



# MUOGRAPHERS '23

*Naples, 19-22 June*

*International workshop on Muography*

## Lava dome rock strength estimation with 3-D muography at La Soufrière de Guadeloupe



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In collaboration with  
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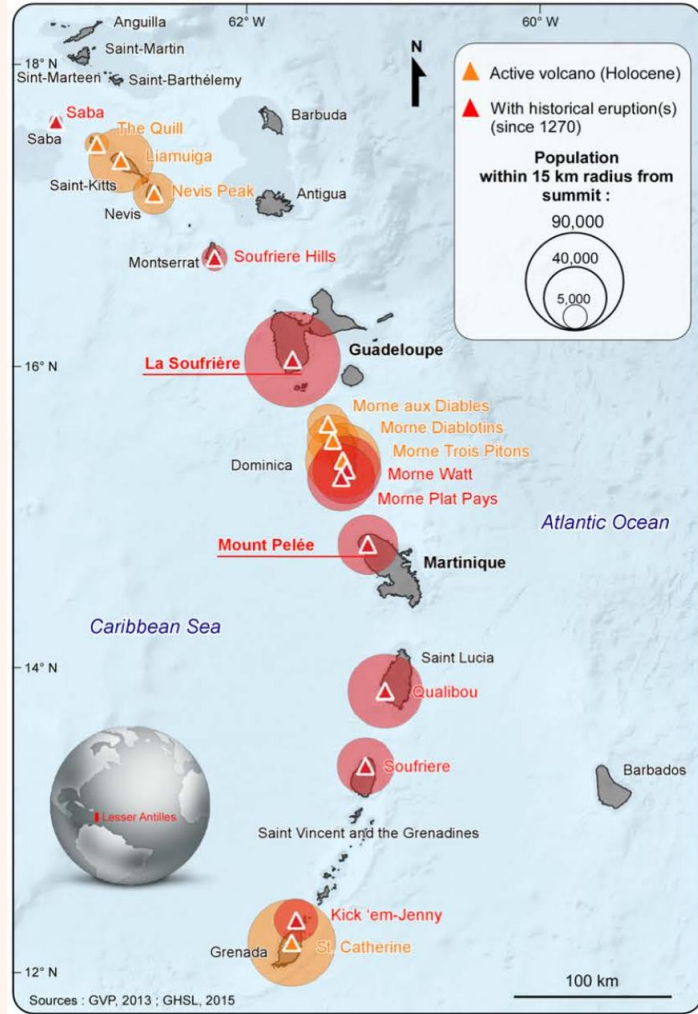
# OUTLINE

- I. La Soufrière de Guadeloupe :  
Motivations and challenges
- II. Muons data reconstruction  
and analysis
- III. 3-D modeling and inversion
- IV. Perspectives and conclusion



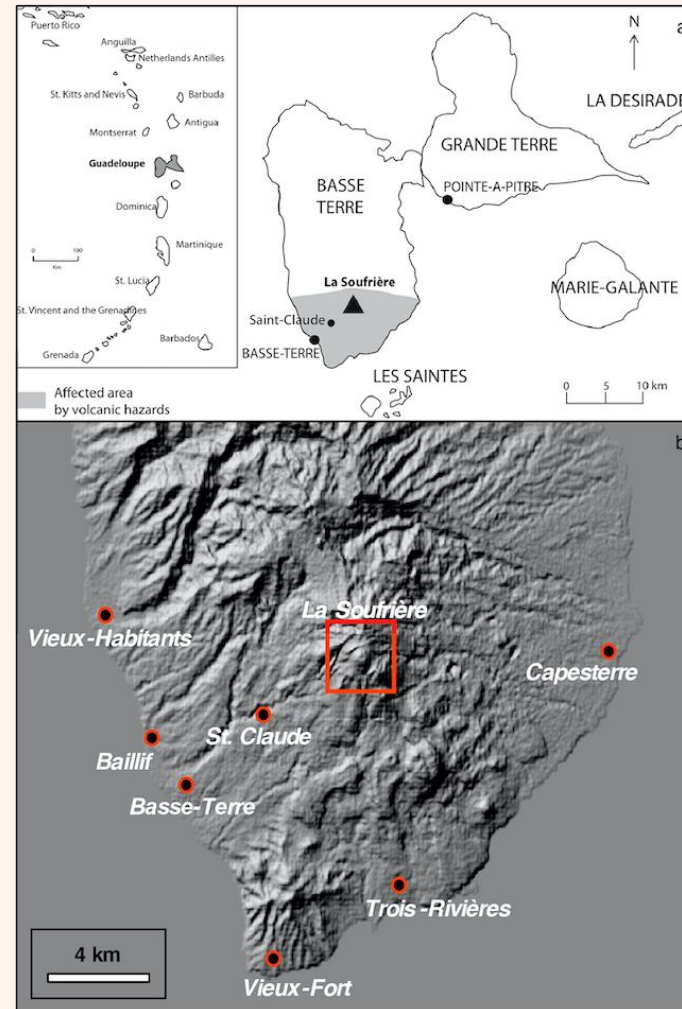
# La Soufrière de Guadeloupe

## Lesser Antilles Volcanic arc



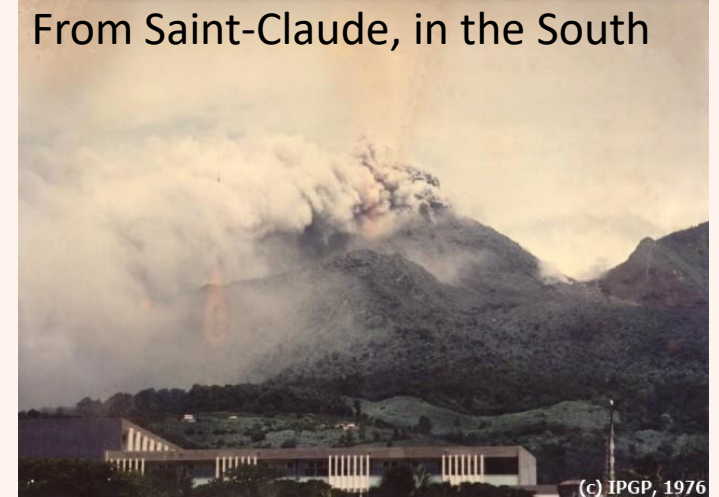
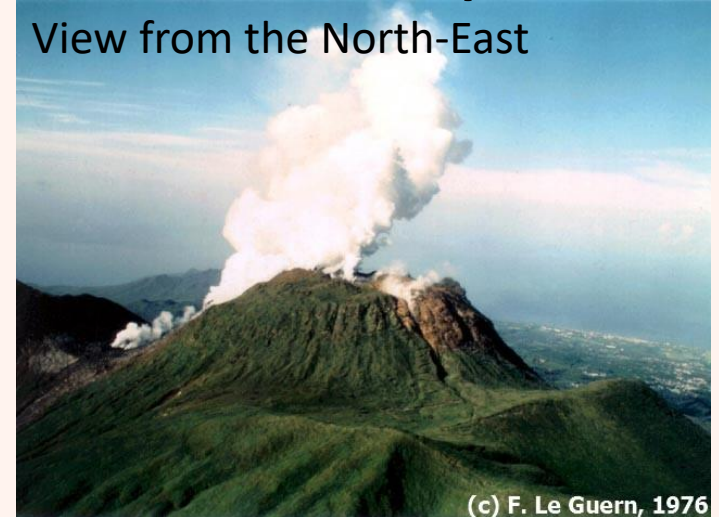
Leone et al., 2019

## Guadeloupe



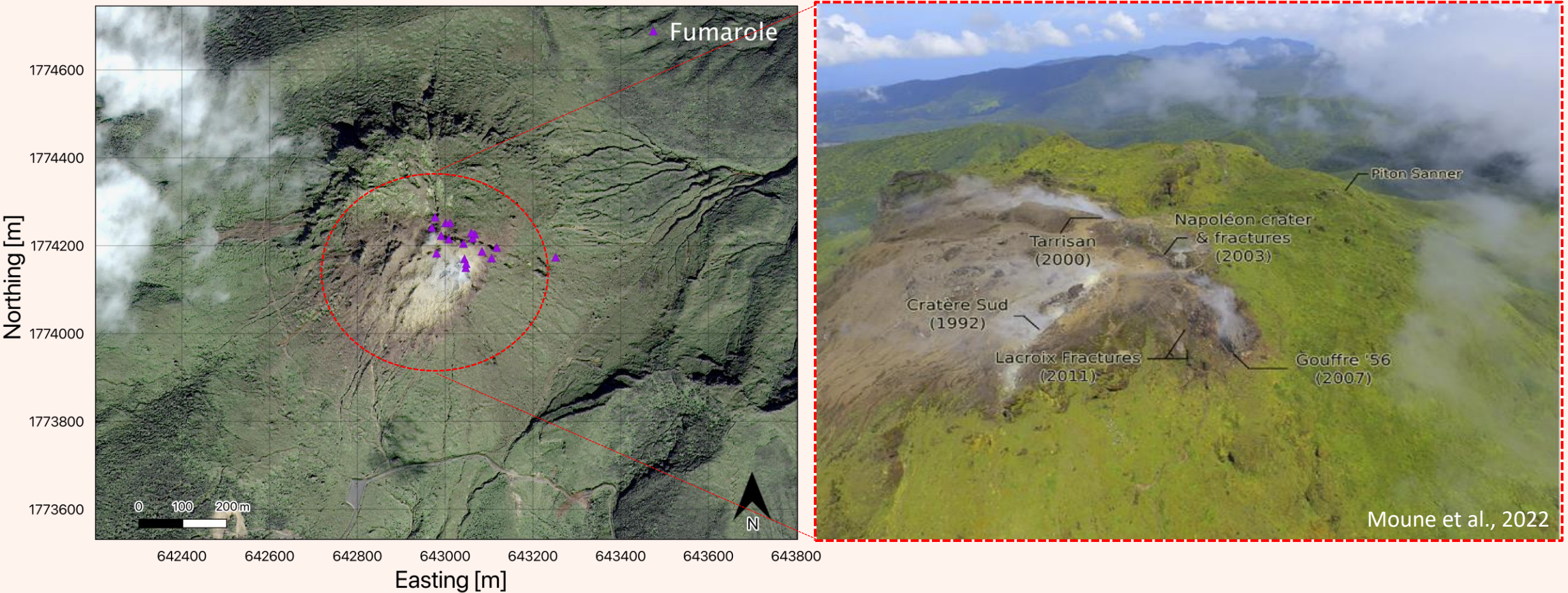
Massaro et al., 2022

## 1976-1977 eruption



Credit: IPGP

# Present : intense Fumarolic Activity at the dome summit



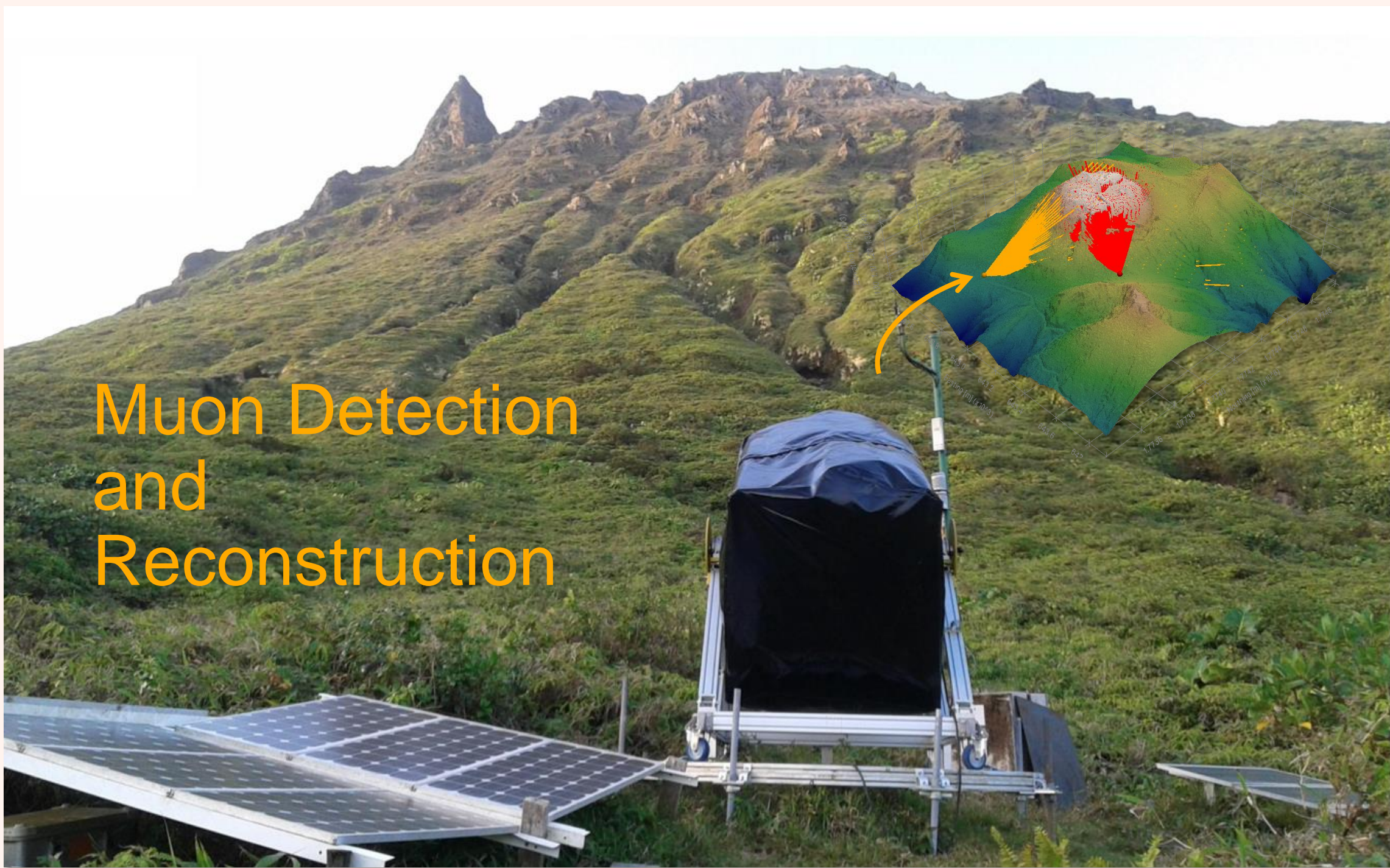
# Natural Hazard at La Soufrière

- **Hydrothermal activity** favours physicochemical process (e.g. **rock alteration**) worsening the dome mechanical stability
  - Potential **partial flank collapse**
- **Structural characterization** with various observables (seismic p-wave velocity, gravimetry, electrical tomography)
  - Input to numerical modeling of the edifice stabilitybut each has limits (e.g. spatial coverage, depth, resolution)...

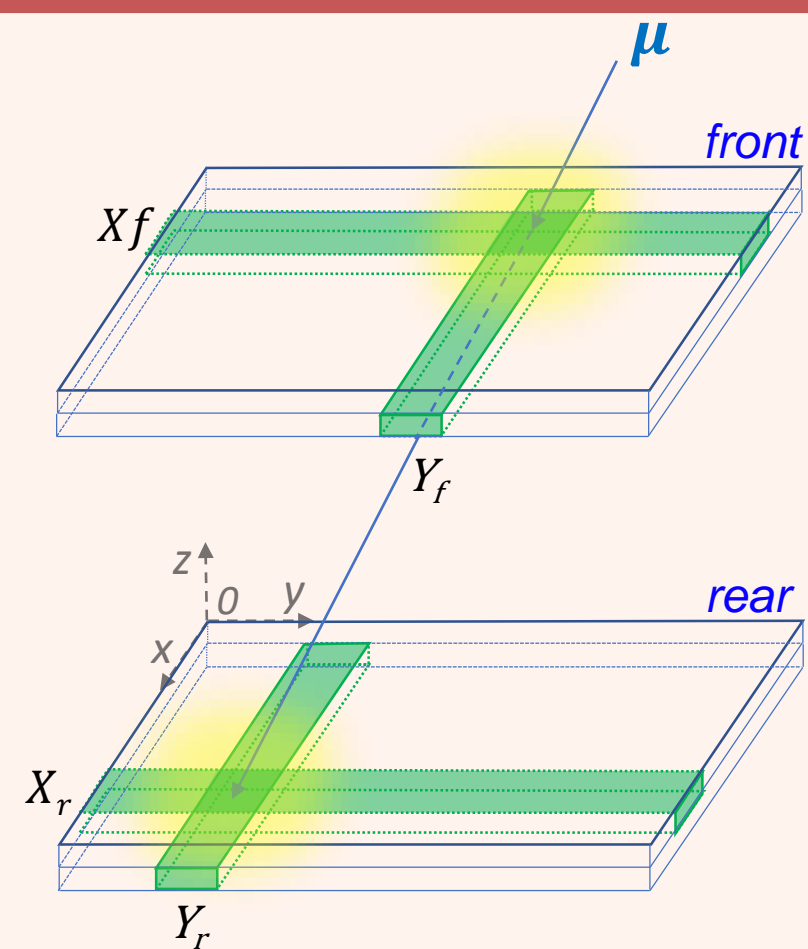
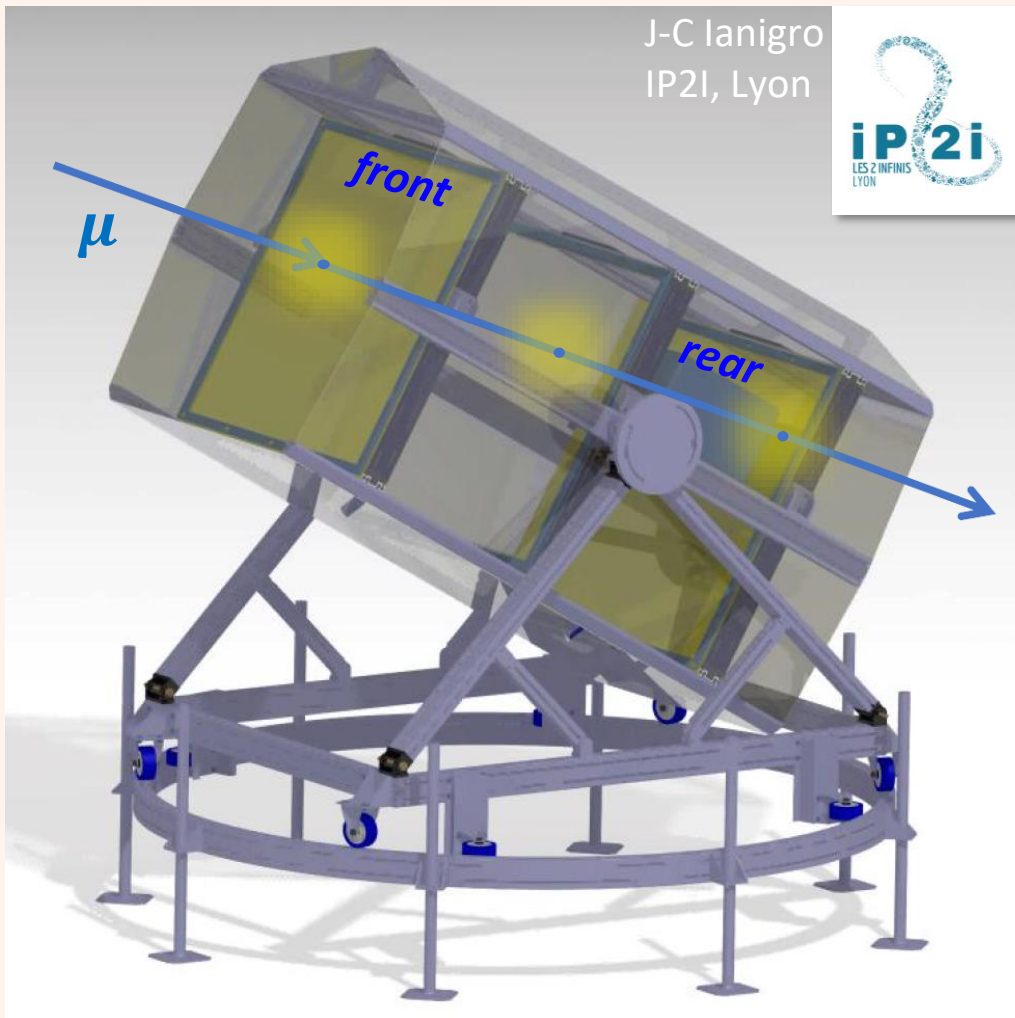
Need for refined characterization of the lava dome density structure ...

⇒ **Muography**

# Muon Detection and Reconstruction



# Muon telescope



More info on the readout system: [Lesparre et al. 2012](#),  
[Marteau et al. 2013](#)

2 detector layouts :

- 3 matrices 32x32 scint. (80x2.5x1 cm<sup>3</sup>) + 7-cm lead/steel shielding panel (SB, BR, OM detectors)
- 4 matrices 16x16 scint. (80x5x1 cm<sup>3</sup>) + shielding panel (SNJ)

# Data Reconstruction

Optimization of a **Random Sampling Consensus (RANSAC)** algorithm.

Comparison to other algorithms:

- **The Hough transform** (Dalitz et al., 2017)
- **Straightness check** (former analysis, Lesparre et al., 2012)

The tracking performances were studied with **GEANT4**.





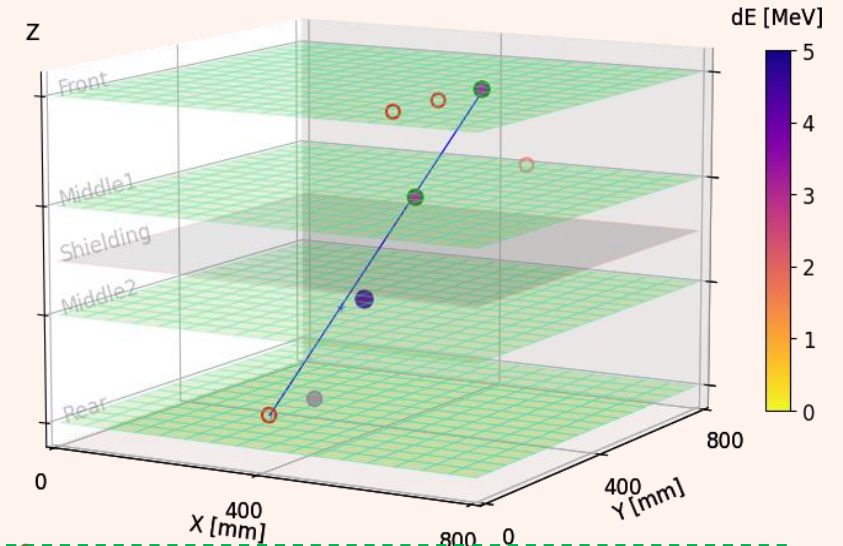
# RANSAC

## Outline:

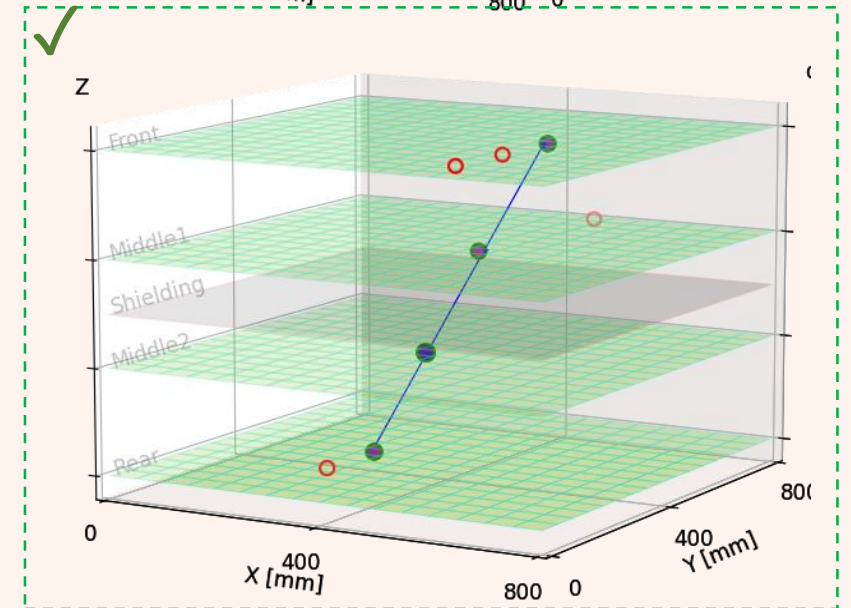
- 1) Randomly select a minimal subset of hits
- 2) Fit a hypothesized model to that subset
- 3) Tag hits that are below a distance **threshold** to the model as « **inliers** » and tag the rest as « **outliers** »
- 4) Iterates N times over the three previous steps and selects the best model that maximizes the number of **inliers**.

## Reconstruction of a 10 GeV muon + noise

TRIALS  
N=2

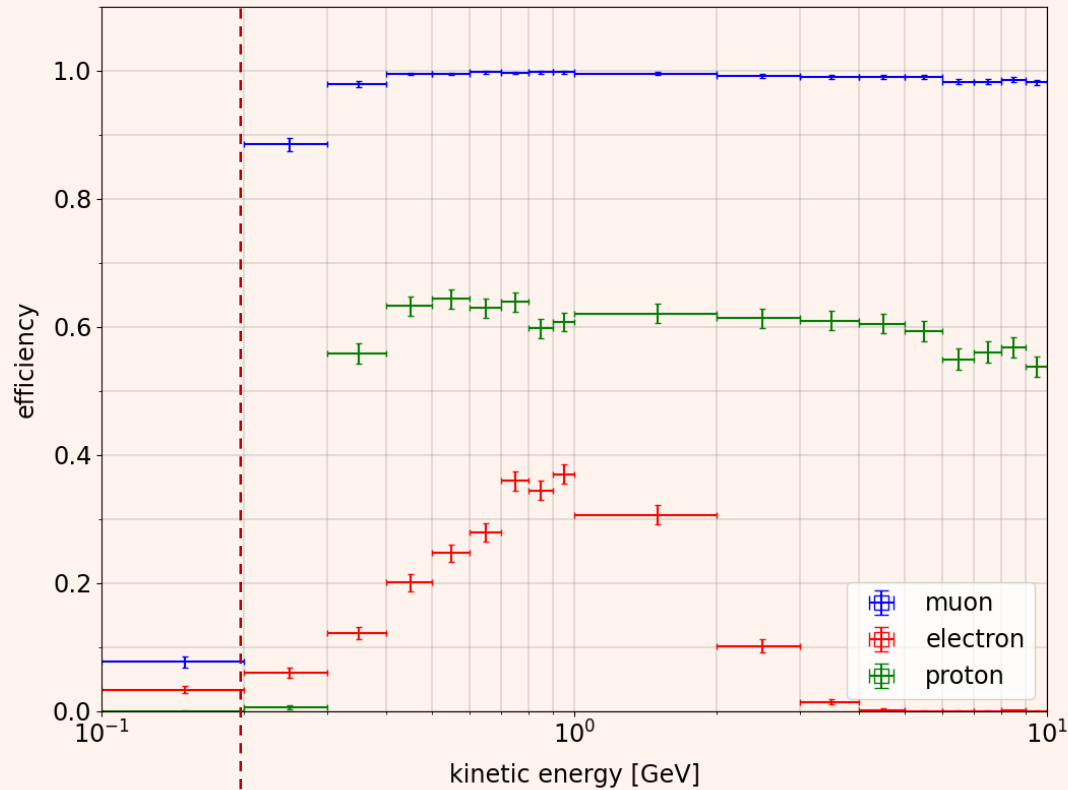


N=20



# Reconstruction efficiency

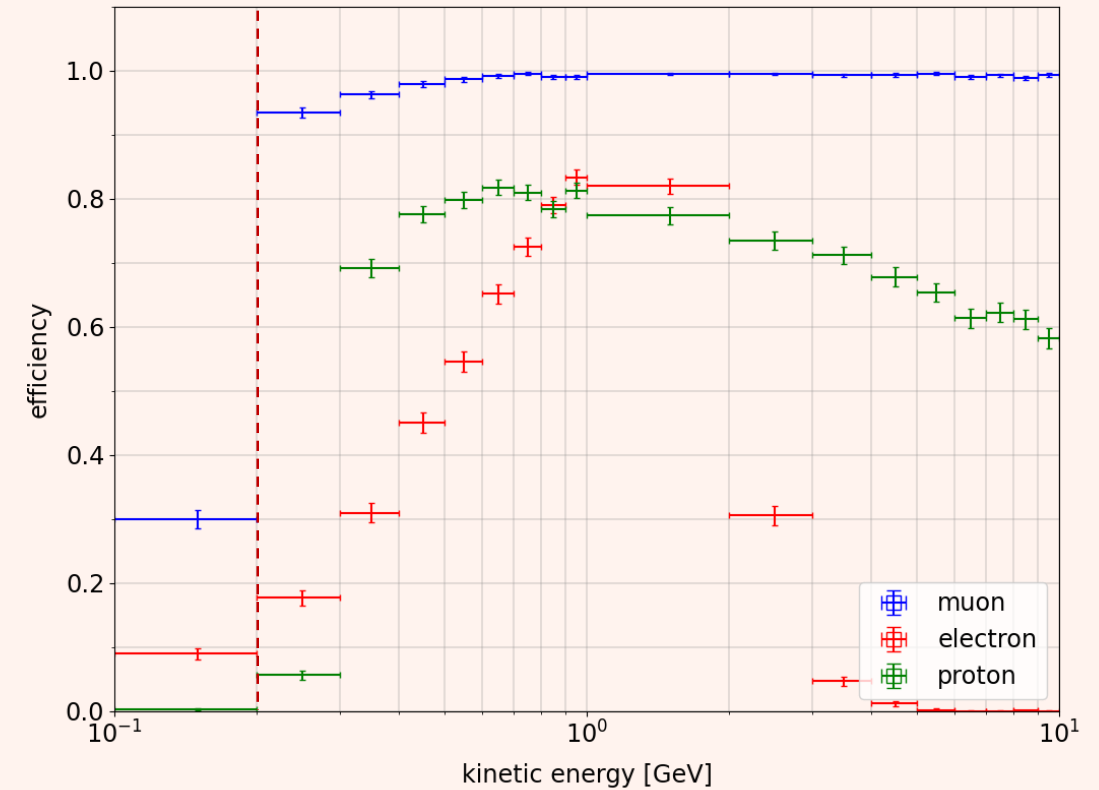
## RANSAC



Shielding cut-off  $\approx 200$  MeV

☛ too low to reject "soft" muons background

## Hough Transform



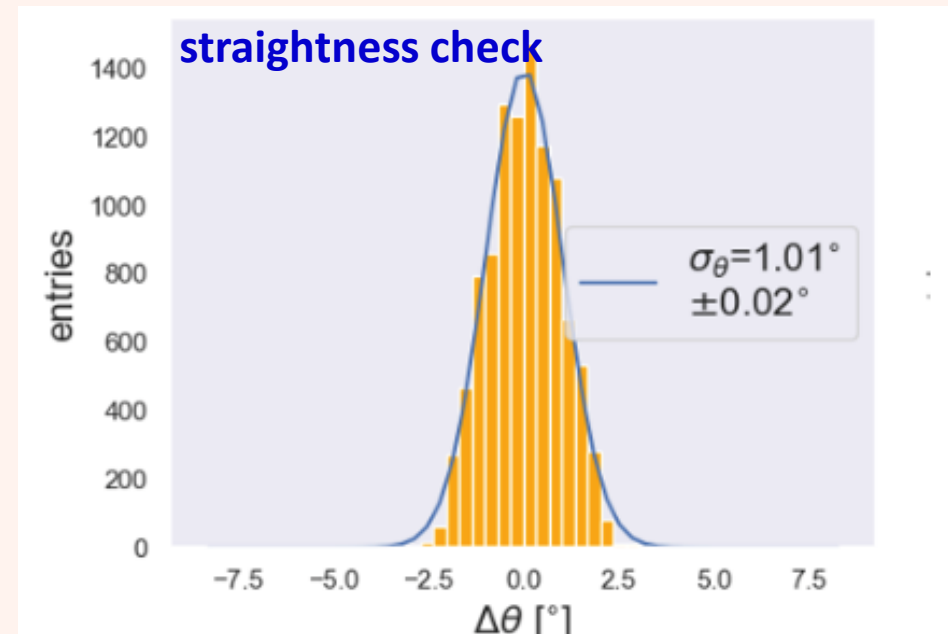
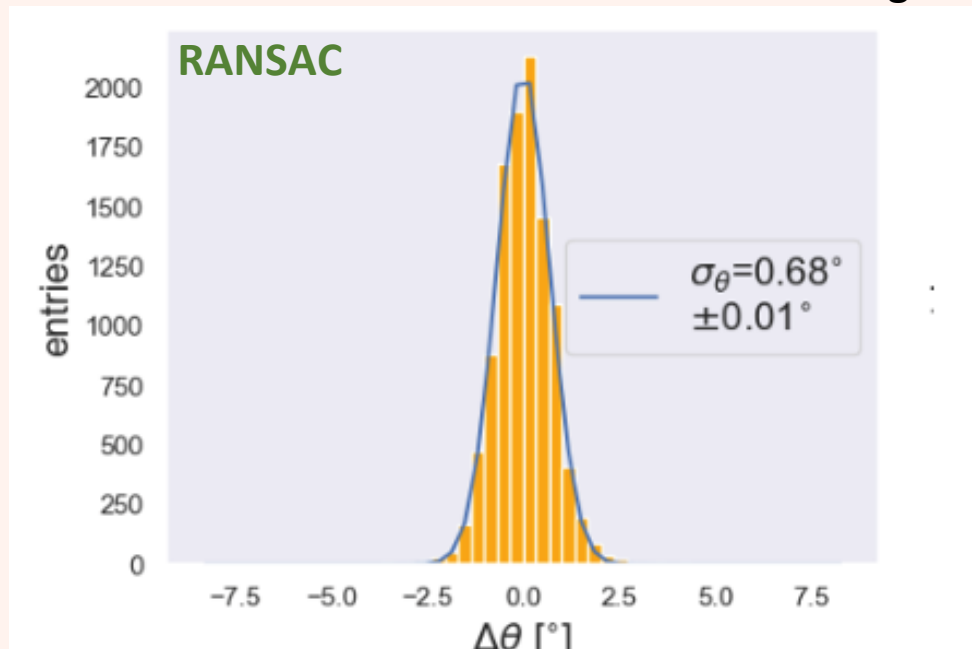
# Summary performances

➤ **RANSAC** exceeds both the **Hough transform** and the **former reconstruction (straightness check)** performances in: angular resolution, electrons and protons rejection and random outlier hits mitigation.



Angular residuals

[Bajou et al., 2021](#)



# Muography of a VHS: application to La Soufrière

From muon counts to dome density estimates

# Workflow

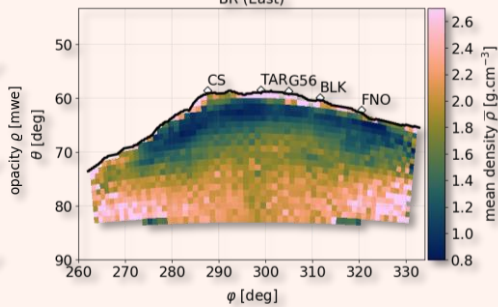
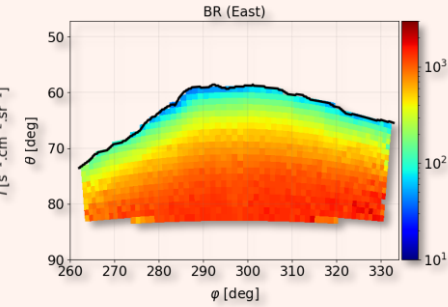
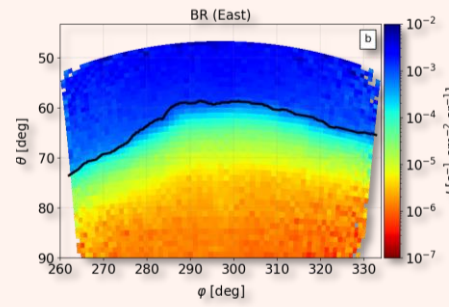
Raw datasets

Tomography

Calibration

Event reconstruction with tracking algorithm &  $\mu$ -candidate selection

Model/  
simulation

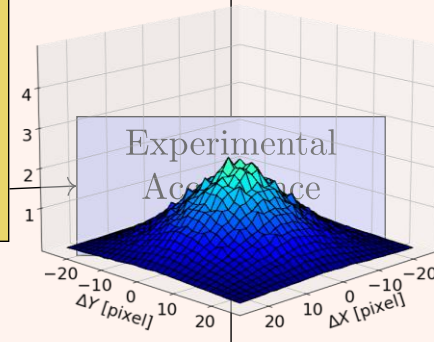


Outgoing flux  
 $I$

Opacity  
 $\rho$

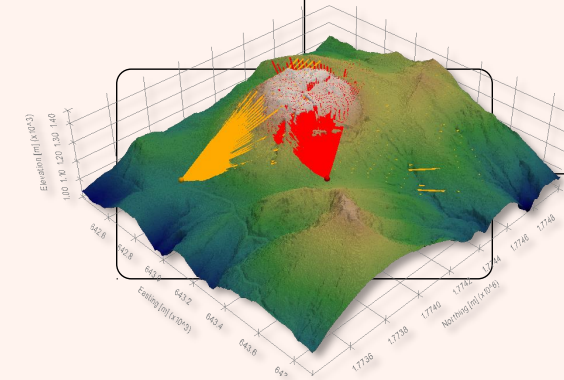
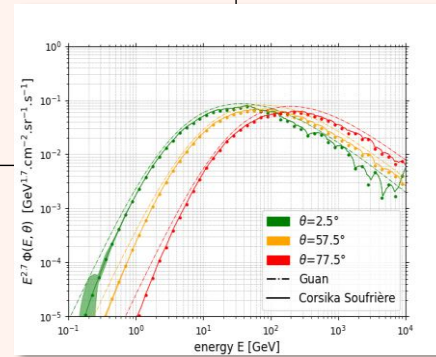
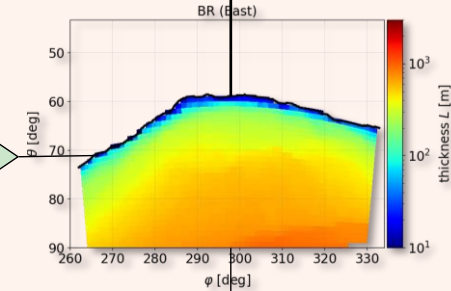
Mean Density  
 $\bar{\rho}$

Inverse problem  
input



Forward problem  
solving

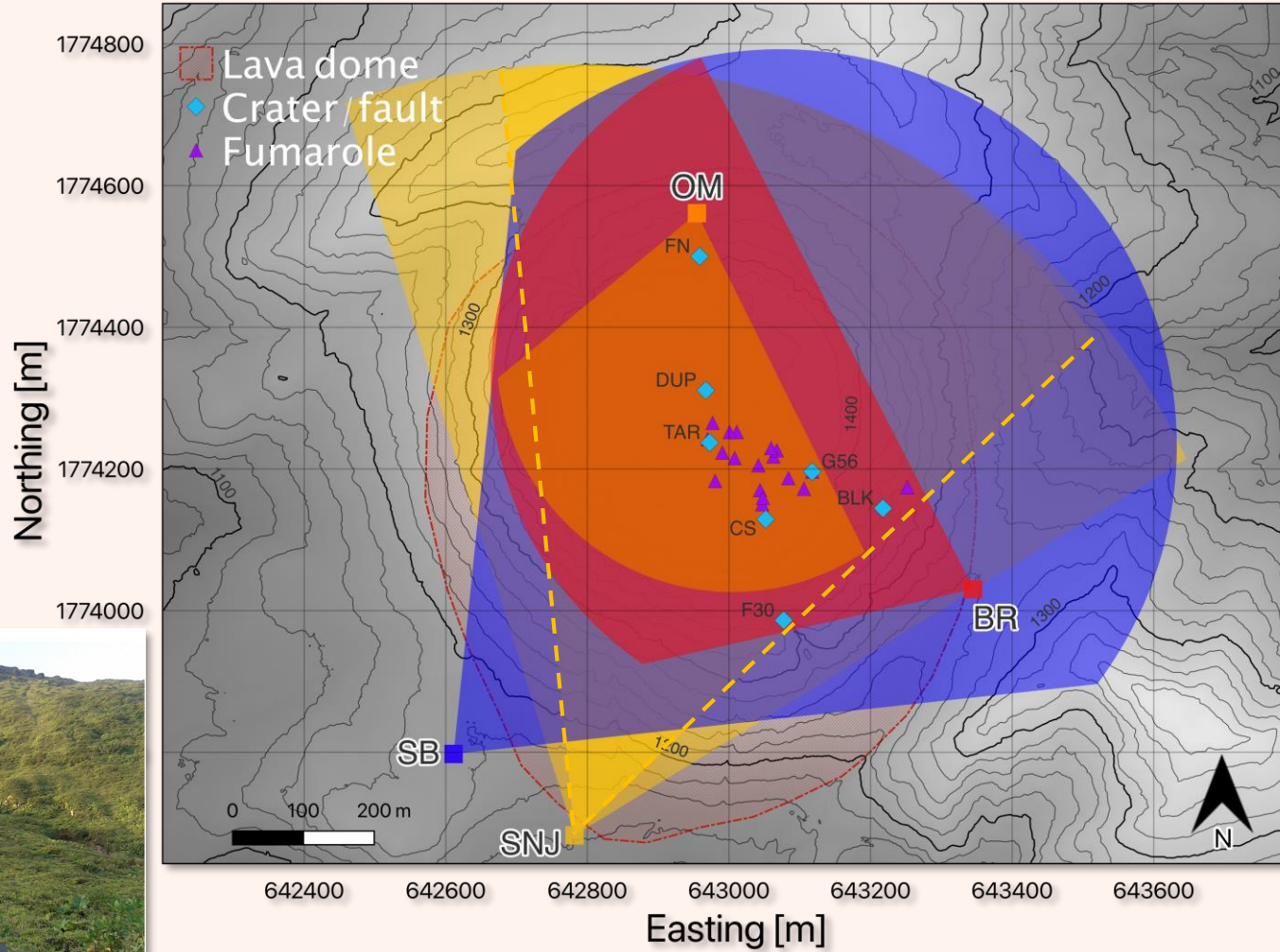
Travel lengths  
 $L(\theta, \varphi)$



Estimated with CORSIKA

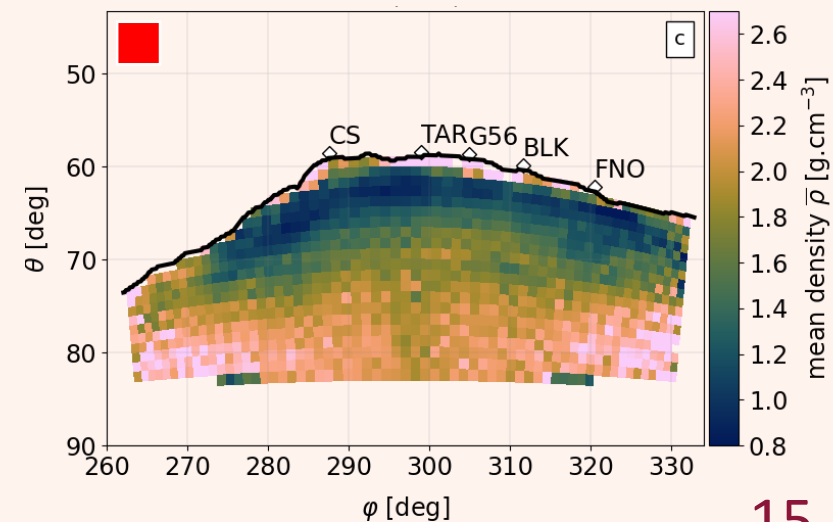
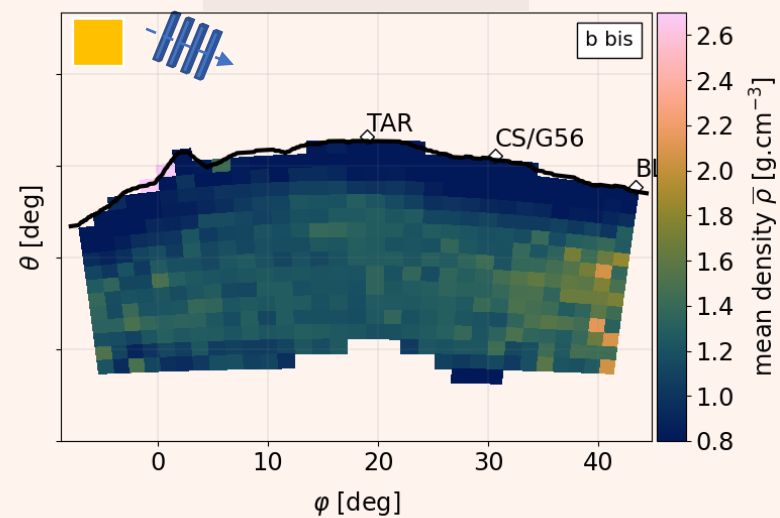
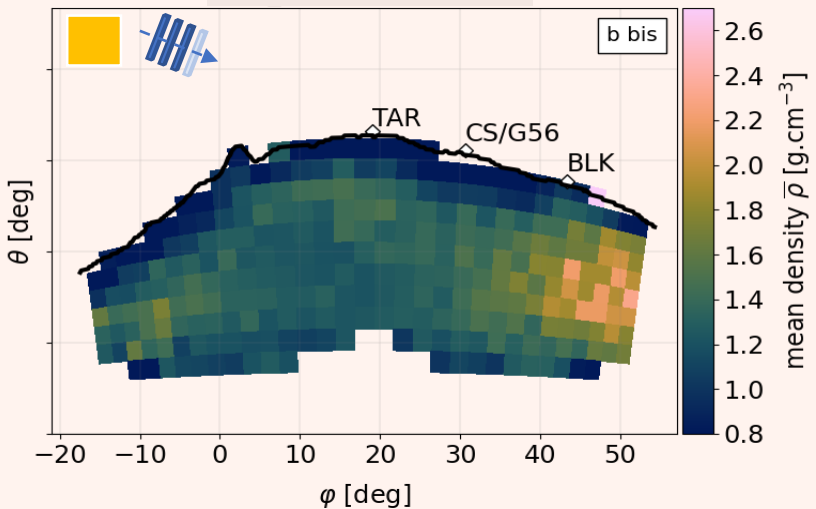
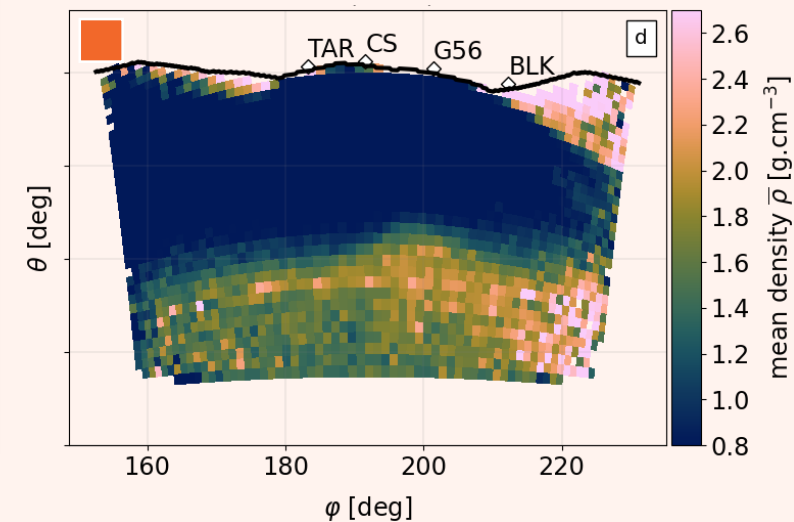
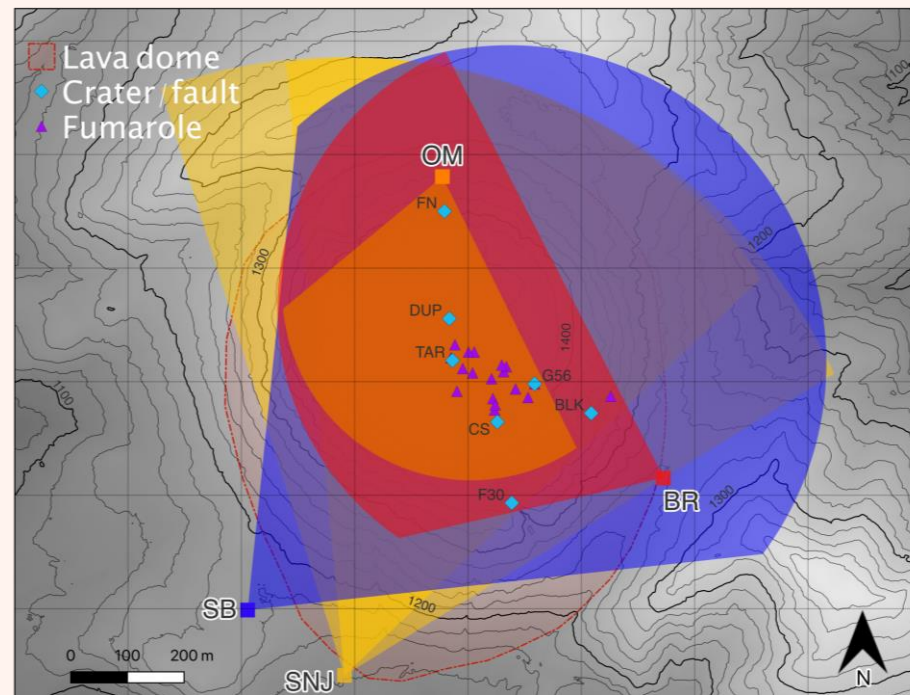
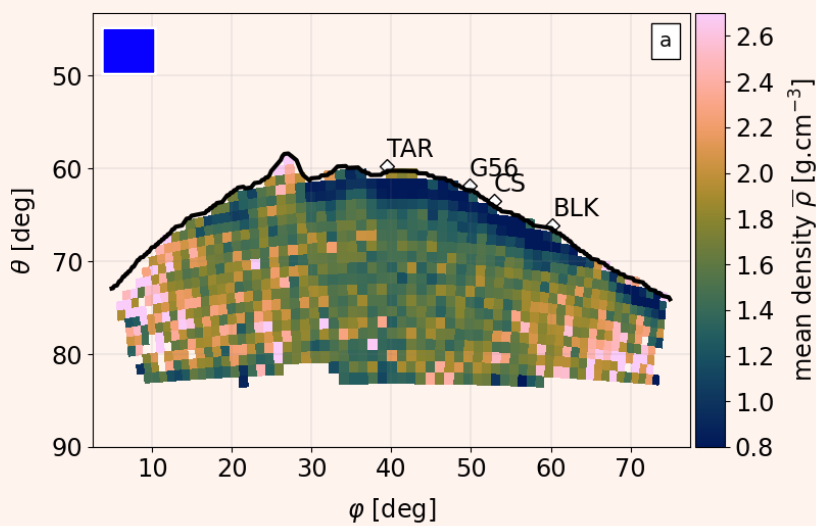


# La Soufrière muon survey

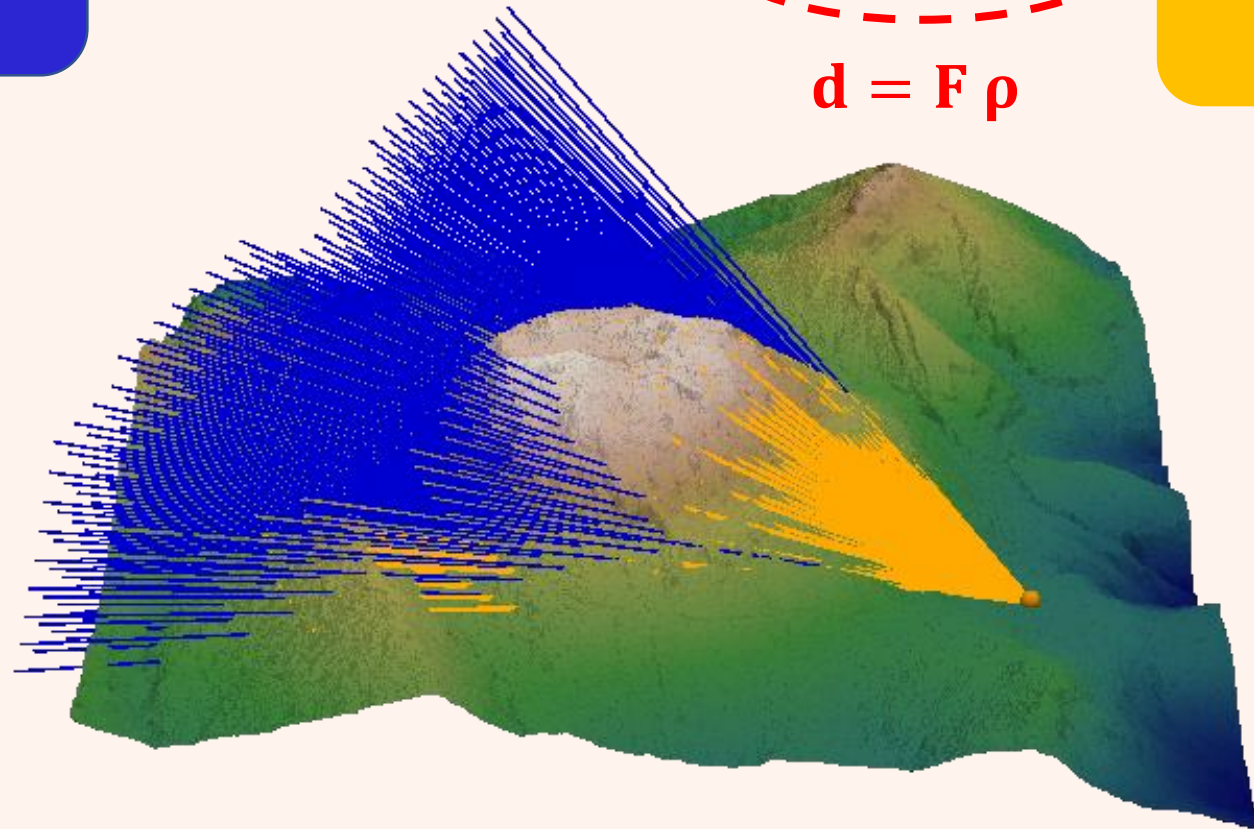
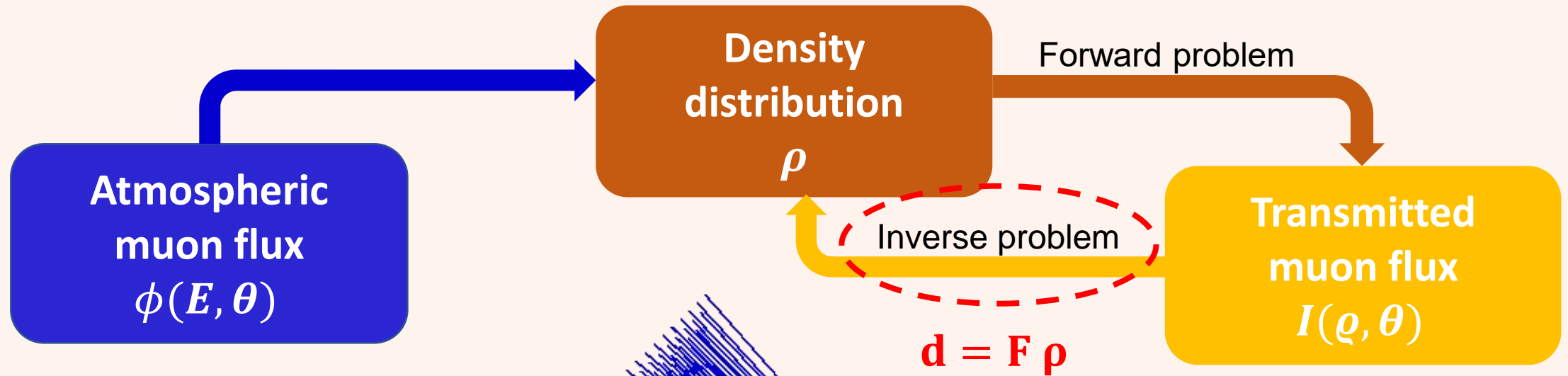


# Mean density maps

Bajou et al., *in rev.*



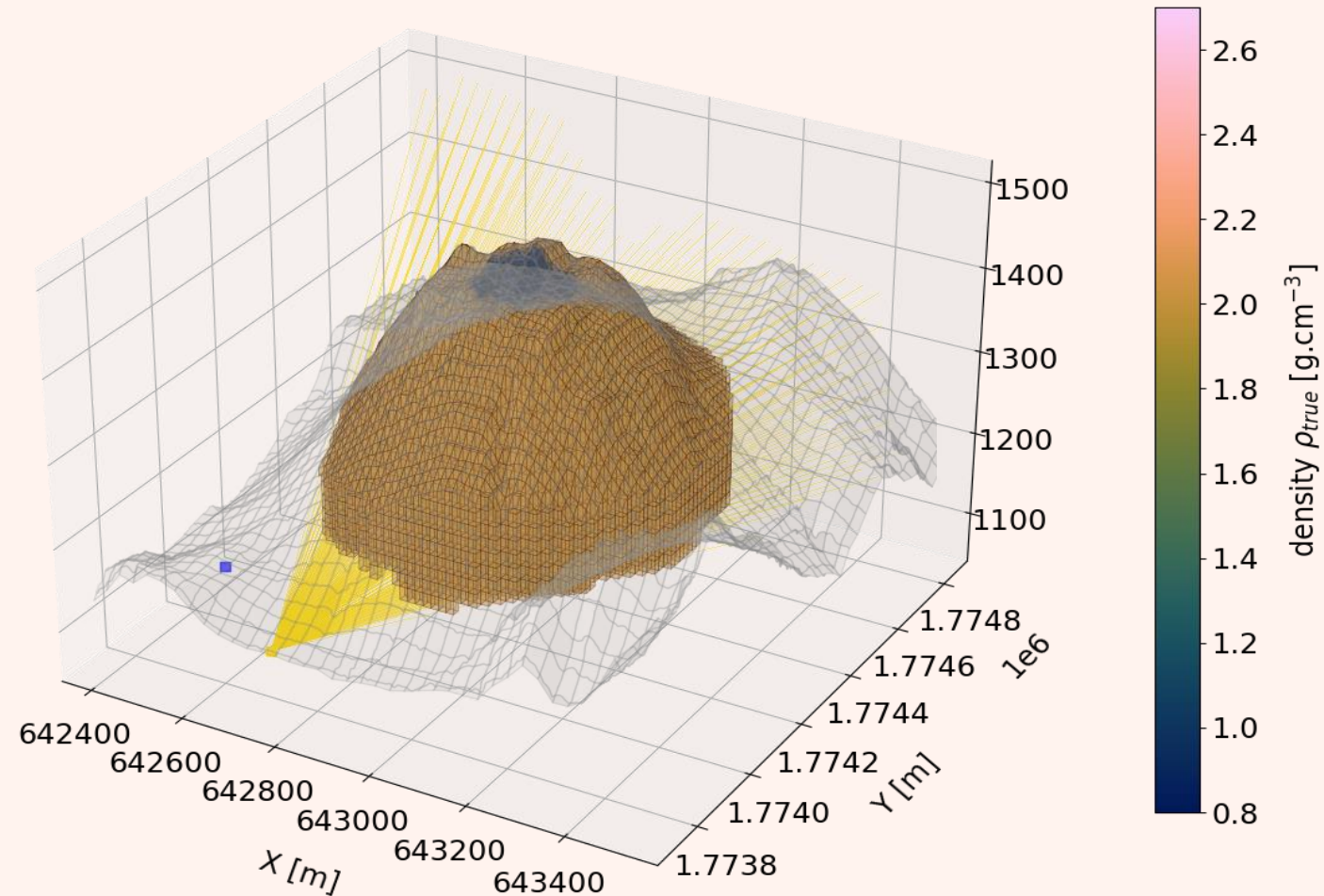
# 3-D Density Modeling





# Lava Dome volume

- La Soufrière digital elevation model (DEM)\*, 1 m resolution
- Discretized into **16032 voxels** of **16 m<sup>3</sup>**
- Total volume covered by the 4 telescopes: **1.5x10<sup>7</sup> m<sup>3</sup>** (~30% of the total dome volume)



\* Obtained from the SHOM campaign: Litto3D (LiDAR)

# Bayesian inversion (Tarantola & Valette)

## Minimization cost function :

$$\phi = (\underset{\text{Data}}{\mathbf{d}} - \underset{\text{Data covariance}}{\mathbf{F}} \underset{\text{Model parameters}}{\boldsymbol{\rho}})^T \underset{\text{Data covariance}}{\mathbf{C}_d}^{-1} (\mathbf{d} - \mathbf{F}\boldsymbol{\rho}) + (\boldsymbol{\rho} - \underset{\text{Prior density}}{\boldsymbol{\rho}_0})^T \underset{\text{Model regularization covariance}}{\mathbf{C}_\rho}^{-1} (\boldsymbol{\rho} - \boldsymbol{\rho}_0)$$

## Posterior density estimates

$$\tilde{\boldsymbol{\rho}} = \boldsymbol{\rho}_0 + (\mathbf{F}^T \mathbf{C}_d^{-1} \mathbf{F} + \mathbf{C}_\rho^{-1})^{-1} \mathbf{F}^T \mathbf{C}_d^{-1} (\mathbf{d} - \mathbf{F}\boldsymbol{\rho}_0)$$

**Model regularization:**  $C_\rho(i, j) = \sigma_\rho^2 e^{-\frac{r(i, j)}{l_c}}$  :

where  $r(i, j)$  : distance inter-voxels,  $\sigma_\rho$  : prior error,  $l_c$  : correlation length

👉 Tuning reg. parameters with synthetic data inversion:

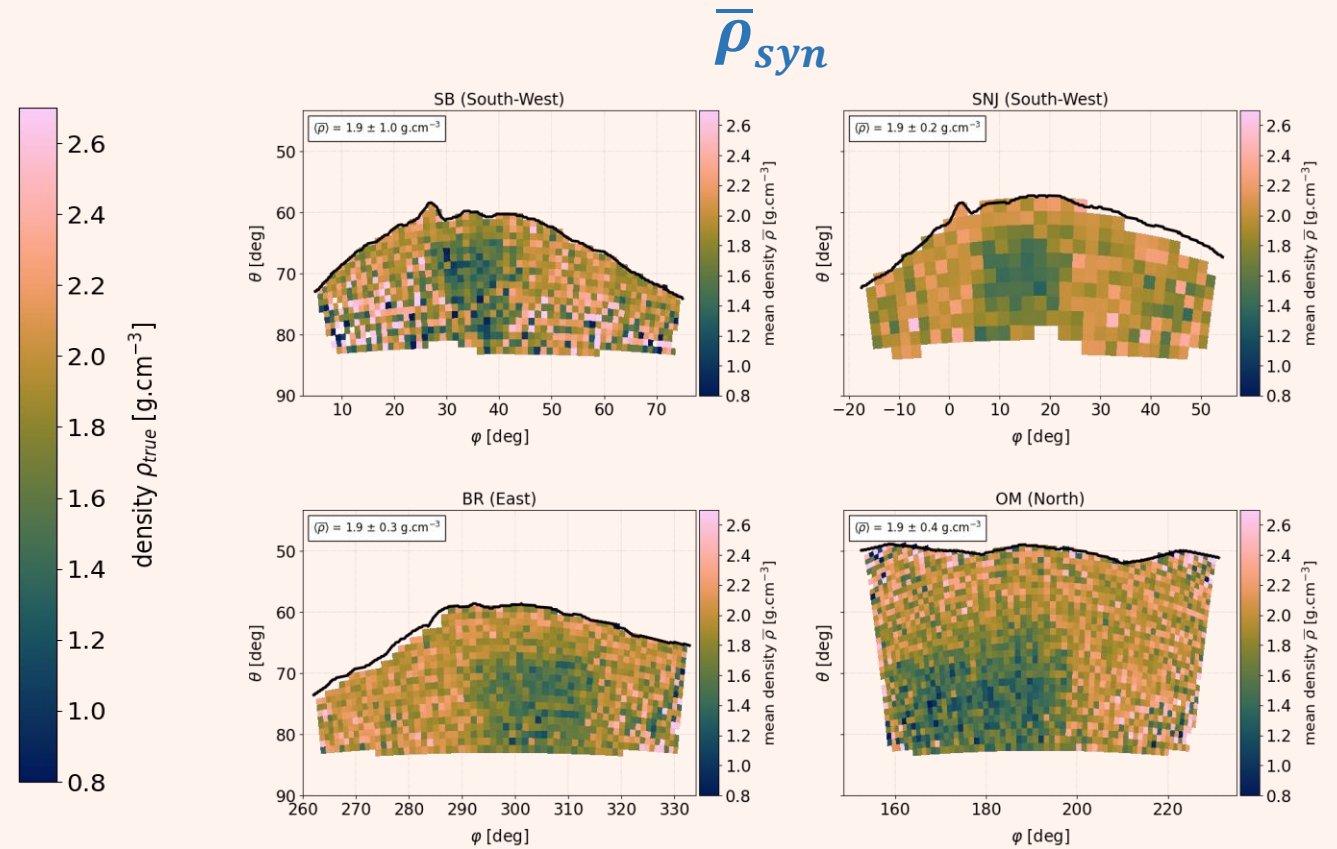
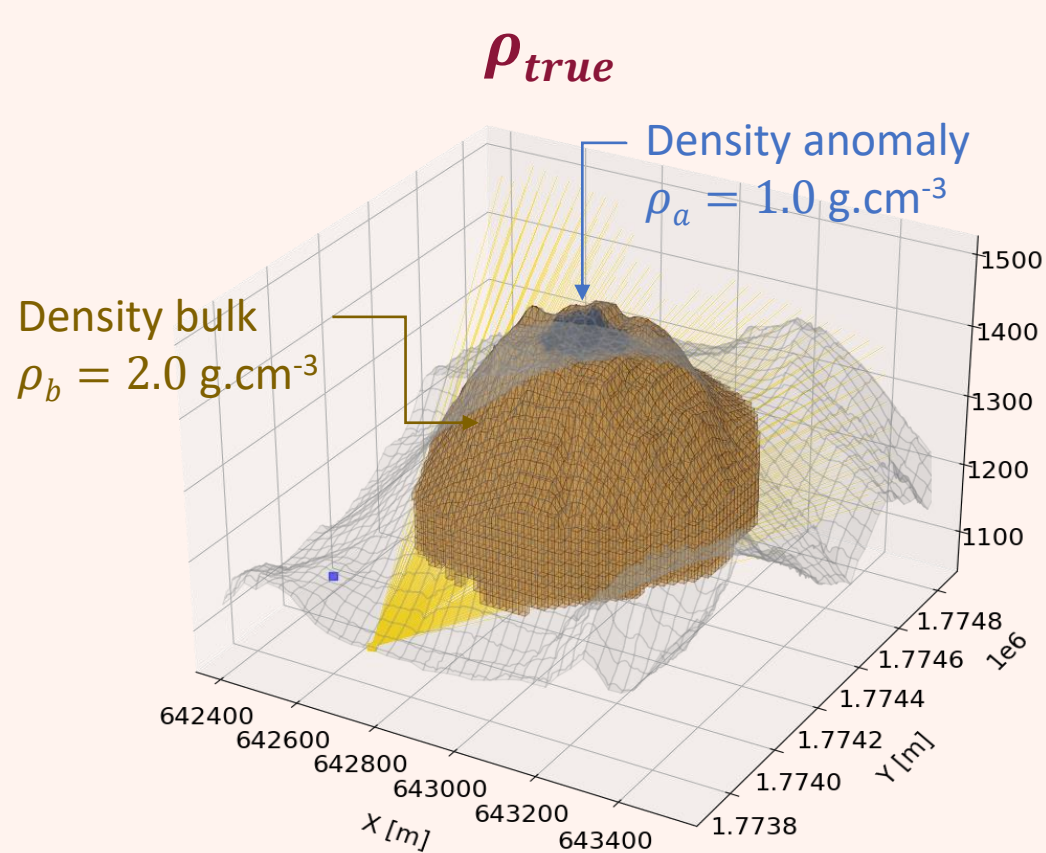
$$(\sigma_\rho, l_c) = (0.3 \text{ g.cm}^{-3}, 200 \text{ m})$$

# Synthetic tests

$$\bar{\rho}_{\text{syn}} = \rho_{\text{true}} + \mathcal{N}(0, \Delta_{\text{real}} \rho_{\text{true}}) : \text{synthetic data set}$$

where  $\mathcal{N}(0, \Delta_{\text{real}} \rho_{\text{true}})$  : Gaussian noise

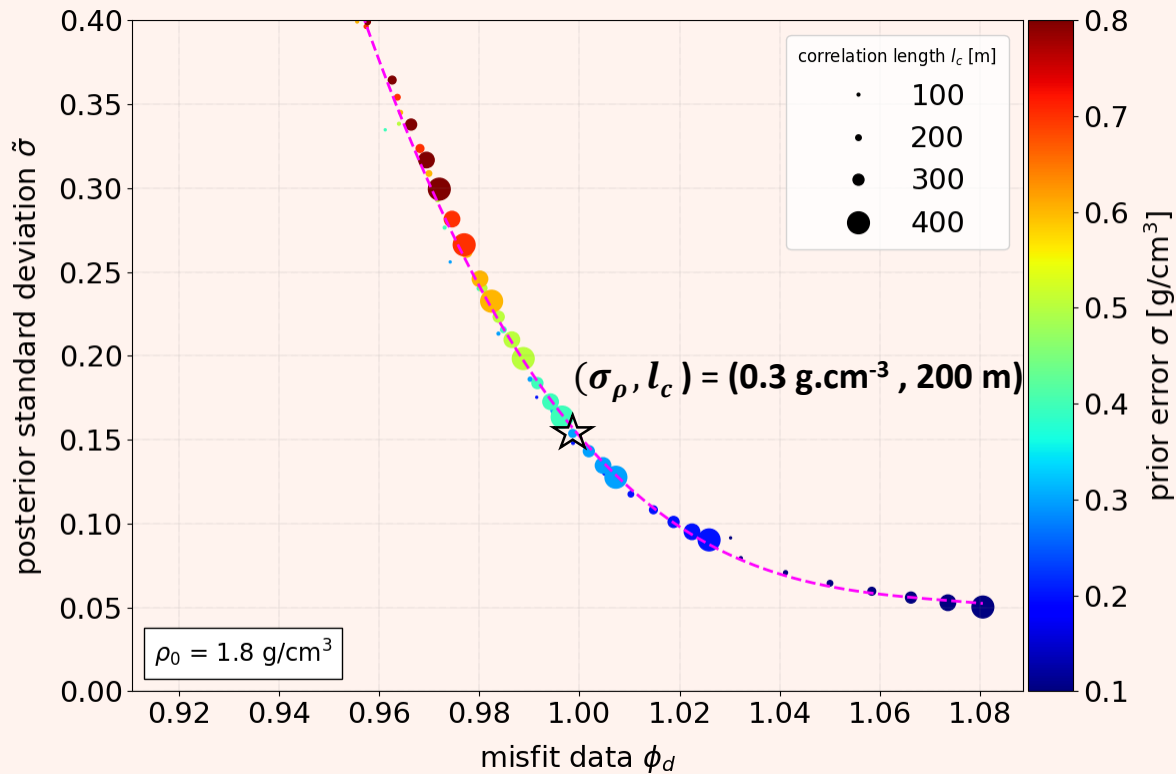
with  $\Delta_{\text{real}}$  : total relative uncertainty on real data estimates  $\bar{\rho}$



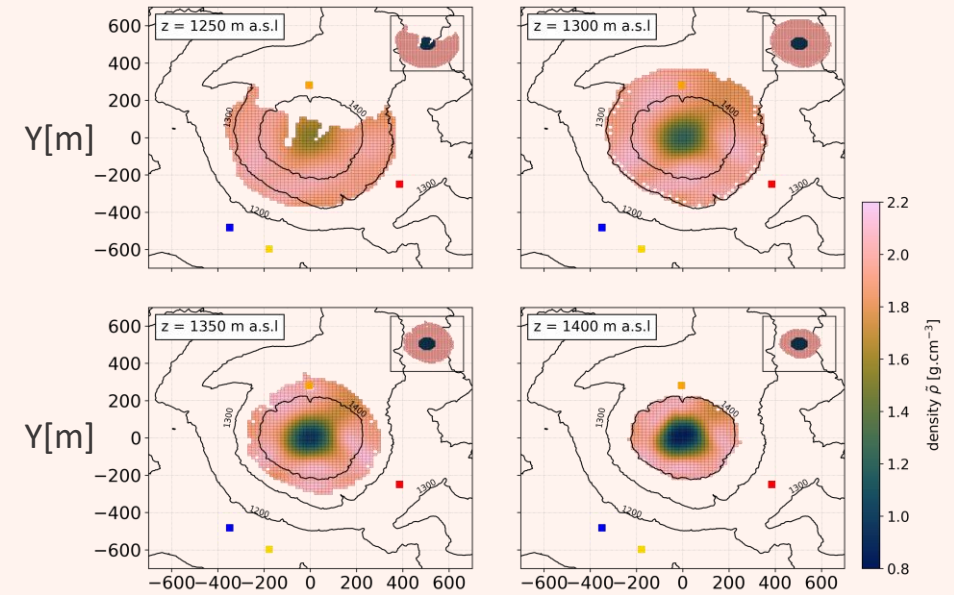
# Synthetic tests

## Tuning regularization parameters

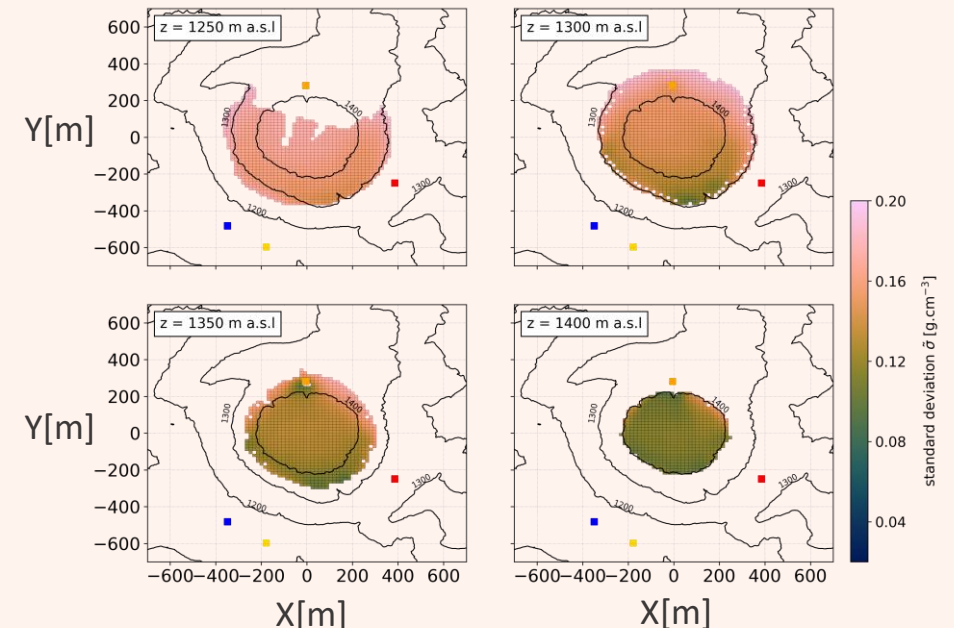
(Harris et al., 1987; Barnoud et al., 2019)



## Density estimates $\tilde{\rho}$

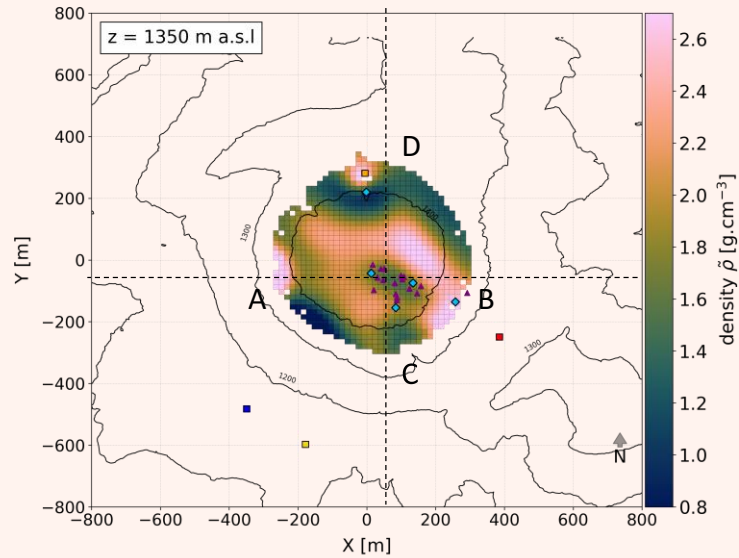


## Standard deviation $\tilde{\sigma}$



# 3-D Density estimates (real data)

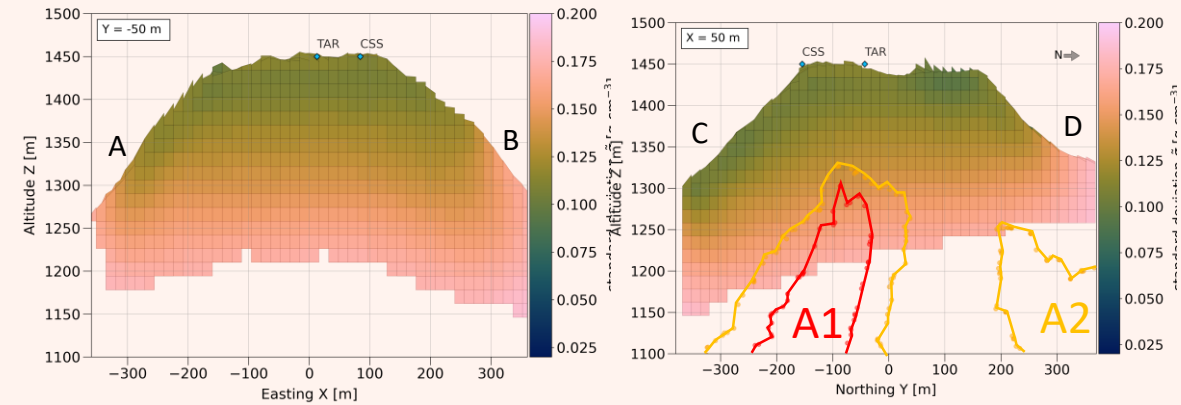
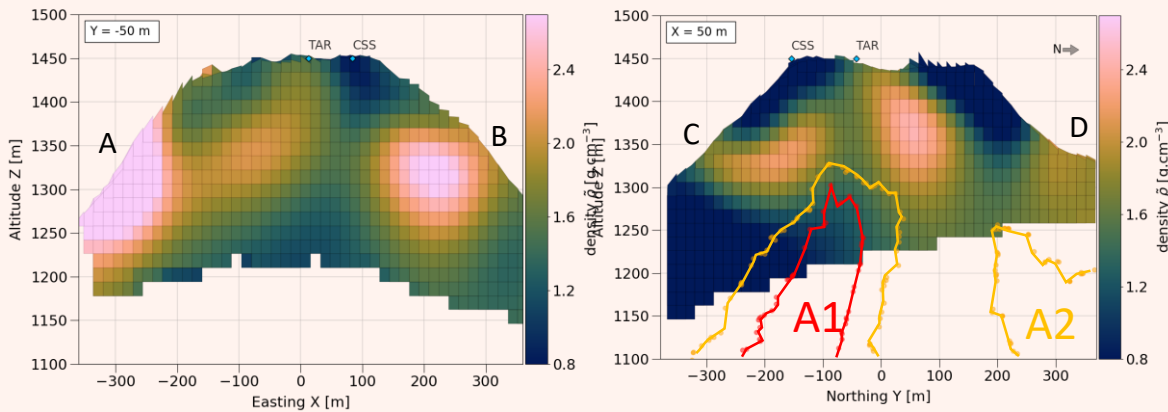
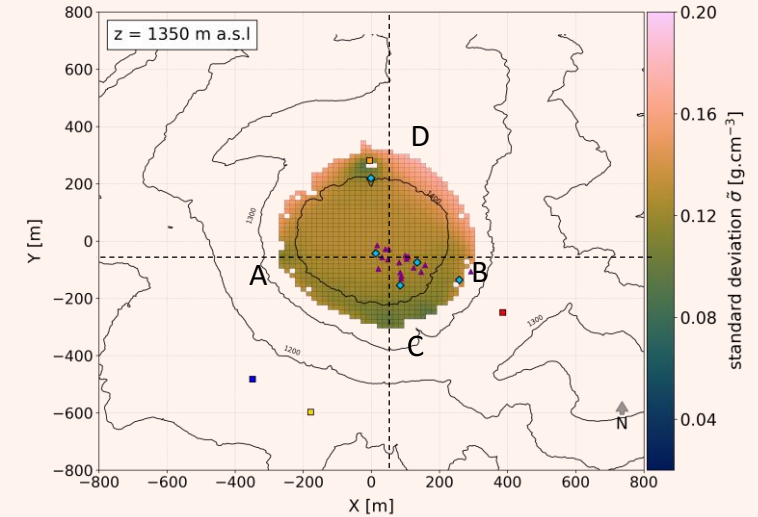
Posterior density  $\tilde{\rho}$



- Highest-density located on the East and West flanks
- Low-density anomaly coincides with high-conductivity conduit

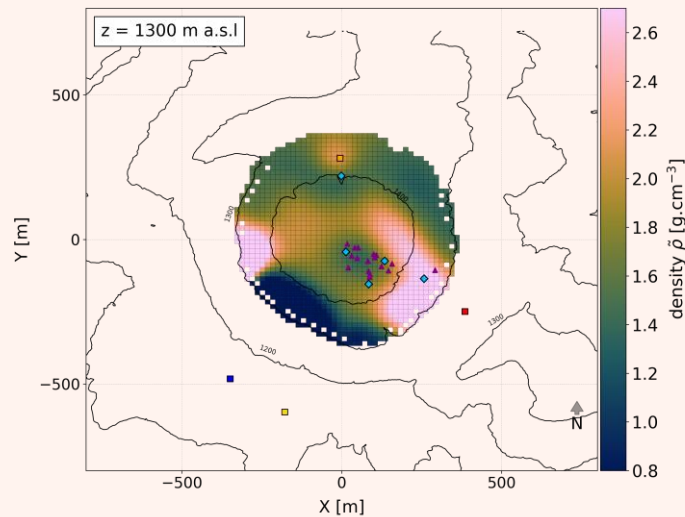
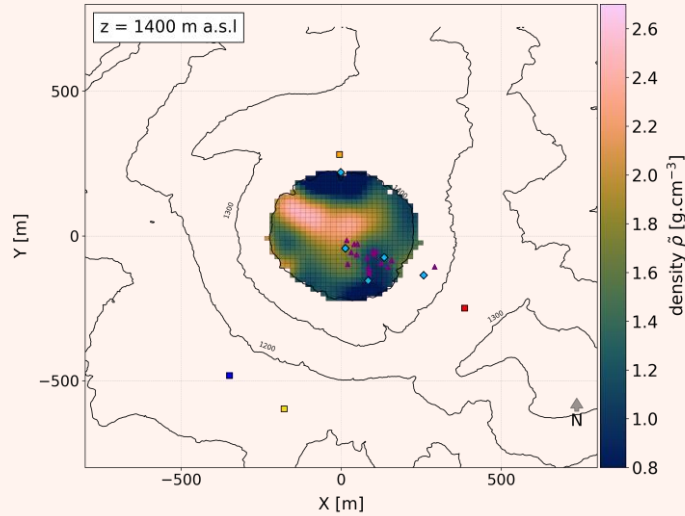


Posterior standard deviation  $\tilde{\sigma}_\rho$

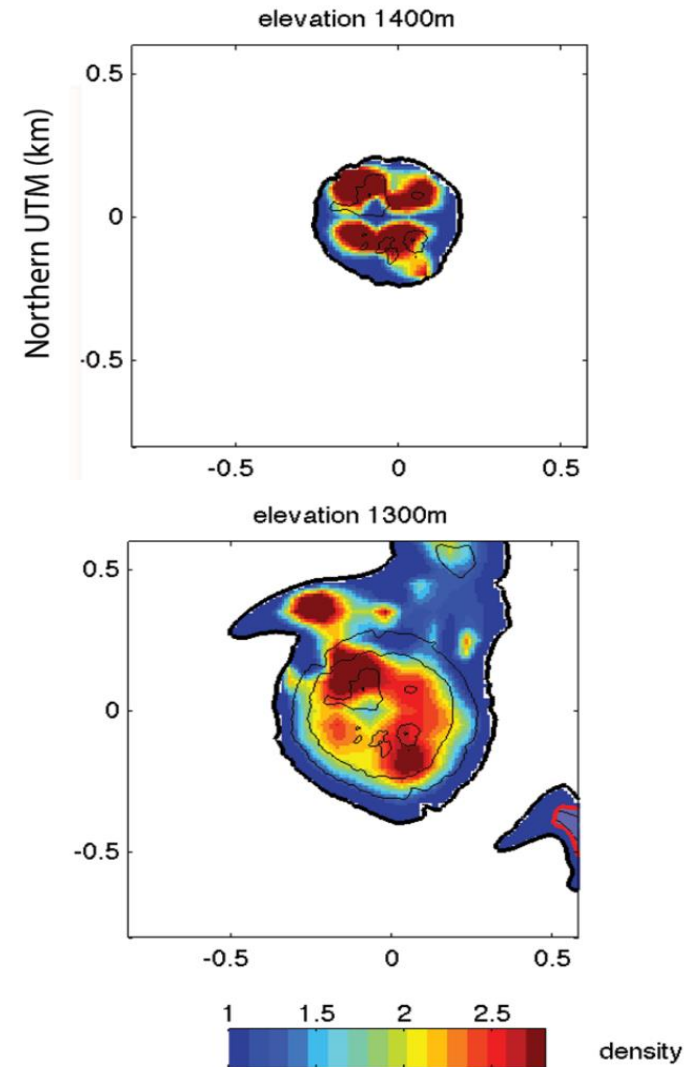


\*A1, A2 : conductivity anomalies from Rosas-Carbajal et al., 2016

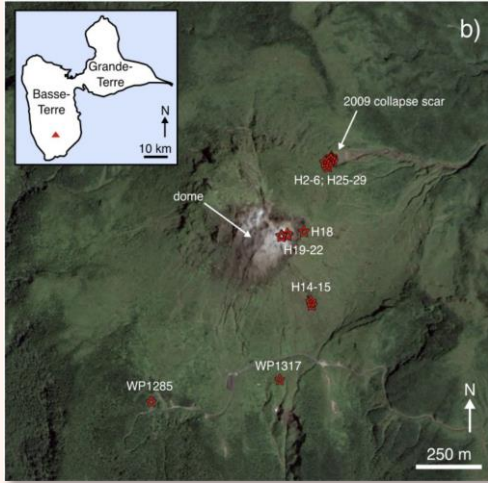
## New model 4 muon telescopes data



## Joint P-wave velocity - gravity data (Coutant et al., 2012)



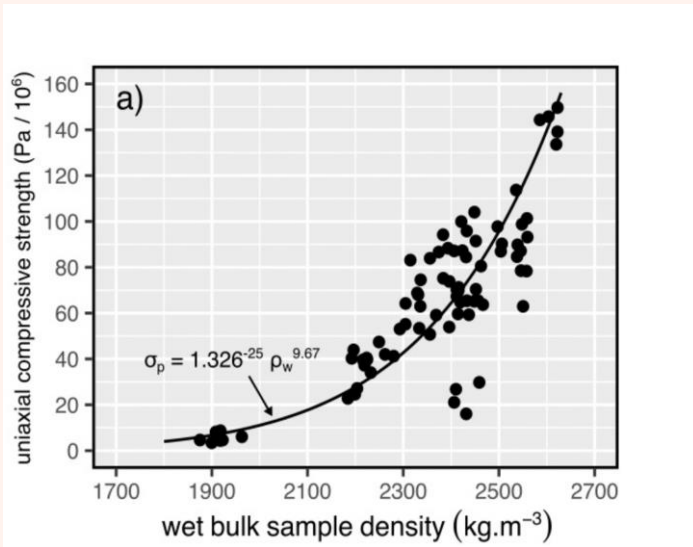
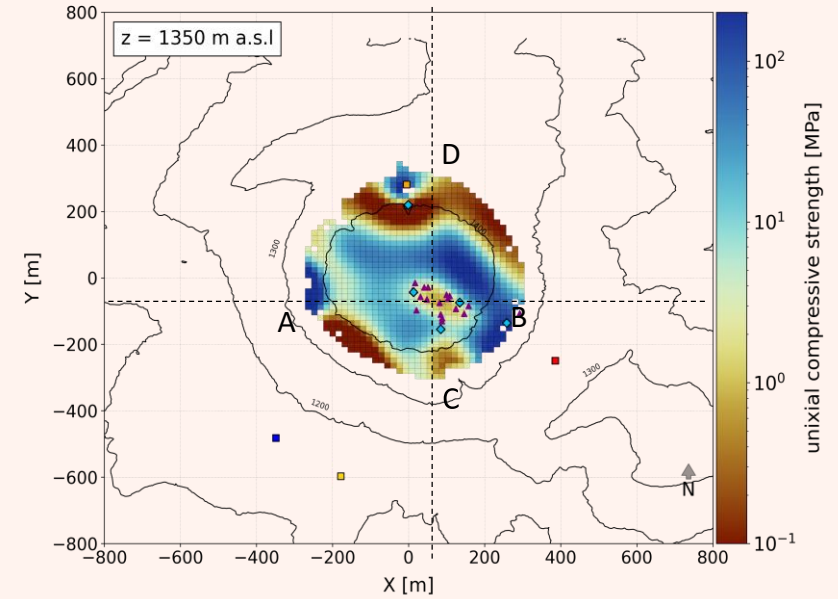
# Rock strength



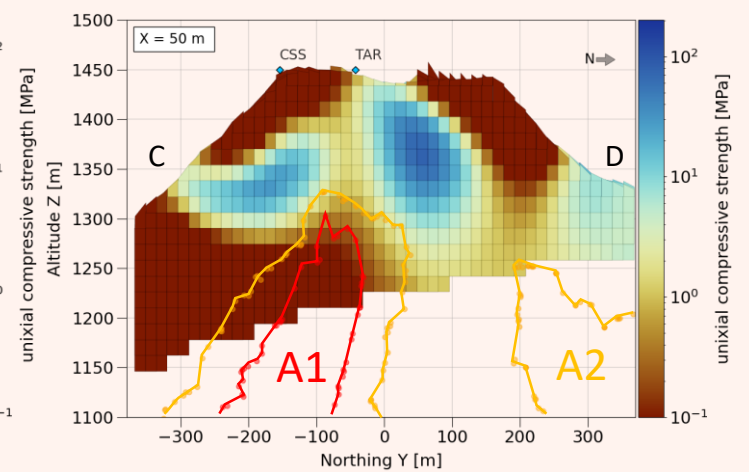
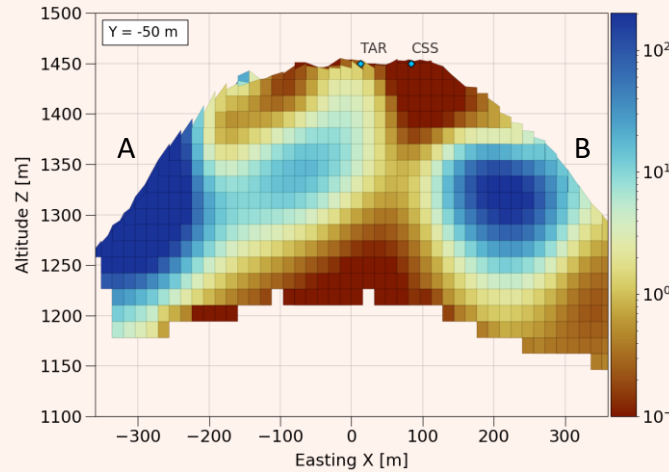
From lab measurements of uniaxial compressive strength (Heap et al. 2021) :

$$\sigma_p = C \rho_w^{9.67}$$

where  $C = 1.326 \cdot 10^{-25} \text{ MPa} \cdot (\text{kg} \cdot \text{m}^3)^{-9.67}$

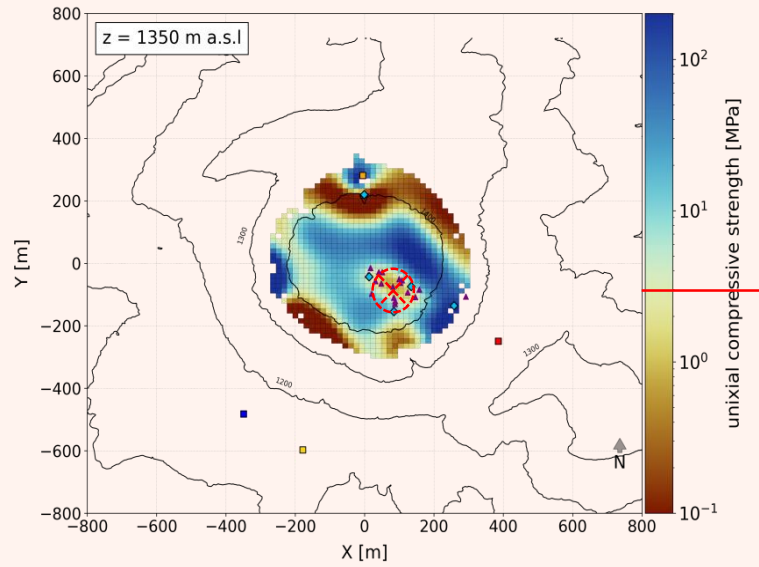


From Heap et al. 2021

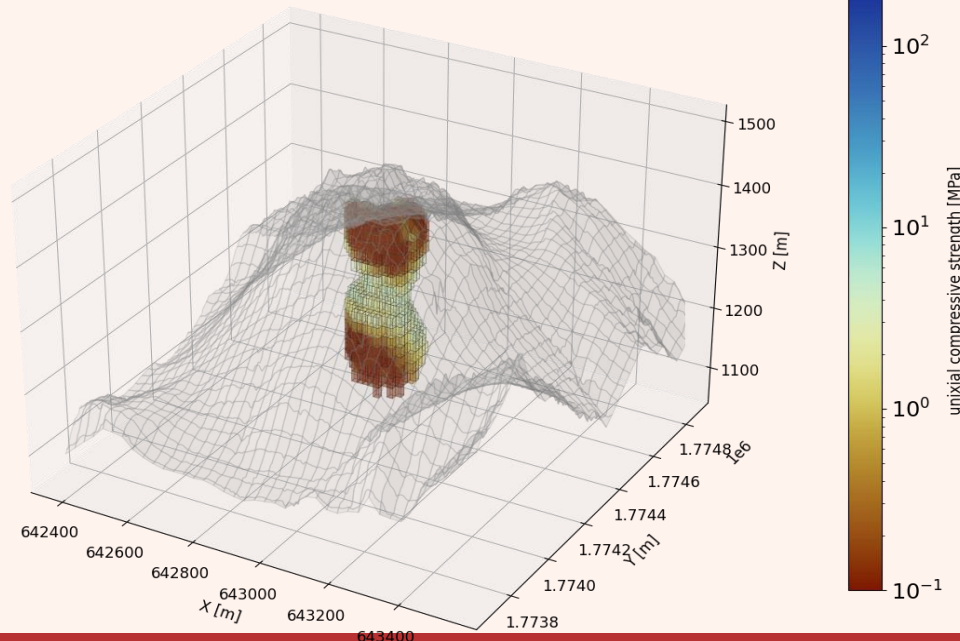
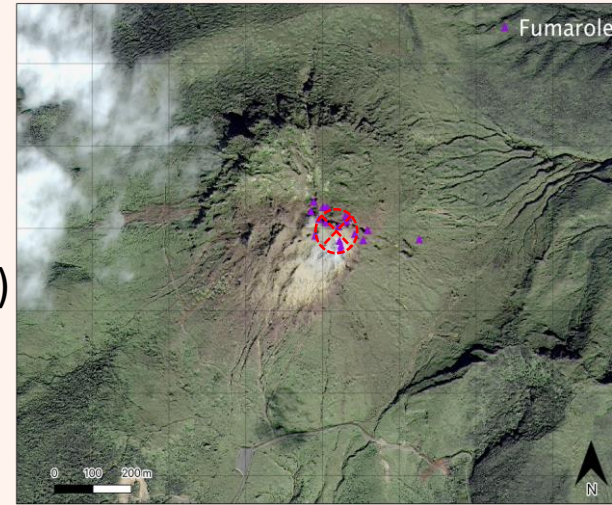


\*A1, A2 : conductivity anomalies

# Cratère Sud anomaly volume



3MPa (\*)



Detection of Southern low-strength anomaly ( $< 3$ MPa, Heap et al. 2021) covering **8.1-12.8%** of the dome's total volume ( $=5.10^7$ m<sup>3</sup>, Boudon et al. 2008)

(\*) Heap et al. 2021

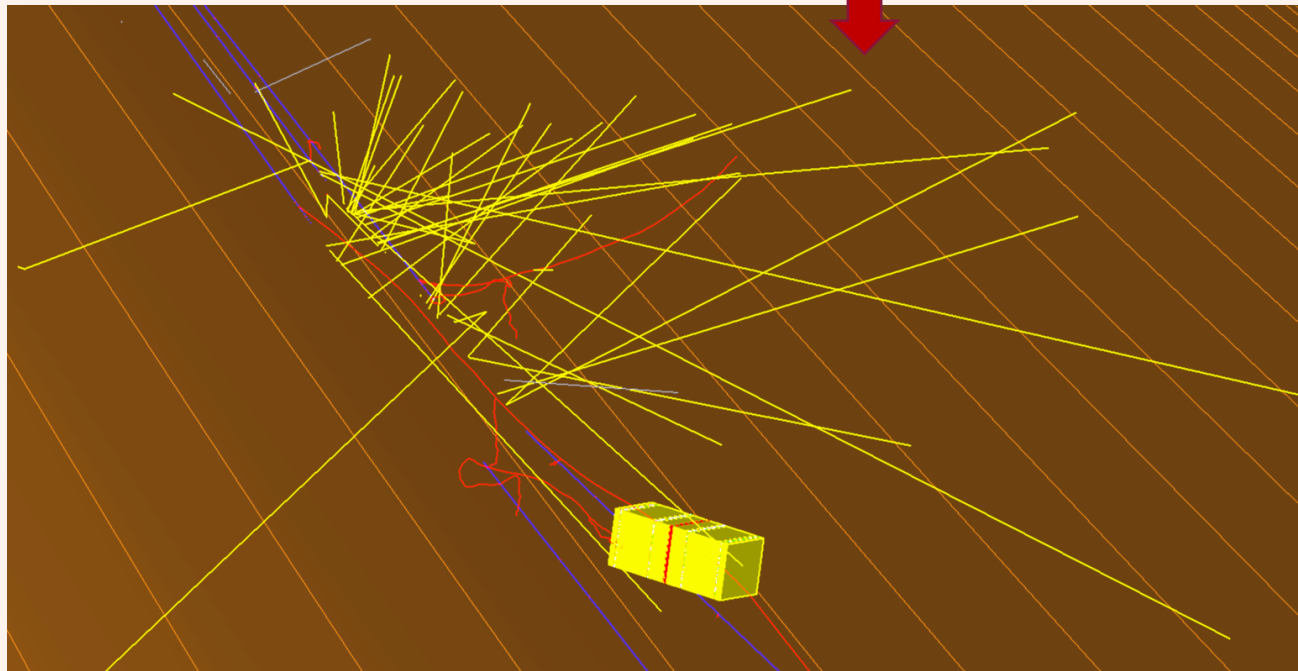


# Perspectives

## Improve absolute density estimation

Background characterization improvement :

- Correction from low-energy particle scattering estimation



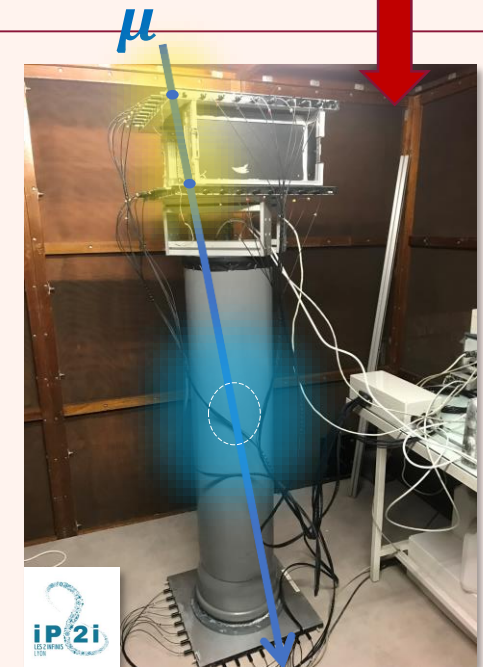
200 MeV muon scattering on standard rock

Future developments

Development of a **water-Cherenkov tank** to replace the passive lead shielding

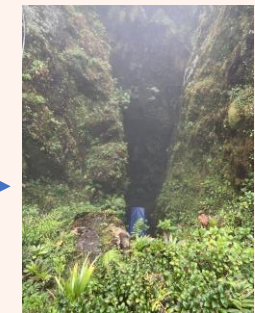
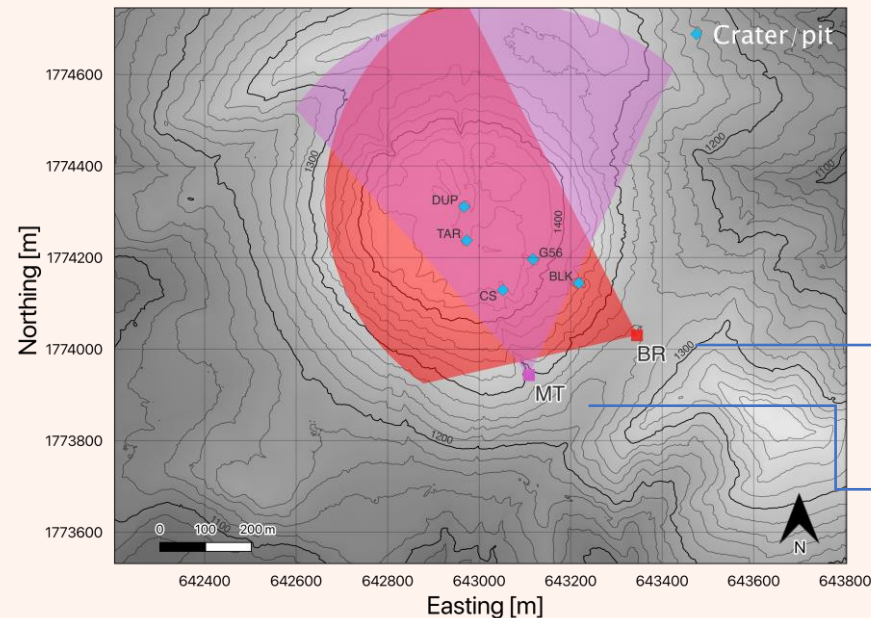
**GOAL: improve the rejection of GeV electron**

(cf. T. Avgitas' presentation)



# Perspectives

- Towards monitoring: temporal flux variations (Jourde et al., 2016)
- Combination with other geophysical data (e.g. gravimetry surveys)
- Current data taking:



# Summary and Outlook

- La Soufrière = ideal natural laboratory to apply and develop muography
- Improved and versatile data processing (tracking) and Monte-Carlo simulation pipelines to assess detector performances
- The first inversion of muon data with a wide 4-telescope coverage
- Low-density anomaly in the Cratère Sud region corresponding to the most active and mechanically weak zone on La Soufrière dome summit (8-12% of the dome total volume)

## Future developments

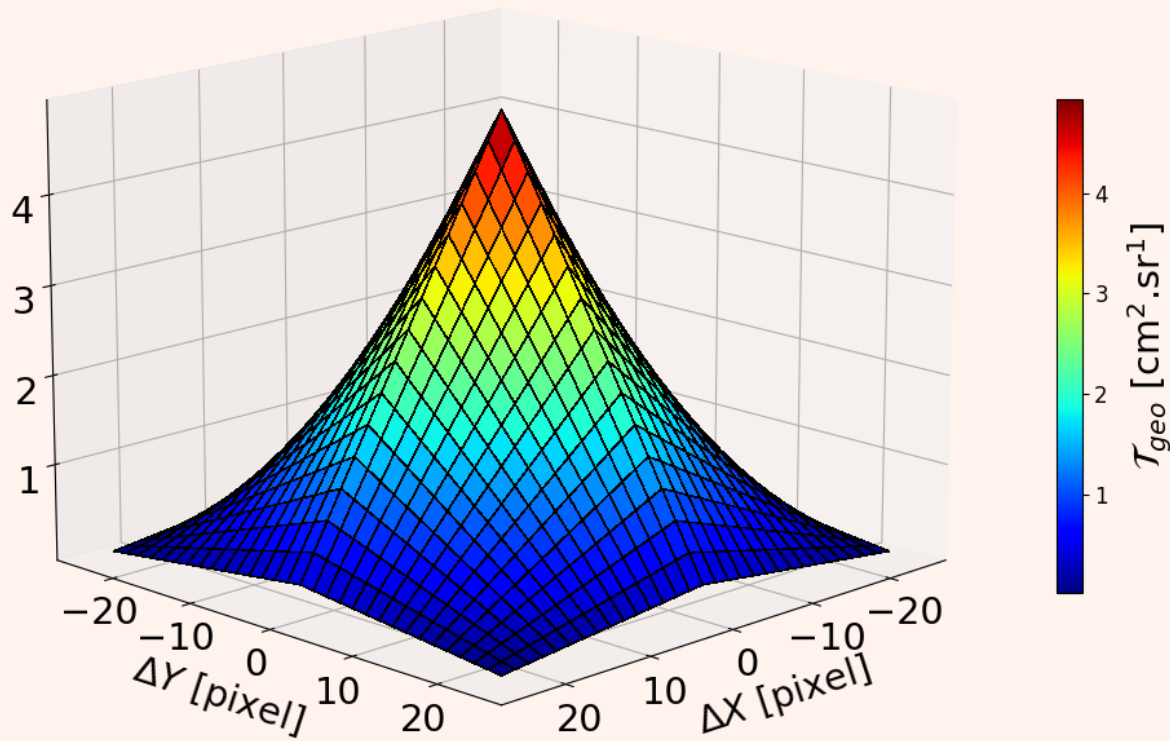
- Instrumental upgrades and better rejection of the low-energy physical background
- Temporal monitoring
- Joint-inversion with gravimetry data

Backup slides

# Experimental acceptance

To correct for the telescope detection efficiency in the transmitted flux estimation, we assess an **experimental acceptance**  $\mathcal{J}_{exp}$  from the ratio between the open-sky event rate and a reference open-sky flux.

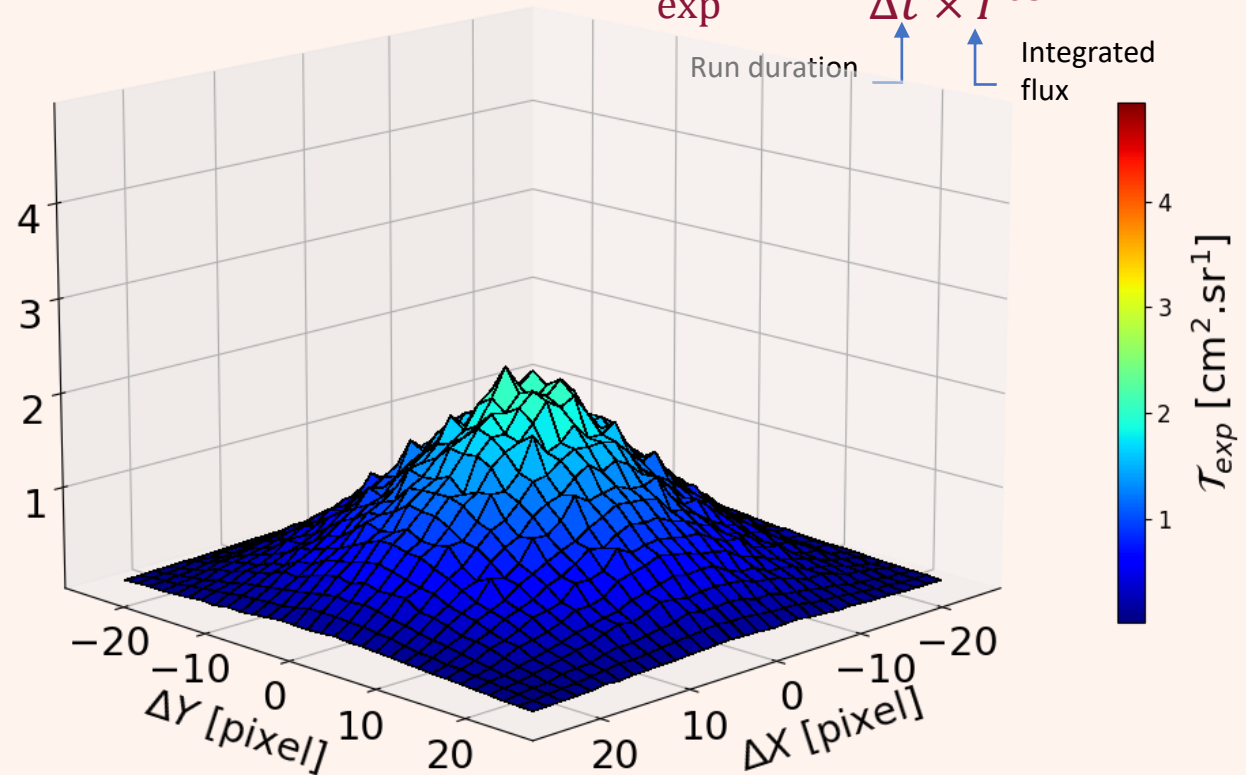
Geometrical acceptance  $\mathcal{J}_{geo}$



Experimental acceptance  $\mathcal{J}_{exp}$

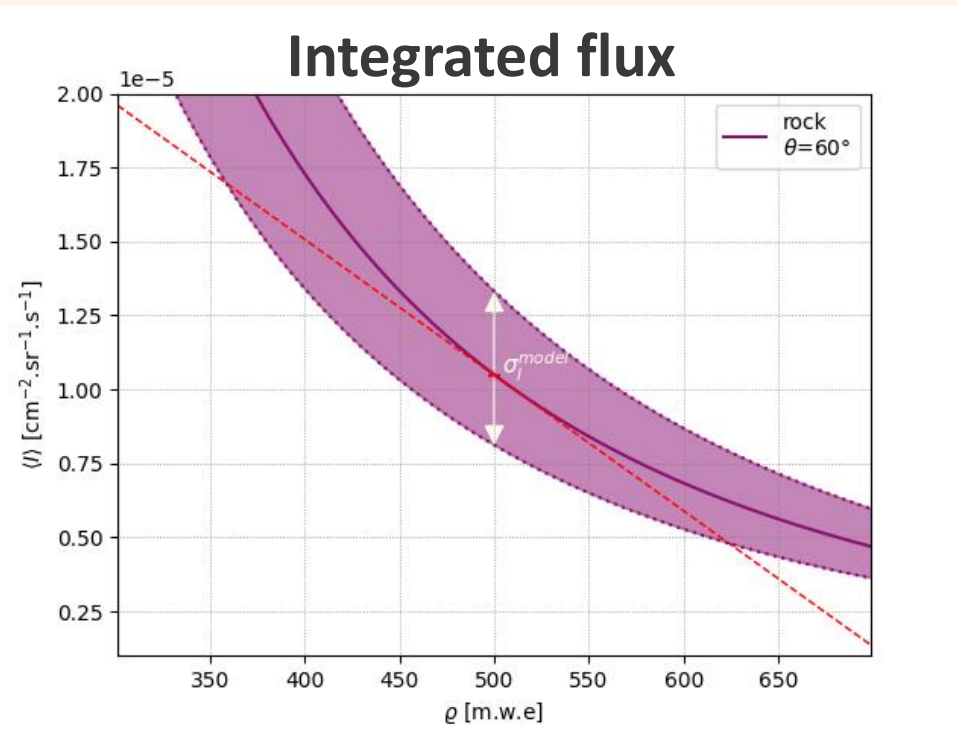
$$\mathcal{J}_{exp} = \frac{N^{OS}}{\Delta t \times I^{OS}}$$

Annotations:  $N^{OS}$  is labeled "Number of detected particles",  $\Delta t$  is labeled "Run duration", and  $I^{OS}$  is labeled "Integrated flux".

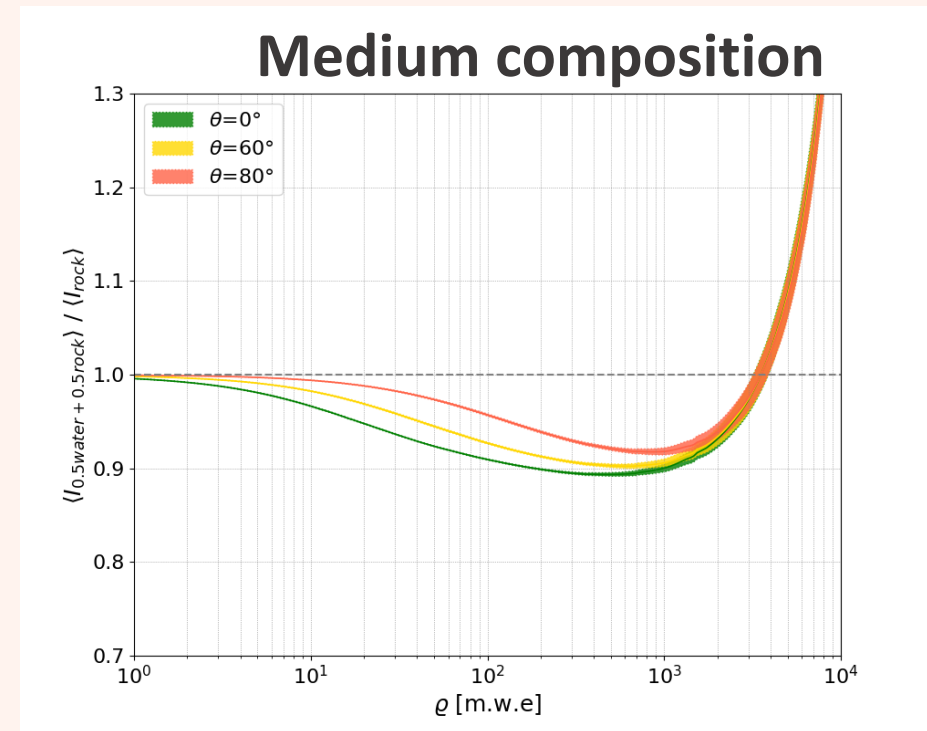


Here for a 4-panel telescope

# Systematic effects



+

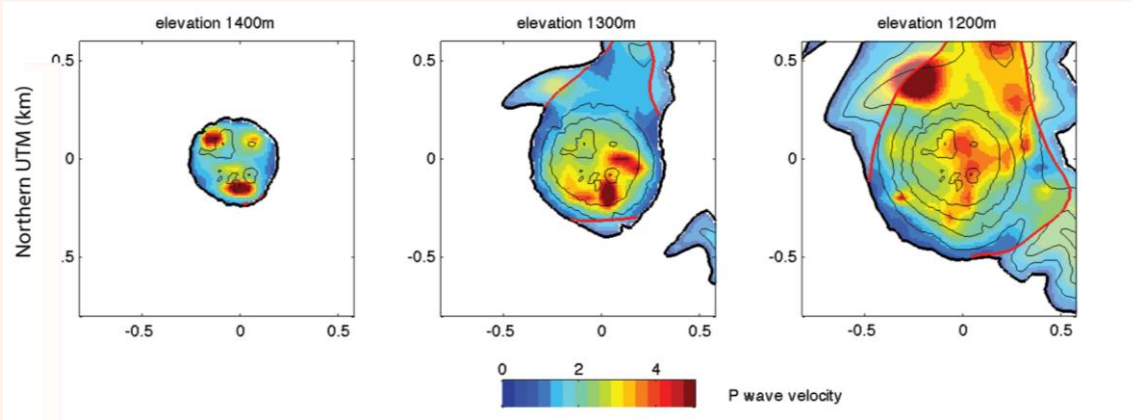


+ Telescope orientation

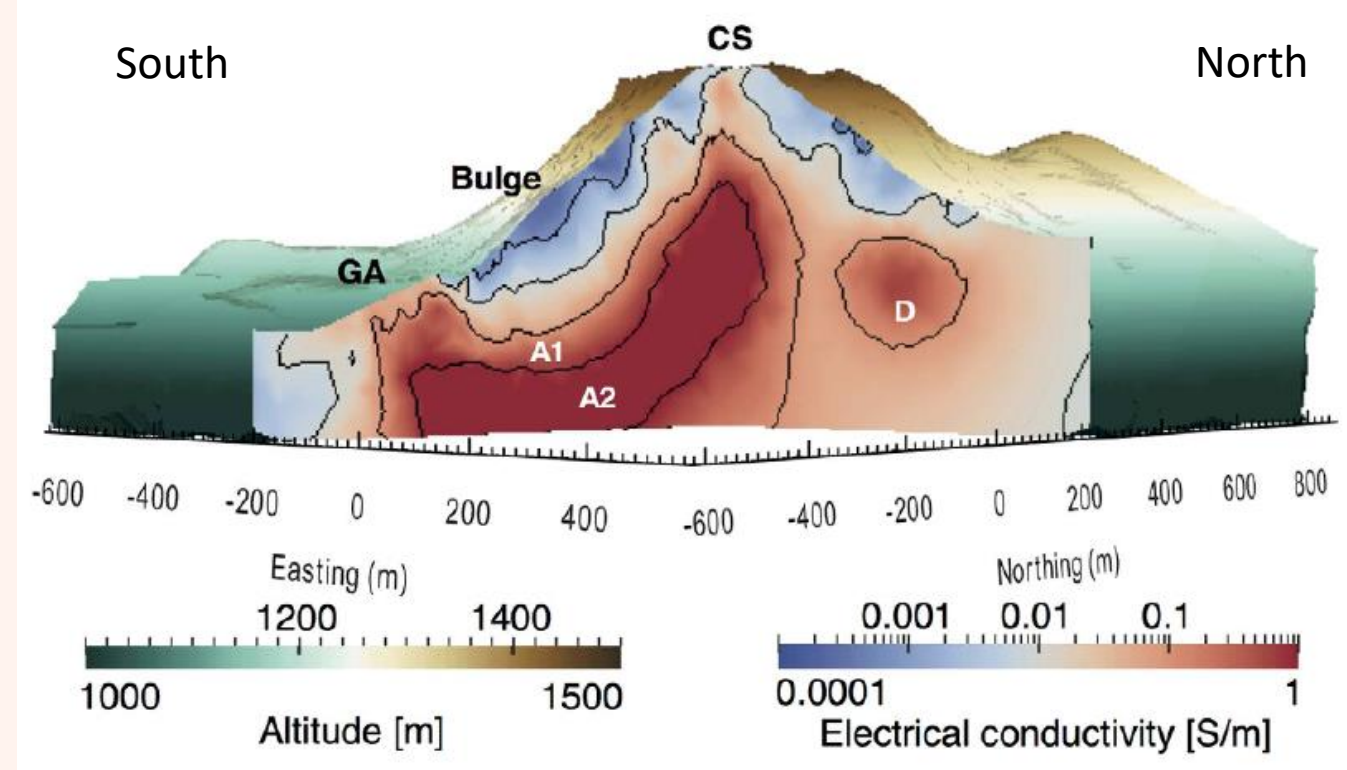
+ Detection efficiency  $\rightarrow$  acceptance  $\mathcal{T}_{\text{exp}}$

# Structural characterization with tomography

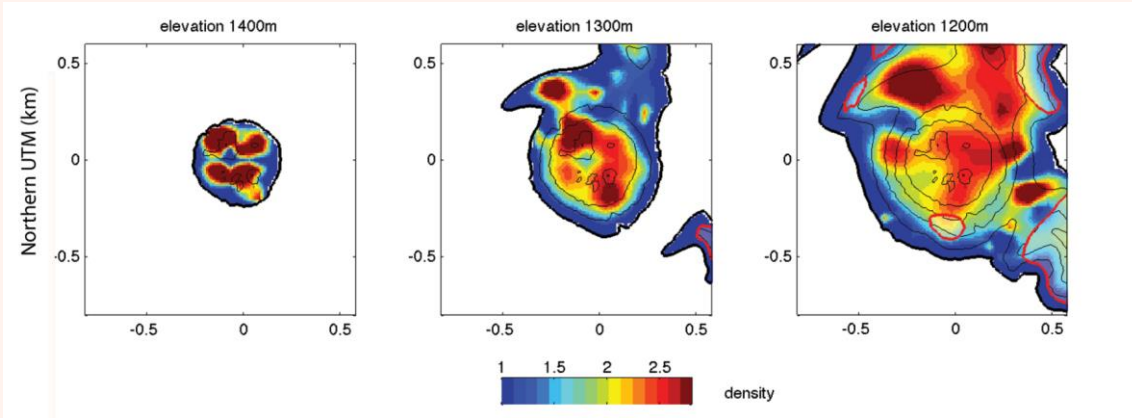
## Seismic P-waves velocity model



## Electrical conductivity model



## Density model (velocity + gravimetry data)



Coutant et al. 2012

Rosas-Carbajal et al. 2016



# Minimal acquisition time

$$\Delta T > \frac{\gamma^2}{\mathcal{J}_{geo}} \frac{I(\varrho)}{\Delta I(\varrho, \Delta\varrho)^2}$$

(Lesparre et al.2010)

where

$\gamma$  : number of Gaussian standard deviations,

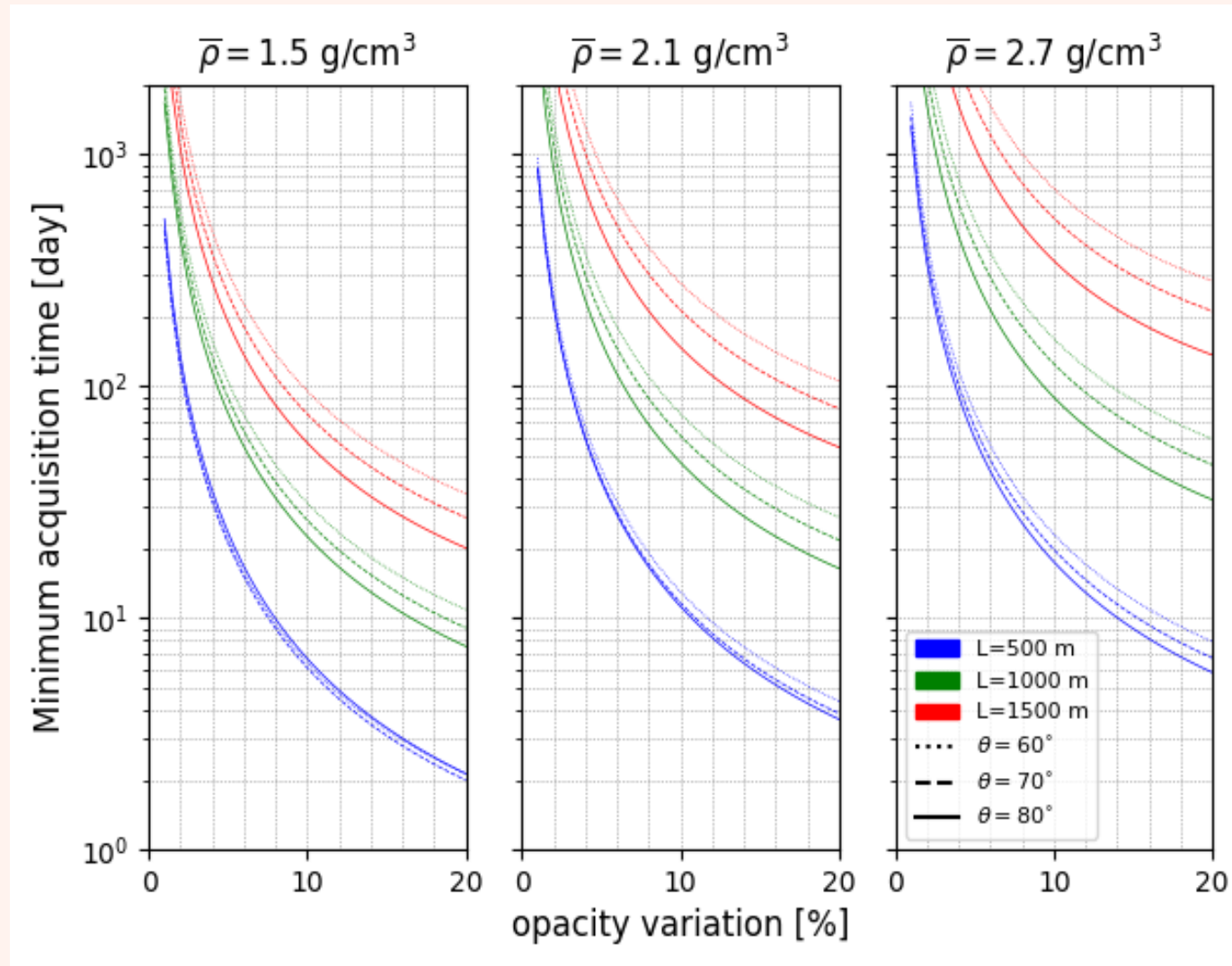
$\mathcal{J}_{geo}$  : the geometrical acceptance,

$I(\varrho)$  : integrated muon flux over

$[E_{min}(\varrho), +\infty[$ ,

and  $\Delta I(\varrho, \Delta\varrho) = I(\varrho + \Delta\varrho) - I(\varrho)$  i.e is the variation of flux linked to a

$\Delta\varrho$  shift in opacity.





# Comparison tracking algorithms

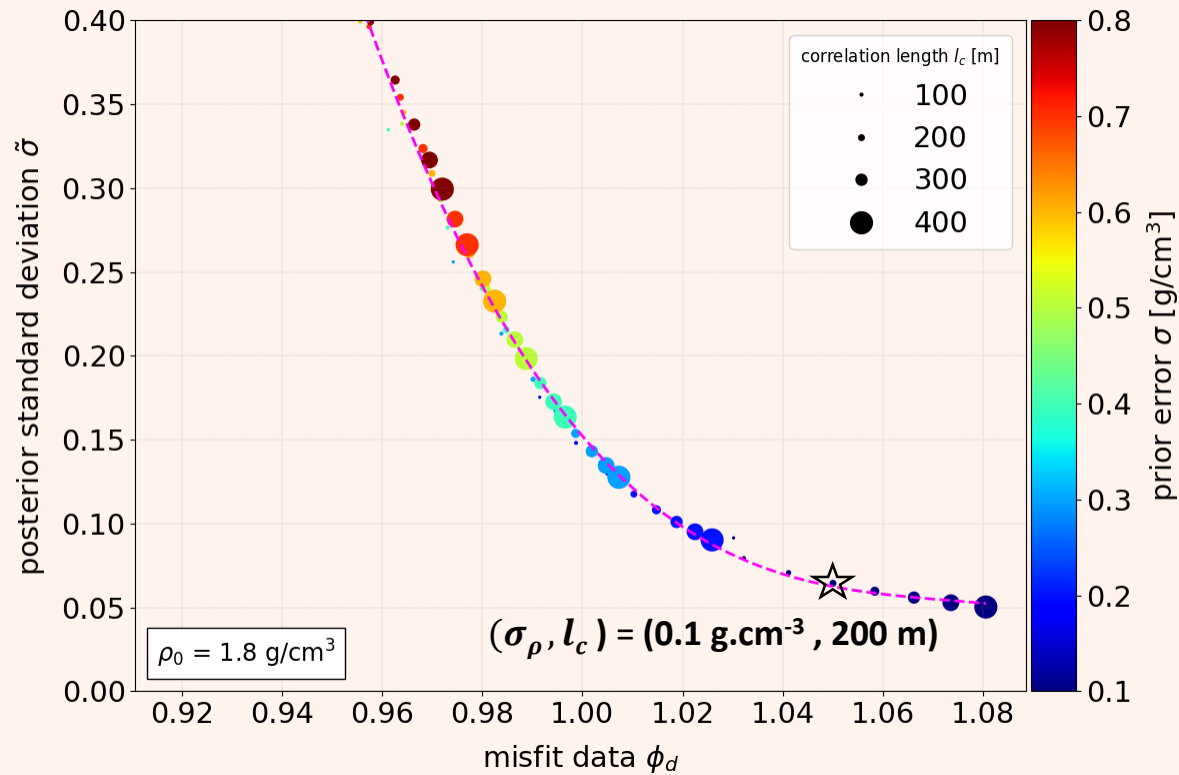
<b>10<sup>6</sup> muons <math>\mu</math></b> E $\in$ [0.1, 10 <sup>5</sup> ] GeV without noise (all $\theta$ )	<b>RANSAC</b>	<b>HOUGH</b>
Reco. Efficiency $\langle \varepsilon \rangle$	0.94 +/- 0.02	0.66 +/- 0.03
Angular resolution (RMSE)	12.47 +/- 0.04	31.09 +/- 0.24
Processing time $\Delta t$ [s]	2122	7042

Reco. Efficiency (vertical $\theta = 0$ )		<b>RANSAC</b>	<b>HOUGH</b>
<0.2 GeV	$\mu$	0.08 +/- 0.01	0.30 +/- 0.01
	$e$	0.03 +/- 0.01	0.09 +/- 0.01
	$p$	0.0 +/- 0.0	0.0 +/- 0.0
0.2 – 1 GeV	$\mu$	0.92 +/- 0.01	0.98 +/- 0.02
	$e$	0.25 +/- 0.11	0.56 +/- 0.23
	$p$	0.54 +/- 0.22	0.69 +/- 0.26
10-100 GeV	$\mu$	0.97 +/- 0.01	0.98 +/- 0.01
	$e$	0.0 +/- 0.0	0.0 +/- 0.0
	$p$	0.54 +/- 0.02	0.56 +/- 0.02

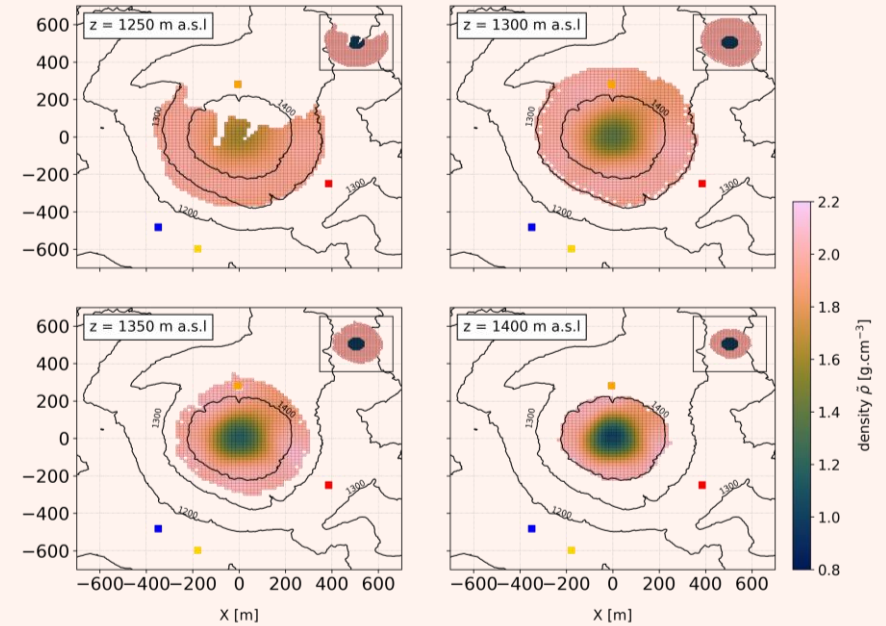
# La Soufrière muon datasets

characteristics		telescopes			
		Sacre bleu (SB)	Super Nain Jaune (SNJ)	Baron Rouge (BR)	Orange Mécanique (OM)
configuration	number of panels	3	4	3	3
	length $L$ [cm]	120	180	120	120
	scint. bars $X \times Y$ (front and rear matrices)	$32 \times 32$	$16 \times 16$	$32 \times 32$	$32 \times 32$
position & orientation	site	Savane South-West	Parking lot	Roche Fendue	Fente du Nord
	X (UTM zone 20)	642611	642782	643346	642955
	Y (UTM zone 20)	1773798	1773683	1774030	1774561
	Z (WGS84)	1185	1145	1268	1345
	elevation [°]	10.0	15.1	19.5	13.9
	azimuth [°]	44.9	20.0	295.0	192.0
run	start	24/08/2016	24/02/2019	09/03/2017	11/03/2017
	stop	01/04/2017	11/08/2019	19/08/2019	01/05/2018
	total duration [days]	202.9	145.0	640.3	191.2
	number of events (raw)	$1.38 \cdot 10^7$	$2.23 \cdot 10^7$	$1.1 \cdot 10^8$	$2.81 \cdot 10^6$
geometrical acceptance	max [cm <sup>2</sup> .sr]	2.78	11.11 (3p) / 6.25 (4p)	2.78	2.78
rock thickness	min [m]	6	14 (3p) / 5 (4p)	1	1
	center [m]	748	797 (3p) / 797 (4p)	607	394
	max [m]	1272	1301 (3p) / 1301 (4p)	1164	1209
scanned volume ( $V_{\text{tot}}^{\text{dome}} \sim 5 \cdot 10^7 \text{ m}^3$ )	estimate [m <sup>3</sup> ]	$1.33 \cdot 10^7$	$1.41 \cdot 10^7$	$9.89 \cdot 10^6$	$3.83 \cdot 10^6$

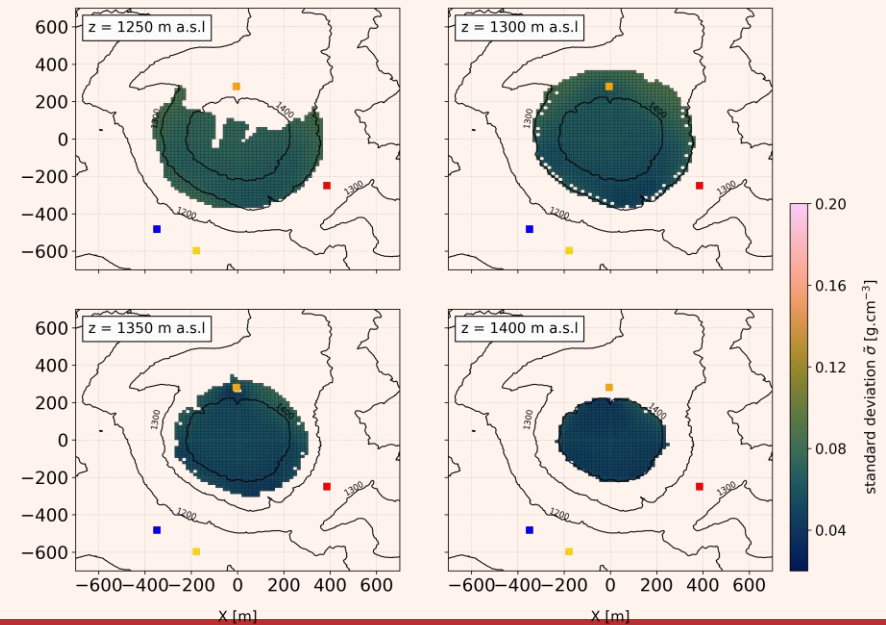
# Synthetic test



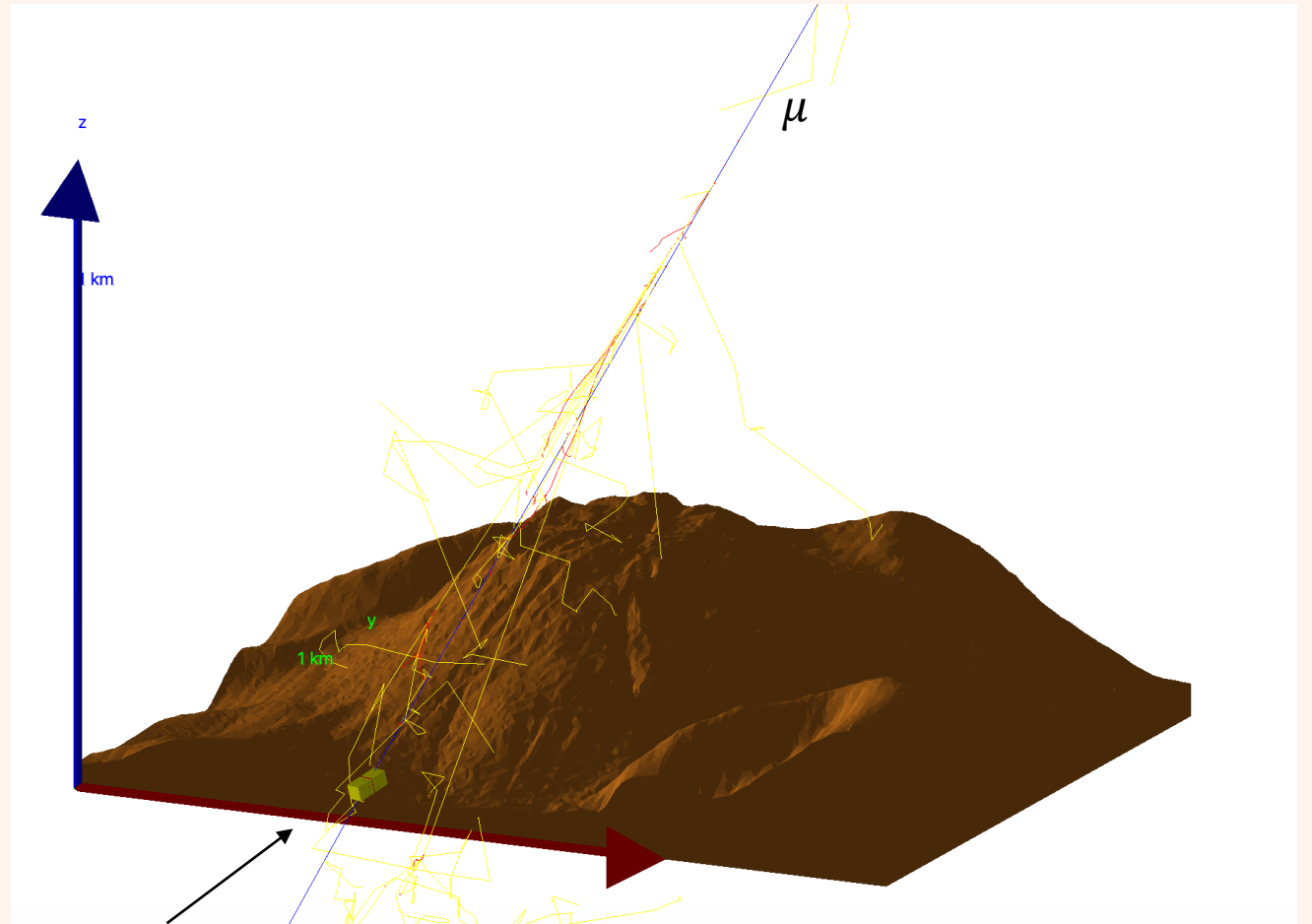
## Density estimates $\tilde{\rho}$



## Standard deviation $\tilde{\sigma}$

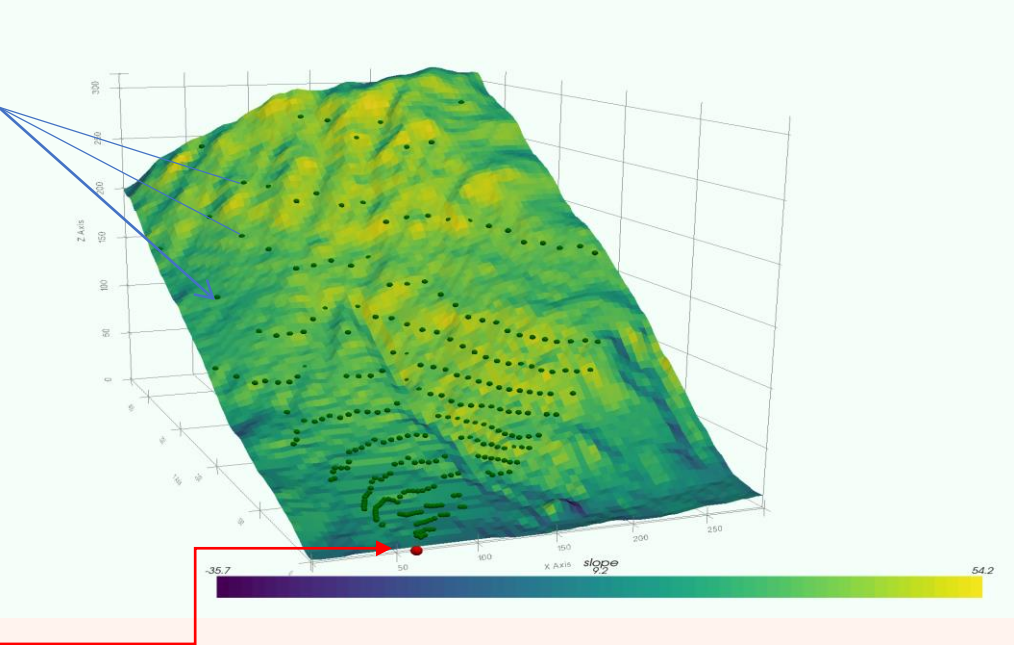
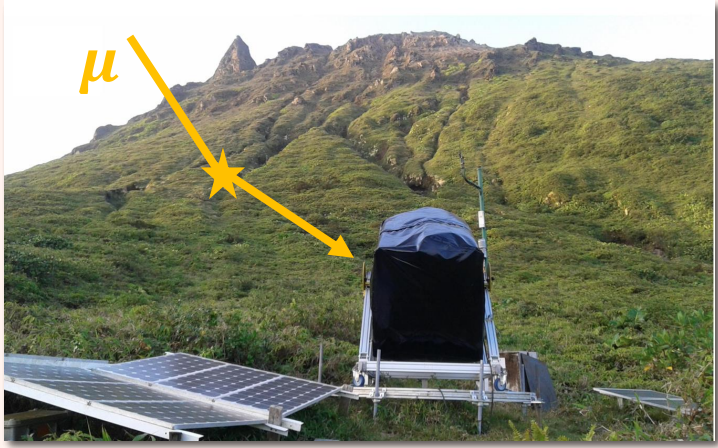


# La Soufrière Dome in Geant4



Detector (size x 30)

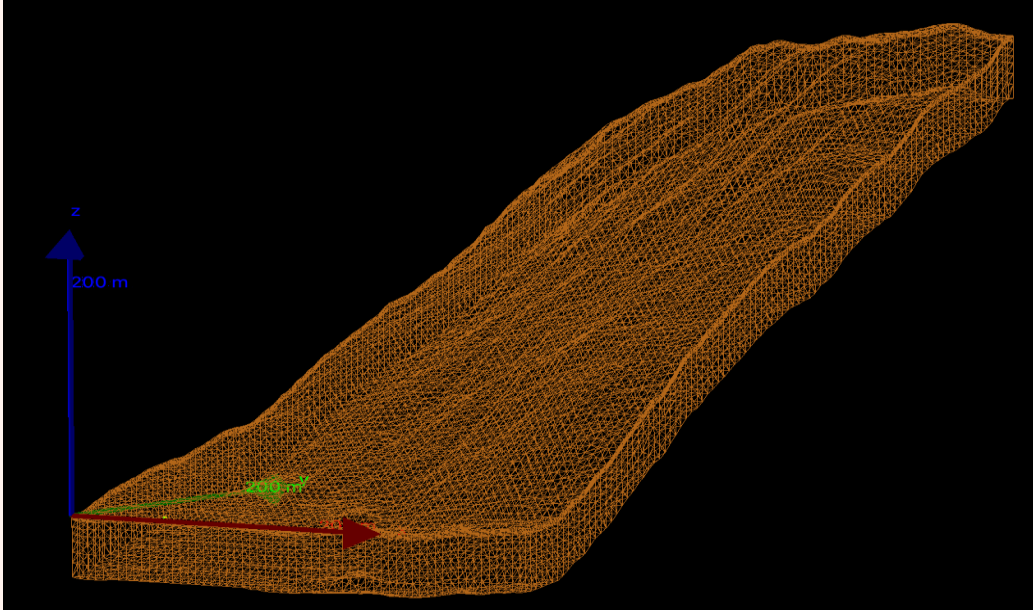
# Scattering simulation setup



Exit points  
line-of-sights

Telescope  
position

Southern flank dome  
with slope  $\in [-35; 54]^\circ$



Same mesh in GEANT4