



LFU and semileptonic B decays at Belle

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On behalf of the Belle collaboration

ALPS 2023

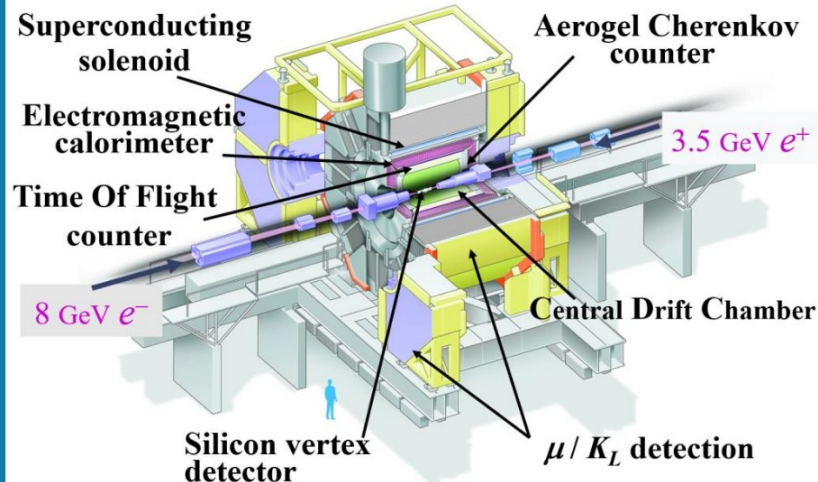
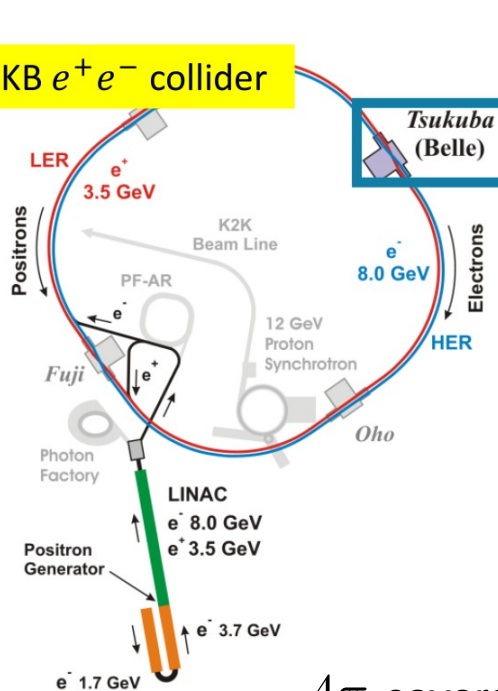
Obergurgl, 26-31.3.2023

Outline

- The Belle experiment
- Summary of $R(D^{(*)})$ measurements
- Semileptonic B decays
 - Simultaneous Determination of Inclusive and Exclusive $|V_{ub}|$
 - Measurement of Differential Distributions of $B \rightarrow D^* \ell \nu_\ell$ and $|V_{cb}|$
- Summary

The Belle experiment

KEKB e^+e^- collider

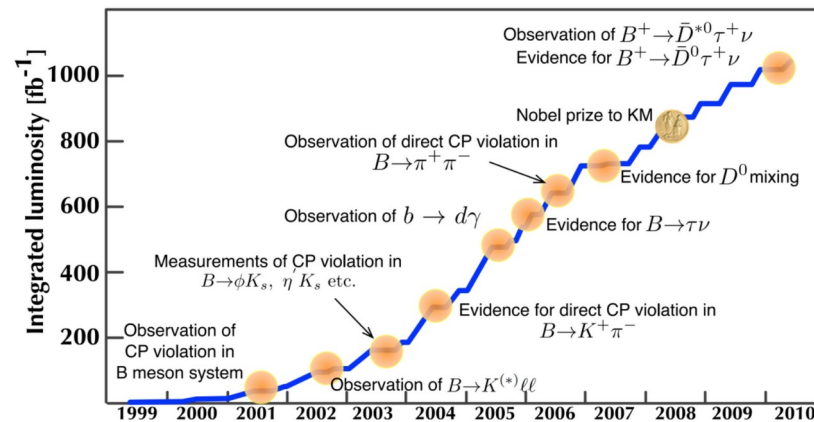


→ operated at the KEKB collider (from 1999-2010)

→ mostly at the energy of $\Upsilon(4S)$
 $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$

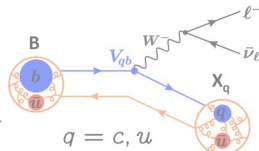
Process	No. of pairs
$e^+e^- \rightarrow \Upsilon(4S)$	$772 \times 10^6 B\bar{B}$
$e^+e^- \rightarrow \Upsilon(5S)$	$7.1 \times 10^6 B_s\bar{B}_s$

- 4π coverage
- clean e^+e^- environment with known initial state ($B\bar{B}$ pair)
- good charged track reconstruction efficiency, particle identification, gamma reconstruction
- excellent vertexing capabilities



LFU tests in semi-tauonic B decays

Lepton flavor universality tests: $\mathcal{R}(H_c) = \frac{\mathcal{B}(B \rightarrow H_c \tau \bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow H_c \ell \bar{\nu}_\ell)}$ $H_C = D^{(*)}, J/\psi$
 $(\ell = e, \mu)$



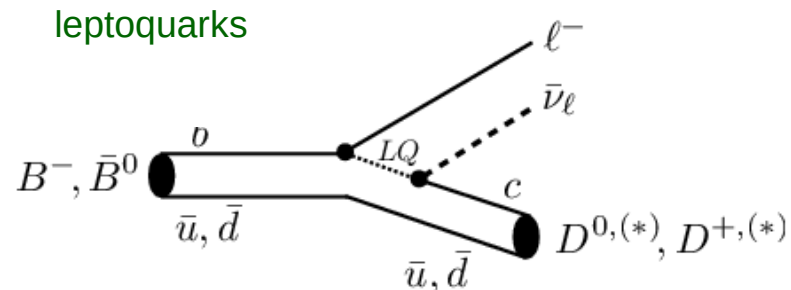
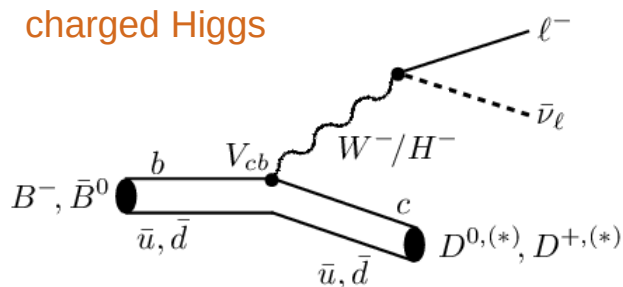
→ experimentally and theoretically convenient due to cancellation of several uncertainties in the ratio

Kinematic variables: e.g. $q^2 = (p_B - p_{D^*})^2$ distributions

Polarization fractions: τ polarization, D^{*-} longitudinal polarization

Uncertainties of the SM predictions for $\mathcal{R}(H_c)$ range from 1% to 3%

→ sensitivity to NP contributions



Decay reconstruction basics

- relatively large branching fractions
- but multiple neutrinos (not detected!) in the final state → challenging decay reconstruction
- determination of initial B momentum allows for evaluation of

$$q^2 = (p_B - p_{D^*})^2$$

momentum transfer to leptons


$$m_{\text{miss}}^2 = (p_B - p_{D^{(*)}} - p_\ell)^2$$


missing mass

$$E_\ell^* = (p_\ell \cdot p_B) / m_B$$

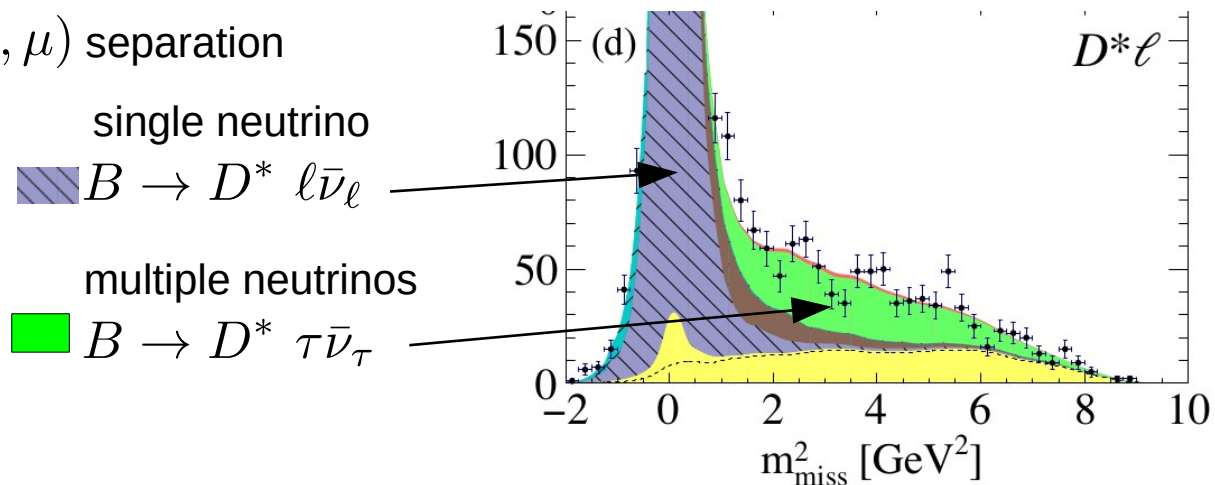
charged lepton energy in B frame

→ basis for SL decays with τ or ℓ (e, μ) separation

single neutrino
 $B \rightarrow D^* \ell \bar{\nu}_\ell$

multiple neutrinos
 $B \rightarrow D^* \tau \bar{\nu}_\tau$

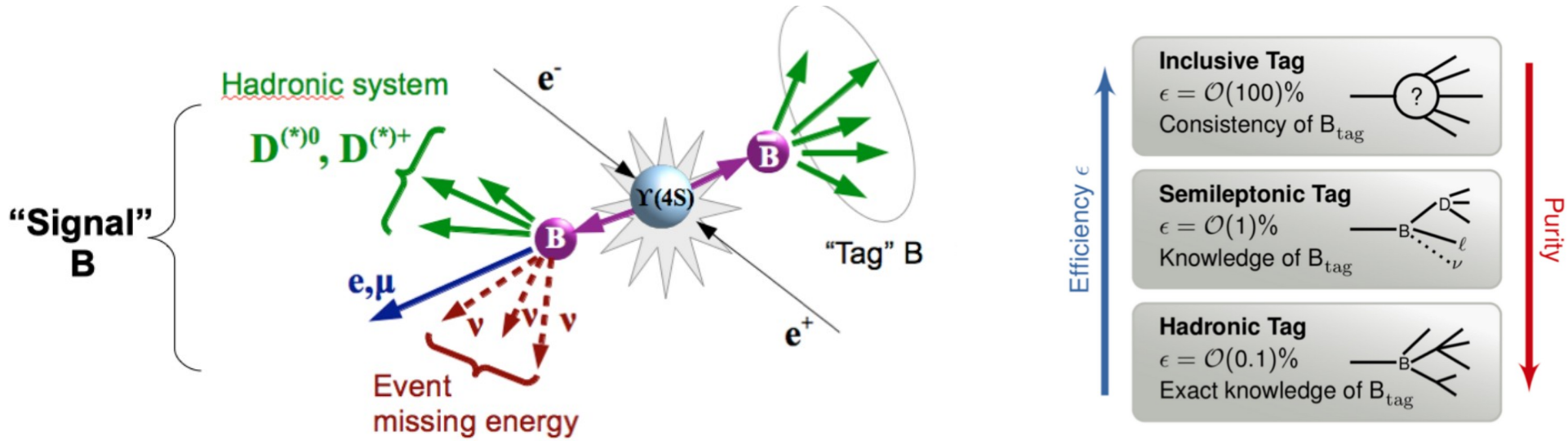
- accessible to **B factories** and **LHCb**



Decay reconstruction basics – Belle (B-factories)

$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$$

- fully known initial state + hermetic detector (4π) \rightarrow **tagging techniques**



\rightarrow in SL decays with correct B_{tag} all particles in an event assigned (to B_{sig} or B_{tag})!

background events: larger E_{ECL}

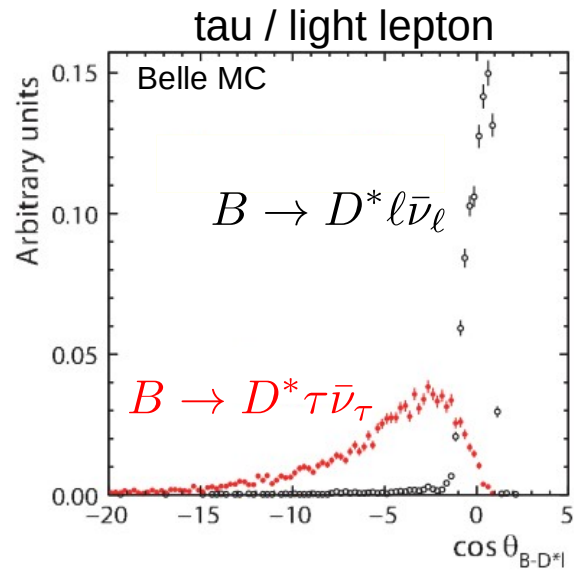
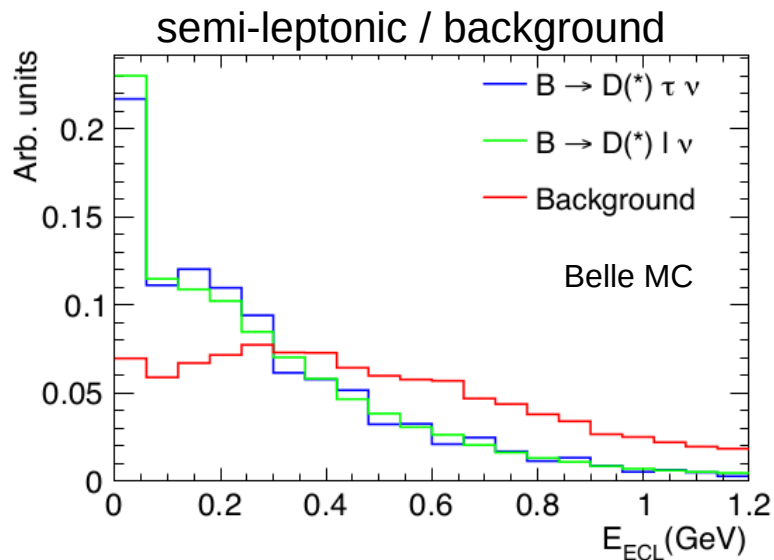
extra energy in the EM calorimeter

τ vs. l (e, μ): $m_{\text{miss}}^2 + \text{kinematics}$

Latest and most precise $\mathcal{R}(D^{(*)})$ from Belle – semi-leptonic tag

Phys. Rev. Lett. 124, 161803, [arXiv:1910.05864](https://arxiv.org/abs/1910.05864)

- using FEI (full event interpretation) for the tag-side $B \rightarrow D^{(*)} l \bar{\nu}_l$ reconstruction
- reconstructed signal modes: $D^+ \ell^-$, $D^0 \ell^-$, $D^{*+} \ell^-$, $D^{*0} \ell^-$ ($\ell = e, \mu$)
- combine kinematic variables using BDT: $(\cos \theta_{B, D^{(*)} l}, m_{\text{miss}}^2, E_{\text{vis}}) \rightarrow \mathcal{O}_{\text{sig}}$



$$\equiv \frac{2E_{\text{beam}}E_{D^{(*)}\ell} - m_B^2 - m_{D^{(*)}\ell}^2}{2|\mathbf{p}_B||\mathbf{p}_{D^{(*)}\ell}|}$$

Latest and most precise $\mathcal{R}(D^{(*)})$ from Belle – semi-leptonic tag

Phys. Rev. Lett. 124, 161803, [arXiv:1910.05864](https://arxiv.org/abs/1910.05864)

- $E_{\text{ECL}} - \mathcal{O}_{\text{sig}}$ distributions of all samples are fit simultaneously, constraining $\mathcal{R}(D^{(*)0}) = \mathcal{R}(D^{(*)+})$

- free parameters: signal yields (τ, ℓ), $B \rightarrow D^{**} l \nu$ yield
feed-down D^* yield

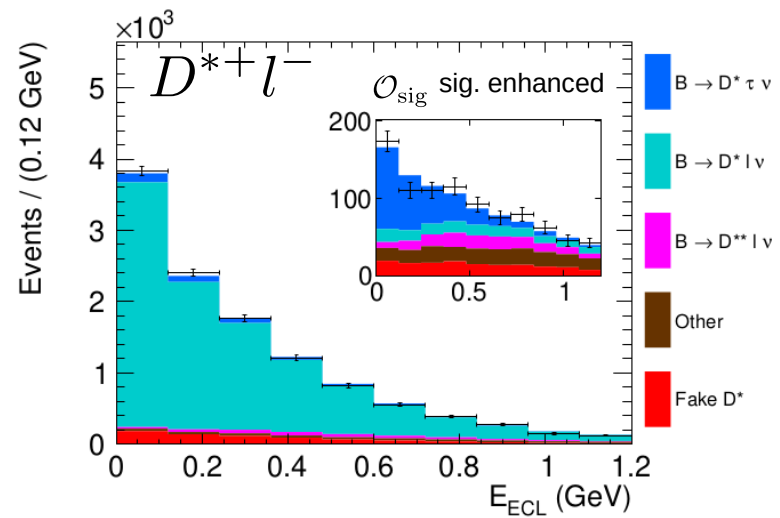
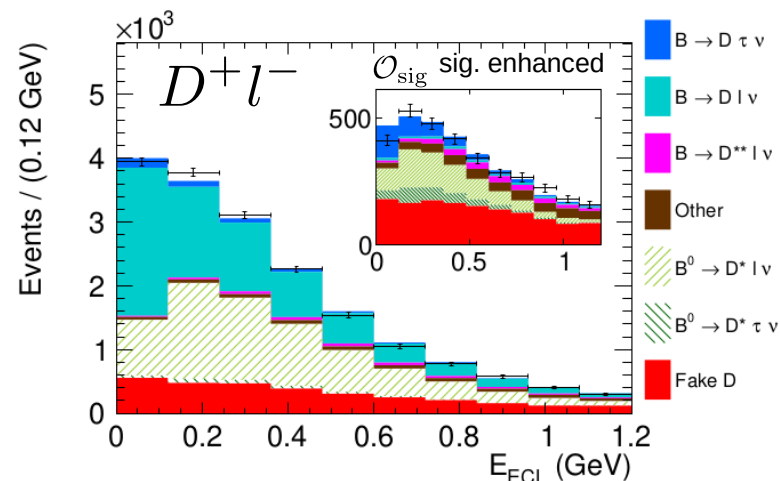
$$\mathcal{R}(D) = 0.307 \pm 0.037 \text{ (stat)} \pm 0.016 \text{ (syst)}$$

$$\mathcal{R}(D^*) = 0.283 \pm 0.018 \text{ (stat)} \pm 0.014 \text{ (syst)}$$

Most precise values to date!

Main systematic uncertainties

Source	$\Delta\mathcal{R}(D)$ (%)	$\Delta\mathcal{R}(D^*)$ (%)
D^{**} composition	0.76	1.41
PDF shapes	4.39	2.25
Feed-down factors	1.69	0.44
Efficiency factors	1.93	4.12



Summary of existing B factory measurements

Hadronic tag with $\tau \rightarrow \ell\nu\bar{\nu}$

BaBar: Phys. Rev. Lett. 109, 101802, [arXiv:1205.5442](https://arxiv.org/abs/1205.5442)

Belle: Phys. Rev. D 92, 072014, [arXiv:1507.03233](https://arxiv.org/abs/1507.03233)

Result	BABAR	Belle
$\mathcal{R}(D)$	$0.440 \pm 0.058 \pm 0.042$	$0.375 \pm 0.064 \pm 0.026$
$\mathcal{R}(D^*)$	$0.332 \pm 0.024 \pm 0.018$	$0.293 \pm 0.038 \pm 0.015$

Semi-leptonic tag with $\tau \rightarrow \ell\nu\bar{\nu}$

Belle: Phys. Rev. Lett. 124, 161803, [arXiv:1910.05864](https://arxiv.org/abs/1910.05864)

$$\mathcal{R}(D) = 0.307 \pm 0.037 (\text{stat}) \pm 0.016 (\text{syst})$$

$$\mathcal{R}(D^*) = 0.283 \pm 0.018 (\text{stat}) \pm 0.014 (\text{syst})$$

Hadronic tag with $\tau \rightarrow \pi\nu, \tau \rightarrow \rho\nu$
Belle τ polarization measurement

Phys. Rev. D 97 (1), 012004, [arXiv:1709.00129](https://arxiv.org/abs/1709.00129)

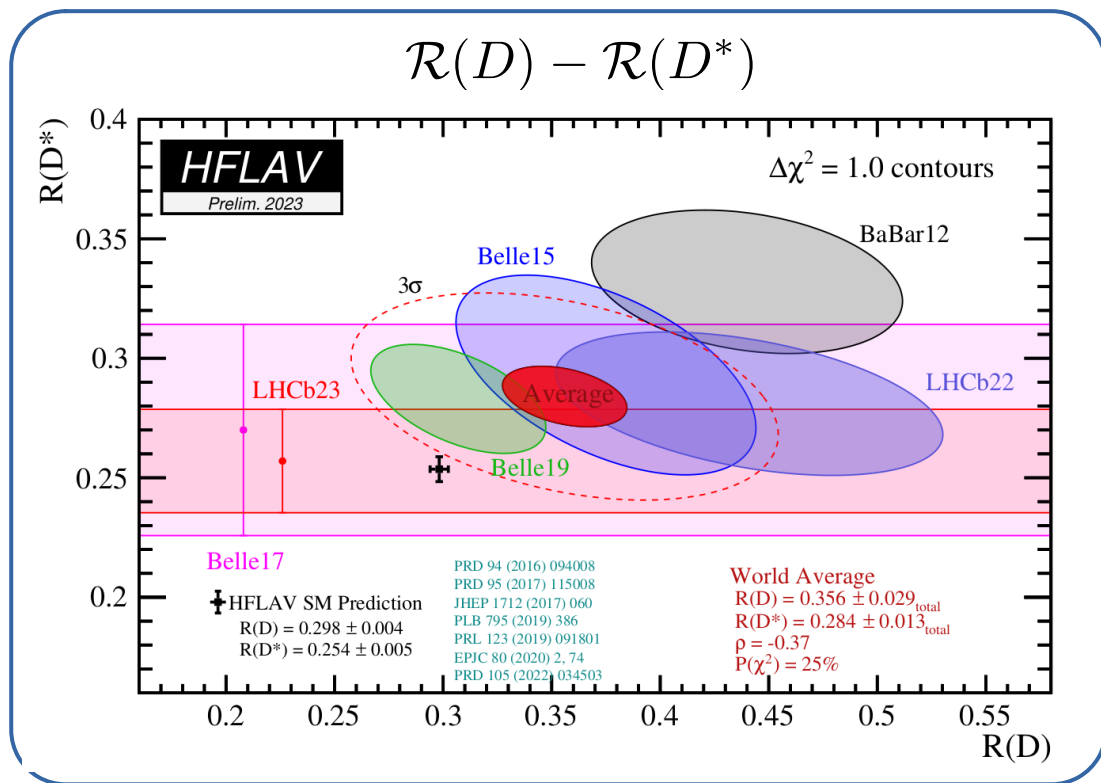
$$R(D^*) = 0.270 \pm 0.035 (\text{stat})_{-0.025}^{+0.028} (\text{syst})$$

$$P_\tau(D^*) = -0.38 \pm 0.51 (\text{stat})_{-0.16}^{+0.21} (\text{syst})$$

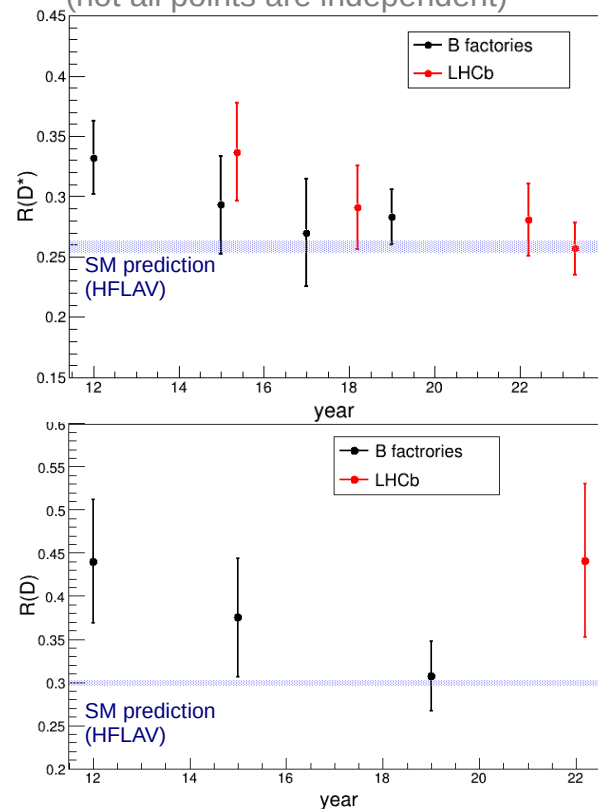
Inclusive tag with $\tau \rightarrow \pi\nu, \tau \rightarrow \ell\nu\bar{\nu}$
Belle D^{*-} polarization measurement
[arXiv:1903.03102](https://arxiv.org/abs/1903.03102)

$$F_{L,\tau}(D^*) = 0.60 \pm 0.08 (\text{stat}) \pm 0.04 (\text{syst})$$

Consistency with the SM predictions



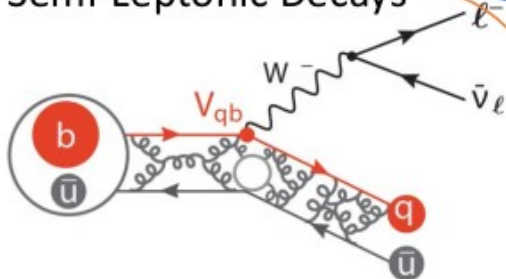
Timeline of published results
(not all points are independent)



→ present world average of $\mathcal{R}(D) - \mathcal{R}(D^*)$ deviates from the SM with significance of $\sim 3.16 \sigma$.

$|V_{ub}|$ and $|V_{cb}|$ from semileptonic B decays

Semi-Leptonic Decays



Inclusive $|V_{ub}|$

$$B \rightarrow X_u \ell \bar{\nu}_\ell$$

Inclusive $|V_{cb}|$

$$B \rightarrow X_c \ell \bar{\nu}_\ell$$

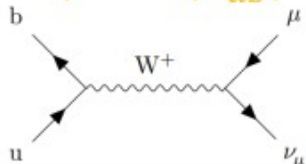
Operator Product Expansion

$$BR \propto |V_{qb}|^2 \left[1 + \frac{c_5(\mu) \langle O_5 \rangle(\mu)}{m_b^2} + \frac{c_6(\mu) \langle O_6 \rangle(\mu)}{m_b^3} + O(m_b^4) \right]$$

+ Shape Function / Fermi Motion

Leptonic Decays

Leptonic $|V_{ub}|$



$$BR \propto |V_{ub}|^2 f_B^2 m_l^2$$

f_B : B-Meson decay constant

Exclusive $|V_{ub}|$

$$B \rightarrow \pi, \rho, \omega \ell \bar{\nu}_\ell$$

$$\Lambda_b \rightarrow p \mu \bar{\nu}_\ell$$

$$B_s \rightarrow K \mu \bar{\nu}_\mu$$

Exclusive $|V_{cb}|$

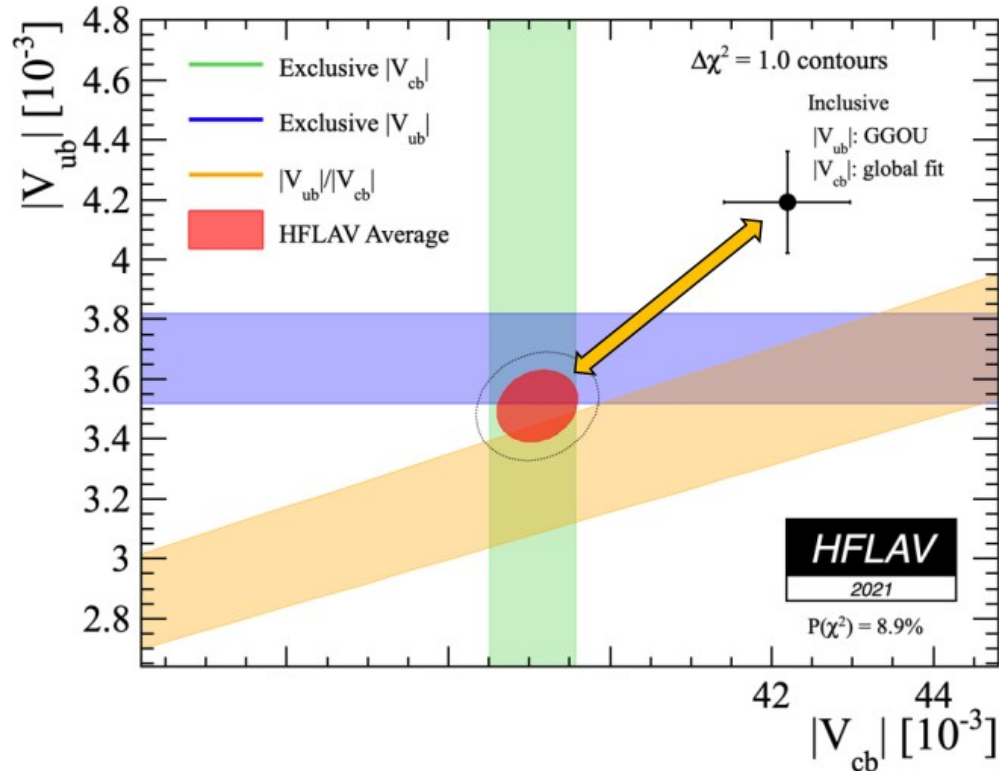
$$B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \bar{\nu}_\ell$$

Form Factors

$$\langle B | H_\mu | P \rangle = (p + p')_\mu f_+$$

$$BR \propto |V_{qb}|^2 f^2$$

Experimental status



- longstanding significant discrepancy between $|V_{ub}|$ and $|V_{cb}|$ as determined from inclusive or exclusive measurements
- several new results on $|V_{xb}|$ by Belle and Belle II recently

First Simultaneous Determination of Inclusive and Exclusive $|V_{ub}|$

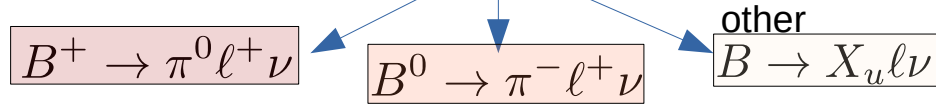
Preliminary

- full Belle dataset (711 fb^{-1})

- hadronic reconstruction of B_{tag}

→ allows for reconstruction of $B \rightarrow X_u \ell \nu$

$$q^2 = (p_B - p_{X_u})^2 \text{ and } N_{\pi^\pm} \text{ in } X_u$$



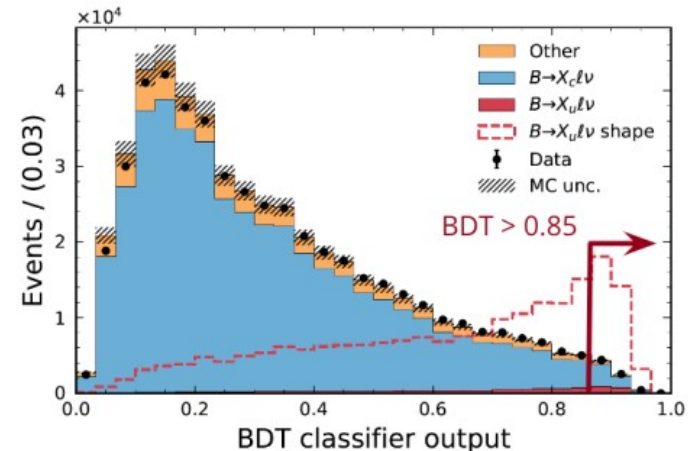
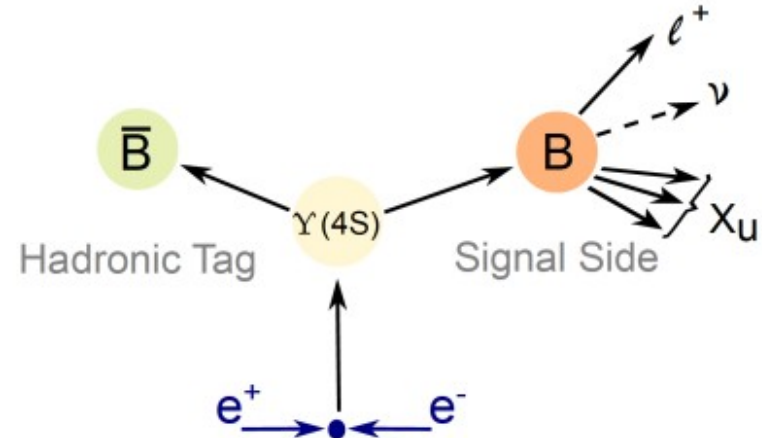
- electron or muon with $E_\ell^B = |\mathbf{p}_\ell^B| > 1 \text{ GeV}$

- BDT with 11 training features to suppress $B \rightarrow X_c \ell \nu$

$$M_{miss}^2, \chi_{\nu tx}^2, \#K$$

- X_u thrust in the CMS to increase $B \rightarrow \pi \ell \nu$ significance

$$|V_{ub}^{\text{excl.}}| / |V_{ub}^{\text{incl.}}| = 0.84 \pm 0.04$$



First Simultaneous Determination of Inclusive and Exclusive $|V_{ub}|$

- Likelihood fit with binned templates' normalisations and $B \rightarrow \pi \ell \nu$ form factor parameters free

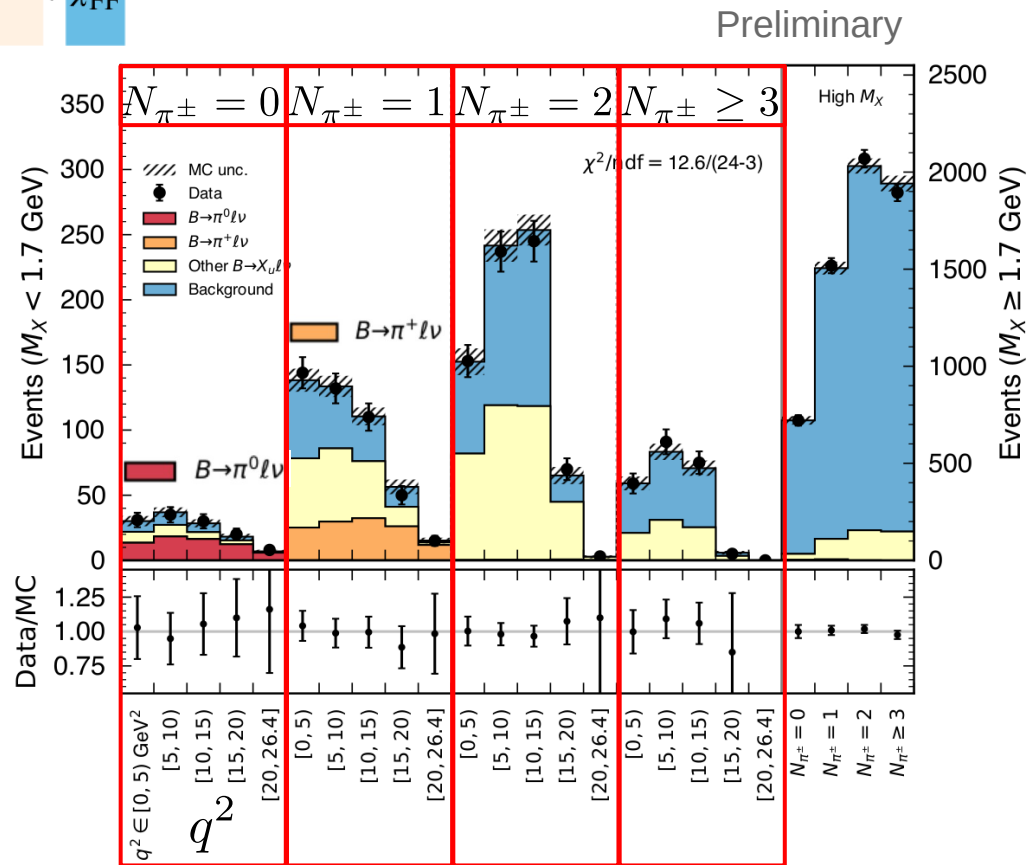
$$-2 \log \mathcal{L} = -2 \log \prod_i \text{Poisson} \left(\eta_{\text{obs}, i}, \eta_{\text{pred}, i} \cdot (1 + \epsilon \cdot \theta) \right) + \theta \rho_{\theta}^{-1} \theta^T + \chi_{\text{FF}}^2$$

- BCL modeling of FF is used with parameters constrained to the LQCD or LQCD+exp. fits.
- all systematics (additive and multiplicative) are added as nuisance parameters for each template.
- dominant systematics are:
 $B \rightarrow X_u \ell \nu$ modeling, $u \rightarrow X_u$ fragmentation and reconstruction efficiencies (tagging, etc.)
- finally $|V_{ub}|$ is extracted via:

$$\mathcal{B}(B \rightarrow X_u \ell \nu) = \mathcal{B}(B \rightarrow \pi^0 \ell \nu) + \mathcal{B}(B \rightarrow \pi^+ \ell \nu) + \mathcal{B}(B \rightarrow X_u^{\text{other}} \ell \nu)$$

$$\Delta \mathcal{B}(B \rightarrow X_u \ell \nu) = \mathcal{B}(B \rightarrow X_u \ell \nu) \cdot \epsilon_{\Delta \text{PS}; E_B^{\text{eff}} > 1 \text{ GeV}}$$

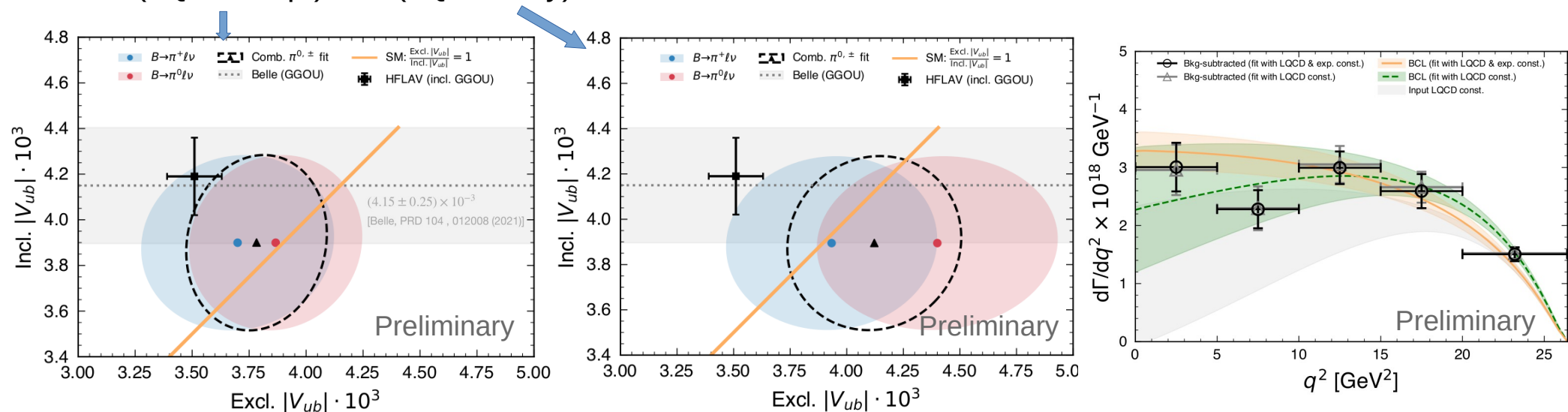
$$|V_{ub}| = \sqrt{\frac{\mathcal{B}}{\tau_B \cdot \Gamma}}$$



First Simultaneous Determination of Inclusive and Exclusive $|V_{ub}|$

Various fit scenarios considered:

- separate $B^+ \rightarrow \pi^0 \ell^+ \nu$ and $B^0 \rightarrow \pi^- \ell^+ \nu$ and combined.
- (LQCD+exp.) and (LQCD only) constraints for $B \rightarrow \pi \ell \nu$ form factors.



$$\left. \begin{aligned} |V_{ub}^{\text{excl.}}| &= (3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3} \\ |V_{ub}^{\text{incl.}}| &= (3.90 \pm 0.20 \pm 0.32 \pm 0.09) \times 10^{-3} \end{aligned} \right\} \text{for (LQCD+exp.) combined fit}$$

Weighted average of excl. & incl.

$$(3.85 \pm 0.26) \times 10^{-3}$$

CKM global fit (w/o $|V_{ub}|$): $(3.64 \pm 0.07) \times 10^{-3}$,
compatible within 0.8 σ

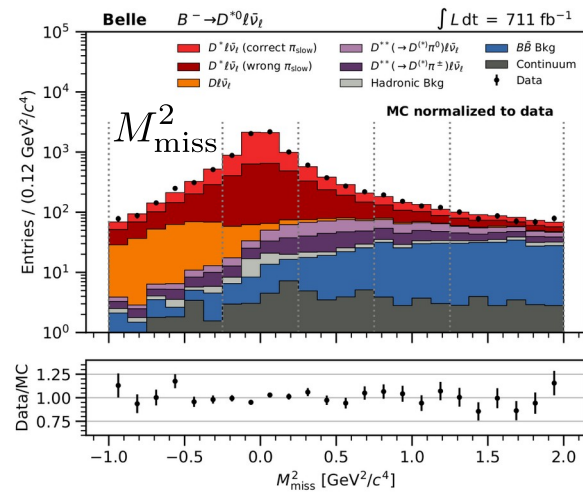
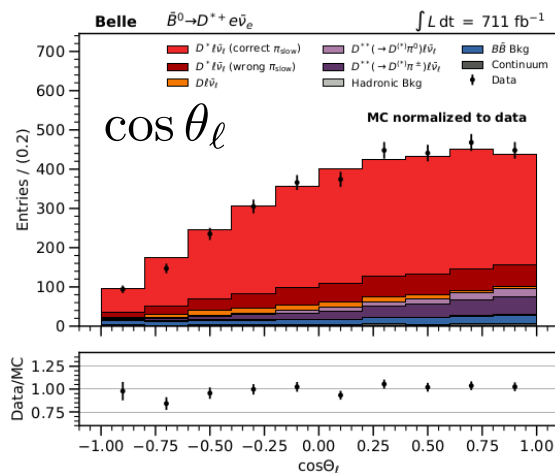
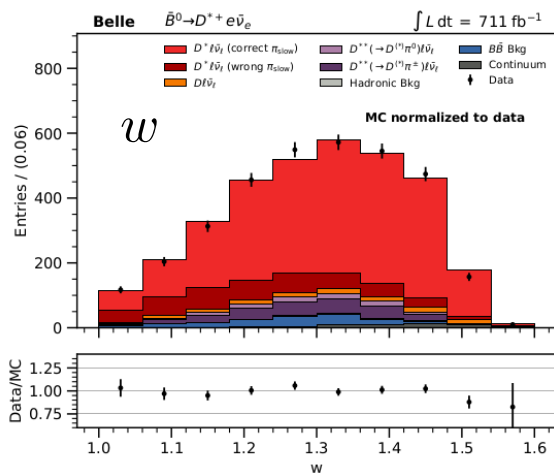
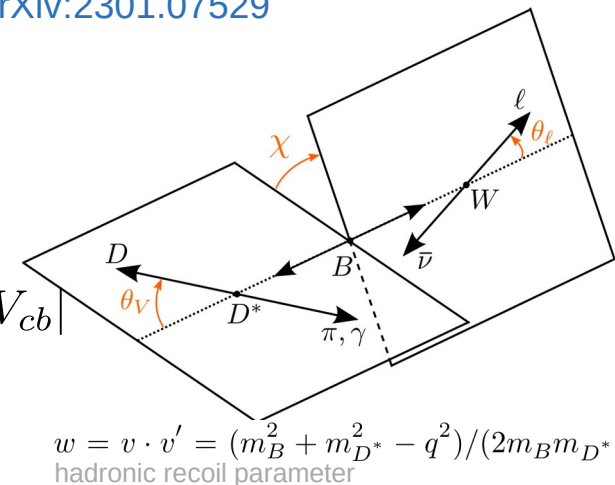
$$\boxed{|V_{ub}^{\text{excl.}}| / |V_{ub}^{\text{incl.}}| = 0.97 \pm 0.12}$$

→ compatible with the
W.A. and with the SM

Measurement of Differential Distributions of $B \rightarrow D^* \ell \nu_\ell$ and $|V_{cb}|$

arXiv:2301.07529

- full Belle dataset (711 fb^{-1}), 4 separate decay modes: $B^{\pm,0}, \ell = e, \mu$
- hadronic reconstruction of B_{tag} using Belle II tools (Full Event Interpretation)
- fit the form factor parametrisation to the measured differential shapes in $w, \cos \theta_\ell, \cos \theta_V, \chi$ and use W.A. branching fraction to determine $|V_{cb}|$
- for each decay in each bin background is subtracted via fitting of model independent variable: $M_{\text{miss}}^2 = p_{\text{miss}}^2 = (p_{e^+e^-} - p_{\text{tag}} - p_{D^*} - p_\ell)^2$
→ resulting in 160 fits



Measurement of Differential Distributions of $B \rightarrow D^* l \bar{\nu}_l$ and $|V_{cb}|$

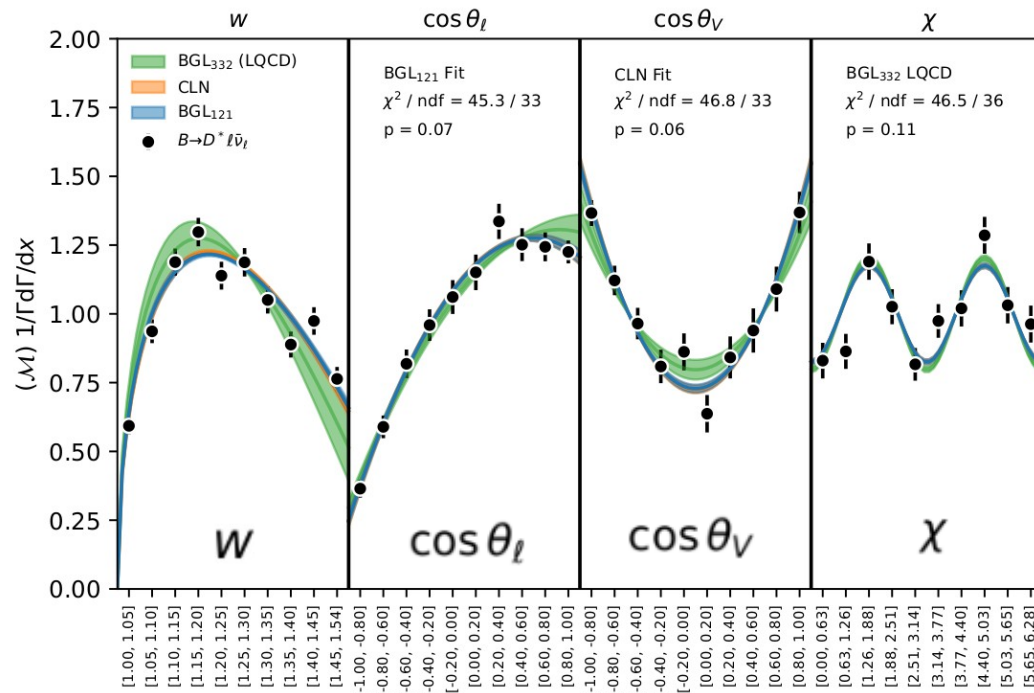
arXiv:2301.07529

- shapes are corrected for resolution and acceptance effects
- averaged shapes are used to fit BGL and CLN parametrization to the data
- External constraints on branching fraction (HFLAV) and $h_X = h_{A_1}(1) = 0.906 \pm 0.013$
FF at zero-recoil (FNAL/MILC)

$$\chi^2 = \left(\frac{\Delta \vec{\Gamma}^m}{\Gamma^m} - \frac{\Delta \vec{\Gamma}^p(\vec{x})}{\Gamma^p(\vec{x})} \right) C_{\text{exp}}^{-1} \left(\frac{\Delta \vec{\Gamma}^m}{\Gamma^m} - \frac{\Delta \vec{\Gamma}^p(\vec{x})}{\Gamma^p(\vec{x})} \right)^T$$

$$+ (\Gamma^{\text{ext}} - \Gamma^p(\vec{x}))^2 / \sigma(\Gamma^{\text{ext}})^2$$

$$+ (h_X - h_X^{\text{LQCD}}) C_{\text{LQCD}}^{-1} (h_X - h_X^{\text{LQCD}})$$



$ V_{cb} $	BGL ₁₂₁	CLN
$B^+ \rightarrow D^{*0} l \bar{\nu}_l$	42.0 ± 1.2	41.4 ± 1.2
$\bar{B}^0 \rightarrow D^{*+} l \bar{\nu}_l$	38.5 ± 1.3	38.3 ± 1.1
$B \rightarrow D^* l \bar{\nu}_l$	40.6 ± 0.9	40.1 ± 0.9

Measurement of Differential Distributions of $B \rightarrow D^* \ell \bar{\nu}_\ell$ and $|V_{cb}|$

arXiv:2301.07529

- additional fit scenarios are tested:
using recent lattice constraints on $B \rightarrow D^*$ FF
beyond zero recoil

- D^* longitudinal polarization fraction

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_V} = \frac{3}{2} \left(F_L \cos^2 \theta_V + \frac{1 - F_L}{2} \sin^2 \theta_V \right)$$

$$B \rightarrow D^* \ell \bar{\nu}_\ell \quad F_L^{D^*} \quad 0.501 \pm 0.012 \pm 0.003$$

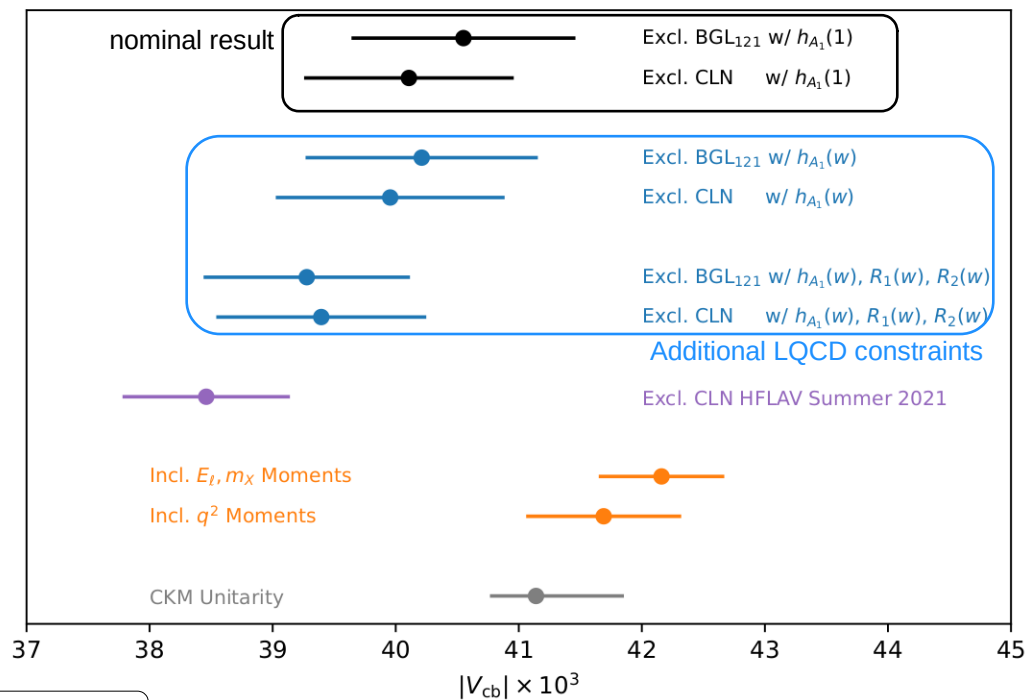
- Forward-Backward asymmetry

$$\Delta A_{FB} = A_{FB}^\mu - A_{FB}^e$$

$$B \rightarrow D^* \ell \bar{\nu}_\ell \quad 0.022 \pm 0.026 \pm 0.007$$

- LFU ratio $R_{e\mu} = \frac{\mathcal{B}(B \rightarrow D^* e \bar{\nu}_e)}{\mathcal{B}(B \rightarrow D^* \mu \bar{\nu}_\mu)} = 0.990 \pm 0.021 \pm 0.023$

- All consistent with no LFU violation.



Overview of recent Belle and Belle II results

$|V_{cb}|$

Belle II $B^0 \rightarrow D^* \ell \nu$ arXiv:2301.04716

Belle II $B \rightarrow D \ell \nu$ arXiv:2210.13143

BELLE $B \rightarrow D^* \ell \nu$
shapes arXiv:2301.07529

BELLE $B \rightarrow X_c \ell \nu$ PRD 104, 112011 (2021)
arXiv:2205.06372

Belle II $\langle q^{2m} \rangle$ JHEP 10 (2022) 068

$|V_{ub}|$

BELLE $\Delta \mathcal{B}(B \rightarrow X_u \ell \nu) / \Delta \mathcal{B}(B \rightarrow X_c \ell \nu)$ preliminary

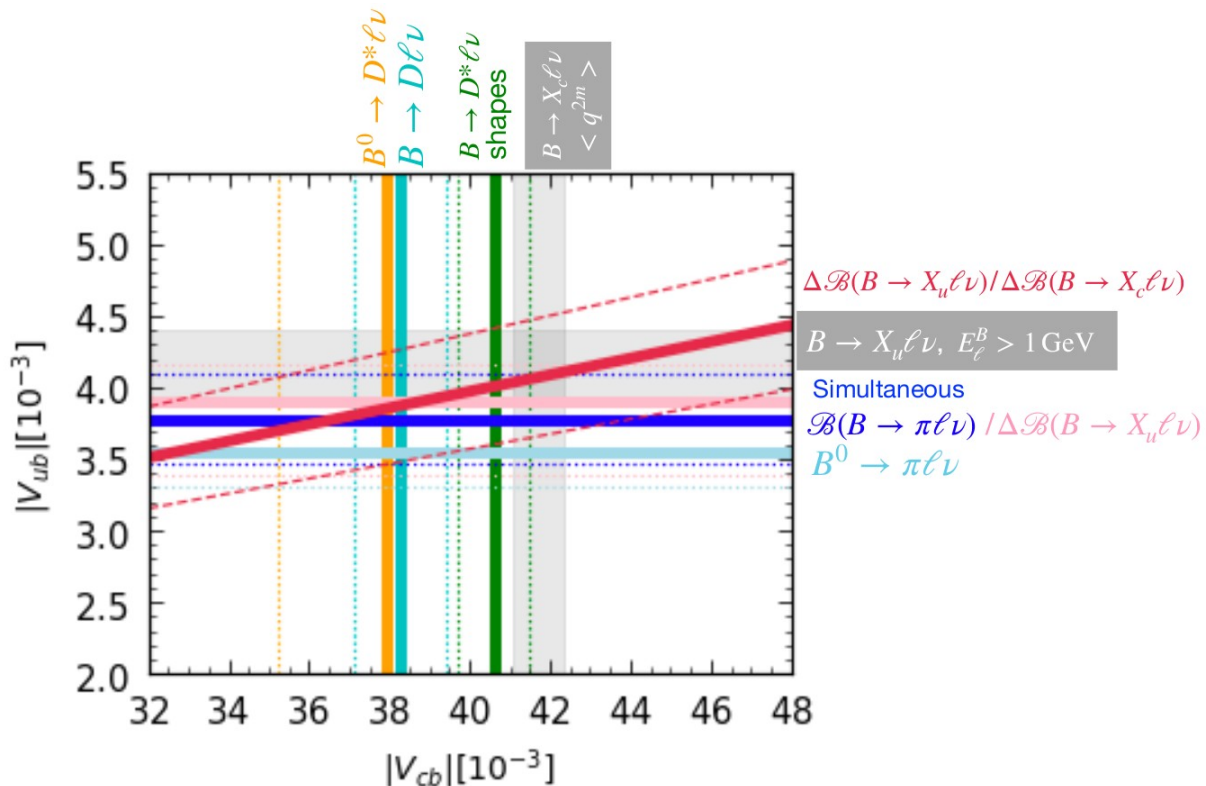
BELLE $B \rightarrow X_u \ell \nu, E_\ell^B > 1 \text{ GeV}$ PRD 104, 012008 (2021)

Simultaneous

BELLE $\mathcal{B}(B \rightarrow \pi \ell \nu) / \Delta \mathcal{B}(B \rightarrow X_u \ell \nu)$ preliminary

Belle II $B^0 \rightarrow \pi \ell \nu$ arXiv:2210.04224

Hadronic tag Untagged



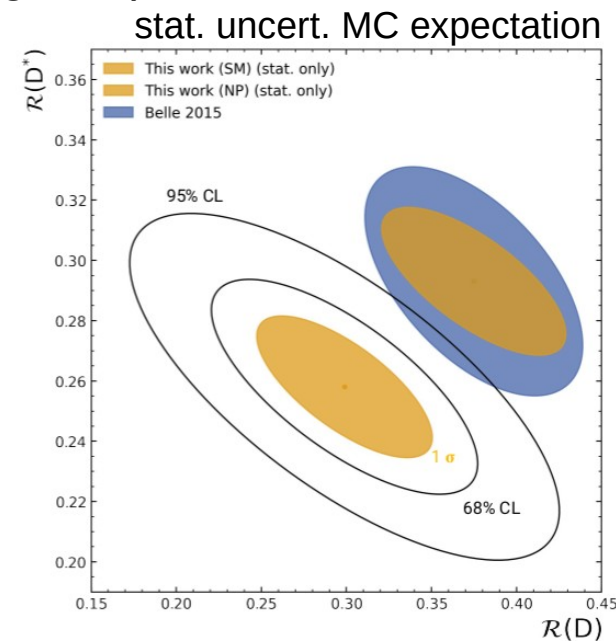
→ more results are yet to come, which will help to resolve the inclusive/exclusive puzzle

→ continuous effort from exp. and theory side still needed

Summary

- even 10+ years after the stop of its operation Belle keeps producing important physics results
- “legacy” $\mathcal{R}(D^{(*)})$ measurement (had. tagging with leptonic tau decays) based on the Belle II tools (FEI \rightarrow increased efficiency, improved MC modeling, etc.) and many other measurements in the pipeline
- Belle II has up to now collected $\sim 1/2$ of Belle data set size and gradual boost is expected in the next year
 - \rightarrow due to detector and data analyses improvements already producing competitive/leading results (stay tuned for the new $\mathcal{R}(D^*)$ and more coming soon)

Thank you!

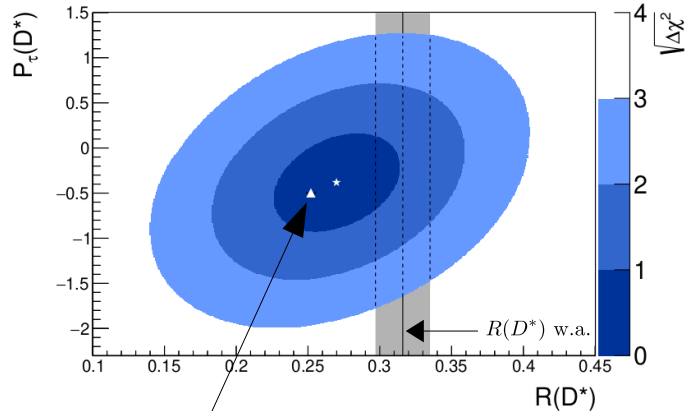


Related observables in semi-tauonic decays

Belle τ polarization measurement

Phys. Rev. D 97 (1), 012004, [arXiv:1709.00129](https://arxiv.org/abs/1709.00129)

$$P_{\tau}(D^{(*)}) = \frac{\Gamma^{+} - \Gamma^{-}}{\Gamma^{+} + \Gamma^{-}} \quad \Gamma^{\pm} - \tau \text{ helicity}$$

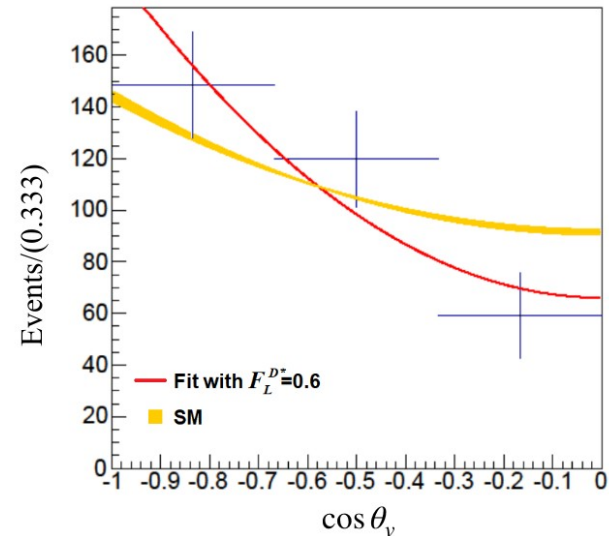


SM expectation

Phys. Rev. D 88, 094012 (2013) [arXiv:1309.0301](https://arxiv.org/abs/1309.0301)

Belle D^{*-} longitudinal polarization fraction

[arXiv:1903.03102](https://arxiv.org/abs/1903.03102)



Phys. Rev. D 98, 095018 (2018) [arXiv:1808.03565](https://arxiv.org/abs/1808.03565)

consistent with the SM at 1.6σ