Ultra Light Dark Matter from Thermal Fermions

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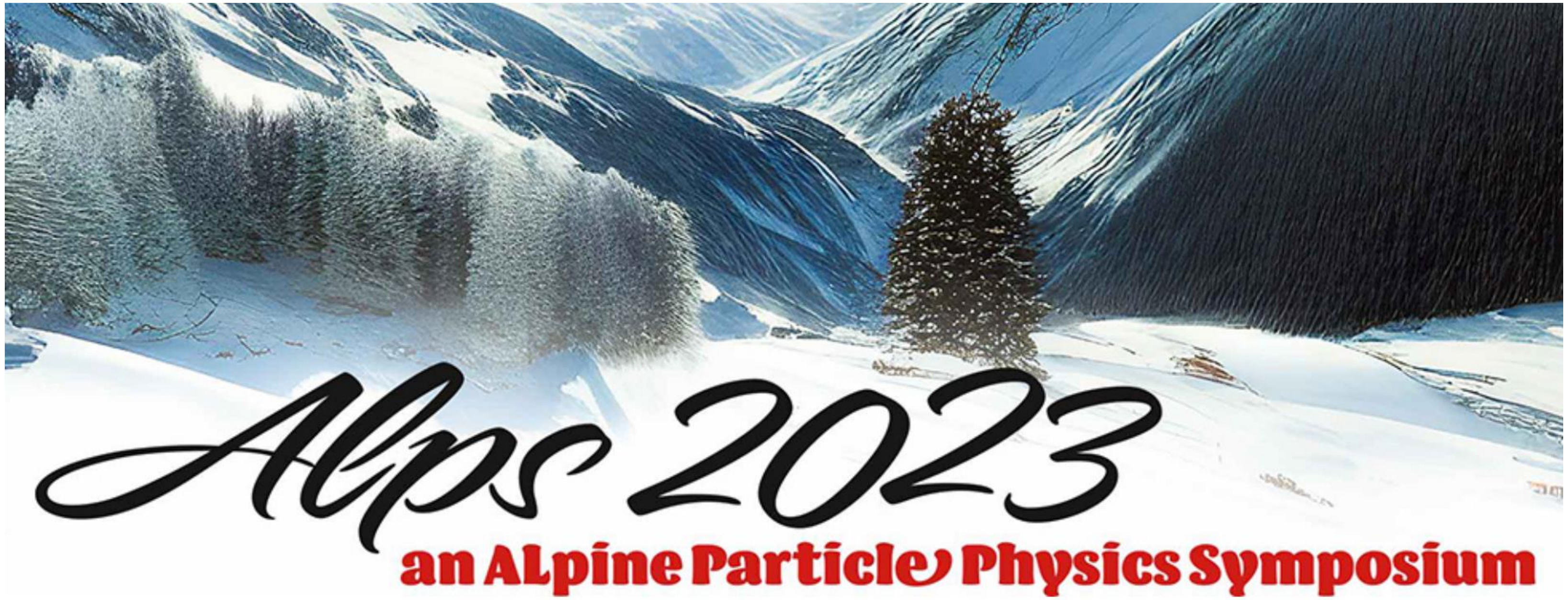


Outline

- Ultralight boson with a huge population can be a cold dark matter candidate

$$\frac{n_\phi}{s} \sim 10^{19}\,\frac{10^{-20}\mathrm{eV}}{m_\phi}$$

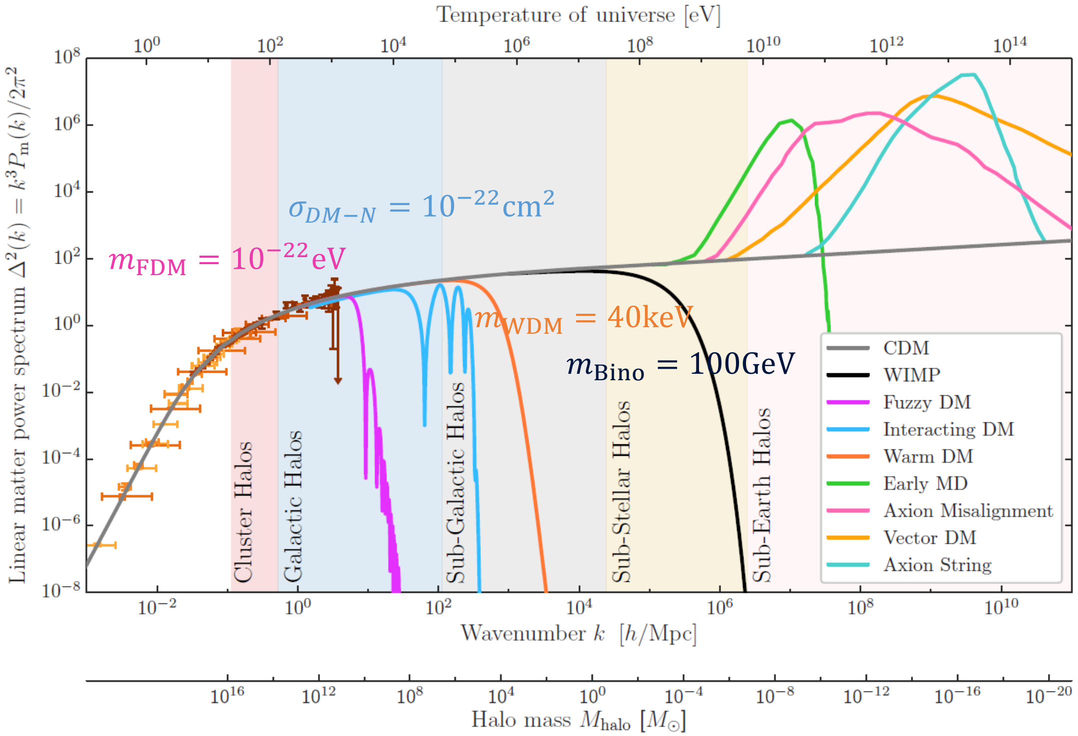
- It can be generated by a misaligned initial amplitude leading to a coherent oscillation (e.g., QCD axion with instanton potential).
- Thermal fermions may be the origin of the ULDM misalignment.
- Its impact on the neutrino mass.

EJC, 2109.07423

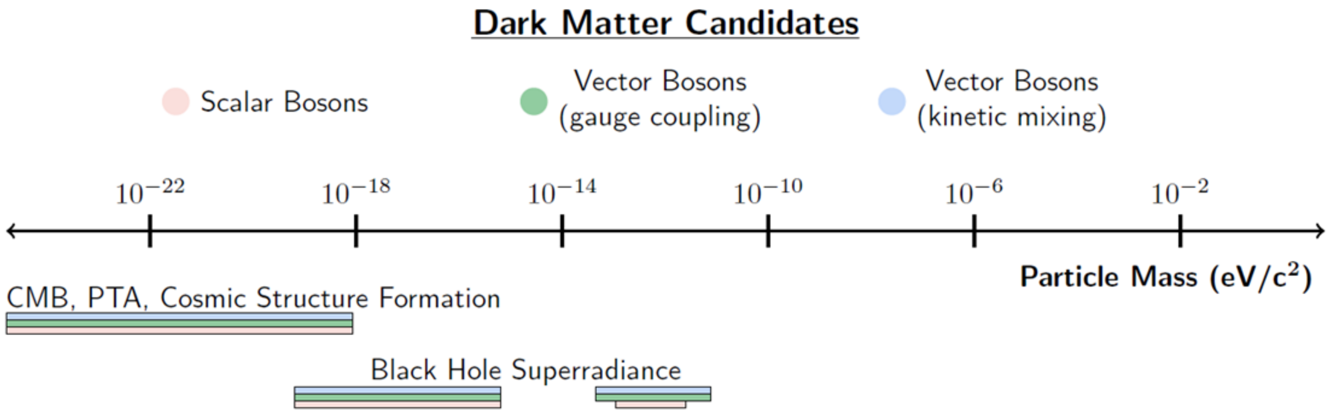
KY Choi, EJC, JK Kim, 2012.09474, 1909.10478

ULDM and cosmic structures

Snowmass, 2203.07354

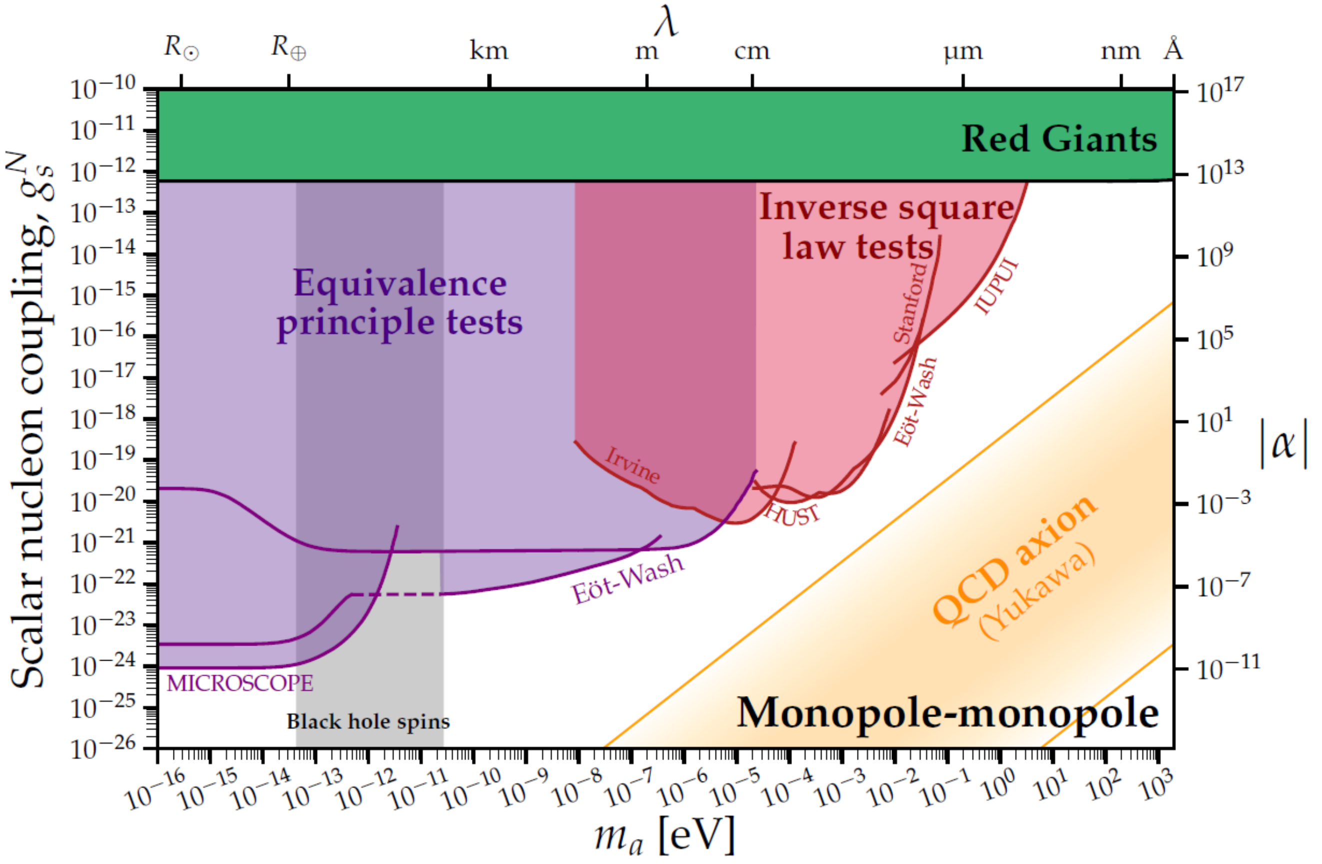


Snowmass, 2203.14915



Bounds from gravity

O'Hair, Vitagliano
2010.03889



Misalignment mechanism

- Evolution in the FLRW universe:

$$\langle\hat{\phi}(x)\rangle_T = \phi(t)$$

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + V'(\phi) = 0$$

- For an approximately free field:

$$\phi''(x) + \frac{3}{2x}\phi'(x) + \phi(x) \approx 0$$

$$x \equiv m_\phi t$$

- Analytic solution

$$\phi(x) = C_1 \frac{J_{1/4}(x)}{x^{1/4}} + C_2 \frac{Y_{1/4}(x)}{x^{1/4}}$$

$$H \gg m_\phi \ (x \ll 1): \quad \phi = \phi_i; \dot{\phi} = 0$$

$$H \ll m_\phi \ (x \gg 1): \quad \phi \sim \phi_i \frac{\sin\left(m_\phi t + \frac{\pi}{8}\right)}{(m_\phi t)^{3/4}}$$

$$m_\phi \gg H_{eq} \approx 3 \cdot 10^{-27}\text{eV}$$

CDM density: $\rho_{DM}(x_{eq}) \approx 0.23 \text{ eV}^4$

$$\rho_\phi(x_{eq}) \sim \frac{m_\phi^2 \phi_i^2}{x_{eq}^{3/2}} \Rightarrow \phi_i \sim 0.01\, M_p \left(\frac{10^{-20}\text{eV}}{m_\phi}\right)^{\frac{1}{4}}$$

Scalar field in thermal background

- Scalar field interacting with thermal fermions:

$$\mathcal{L}' = y_\phi\, \hat{\phi}\, (\bar{f}_R f_L + \bar{f}_L f_R)$$

$$\rightarrow V_{T,\mathrm{eff}}(\phi) = -\frac{g_f}{2\pi^2} T^4 J_F\left(\frac{(m_f + y_\phi \phi)^2}{T^2}\right)$$

Dolan+Jackiw, 1974
Weinberg, 1974

- Leading thermal effects in cosmological evolution:

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + (m_\phi^2 + m_T^2)\phi(t) \approx \frac{\partial}{\partial\phi}\langle\mathcal{L}'\rangle_T$$

$$m_T^2 = \frac{g_f}{24} y_\phi^2 T^2, \quad \langle\mathcal{L}'\rangle_T = y_\phi \phi \frac{g_f m_f T^2}{24}$$

$$g_f = 4N_c \ (2) \ \text{for} \ f = q, l\ (\nu)$$

Esteban+Salvado, 2101.05804

Batell+Ghalsasi, 2109.04476

General features I

- Evolution from $T_{ew} \approx 100$ GeV down to $T_{eq} \approx 0.67$ eV:

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \left(1 + \frac{x_1}{x}\right)\tilde{\phi}(x) = \frac{x_S}{x}$$

$$x_1 \equiv y_\phi^2 \frac{c_t^2 g_f}{48}\frac{M_P}{m_\phi}, \qquad x_S \equiv y_\phi \frac{c_t^2 g_f}{48}\frac{m_f}{m_\phi}$$

(Notation) $T = c_t\sqrt{M_P/2t}$ $\quad c_t = 1.74/g_*^{1/4}$

$$x = m_\phi t = \frac{c_t^2 m_\phi M_P}{2T^2},$$

$$\tilde{\phi} = \phi/M_P, \quad m_{20} \equiv m_\phi/10^{-20}\text{eV}$$

- Vanishing initial condition at x_{ew}: $\phi = 0, \phi' = 0$.
- The resulting DM density will be

$$\rho_\phi(x_{eq}) \approx \frac{c_f^2 x_S^2}{\pi}\frac{m_\phi^2 M_P^2}{x_{eq}^{3/2}}$$

$$\rho_\phi = \rho_{DM} \Rightarrow c_f^2 x_S^2 \approx 2.5 \times 10^{-4} m_{20}^{-1/2}$$

$$x_{ew} \approx 10^{-15} m_{20} \qquad x_{eq} \approx 2\times\ 10^7 m_{20}$$

$$x_1 \approx 10^{47} y_\phi^2 m_{20}^{-1} \qquad x_S \approx 2\cdot 10^{24}\ y_\phi\ m_{20}^{-1}\left(\frac{m_f}{m_e}\right)$$

General features II

- Nontrivial evolution with $x_{1,S}$ from T_{ew} (x_{ew}) to $T_f = m_f(x_f)$, and then free evolution ($x_{1,S} = 0$) from $T_f(x_f)$ to T_{eq} (x_{eq})

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \left(1 + \frac{x_1}{x}\right)\tilde{\phi}(x) = \frac{x_S}{x}$$

$$x_1 \equiv y_\phi^2 \frac{c_t^2 g_f}{48}\frac{M_P}{m_\phi}, \qquad x_S \equiv y_\phi \frac{c_t^2 g_f}{48}\frac{m_f}{m_\phi}$$

- Different types of solutions depending on the location of T_1 (x_1):

$$x_{ew} \dashrightarrow x_f \dashrightarrow x_{eq}$$

$$x_1$$

Analytic solutions in various limits

$x \ll x_1$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) = \frac{x_S}{x}$$

$x \gg x_1$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

$x > x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

$$\tilde{\phi}(x) = \frac{x_S}{x_1} + C_1\frac{e^{2i\sqrt{xx_1}}}{\sqrt{x}} + iC_2\frac{e^{-2i\sqrt{xx_1}}}{\sqrt{xx_1}}$$

$$\tilde{\phi}(x) = \frac{\pi x_S}{(2x)^{1/4}}\Bigg(\frac{x}{\Gamma(3/4)} J_{1/4}(x)\, F_{1,2}\left(\frac{1}{2};\frac{3}{4},\frac{3}{2};-\frac{x^2}{4}\right) - \frac{x^{3/2}}{3\Gamma(5/4)}\left(J_{1/4}(x) - Y_{1/4}(x)\right) F_{1,2}\left(\frac{3}{4};\frac{5}{4},\frac{7}{4};-\frac{x^2}{4}\right)\Bigg) + C_1\frac{J_{1/4}(x)}{x^{1/4}} + C_2\frac{Y_{1/4}(x)}{x^{1/4}}$$

$$\tilde{\phi}(x) = C_1\frac{J_{1/4}(x)}{x^{1/4}} + C_2\frac{Y_{1/4}(x)}{x^{1/4}}$$

Scalar DM from q/l

i) $x_1 < x_{\rm ew} < x_f$

$$x < x_f$$

$$\tilde\phi''(x) + \frac{3}{2x}\tilde\phi'(x) + \frac{x_1}{x}\tilde\phi(x) = \frac{x_S}{x}$$

$$x > x_f$$

$$\tilde\phi''(x) + \frac{3}{2x}\tilde\phi'(x) + \tilde\phi(x) = 0$$

$$\rho_\phi(x_{eq}) \approx \frac{c_f^2 x_S^2}{\pi}\frac{m_\phi^2 M_P^2}{x_{eq}^{3/2}}$$

$$c_f^2 = \begin{cases} 4\sqrt{2}\Gamma\left(\frac{5}{4}\right)^2 x_f^2 & \text{for } x_f \ll 1 \\ \dfrac{\pi\Gamma\left(\frac{3}{4}\right)^2}{2\sqrt{2}} & \text{for } 1 \ll x_f \end{cases}$$

$$y_\phi \approx 3.6\cdot 10^{-24}\,\frac{m_{20}^{-\frac{1}{4}}}{N_c}\left(\frac{m_f}{m_e}\right) \text{ with } \frac{550}{N_c^{\frac{2}{5}}}\left(\frac{m_f}{m_e}\right)^{\frac{4}{5}} \ll m_{20} \ll 2\cdot\ 10^3\left(\frac{m_f}{m_e}\right)^2 \quad \text{for } x_f \ll 1$$

$$y_\phi \approx 3\cdot 10^{-27}\,\frac{m_{20}^{\frac{3}{4}}}{N_c}\left(\frac{m_e}{m_f}\right) \text{ with } m_{20} \gg \text{Max}\left[2\frac{10^{16}}{N_c^2}\left(\frac{m_f}{m_e}\right)^4, 2\cdot 10^3\left(\frac{m_f}{m_e}\right)^2\right] \text{ for } 1 \ll x_f$$

(*) Neglecting Freeze-In: $m_\phi \ll 46\ \text{eV}\ (5.7\ \text{MeV})$ for $f = e\ (b)$, $y_\phi \ll 5.3\,(1.4)\times 10^{-11}$.

Scalar DM from q/l

ii) $x_{ew} < x_f < x_1$

$$x < x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) = \frac{x_S}{x}$$

$$x > x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

$$\rho_\phi(x_{eq}) \approx \frac{c_f^2 x_S^2}{\pi}\frac{m_\phi^2 M_P^2}{x_{eq}^{3/2}}$$

$$c_f^2 = \begin{cases} 4\sqrt{2}\Gamma\left(\frac{5}{4}\right)^2 x_f^2 \ \text{ for } x_f \ll x_1 \ll 1 \\ \frac{\pi x_f^3}{2x_1^2} \ \text{ for } \ 1 \ll x_f \ll x_1 \end{cases}$$

$$y_\phi \approx 3.6 \cdot 10^{-24}\,\frac{m_{20}^{-\frac{1}{4}}}{N_c}\left(\frac{m_f}{m_e}\right) \text{ with } m_{20} \ll 31\,N_c^{-\frac{2}{5}}\left(\frac{m_f}{m_e}\right)^{8/5} \quad \text{for } \ x_f \ll x_1 \ll 1$$

$$y_\phi \approx 5.3 \cdot 10^{-24}\,m_{20}\left(\frac{m_e}{m_f}\right)^{\frac{1}{2}} \text{ with } m_{20} \gg 2 \cdot 10^3\left(\frac{m_f}{m_e}\right)^2 \quad \text{for } \ 1 \ll x_f \ll x_1$$

(*) BBN constraint: $m_\phi \ll 10^{-6}\,(2.5 \times 10^{-3})\ \text{eV} \ \ \text{for} \ \ f = e\ (b),$

$y_\phi \ll 5.7 \times 10^{-10}\,(1.5 \times 10^{-8}).$

Scalar DM from q/l

iii) $x_{ew} < x_1 < x_f$

$x < x_1$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) = \frac{x_S}{x}$$

$x_1 < x < x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

$x_f < x$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

(i) $x_1 \ll x_f \ll 1$

$$y_\phi \approx 3.6 \cdot 10^{-24} \frac{m_{20}^{-\frac{1}{4}}}{N_c} \frac{m_f}{m_e} \quad \text{with } \frac{28}{N_c^{\frac{2}{5}}}\left(\frac{m_f}{m_e}\right)^{\frac{8}{5}} \ll m_{20} \ll \text{Min}\left[2 \cdot 10^3 \left(\frac{m_f}{m_e}\right)^2, \frac{6 \cdot 10^5}{N_c^{\frac{2}{5}}}\left(\frac{m_f}{m_e}\right)^{\frac{4}{5}}\right]$$

(ii) $x_1 \ll 1 \ll x_f$

$$y_\phi \approx 3 \cdot 10^{-27} \frac{m_{20}^{\frac{3}{4}}}{N_c} \frac{m_e}{m_f} \quad \text{with } 2 \cdot 10^3 \left(\frac{m_f}{m_e}\right)^2 \ll m_{20} \ll \text{Min}\left[3 \cdot 10^{11} N_c^2 \left(\frac{m_f}{m_e}\right)^4, \frac{2 \cdot 10^{16}}{N_c^2}\left(\frac{m_e}{m_f}\right)^4\right]$$

(iii) $1 \ll x_1 \ll x_f$

$$y_\phi \approx 2 \cdot 10^{-22} \frac{m_{20}^{\frac{1}{3}}}{N_c^{\frac{1}{6}}} \left(\frac{m_f}{m_e}\right)^{\frac{2}{3}} \quad \text{with } 10^8 N_c^{\frac{5}{4}} \left(\frac{m_f}{m_e}\right)^{\frac{13}{4}} \ll m_{20} \ll 4 \cdot 10^{11} N_c^2 \left(\frac{m_f}{m_e}\right) \quad \text{for } x_1^5 \ll x_f$$

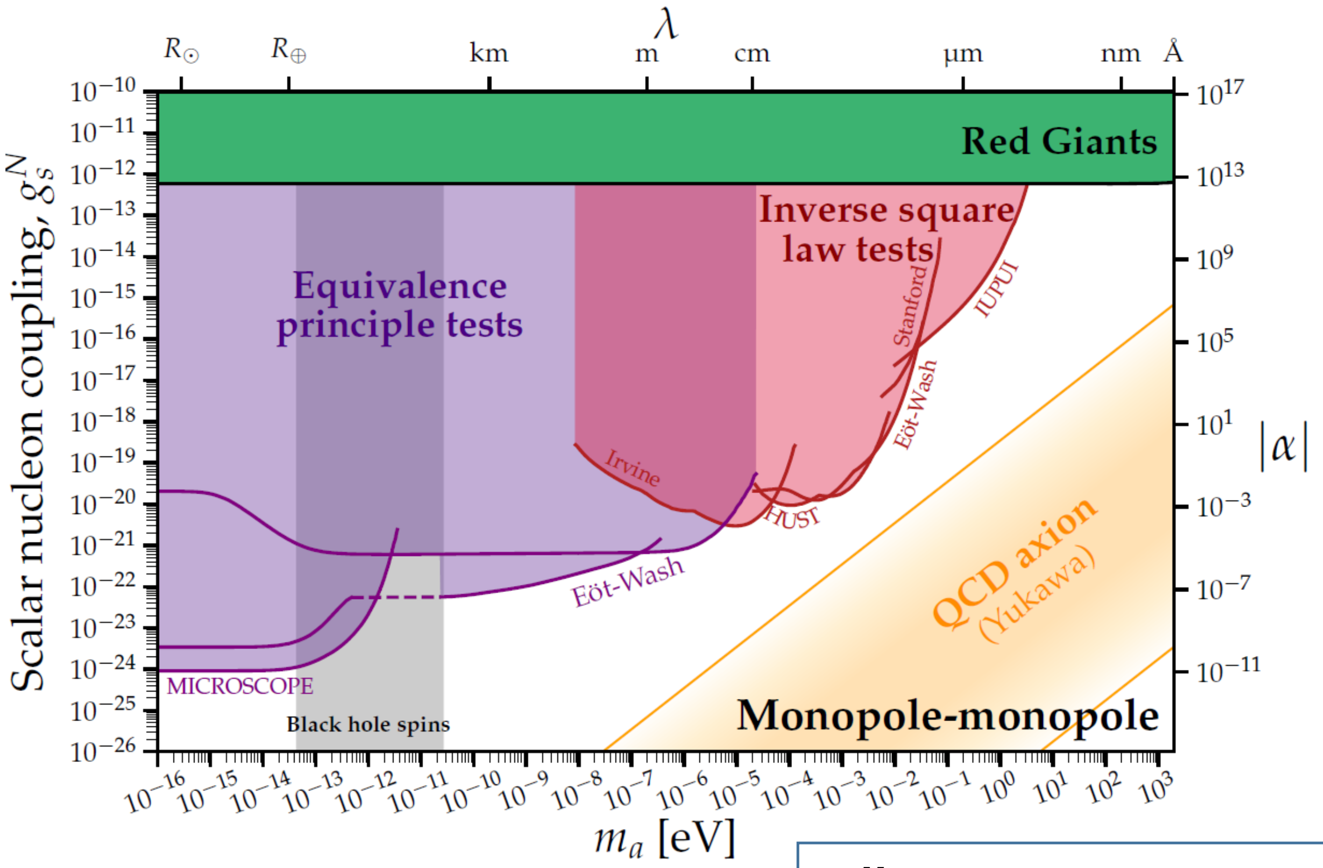
$$y_\phi \approx 4.5 \cdot 10^{-28} \frac{m_{20}}{N_c} \left(\frac{m_e}{m_f}\right)^{\frac{2}{3}} \quad \text{with } m_{20} \gg 5 \cdot 10^8 N_c^{\frac{5}{4}} \left(\frac{m_f}{m_e}\right)^{\frac{13}{4}} \quad \text{for } x_1^5 \gg x_f$$

(*) BBN+Feeze-In: $y_\phi \ll 5.7\,(1.2) \times 10^{-10}$,

$m_\phi \ll 1.3 \times 10^{-2}\,(6.1 \times 10^3)$ eV for $f = e\,(b)$

Bounds from gravity

O'Hair, Vitagliano
2010.03889



$$g_s^N \sim (1-10)y_{\phi ff} \text{ for } f = u,d,s,e \Rightarrow \text{Excluded}$$

$$g_s^N \sim (0.01-0.05)y_{\phi ff} \text{ for } f = c,b$$

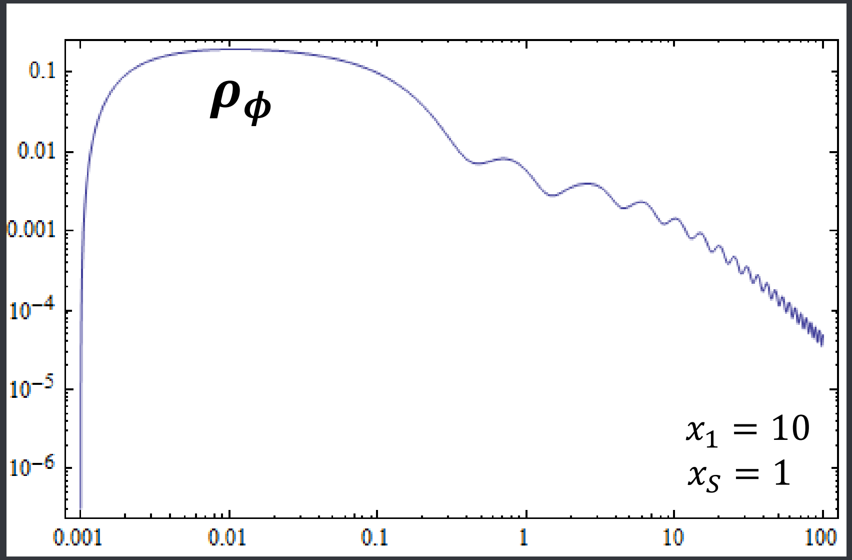
Scalar DM from ν

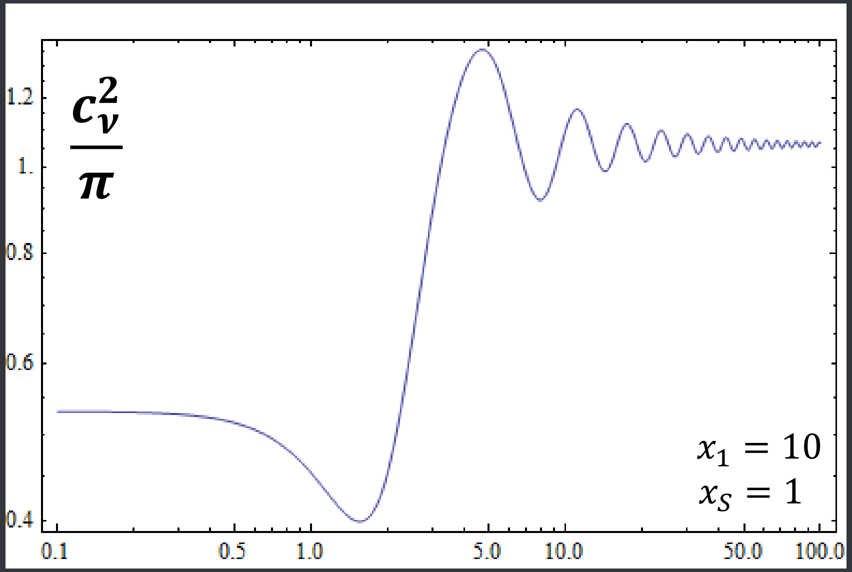
- $x_{ew} < x_1 < x_{eq}$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \left(1 + \frac{x_1}{x}\right)\tilde{\phi}(x) = \frac{x_S}{x}$$

Analytic Solution

$$\tilde{\phi}(x) = \begin{cases} \frac{x_S}{x_1}\left(1 - \sqrt{\frac{x_{ew}}{x}}\cos\left[2\left(\sqrt{x x_1} - \sqrt{x_{ew} x_1}\right)\right] - \frac{1}{2\sqrt{x x_1}}\sin\left[2\left(\sqrt{x x_1} - \sqrt{x_{ew} x_1}\right)\right]\right) \\ C_1 \frac{J_{1/4}(x)}{x^{1/4}} + C_2 \frac{Y_{1/4}(x)}{x^{1/4}} + \frac{x_S}{(2x)^{1/4}}\left(\frac{\pi}{\Gamma\left(\frac{3}{4}\right)} J_{1/4}(x) G_1(x) - \frac{4\Gamma\left(\frac{3}{4}\right)}{3} J_{-1/4}(x) G_2(x)\right) \end{cases}$$





$$\rho_\phi(x_{eq}) \approx \frac{c_\nu^2 x_S^2}{\pi} \frac{m_\phi^2 M_P^2}{x_{eq}^{3/2}}$$

$$c_\nu^2 = \frac{\pi \Gamma\left(\frac{3}{4}\right)^2}{2\sqrt{2}} + \frac{2^{\frac{3}{4}} \pi^{\frac{3}{2}} C_1}{\Gamma\left(\frac{1}{4}\right) x_S} + \frac{C_1^2 + C_2^2}{x_S^2} \approx 3.34 \text{ for } x_1 \gg 1$$

$$y_\phi \approx 3 \cdot 10^{-20} \, m_{20}^{\frac{3}{4}} \left(\frac{0.05\text{eV}}{m_\nu}\right)$$

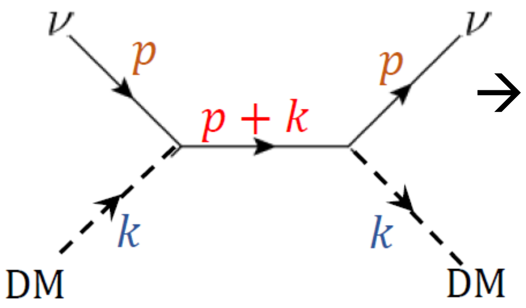
$$\frac{x_S}{x_1} \approx 6.8 \cdot 10^{-10} \, m_{20}^{-\frac{3}{4}} \left(\frac{m_\nu}{0.05\text{eV}}\right)^2$$

$$\frac{x_1}{x_{eq}} \sim 0.34 \, m_{20}^{-\frac{1}{2}} \left(\frac{0.05\text{eV}}{m_\nu}\right)^2 < 1$$

(*) BBN: $y_\phi \lesssim 7 \times 10^{-6}$ $\quad m_\phi \lesssim 0.092 \text{ eV} \left(\frac{m_\nu}{0.05 \text{ eV}}\right)^{4/3}$

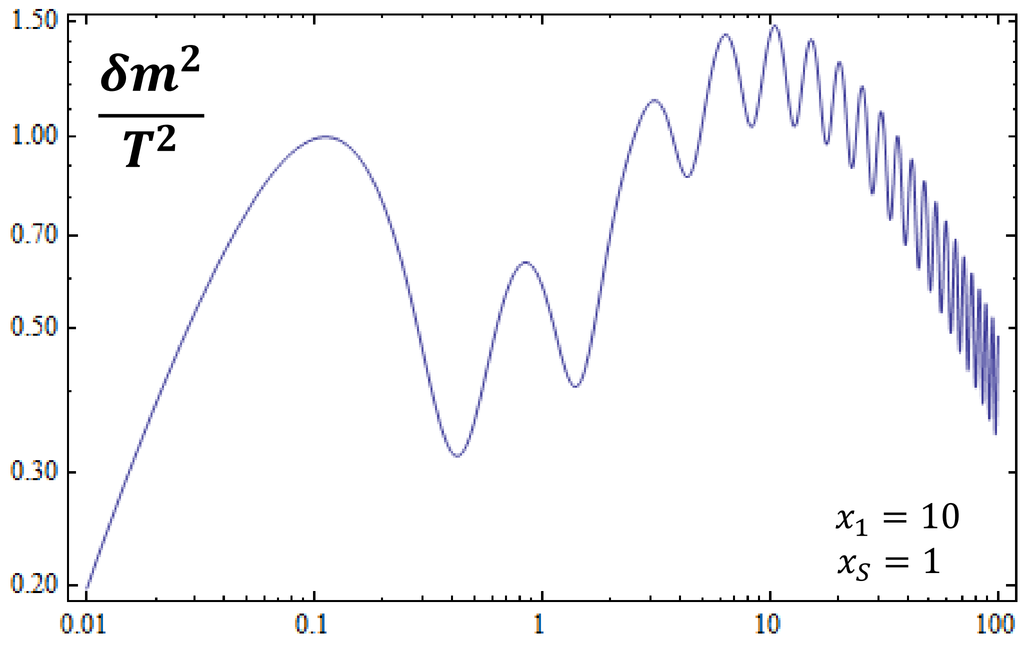
Neutrinos propagating in ULDM

Dispersion relation of Neutrinos travelling through ULDM background gets modified → the medium-induced mass-squared can be large.



$$\rightarrow E \approx \sqrt{\mathrm{p}^2 + m_\nu^2 + \delta m^2}$$

$$\delta m^2 = y_\phi^2 \frac{\rho_\phi}{2m_\phi^2}$$



Neutrino mass correction by ULDM

During DM genesis at $T_1(x_1)$

Demanding for the consistency:

$$\frac{\delta m^2(T)}{T^2} < 0.1 \text{ at } T = T_1$$

we get $m_{20} > 2.2 \times 10^6 \left(\frac{0.05\text{eV}}{m_\nu}\right)^4$

or $T_1 > 70\, T_{eq}$

Now around us

Neutrinos around us get negligible corrections:

$$\delta m^2_{local} = y_\phi^2 \frac{\rho_{\rm DM}^{local}}{2m_\phi^2} < 7.2 \cdot 10^{-9}\text{eV}^2$$

($\rho_{\rm DM}^{local} = 0.3\text{GeV/cm}^3$)

Summary

- A huge population of ULDM can be originated from its tiny coupling to SM fermions. Gravity bounds excludes u, d, s, e as the main source of the DM generation.

- For the $\phi\nu\nu$ coupling, DM genesis requires $m_\phi \sim (10^{-11}, 10^{-2})\mathrm{eV}$ and $y_\phi \sim (10^{-12}, 10^{-6})$, and can occur very late at around $T > 70T_{eq}$.

- Tiny correction to the neutrino mass-squared ($< 10^{-8}\ \mathrm{eV}^2$) at present.