3D muon-tomography of an underground crack zone
inversion methodology and validation by drills

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Outline

• Crack zone imaging introduction
• Formulation of a Bayesian reconstruction algorithm
• Inversion results on a real case („Kiráylak” measurements)
• Outlook
Why crack zone imaging is interesting

- Crack zones are low density regions (created by termohydraulic erosion or tectonic movement)
- Dangerous for the civil infrastructure of construction

Landslide (image: NASA)

Tunnelling through fault zone
Mathematical background of density reconstruction

• Base equation of the (linearized) inverse problem: \( \gamma = F\rho \)

• Issues: imperfect mapping to voxel grid, underdetermination, limited detector positioning, inhomogeneous statistics, systematic uncertainties, etc...

• Regularization: Bayes criterion, Maximum Likelihood, linearization, 2D simplification → parameter bias and artifacts, but the back-projection of the measurement uncertainty weights can be used for filtering. Weigth matrix: \( W \)

• Functional to be minimized follows from the weighted least squares:
  \[
  Q^{(0)} = Q^{(0)}_\gamma + Q^{(0)}_\rho = (\gamma - F\rho)^T W_\gamma (\gamma - F\rho) + (\rho - \rho^{(0)})^T W^{(0)}_\rho (\rho - \rho^{(0)})
  \]

  Maximum Likelihood Bayes parameters to be fitted

• Estimation of the density distribution:
  \[
  \rho^{(1)} = \left( R + W^{(0)}_\rho \right)^{-1} \left( F^T W_\gamma \gamma + W^{(0)}_\rho \rho^{(0)} \right)
  \]

  where \( R = F^T W_d F \) is the Fischer matrix.

Gábor Nyitrai
Test on synthetic data

- 5 underground measurement assumed
- 3 m diameter cavity in the middle
Imaging of crack zones in Budapest from the „Királylaki” tunnels

- Multiple anomalies found -> cavities?
- Beginning with „triangulation”
- Where to drill? (closest point of the anomalies)
Configuration of the measurements

- Bayesian inversion applied on the measurements
- Positions along a straight line -> 2+1D slicing (stability, convergence, comp. time)
Inverse solution

- Density distribution results showing significant anomalies crosswise
- No anomaly in south (-0.2) -> homogeneous density
- Validation drills indicated
Parameter distribution and sensitivity

• Estimated density uncertainty converges to the Bayesian constraint (in low-sensitivity region)
• The distribution of density values peak around the assumed solid rock density (extra humb due to the anomalies)
• The distribution of estimated errors (residual distribution) has an almost zero mean Gaussian shape distribution (minimal Bayesian bias)
• Results are not sensitive to the Bayes condition
Validation by core drills

- Exploratory drills (5—10 m length) into the anomalies
- The altered dolomite powder found (~1.8 g/cm³) besides the base rock (~2.6 g/cm³)
History and perspectives of the results

Hungarian geological book from 1929 reported landslides in the region
Schafarzik, Vendl: Geológiai kirándulások Budapest környékén

New housing estates next to the entrance of the Királylaki tunnels
-> Possibility of danger should be re-examined?

Secret documentation from the 60s for the construction of a gas reservoir
Further possible landslide imaging project: Santorini (Greece)
Towards 3D inversion and further applications

• Direct 3D inversion implementation on the way
• Test on an ideal case of a vertical shaft
• Inversion results (central slices)

Applications expected i.a. Buda Castle anomalies (see G. Surányi’s presentation)
Summary

• Crack zone imaging important for civil engineering (landslides, tunneling)
• Possible by muography („Királylak” measurement case, validated by drills)
• A 3D inversion method demonstrated (applying Maximum Likelihood and Bayesian approach)
• Paper submitted by L. Balázs et al.
Thank you for your attention!

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Backup slides
Point-response mapping