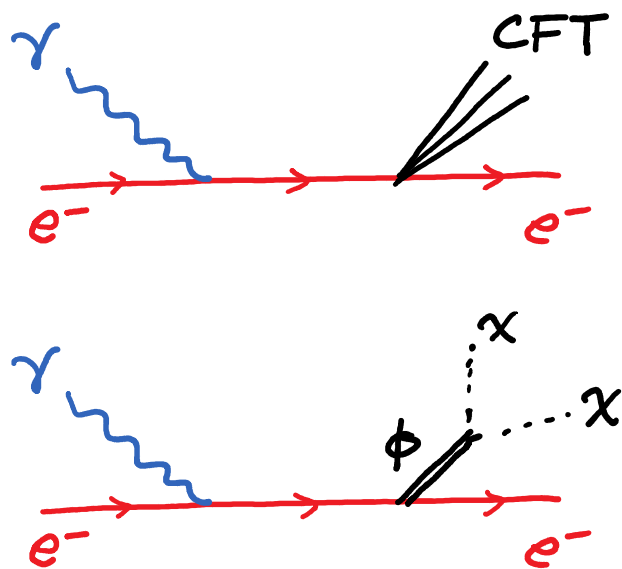
$$T \sim 30 \text{ MeV}, \quad \epsilon \leq 10^{19} \text{ erg g}^{-1} \text{s}^{-1}, \quad e^-, p \text{ degenerate}$$

# COFI Phenomenology

## (3) Stellar evolution

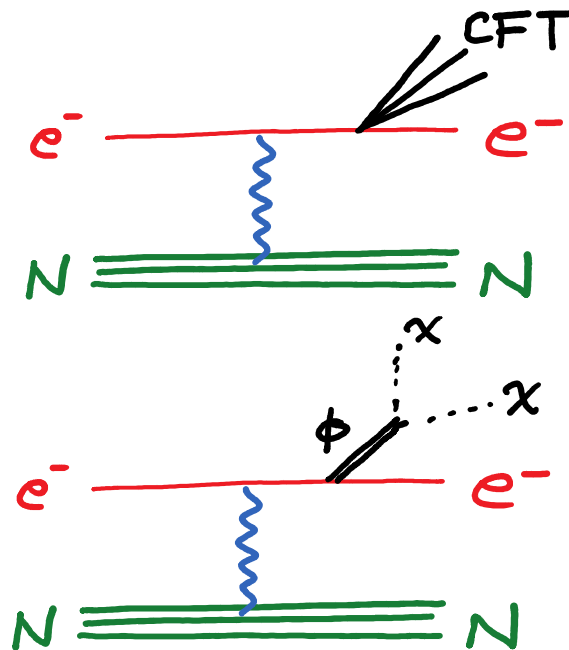
E.g.  $\mathcal{O}_{SM} = H \bar{L} \ell$

MS and HB



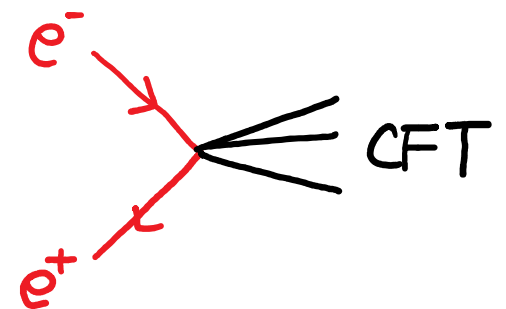
Compton

RGB



Bremsstrahlung

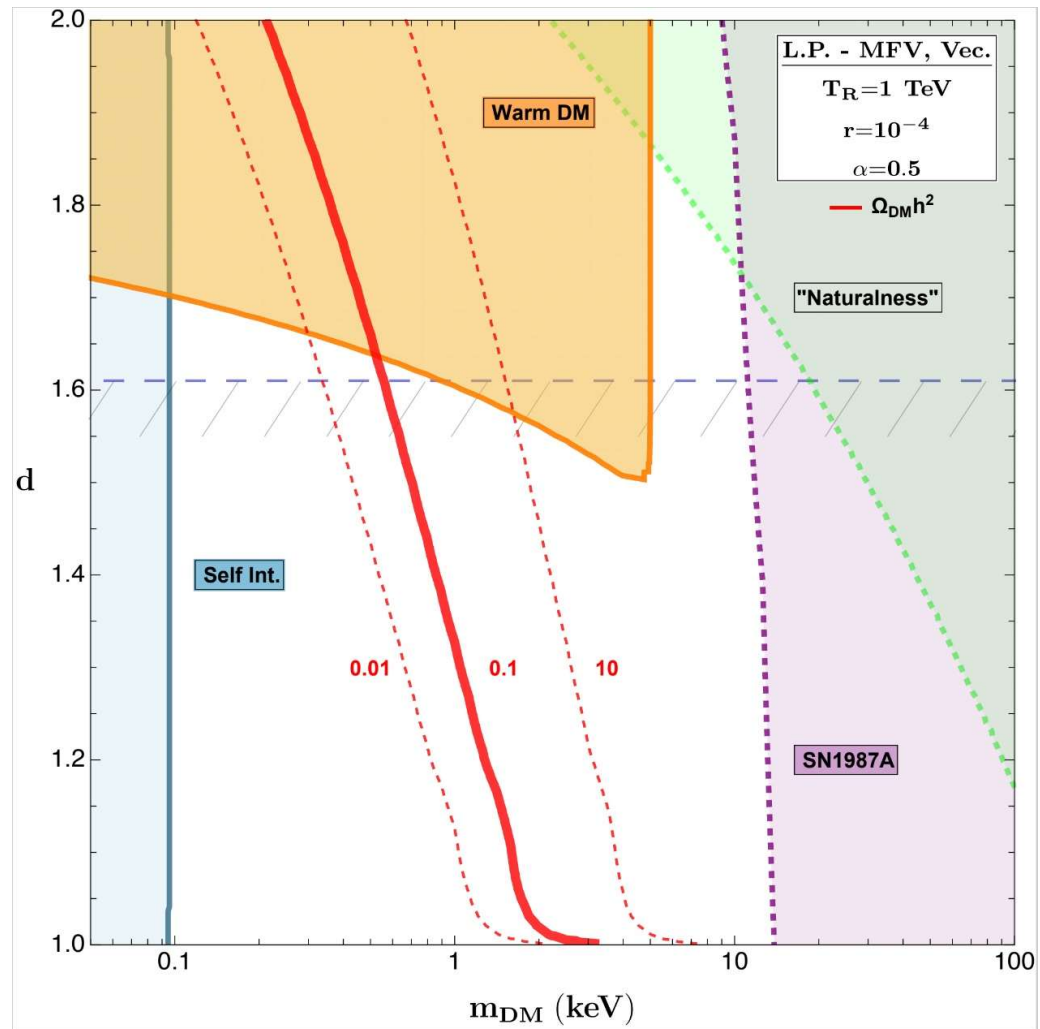
SN



Pair annihilation

# COFI Phenomenology

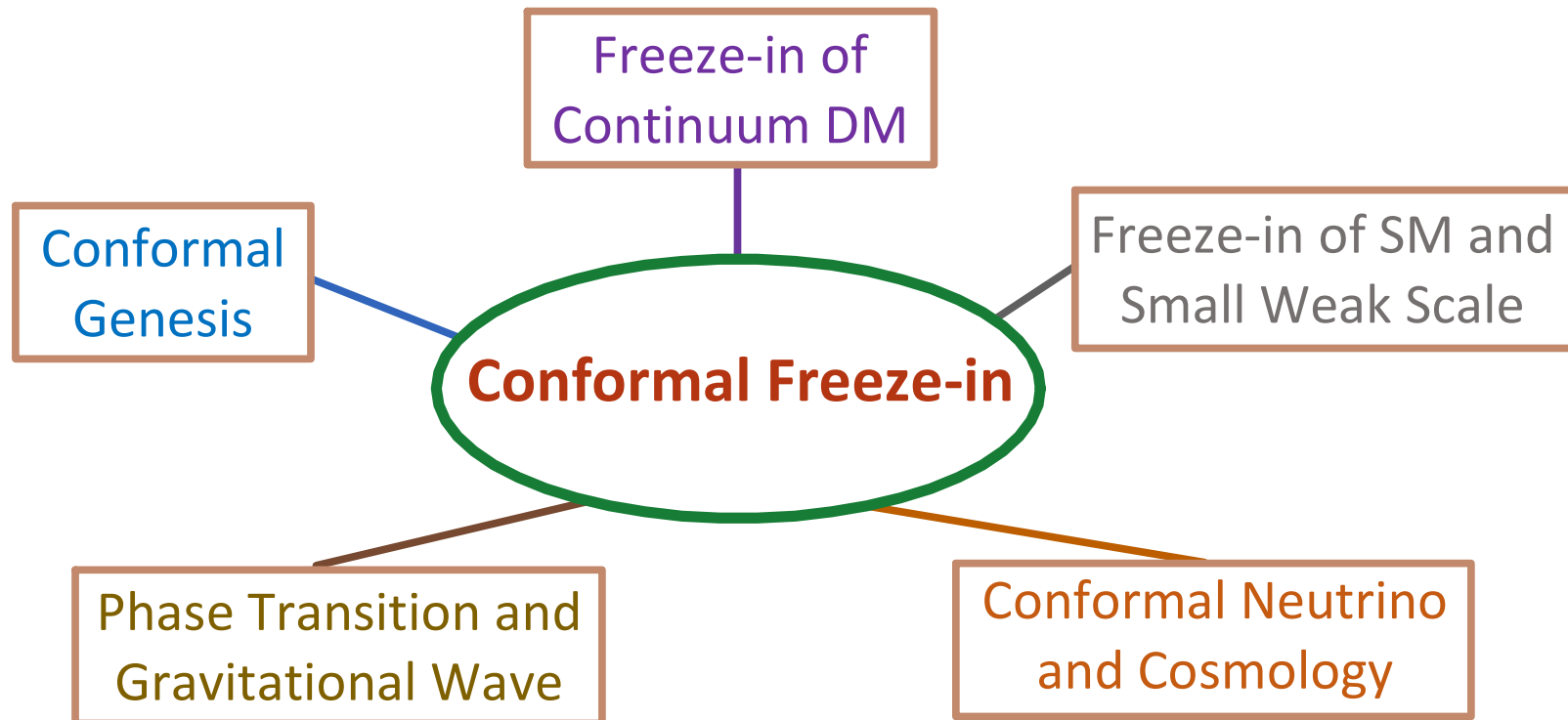
## (3) Stellar evolution



# **Outline**

1. Introduction to Conformal Freeze-In
2. Robustness of COFI mechanism
3. COFI Phenomenology
  - DM Self-Interaction, WDM, Star Cooling
4. Conclusion and Outlook

# Conformal Freeze-In Physics!



Thank you!

## App-1. Why COFI?

---

(Q) Imagine there is a sector of CFT.

Under what conditions can this "conformal dark sector" turn into a "dark matter sector" ?

(A) Most likely\* answer: Conformal Freeze-In (COFI)

---

(i) CFT = no massless = "radiation"

→ so for DM, we need to break the conformal invariance

→ We do it by coupling CFT to SM

$$\mathcal{L} \sim \mathcal{O}_{SM} \mathcal{O}_{CFT}$$

→ In fact, it is technically natural that this is the only CFT breaking (up to operator-mixing and OPE)



## App-1. Why COFI?

(ii) If CFT **thermalizes** with SM and freezes-out around  $T \sim m_g$

$$\rho_d(m_g) \sim m_g n_d, \quad n_d \sim \frac{1}{\pi} g_{CFT} m_g^3$$

$$\rho_0 \approx \rho_d(m_g) \left( \frac{T_0}{m_g} \right)^3$$

$$\rightarrow \rho_0 = \rho_{0,crit} = 8 \times 10^{-47} h^2 \text{ GeV}^4$$

$$\rightarrow m_g \sim \mathcal{O}(1) \text{ eV} : \text{ruled out by WDM}$$

## App-1. Why COFI?

(iii) Why not just freeze-in in **hadronic phase** of CFT

- This will be the usual "**particle**" freeze-in
- However, it is mostly **UV-dominant** and **model-dependent**
- COFI, on the other hand, requires only **CFT 2-point function**
  - COFI production is independent of details of CFT  
i.e. **universal!**

## App-1. Why COFI?

(iv) Something in between?

- This would need to include important effects of  $CFT \rightarrow SM$
- "Hot" CFT is an interesting but challenging topic  
: currently getting developed
- Maybe AdS/CFT helps : finite T CFT  $\leftrightarrow$  AdS with BH

## App-2. Small $\lambda$ from UV Banks-Zaks Phase

I. At  $v < \Lambda < E < M$

$$\mathcal{L} \sim \frac{\lambda_0}{M^{D_0-4}} \mathcal{O}_{SM} \mathcal{O}_{BZ}, \quad D_0 = d_{SM} + d_{BZ}$$

- "BZ" = gauge theory with (strongly) interacting **IR-fixed point**
- $\lambda_0 \sim \mathcal{O}(1)$ ,  $M < M_p$
- $\mathcal{O}_{BZ} = \bar{Q}Q, V_{\mu\nu}^a V^{a\mu\nu}, \dots$

## App-2. Small $\lambda$ from UV Banks-Zaks Phase

I. At  $v < \Lambda < E < M$

$$\mathcal{L} \sim \frac{\lambda_0}{M^{D_0-4}} \mathcal{O}_{SM} \mathcal{O}_{BZ}, \quad D_0 = d_{SM} + d_{BZ}$$

- "BZ" = gauge theory with (strongly) interacting **IR-fixed point**
- $\lambda_0 \sim \mathcal{O}(1)$ ,  $M < M_p$
- $\mathcal{O}_{BZ} = \bar{Q}Q, V_{\mu\nu}^a V^{a\mu\nu}, \dots$

II. At  $E \rightarrow \Lambda$  (slow RG-"walking")

$$\mathcal{O}_{BZ} \sim \Lambda^{d-d_{BZ}} \mathcal{O}_{CFT}$$

## App-2. Small $\lambda$ from UV Banks-Zaks Phase

I. At  $v < \Lambda < E < M$

$$\mathcal{L} \sim \frac{\lambda_0}{M^{D_0-4}} \mathcal{O}_{SM} \mathcal{O}_{BZ}, \quad D_0 = d_{SM} + d_{BZ}$$

- "BZ" = gauge theory with (strongly) interacting **IR-fixed point**
- $\lambda_0 \sim \mathcal{O}(1)$ ,  $M < M_p$
- $\mathcal{O}_{BZ} = \bar{Q}Q, V_{\mu\nu}^a V^{a\mu\nu}, \dots$

II. At  $E \rightarrow \Lambda$  (slow RG-"walking")

$$\mathcal{L} \sim \lambda_0 \left(\frac{\Lambda}{M}\right)^{D_0-4} \frac{1}{\Lambda^{D-4}} \mathcal{O}_{SM} \mathcal{O}_{CFT}, \quad D = d_{SM} + d_{CFT}$$

$$\lambda = \lambda_0 \left(\frac{\Lambda}{M}\right)^{D_0-4} \ll 1$$