The Cabibbo angle anomaly, electroweak fits, and $(g-2)_\tau$

Martin Hoferichter

Albert Einstein Center for Fundamental Physics,
Institute for Theoretical Physics, University of Bern

UNIVERSITÄT BERN

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CKM unitarity

Benchmarks numbers for CKM tests from PDG "12. CKM Quark-Mixing Matrix"

first row:	$\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2 + \lvert V_{ub}\rvert^2 = 0.9985(5)$
second row:	$\lvert V_{cd}\rvert^2 + \lvert V_{cs}\rvert^2 + \lvert V_{cb}\rvert^2 = 1.025(22)$
first column:	$\lvert V_{ud}\rvert^2 + \lvert V_{cd}\rvert^2 + \lvert V_{td}\rvert^2 = 0.9970(18)$
second column:	$\lvert V_{us}\rvert^2 + \lvert V_{cs}\rvert^2 + \lvert V_{ts}\rvert^2 = 1.026(22)$

- **First-row unitarity test**
  - Testing consistency of V_{ud} and V_{us} at precision of a few times 10^{-4}
  - $|V_{ub}|^2 \simeq 1.5 \times 10^{-5}$
  - Deficit of $(2–3)\sigma$ (also deficit in first-column test, but less sensitive)

    $\hookrightarrow$ "**Cabibbo angle anomaly**"
  - Second row/column more than an order of magnitude away; third row/column $\mathcal{O}(\lambda^4)$
- First part of this talk:
  - Review inputs to first-row test, focus on uncertainties
  - Discuss prospects for improvements

Determination of V_{ud} from superallowed β decays

- Master formula Hardy, Towner 2018

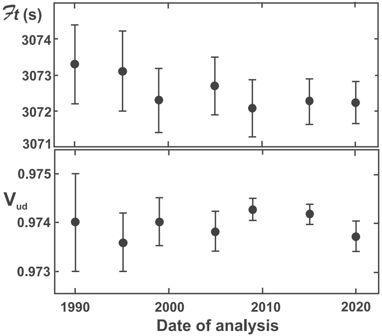
$$|V_{ud}|^2 = \frac{2984.432(3)\,\text{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

with (universal) radiative corrections Δ_R^V

- Value of V_{ud} crucially depends on Δ_R^V:

Ref.	Δ_R^V
Marciano, Sirlin 2006	0.02361(38)
Seng, Gorchtein, Patel, Ramsey-Musolf 2018	0.02467(22)
Czarnecki, Marciano, Sirlin 2019	0.02426(32)
Seng, Feng, Gorchtein, Jin 2020	0.02477(24)
Hayen 2020	0.02474(31)
Shiells, Blunden, Melnitchouk 2021	0.02472(18)
Cirigliano, Crivellin, MH, Moulson 2022	0.02467(27)

$\hookrightarrow$ main uncertainty from Regge region, lattice QCD to improve?



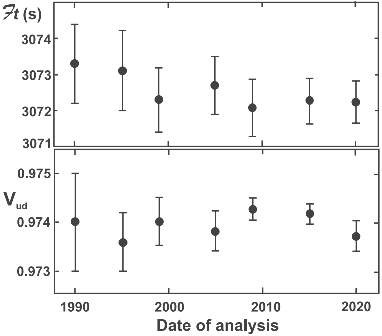
Hardy, Towner 2020

Determination of V_{ud} from superallowed β decays

- Further corrections
  - Isospin breaking Miller, Schwenk 2008, 2009, Condren, Miller 2022, Seng, Gorchtein 2022, Crawford, Miller 2022
  - Nuclear corrections Seng, Gorchtein, Ramsey-Musolf 2018, Gorchtein 2018, Seng, Gorchtein 2022
- Estimate from Gorchtein 2018 becomes dominant source of uncertainty

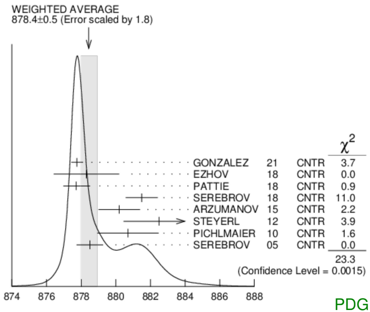
$$V_{ud}^{0^+\to 0^+} = 0.97367(11)_{\text{exp}}(13)_{\Delta_V^R}(27)_{\text{NS}}[32]_{\text{total}}$$

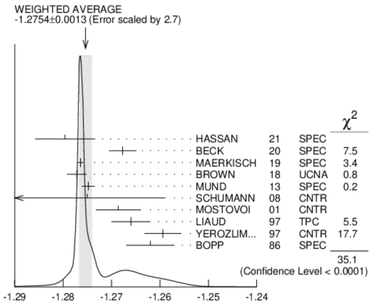
- Improvements from ab-initio nuclear structure? Martin, Stroberg, Holt, Leach 2021



Hardy, Towner 2020

Determination of V_{ud} from neutron decay





PDG 2022

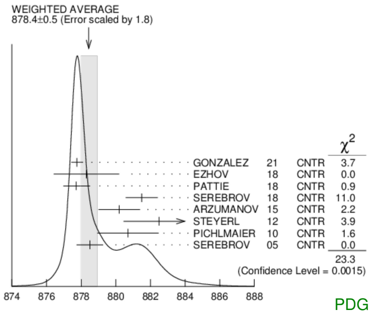
- Master formula Czarnecki, Marciano, Sirlin 2018

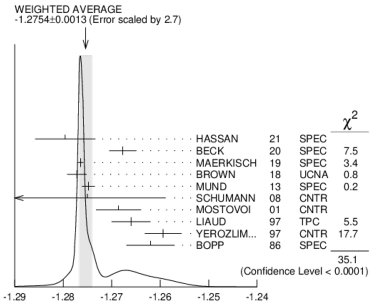
$$|V_{ud}|^2\tau_n(1+3g_A^2)(1+\Delta_{\rm RC}) = 5099.3(3)\,\text{s}$$

with radiative corrections $\Delta_{\rm RC}$

$\hookrightarrow$ need lifetime τ_n and asymmetry $\lambda = g_A/g_V$

- PDG average especially for g_A includes large scale factors





PDG 2022

- Results for V_{ud}

$$V_{ud}^{\text{n, PDG}} = 0.97441(3)_f(13)_{\Delta_R}(82)_\lambda(28)_{\tau_n}[88]_{\text{total}}$$

$$V_{ud}^{\text{n, best}} = 0.97413(3)_f(13)_{\Delta_R}(35)_\lambda(20)_{\tau_n}[43]_{\text{total}}$$

$\hookrightarrow$ average of $V_{ud}^{0^+\to 0^+}$ with $V_{ud}^{\text{n, best}}$ gives $V_{ud}^{\beta} = 0.97384(26)$

- Need improved measurements especially for g_A to make progress

Determination of V_{ud} from pion β decay

- Master formula Cirigliano, Knecht, Neufeld, Pichl 2003, Czarnecki, Marciano, Sirlin 2020, Feng et al. 2020

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu_e(\gamma)) = \frac{G_F^2 |V_{ud}|^2 M_{\pi^\pm}^5 |f_+^\pi(0)|^2}{64\pi^3}(1 + \Delta_{\text{RC}}^{\pi\ell}) I_{\pi\ell}$$

$\hookrightarrow$ need branching fraction and pion life time from experiment

- (Theory) inputs
  - Phase space $I_{\pi\ell} = 7.3766(43) \times 10^{-8}$
  - Form factor $f_+^\pi(0) = 1 - 7 \times 10^{-6}$

    $\hookrightarrow$ protected by $SU(2)$ Ademollo–Gatto theorem (Behrends–Sirlin)
  - Radiative corrections $\Delta_{\text{RC}}^{\pi\ell} = 0.0334(10)$ ChPT, Cirigliano et al., $\Delta_{\text{RC}}^{\pi\ell} = 0.0332(3)$ lattice QCD, Feng et al.

- Resulting V_{ud} extracted from PIBETA 2004

$$V_{ud}^{\pi,\text{ChPT}} = 0.97376(281)_{\text{BR}}(9)_{\tau_\pi}(47)_{\Delta_{\text{RC}}^{\pi\ell}}(28)_{I_{\pi\ell}}[287]_{\text{total}}$$

$$V_{ud}^{\pi,\text{lattice}} = 0.97386(281)_{\text{BR}}(9)_{\tau_\pi}(14)_{\Delta_{\text{RC}}^{\pi\ell}}(28)_{I_{\pi\ell}}[283]_{\text{total}}$$

$\hookrightarrow$ factor 10 possible before other errors creep in, aim for **PIONEER experiment**

Determination of V_{us}/V_{ud} from kaon decays: $K_{\ell 2}/\pi_{\ell 2}$

- **$K_{\ell 2}$ decays**: $K \to \ell \nu_\ell$

$$\frac{V_{us}}{V_{ud}}\frac{F_K}{F_\pi} = \left(\frac{\Gamma(K^+ \to \mu^+\nu_\mu(\gamma)M_\pi}{\Gamma(\pi^+ \to \mu^+\nu_\mu(\gamma)M_K}\right)^{1/2} \frac{1-\frac{m_\mu^2}{M_\pi^2}}{1-\frac{m_\mu^2}{M_K^2}}\left(1-\underbrace{\frac{\Delta_{RC}^K-\Delta_{RC}^\pi}{2}}_{\Delta_{RC}^{K\pi}/2}\right)$$

- Consider the ratio over $\pi_{\mu 2}$ because
  - Only need ratio of decay constant
  - Certain structure-dependent radiative corrections cancel
- Need theory input for:
  - **Decay constants** in isospin limit: $F_K/F_\pi = 1.1978(22)$ HPQCD 2013, Fermilab/MILC 2017, CalLat 2020, ETMC 2021
  - **Isospin-breaking corrections**: $\Delta_{RC}^{K\pi} = -0.0112(21)$ ChPT, Cirigliano, Neufeld 2011, $\Delta_{RC}^{K\pi} = -0.0126(14)$ lattice, Di Carlo et al. 2019
- Result:

$$\frac{V_{us}}{V_{ud}}\bigg|_{K_{\ell 2}/\pi_{\ell 2}} = 0.23108(23)_{\text{exp}}(42)_{F_K/F_\pi}(16)_{\text{IB}}[51]_{\text{total}}$$

Determination of V_{us} from kaon decays: $K_{\ell 3}$

- **$K_{\ell 3}$ decays**: $K \to \pi \ell \nu_\ell$

$$\Gamma(K \to \pi \ell \nu_\ell(\gamma)) = \frac{C_K^2 G_F^2 |V_{us}|^2 M_K^5 |f_+^{K\pi}(0)|^2}{192\pi^3} \Big(1 + \underbrace{\Delta_{\text{RC}}^{K\ell}}_{\Delta_{\text{EM}}^{K\ell} + \Delta_{SU(2)}}\Big) I_{K\ell}$$

$\hookrightarrow \ell = \mu, e$ and two charge channels

- Need theory input for:
  - **Form factor**: $f_+^{K\pi}(0) = 0.9698(17)$ ETMC 2016, Fermilab/MILC 2019
  - **Radiative corrections**: $\Delta_{SU(2)} = 0.0252(11)$ Cirigliano et al. 2002, $\Delta_{\text{EM}}^{K^0 e} = 0.0116(3)$, $\Delta_{\text{EM}}^{K^+ e} = 0.0021(5)$, $\Delta_{\text{EM}}^{K^0 \mu} = 0.0154(4)$, $\Delta_{\text{EM}}^{K^+ \mu} = 0.0005(5)$ Seng et al. 2022
- Result:

$$V_{us}^{K_{\ell 3}} = 0.22330(35)_{\text{exp}}(39)_{f_+}(8)_{\text{IB}}[53]_{\text{total}}$$

Tensions in the V_{ud}–V_{us} plane

- Global-fit point away from unitarity line

  ($\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 - 1$)

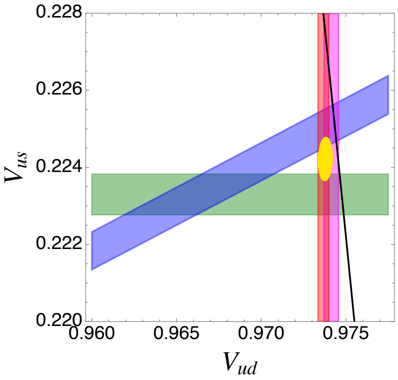
$$V_{ud} = 0.97378(26) \qquad V_{us} = 0.22422(36)$$

$$\Delta_{\text{CKM}} = -1.48(53) \times 10^{-3} \quad [2.8\sigma]$$

- Three possible measures of the CKM tension

$$\Delta^{(1)}_{\text{CKM}} = \big|V^{\beta}_{ud}\big|^2 + \big|V^{K_{\ell 3}}_{us}\big|^2 - 1$$
$$= -1.76(56) \times 10^{-3} \quad [3.1\sigma]$$
$$\Delta^{(2)}_{\text{CKM}} = \big|V^{\beta}_{ud}\big|^2 + \big|V^{K_{\ell 2}/\pi_{\ell 2},\,\beta}_{us}\big|^2 - 1$$
$$= -0.98(58) \times 10^{-3} \quad [1.7\sigma]$$
$$\Delta^{(3)}_{\text{CKM}} = \big|V^{K_{\ell 2}/\pi_{\ell 2},\,K_{\ell 3}}_{ud}\big|^2 + \big|V^{K_{\ell 3}}_{us}\big|^2 - 1$$
$$= -1.64(63) \times 10^{-2} \quad [2.6\sigma]$$

$\hookrightarrow$ already tension in kaon sector alone 2.6σ



Cirigliano, Crivellin, MH, Moulson 2022

What can we do to clarify the situation?

- Corroborating V_{ud}
  - Nuclear-structure corrections for superallowed β decays
  - Improved neutron-decay measurements (g_A, τ_n)
  - **Pion β decay** with PIONEER
- Corroborating V_{us}
  - Improved lattice calculations of F_K/F_π
  - **A new measurement of $K_{\mu 3}/K_{\mu 2}$**, possible at NA62
  - τ and hyperon decays sensitive to V_{us}, but feasible at the relevant level of accuracy?

A new measurement of $K_{\mu 3}/K_{\mu 2}$, why?

	current fit	$K_{\mu 3}/K_{\mu 2}$ BR at 0.5%			$K_{\mu 3}/K_{\mu 2}$ BR at 0.2%		
		central	$+2\sigma$	-2σ	central	$+2\sigma$	-2σ
$\frac{V_{us}}{V_{ud}}\Big\vert_{K_{\ell 2}/\pi_{\ell 2}}$	0.23108(51)	0.23108(50)	0.23085(51)	0.23133(51)	0.23108(49)	0.23071(51)	0.23147(52)
$V_{us}^{K_{\ell 3}}$	0.22330(53)	0.22337(51)	0.22360(52)	0.22309(54)	0.22342(49)	0.22386(52)	0.22287(52)
$10^2\,\Delta^{(3)}_{\text{CKM}}$	$-1.64(63)$	$-1.57(60)$	$-1.18(62)$	$-2.02(63)$	$-1.53(59)$	$-0.83(62)$	$-2.33(62)$
	-2.6σ	-2.6σ	-1.9σ	-3.2σ	-2.6σ	-1.4σ	-3.8σ

- Is the $K_{\ell 3}$ vs. $K_{\ell 2}$ tension real or an experimental problem?
  - $K_{\ell 2}$ data base completely dominated by KLOE 2006
  - Global fit to kaon data not great, p-value $\simeq$ 1%
- This can be clarified with **a new precision measurement of $K_{\mu 3}/K_{\mu 2}$**:
  - In case the tension were of experimental origin, there should be a positive shift compared to current fit

    $\hookrightarrow$ $\Delta^{(3)}_{\text{CKM}}$ would move from -2.6σ to -1.4σ for a $+2\sigma$ shift with a 0.2% measurement
  - In case the tension were of BSM origin, the current value would be confirmed (or move further in the other direction)

$\hookrightarrow$ **a single new precision measurement would have a huge impact!**

An interpretation in terms of right-handed currents

- Modify **right-handed current**
  $\hookrightarrow$ vector $\sim 1 + \varepsilon_R$, axial-vector $\sim 1 - \varepsilon_R$

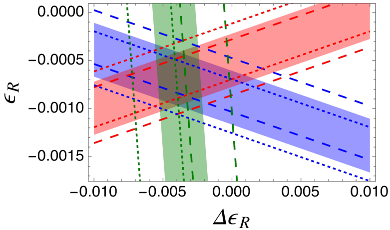
$$\Delta^{(1)}_{\text{CKM}} = 2\varepsilon_R + 2\Delta\varepsilon_R V_{us}^2 \qquad \text{(blue)}$$
$$\Delta^{(2)}_{\text{CKM}} = 2\varepsilon_R - 2\Delta\varepsilon_R V_{us}^2 \qquad \text{(red)}$$
$$\Delta^{(3)}_{\text{CKM}} = 2\varepsilon_R + 2\Delta\varepsilon_R (2 - V_{us}^2) \qquad \text{(green)}$$

  where $\Delta\varepsilon_R \equiv \varepsilon_R^{(s)} - \varepsilon_R$

- Current fit

$$\varepsilon_R = -0.69(27) \times 10^{-3} \qquad [2.5\sigma]$$
$$\Delta\varepsilon_R = -3.9(1.6) \times 10^{-3} \qquad [2.4\sigma]$$

- Impact of new $K_{\mu 3}/K_{\mu 2}$ measurement mainly on $\Delta\varepsilon_R$ (dashed and dotted lines $\pm 2\sigma$ benchmark)



Cirigliano, Crivellin, MH, Moulson 2022

Modification of the Fermi constant

- **Fermi constant**
  - Best determination from muon decay MuLan 2013

$$G_F^\mu = 1.1663787(6) \times 10^{-5}\,\text{GeV}^{-2}$$

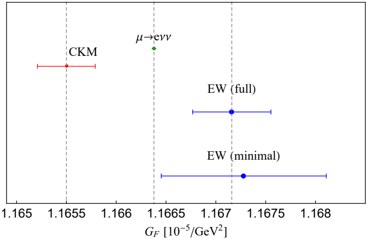
  - Electroweak fit Marciano 1999, update using HEPFit

$$G_F^{\text{EW}}\Big|_{\text{full}} = 1.16716(39) \times 10^{-5}\,\text{GeV}^{-2}$$

  - CKM deficit interpreted as modification of G_F in β decays

$$G_F^{\text{CKM}} = 1.16550(29) \times 10^{-5}\,\text{GeV}^{-2}$$

  - Does not explain tension in kaon sector



Crivellin, MH, Manzari 2021

SMEFT analysis of G_F tensions

- Possible explanations in terms of effective operators

  A. four-fermion operators in $\mu \to e\nu\nu$: only viable for SM operator $Q_{\ell\ell}^{2112} = \bar{\ell}_2\gamma^\mu\ell_1\bar{\ell}_1\gamma_\mu\ell_2$

  B. four-fermion operators in $u \to de\nu$: now excluded by LHC bounds

  C. modified W–u–d couplings: possible in terms of Belfatto, Berezhiani 2021

  $$Q_{\phi q}^{(3)ij} = \phi^\dagger i\overleftrightarrow{D}_\mu^I \phi\bar{q}_i\gamma^\mu\tau^I q_j \qquad Q_{\phi ud}^{ij} = \tilde{\phi}^\dagger iD_\mu\phi\bar{u}_i\gamma^\mu d_j$$

  $\hookrightarrow$ generate left- and right-handed currents, respectively

  D. modified W–ℓ–ν couplings: operator

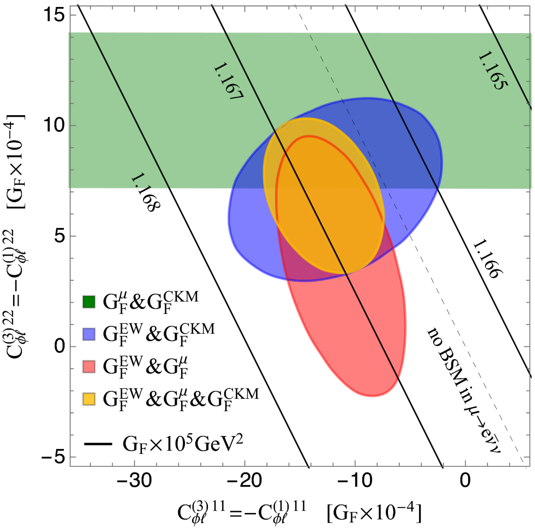
  $$Q_{\phi\ell}^{(3)ij} = \phi^\dagger i\overleftrightarrow{D}_\mu^I \phi\bar{\ell}_i\gamma^\mu\tau^I\ell_j$$

  leads to interpretation in terms of LFUV Crivellin, MH 2020

  E. other operators affecting the EW fit see also talk by D. Stöckinger, $Q_{\phi\ell}^{(3)ij}$ and

  $$Q_{\phi\ell}^{(1)ij} = \phi^\dagger i\overleftrightarrow{D}_\mu\phi\bar{\ell}_i\gamma^\mu\ell_j$$

  $\hookrightarrow$ effect can be minimized by turning off $Z \to \ell\ell$ with $C_{\phi\ell}^{(1)ij} = -C_{\phi\ell}^{(3)ij}$



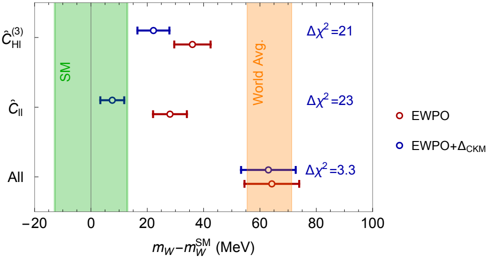
Crivellin, MH, Manzari 2021

- Common explanation in terms of $C_{\phi\ell}^{(1)11} = -C_{\phi\ell}^{(3)11}$ and $C_{\phi\ell}^{(1)22} = -C_{\phi\ell}^{(3)22}$ possible
- For BSM sensitivity the **second-most-precise determination of G_F is crucial**

Impact of CKM unitarity on explanations of W-boson mass

	Result	Result with CKM
$\hat{C}_{\varphi l}^{(1)}$	-0.007 ± 0.011	-0.013 ± 0.009
$\hat{C}_{\varphi l}^{(3)}$	-0.042 ± 0.015	-0.034 ± 0.014
$\hat{C}_{\varphi e}$	-0.017 ± 0.009	-0.021 ± 0.009
$\hat{C}_{\varphi q}^{(1)}$	-0.0181 ± 0.044	-0.048 ± 0.04
$\hat{C}_{\varphi q}^{(3)}$	-0.114 ± 0.043	-0.041 ± 0.015
$\hat{C}_{\varphi u}$	0.086 ± 0.154	-0.12 ± 0.11
$\hat{C}_{\varphi d}$	-0.626 ± 0.248	-0.38 ± 0.22
C_{Δ}	-0.19 ± 0.09	-0.027 ± 0.011

Out[∘]=



Cirigliano, Dekens, de Vries, Mereghetti, Tong 2022

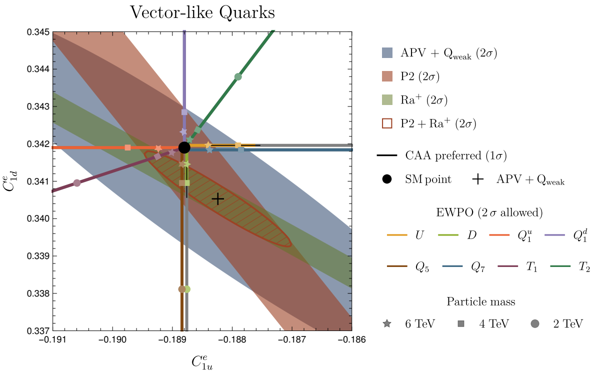
Falkowski, Gonzáles-Alonso, . . .

- Δ_{CKM} excludes certain explanations of M_W

  $\hookrightarrow$ should be included in EW fit

- Otherwise, generic explanations tend to produce a percent-level Δ_{CKM}

Correlations with parity violation in simplified models



Crivellin, MH, Kirk, Manzari, Schnell 2021

- **Low-energy parity violation** conventionally parameterized in terms of

$$\mathcal{L}_{\text{eff}}^{ee} = \frac{G_F}{\sqrt{2}} \sum_{q=u,d,s} \left(C_{1q}^e \left[\bar{q}\gamma^\mu q\right]\left[\bar{e}\gamma_\mu\gamma_5 e\right] + C_{2q}^e \left[\bar{q}\gamma^\mu\gamma_5 q\right]\left[\bar{e}\gamma_\mu e\right] \right)$$

- In simplified models, Cabibbo angle anomaly defines a preferred parameter range

  $\hookrightarrow$ can be tested in parity-violating electron scattering and atomic parity violation

Lepton flavor universality violation

- Let us parameterize the **W couplings** as $\mathcal{L} = -i\frac{g_2}{\sqrt{2}}\bar{\ell}_i\gamma^\mu P_L\nu_j W_\mu\big(\delta_{ij} + \varepsilon_{ij}\big)$
- Modifies Fermi constant in **muon decay**

$$\frac{1}{\tau_\mu} = \frac{(G_F^{\mathcal{L}})^2 m_\mu^5}{192\pi^3}(1+\Delta q)(1+\varepsilon_{ee}+\varepsilon_{\mu\mu})^2$$

$\hookrightarrow$ measured Fermi constant $G_F = G_F^{\mathcal{L}}(1+\varepsilon_{ee}+\varepsilon_{\mu\mu})$

- All β-decay observables affected according to

$$V_{ud} \to V_{ud}^{\beta} = V_{ud}^{\mathcal{L}}(1-\varepsilon_{\mu\mu})$$

where $V_{ij}^{\mathcal{L}}$ fulfill CKM unitarity

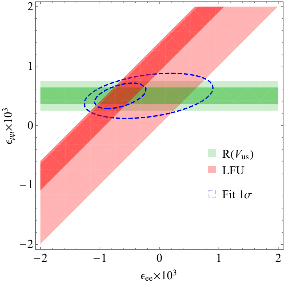
- Construct ratio Crivellin, MH 2020

$$R(V_{us}) \equiv \frac{V_{us}^{K_{\mu 2}}}{V_{us}^{\beta}} \equiv \frac{V_{us}^{K_{\mu 2}}}{\sqrt{1-(V_{ud}^{\beta})^2-|V_{ub}|^2}} = 1 - \left(\frac{V_{ud}}{V_{us}}\right)^2\varepsilon_{\mu\mu} + \mathcal{O}(\varepsilon^2)$$

$\hookrightarrow$ LFUV effect enhanced by $(V_{ud}/V_{us})^2 \sim 20$!

Lepton flavor universality violation

Observable	Measurement	Constraint $\times 10^3$
$\frac{K\to\pi\mu\bar\nu}{K\to\pi e\bar\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	1.0010(25)	1.0(2.5)
$\frac{K\to\mu\nu}{K\to e\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	0.9978(18)	−2.2(1.8)
$\frac{\pi\to\mu\nu}{\pi\to e\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	1.0010(9)	1.0(9)
$\frac{\tau\to\mu\nu\bar\nu}{\tau\to e\nu\bar\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	1.0018(14)	1.8(1.4)
$\frac{W\to\mu\bar\nu}{W\to e\bar\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	0.9960(100)	−4(10)
$\frac{B\to D^{(*)}\mu\nu}{B\to D^{(*)}e\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	0.9890(120)	−11(12)
$R(V_{us}) \simeq 1-\left(\frac{V_{ud}}{V_{us}}\right)^2\varepsilon_{\mu\mu}$	0.9891(35)	0.58(19)



Crivellin, MH 2020

- Most stringent constraint on $\varepsilon_{\mu\mu}$ thanks to **CKM enhancement**
- Also does not explain tension in kaon sector
- Best constraint on $\varepsilon_{\mu\mu}-\varepsilon_{ee}$ from

$$R^\pi_{e/\mu} = \frac{\Gamma(\pi\to e\nu_e(\gamma))}{\Gamma(\pi\to\mu\nu_\mu(\gamma))}$$

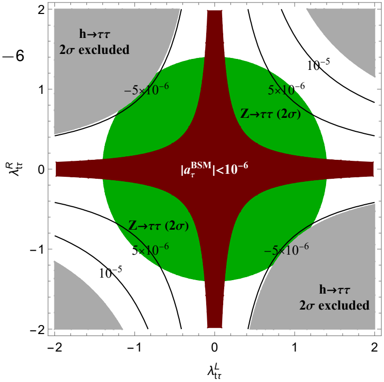
- Factor 3 (10) from PEN/PiENu (PIONEER), factor 3 for τ decays from Belle II

What about $(g-2)_\tau$?

- Current status Abdallah et al. 2004, Keshavarzi et al. 2020

$$a_\tau^{\text{exp}} = -0.018(17) \quad \textit{vs.} \quad a_\tau^{\text{SM}} = 1{,}177.171(39)\times 10^{-6}$$

- **Scaling arguments**:
  - Minimal flavor violation: $a_\tau^{\text{BSM}} \simeq a_\mu^{\text{BSM}} \left(\frac{m_\tau}{m_\mu}\right)^2 \simeq 0.7\times 10^{-6}$
  - Electroweak contribution: $a_\tau^{\text{EW}} \simeq 0.5\times 10^{-6}$
- **Concrete models**:
  - S_1 leptoquark model promising due to **chiral enhancement** with $\frac{m_t}{m_\tau}$ $\hookrightarrow$ can get $a_\tau^{\text{BSM}} \simeq$ (few) $\times 10^{-6}$ without violating $h\to\tau\tau$ and $Z\to\tau\tau$



Crivellin, MH, Roney 2021

- Ultimate target has to be a measurement of a_τ at the level of 10^{-6}

$\hookrightarrow$ requires two-loop accuracy for theory throughout

Experimental prospects for $(g-2)_\tau$

- Many recent proposals, none of which seem to reach much beyond the Schwinger term
- Exception: **$e^+e^- \to \tau^+\tau^-$ at Υ resonances** Bernabéu et al. 2007
  $\hookrightarrow$ quotes projections at 10^{-6} level
- Idea: study $e^+e^- \to \tau^+\tau^-$ cross section and asymmetries
  $\hookrightarrow$ could this be realized at Belle II Crivellin, MH, Roney 2021?
- Answer: yes, but requires **polarization upgrade of SuperKEK** to get access to transverse and longitudinal asymmetries
  $\hookrightarrow$ Hiroshima Workshop on Beam Polarization Feb 8+9, https://indico.belle2.org/event/7500/
- Idea: extract $F_2(s)$ at $s \simeq (10\,\text{GeV})^2$, but heavy new physics decouples
  $\hookrightarrow$ $a_\tau^\text{BSM} = F_2^\text{exp}(s) - F_2^\text{SM}(s)$ as long as $s \ll \Lambda^2_\text{BSM}$
- Bounds on light BSM become model dependent, but anyway better constrained in other processes

First attempt: total cross section

- **Differential cross section** for $e^+e^- \to \tau^+\tau^-$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4s}\Big[(2-\beta^2\sin^2\theta)\big(|F_1|^2-\gamma^2|F_2|^2\big) + 4\text{Re}\,\big(F_1F_2^*\big) + 2(1+\gamma^2)|F_2|^2\Big]$$

with scattering angle θ, $\beta = \sqrt{1-4m_\tau^2/s}$, $\gamma = \sqrt{s}/(2m_\tau)$

- Interference term $4\text{Re}\,\big(F_1F_2^*\big)$ sensitive to the sought two-loop effects
- Could be determined by fit to θ dependence
- But: need to measure total cross section at 10^{-6}

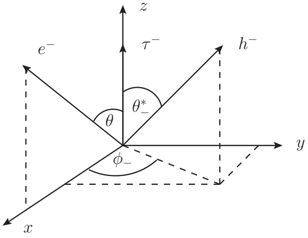
$\hookrightarrow$ **can we use asymmetries instead**?

- Usual forward–backward asymmetry ($z = \cos\theta$)

$$\sigma_{\text{FB}} = 2\pi\bigg[\int_0^1 dz\,\frac{d\sigma}{d\Omega} - \int_{-1}^0 dz\,\frac{d\sigma}{d\Omega}\bigg]$$

alone does not help

Second attempt: normal asymmetry



- Idea: use **polarization information of the $\tau^\pm$**

  $\hookrightarrow$ semileptonic decays $\tau^\pm \to h^\pm \overset{(-)}{\nu}_\tau$, $h = \pi, \rho, \ldots$

  Bernabéu et al. 2007

- Polarization characterized by

$$\mathbf{n}^*_\pm = \mp\alpha_\pm \begin{pmatrix} \sin\theta^*_\pm \cos\phi_\pm \\ \sin\theta^*_\pm \sin\phi_\pm \\ \cos\theta^*_\pm \end{pmatrix} \qquad \alpha_\pm \equiv \frac{m_\tau^2 - 2m_{h^\pm}^2}{m_\tau^2 + 2m_{h^\pm}^2} = \begin{cases} 0.97 & h^\pm = \pi^\pm \\ 0.46 & h^\pm = \rho^\pm \end{cases}$$

  $\hookrightarrow$ angles in $\tau^\pm$ rest frame

- **Normal asymmetry**

$$A_N^\pm = \frac{\sigma_L^\pm - \sigma_R^\pm}{\sigma} \propto \operatorname{Im} F_2(s) \qquad \sigma_L^\pm = \int_\pi^{2\pi} d\phi_\pm \frac{d\sigma_{FB}}{d\phi_\pm} \quad \sigma_R^\pm = \int_0^\pi d\phi_\pm \frac{d\sigma_{FB}}{d\phi_\pm}$$

  $\hookrightarrow$ only get the imaginary part, **need electron polarization**

Third attempt: electron polarization

- **Transverse and longitudinal asymmetries** Bernabéu et al. 2007

$$A_T^{\pm} = \frac{\sigma_R^{\pm} - \sigma_L^{\pm}}{\sigma} \qquad A_L^{\pm} = \frac{\sigma_{\text{FB},\,R}^{\pm} - \sigma_{\text{FB},\,L}^{\pm}}{\sigma}$$

- Constructed based on helicity difference

$$d\sigma_{\text{pol}}^{S} = \frac{1}{2}\Big(d\sigma^{S\lambda}\big|_{\lambda=1} - d\sigma^{S\lambda}\big|_{\lambda=-1}\Big)$$

  and then integrating over angles

$$\sigma_R^{\pm} = \int_{-\pi/2}^{\pi/2} d\phi_{\pm} \frac{d\sigma_{\text{pol}}^{S}}{d\phi_{\pm}} \qquad \sigma_L^{\pm} = \int_{\pi/2}^{3\pi/2} d\phi_{\pm} \frac{d\sigma_{\text{pol}}^{S}}{d\phi_{\pm}} \qquad \sigma_{\text{FB},\,R}^{\pm} = \int_0^1 dz_{\pm}^{*} \frac{d\sigma_{\text{FB,pol}}^{S}}{dz_{\pm}^{*}} \qquad \sigma_{\text{FB},\,L}^{\pm} = \int_{-1}^{0} dz_{\pm}^{*} \frac{d\sigma_{\text{FB,pol}}^{S}}{dz_{\pm}^{*}}$$

- Linear combination

$$A_T^{\pm} - \frac{\pi}{2\gamma} A_L^{\pm} = \mp\alpha_{\pm} \frac{\pi^2\alpha^2\beta^3\gamma}{4s\sigma}\Big[\text{Re}\,(F_2F_1^{*}) + |F_2|^2\Big]$$

  isolates the interesting interference effect

How to make use of this?

Contributions to Re $F_2^{\text{eff}}(s)$	$s=0$	$s=(10\,\text{GeV})^2$
1-loop QED	1161.41	-265.90
e loop	10.92	-2.43
μ loop	1.95	-0.34
2-loop QED (mass independent)	-0.42	-0.24
HVP	3.33	-0.33
EW	0.47	0.47
total	1177.66	-268.77

$$\text{Re}\, F_2^{\text{eff}}((10\,\text{GeV})^2) \simeq \mp\frac{0.73}{\alpha_\pm}\left(A_T^\pm - 0.56 A_L^\pm\right)$$

- **Strategy**:
  - Measure effective $F_2(s)$

$$\text{Re}\, F_2^{\text{eff}} = \mp\frac{8(3-\beta^2)}{3\pi\gamma\beta^2\alpha_\pm}\left(A_T^\pm - \frac{\pi}{2\gamma}A_L^\pm\right)$$

  - Compare measurement to SM prediction for Re F_2^{eff}
  - Difference gives constraint on a_τ^{BSM}
  - A measurement of $A_T^\pm - \frac{\pi}{2\gamma}A_L^\pm$ at $\lesssim 1\%$ would already be competitive with current limits

How to make use of this?

- **Challenges**:
  - Cancellation in $A_T^{\pm} - \frac{\pi}{2\gamma} A_L^{\pm}$: $A_{T,L}^{\pm} = \mathcal{O}(1)$, difference $\mathcal{O}(\alpha)$
  - Two-loop calculation in SM see 2111.10378 for form factor and radiative corrections
  - Form factor only dominates for resonant $\tau^+\tau^-$ pairs

$$|H(M_\Upsilon)|^2 = \left(\frac{3}{\alpha}\text{Br}(\Upsilon \to e^+e^-)\right)^2 \simeq 100$$

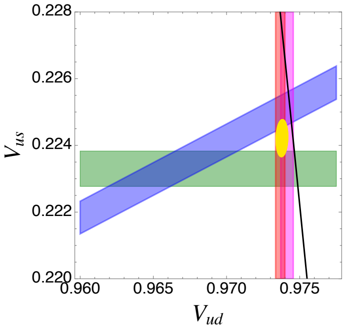
  - However: continuum pairs dominate even at $\Upsilon(nS)$, $n = 1, 2, 3$, due to energy spread
  - Should consider $A_T^{\pm}$, $A_L^{\pm}$ also for nonresonant $\tau^+\tau^-$, but requires substantial investment in theory for SM prediction (box diagrams, ...)

Conclusions

- Tensions among β decays and kaon decays point to the **apparent violation of CKM unitarity**
- Tension at the level of $(2\text{–}3)\sigma$

  $\hookrightarrow$ more work needed to corroborate or resolve
- **Pion β decay** clean, competitive probe of V_{ud} if branching fraction improved by a factor 10
- **New precision measurement of $K_{\mu3}/K_{\mu2}$** to clarify situation in kaon sector
- Interesting interplay with **electroweak fit** and tests of **lepton flavor universality**
- BSM search with $(g-2)_\tau$ via $e^+e^- \to \tau^+\tau^-$ asymmetries using polarized electrons

  SuperKEKB with electron polarization upgrade?



Back to pion β decay

- For $K_{\ell 2}$ and $\pi_{\ell 2}$ decays one uses the ratio

$$R_A = \frac{\Gamma(K^+ \to \mu^+\nu_\mu(\gamma))}{\Gamma(\pi^+ \to \mu^+\nu_\mu(\gamma))} = \left(\frac{V_{us}}{V_{ud}}\frac{F_K}{F_\pi}\right)^2 \frac{M_K}{M_\pi}\left(\frac{1-\frac{m_\mu^2}{M_K^2}}{1-\frac{m_\mu^2}{M_\pi^2}}\right)^2 \left(1+\Delta_{RC}^K-\Delta_{RC}^\pi\right)$$

  to cancel uncertainties and extract V_{us}/V_{ud}

- Can do the same for $K_{\ell 3}$ and $\pi_{\ell 3}$ Czarnecki, Marciano, Sirlin 2020

$$R_V = \frac{\Gamma(K \to \pi\ell\nu_\ell(\gamma))}{\Gamma(\pi^+ \to \pi^0 e^+\nu_e(\gamma))}$$

- Need a factor 2–3 to obtain a competitive value of V_{us}/V_{ud}, first goal for PIONEER

- Caveats: contrary to R_A no cancellation of structure-dependent radiative corrections nor gains in form-factor determination

  $\hookrightarrow$ **need factor 10 of Phase III to unleash full potential**

BSM searches with pion β decay

- Generalize master formula to include **effective operators** not present in SM

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu_e(\gamma)) = \frac{G_F^2 |V_{ud}|^2}{192\pi^3 M_\pi^3}(1 + \Delta_{\text{RC}}^{\pi\ell}) \int_{m_e^2}^{(M_\pi - M_{\pi^0})^2} ds\, \lambda^{3/2}(s)\left(1 + \frac{m_e^2}{2s}\right)\left(1 - \frac{m_e^2}{s}\right)^2$$
$$\times \left[|V(s)|^2 + |A(s)|^2 + \frac{4(s - m_e^2)^2}{9sm_e^2}|T(s)|^2 + \frac{3m_e^2(M_\pi^2 - M_{\pi^0}^2)^2}{(2s + m_e^2)\lambda(s)}\left(|S(s)|^2 + |P(s)|^2\right)\right]$$

  with $V(s)$, $A(s)$, ... depending on Wilson coefficients c_V, c_A, ...

- **Tensor**: $T(s) = \frac{3s}{2s+m_e^2}\frac{m_e}{M_\pi} c_T B_T^\pi(s)$

  $\hookrightarrow$ suppressed by electron mass and tensor form factor

- **Scalar**: more competitive constraints, but still not at the same level as other β decays Falkowski, Gonzáles-Alonso, Naviliat-Cuncic 2020