Automatized One-Loop Matching of BSM Models

Javier Fuentes-Martín
University of Granada

Based on work with M. König, A. E. Thomsen, J. Pagès, and F. Wilsch

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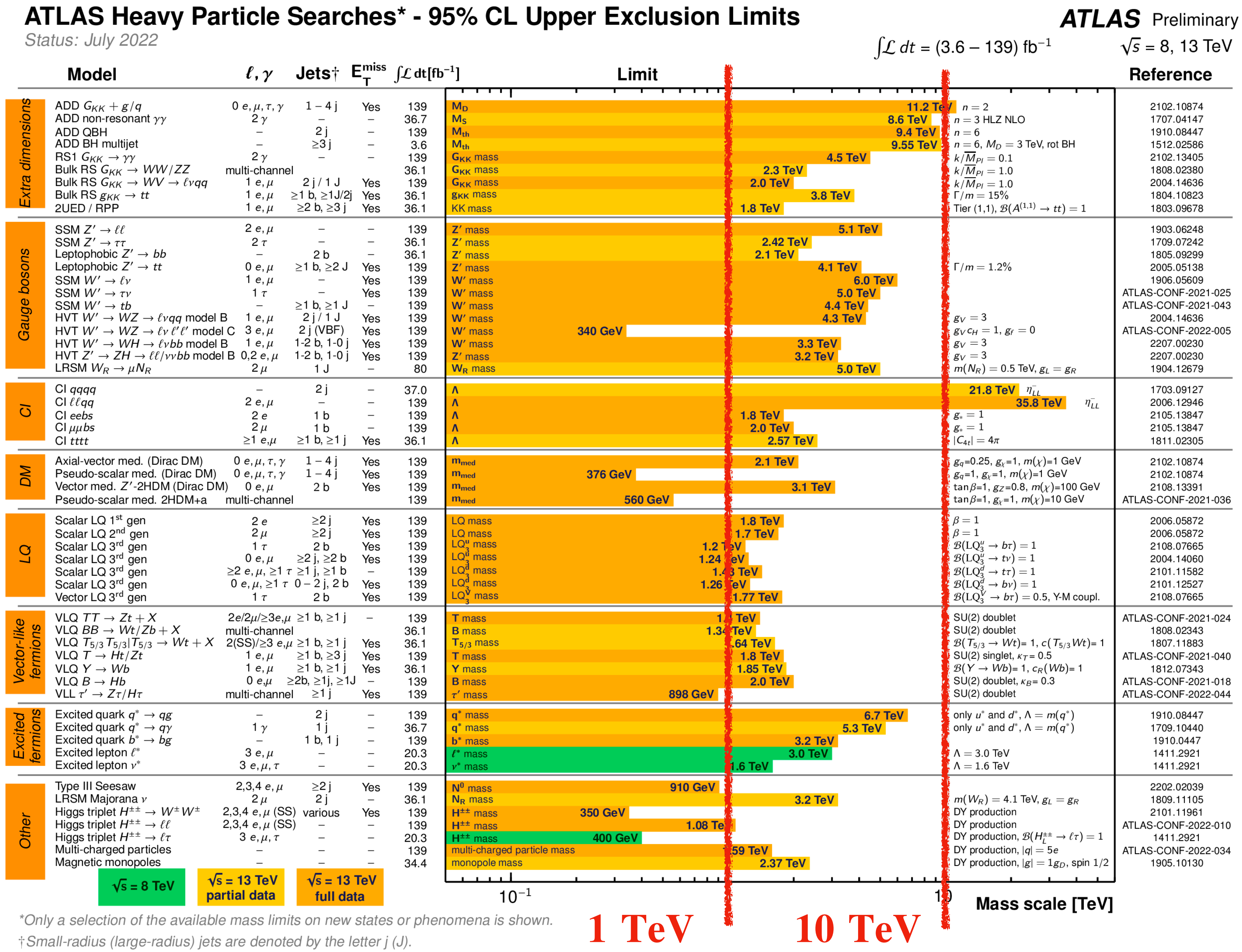
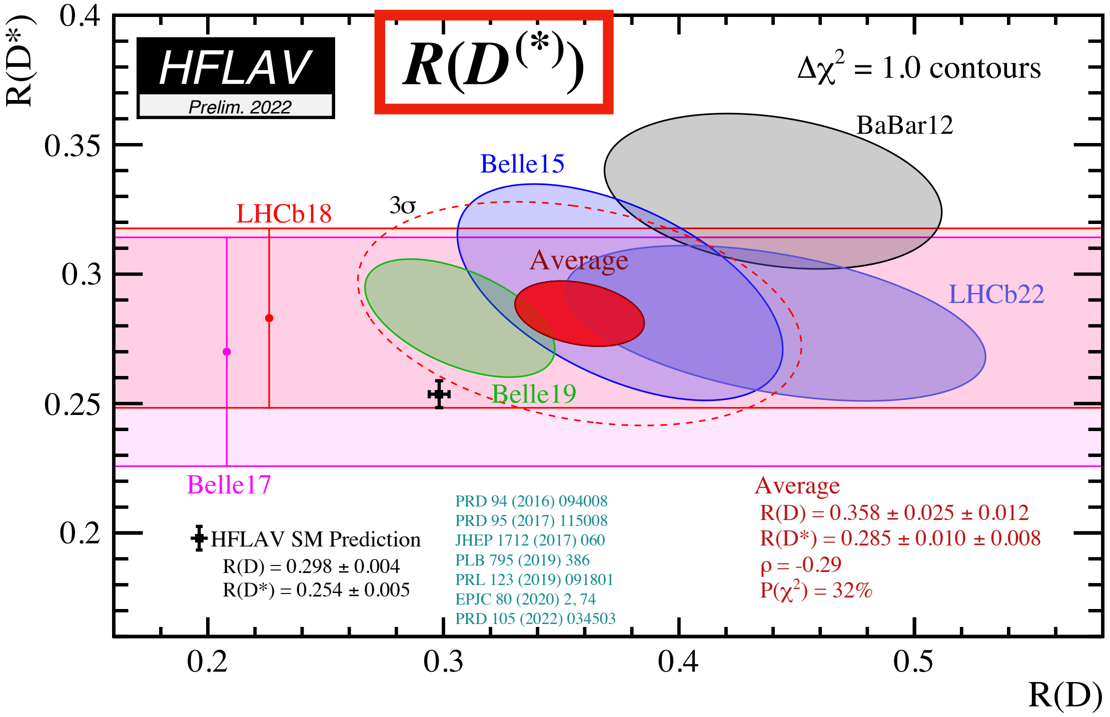
What is experiment telling us?

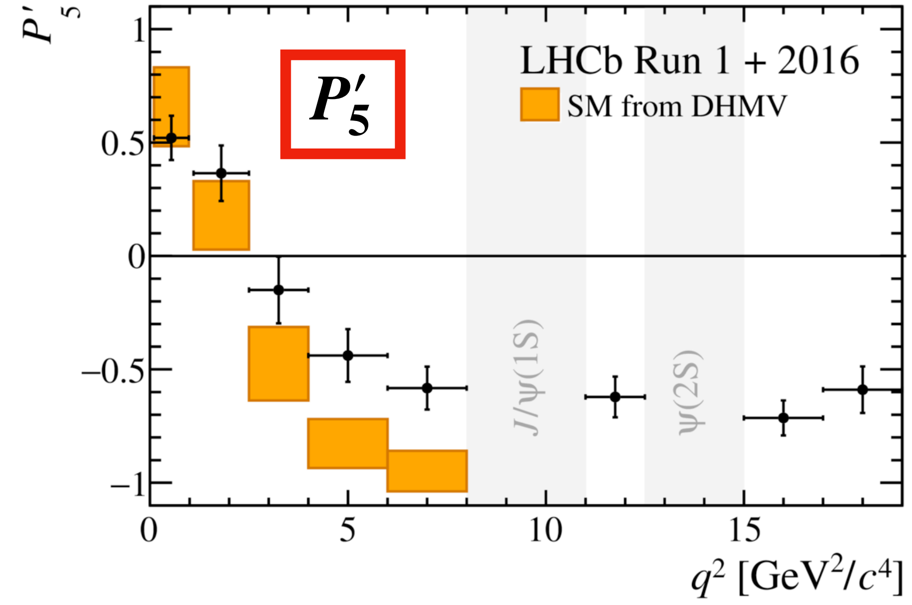
No **direct evidence** for NP despite the many reasons for it [**presence of a mass gap?**]

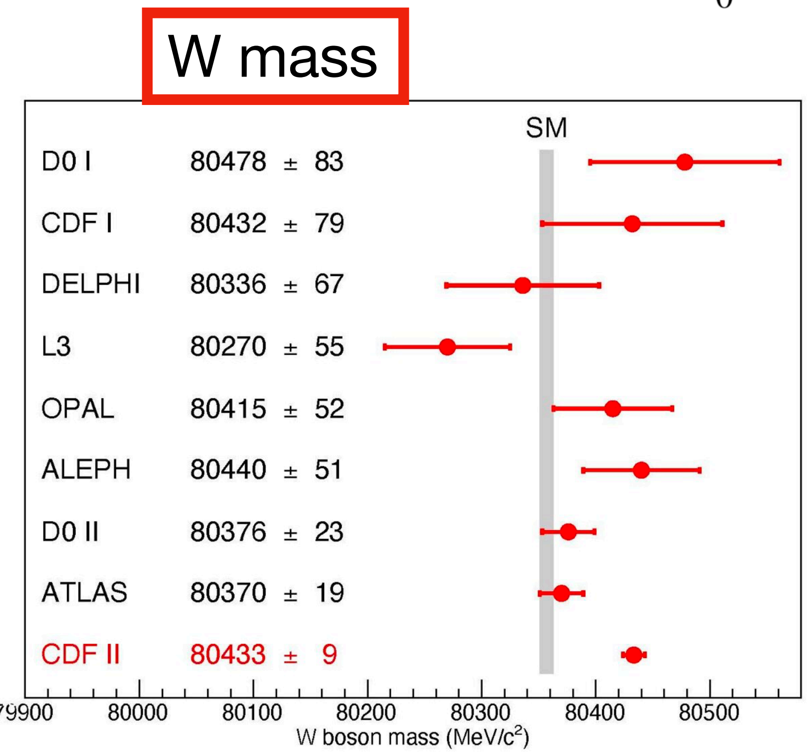


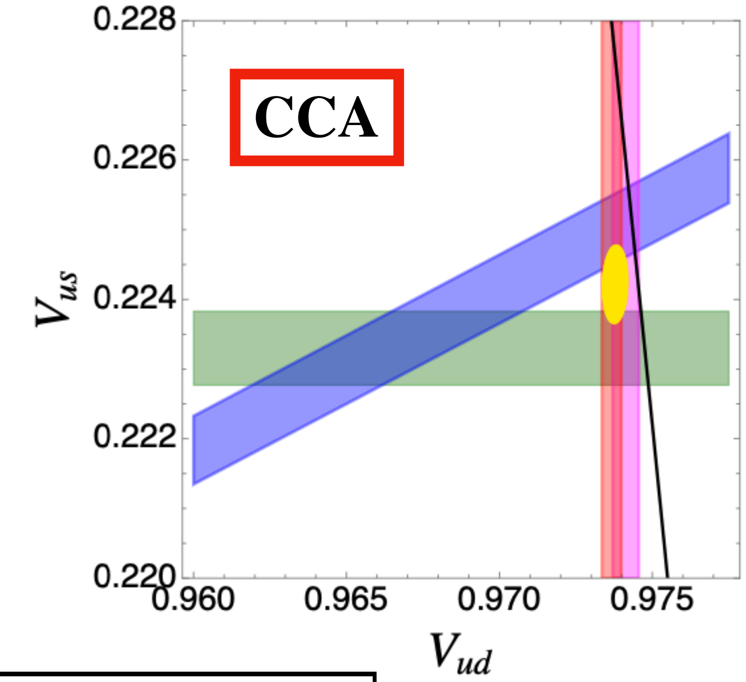
What is experiment telling us?

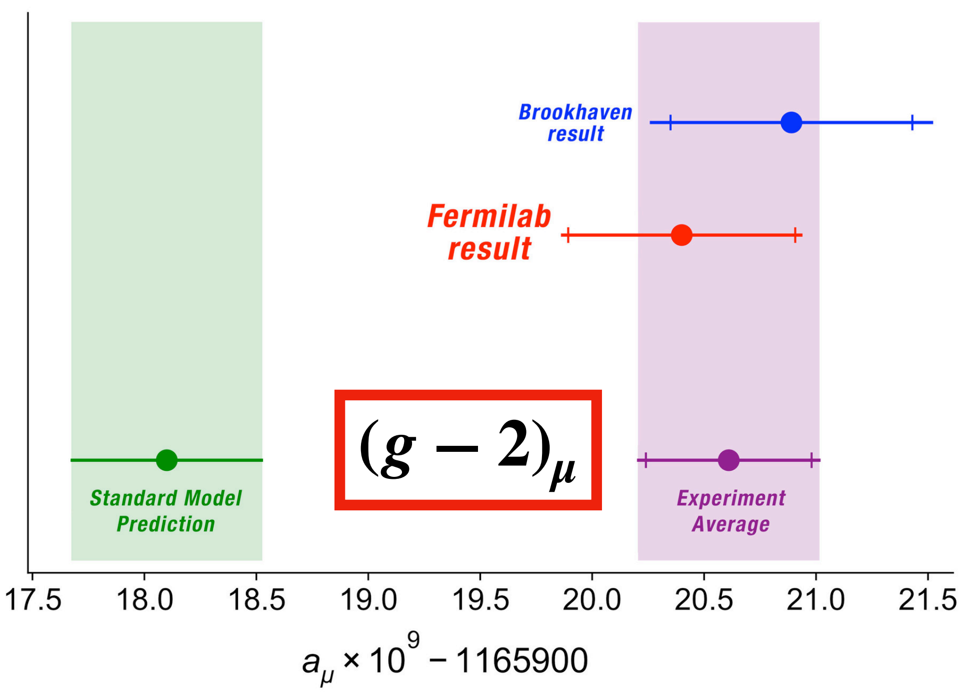
Still several **anomalies** out there

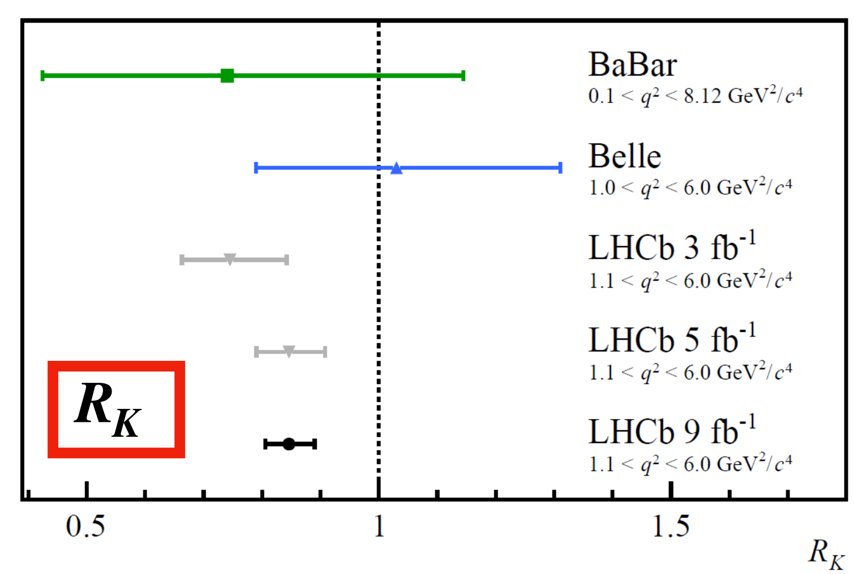












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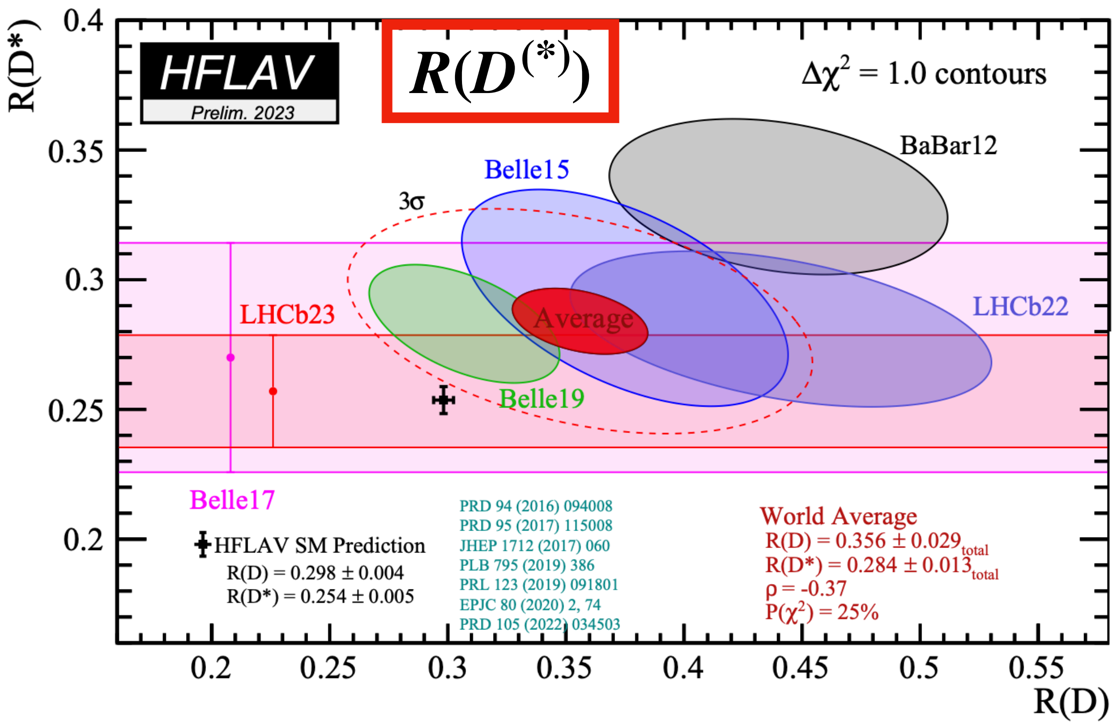
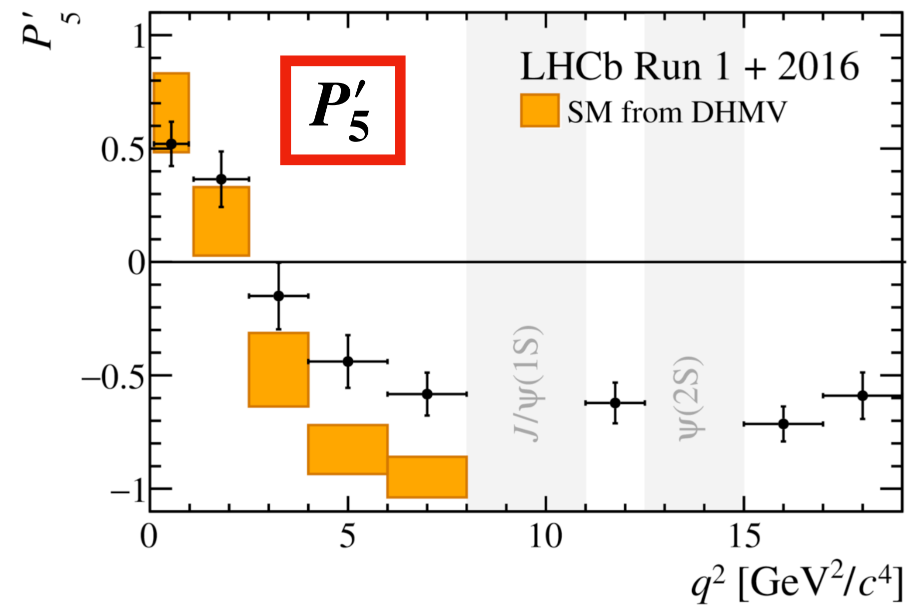
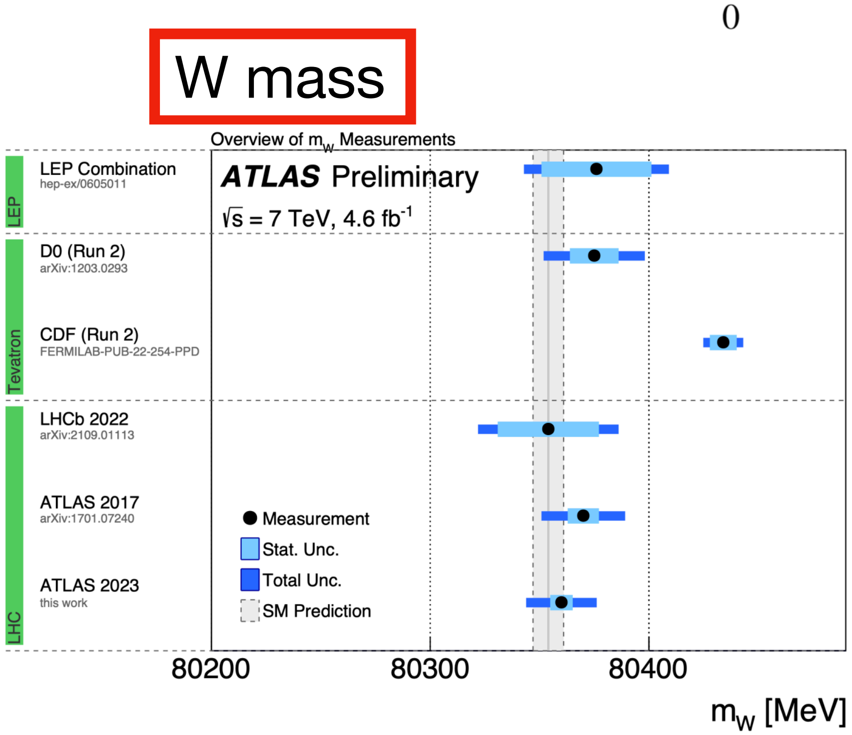
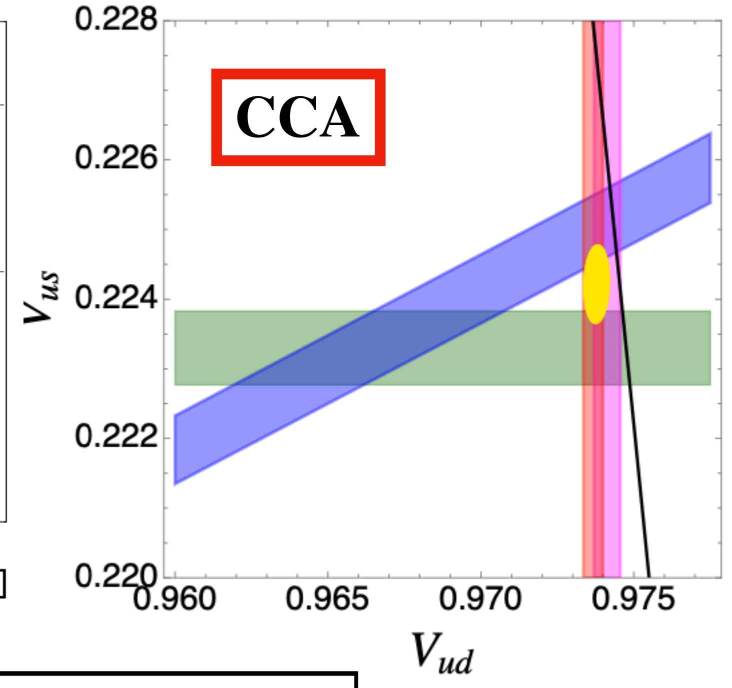
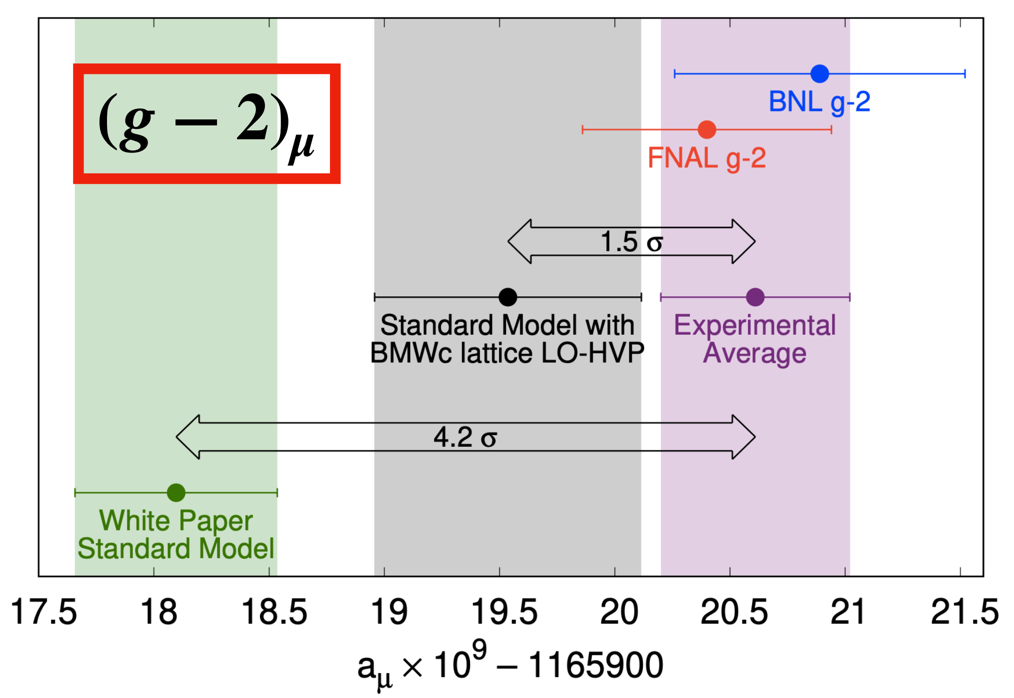
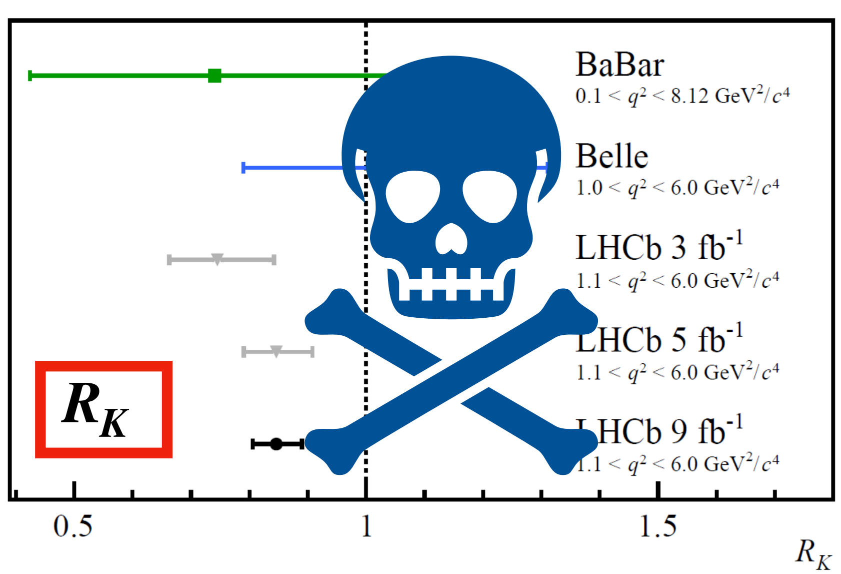






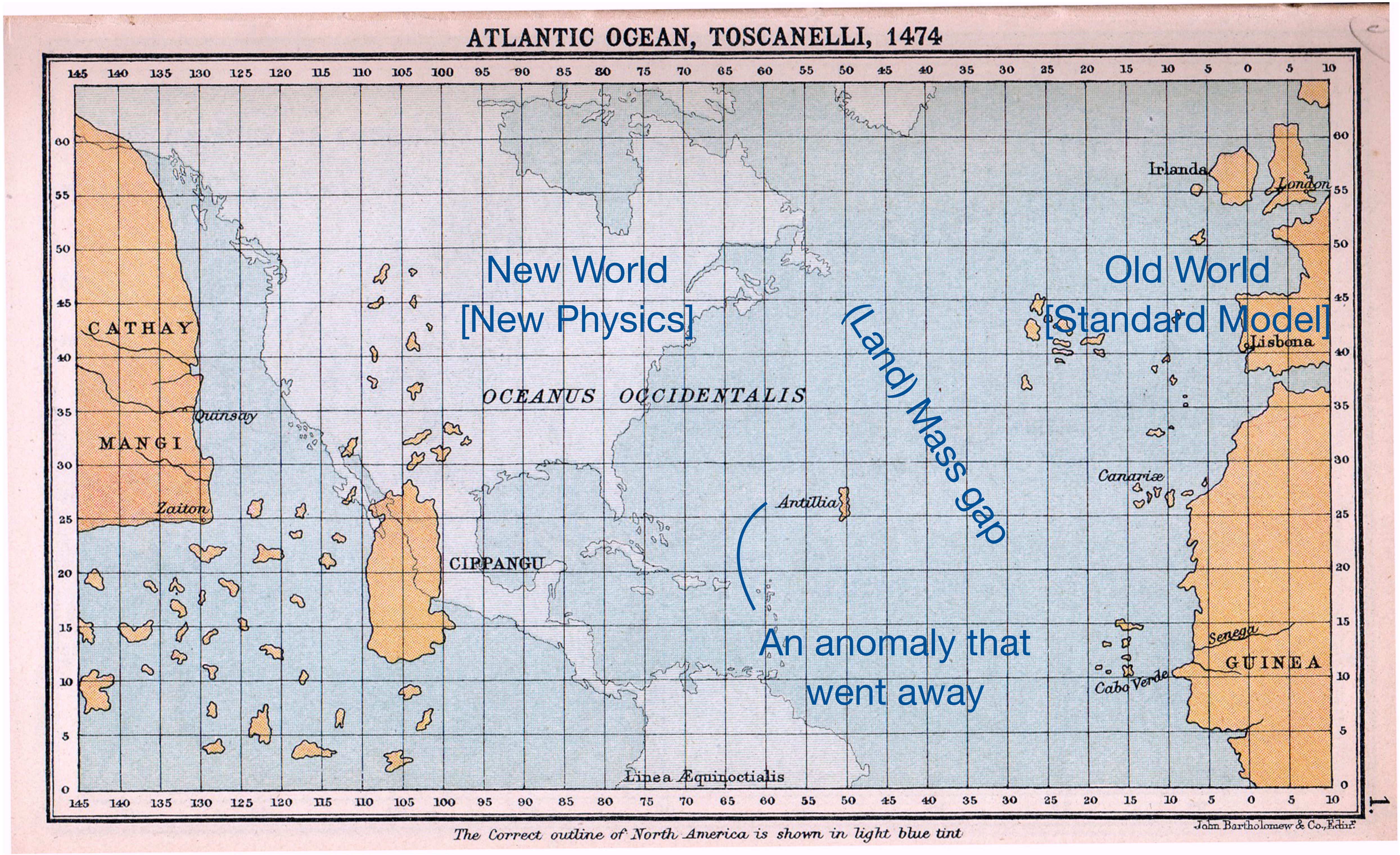






The search for Terra Incognita

Particle Physics has entered an age of exploration



The EFT approach

EFTs are essential to interpret experimental observations

UV physics

$$\mathscr{L}_{\mathrm{EFT}}(\eta_L) = \mathscr{L}_{d=4}(\eta_L) + \sum_{\ell=0}^{\infty} \sum_{n=5}^{\infty} \sum_{k} \frac{C_{n,k}^{(\ell)}}{(16\pi^2)^{\ell}\, \Lambda^{n-4}} O_{n,k}(\eta_L)$$

- Bottom → Up

  Comprehensive approach: Large classes of UV physics covered by a single theory (“model independence”)

- Top → Down

  Reusability: EFT computations can be shared among different BSM models (“compute once for all”)

  (B)SM computations of experimental observables are multi-scale problems: Precision requires the use of EFTs (RG resummation of large logs)

The vast landscape of BSM models and the repetitive nature of EFT computations call for automated solutions

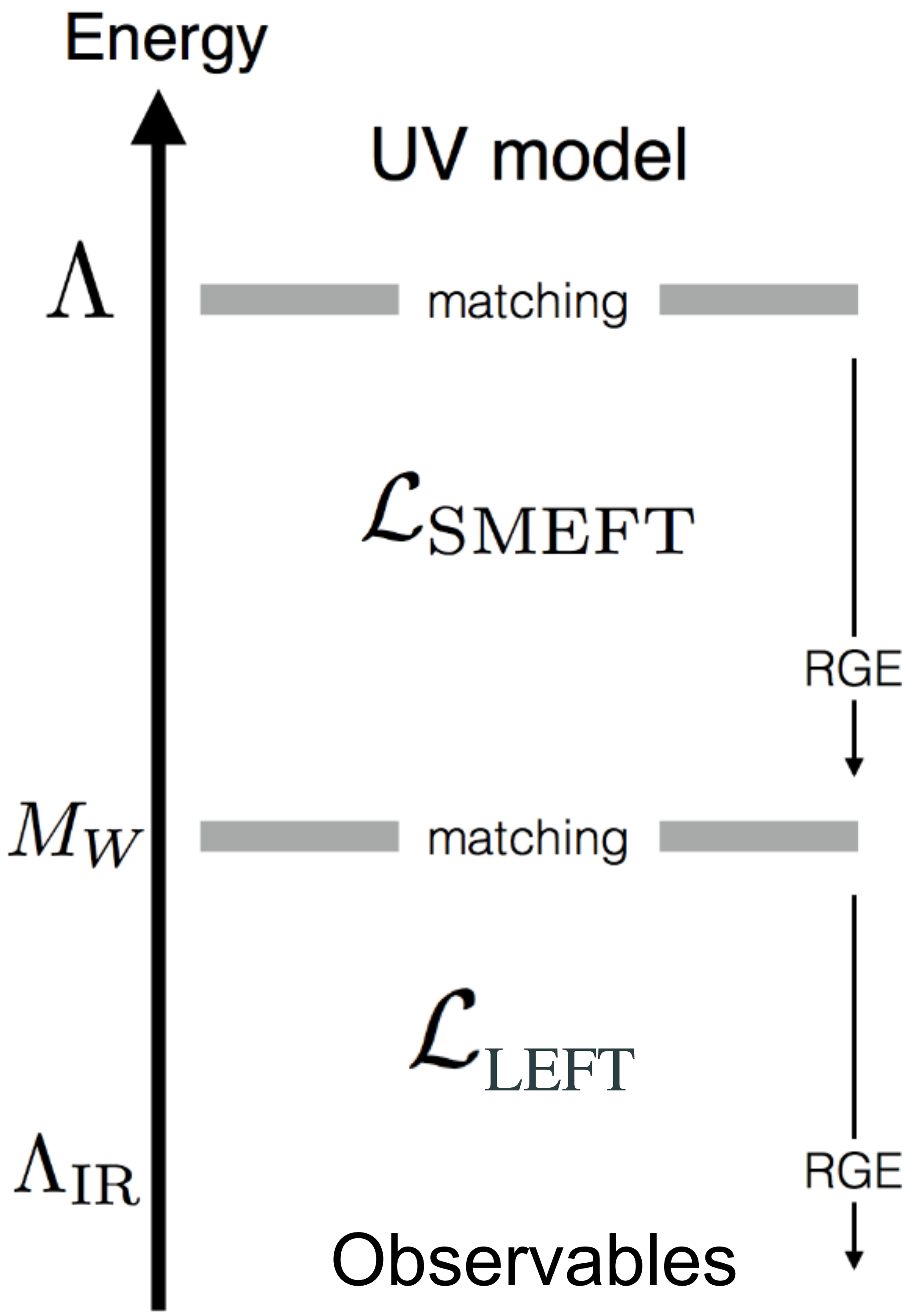
The EFT approach: recent progress

Much progress has been made:

- Tree-level matching to the SMEFT is a solved problem

  [de Blas, Criado, Pérez-Victoria, Santiago, 1711.10391]

  MatchingTools: [Criado, 1710.06445]



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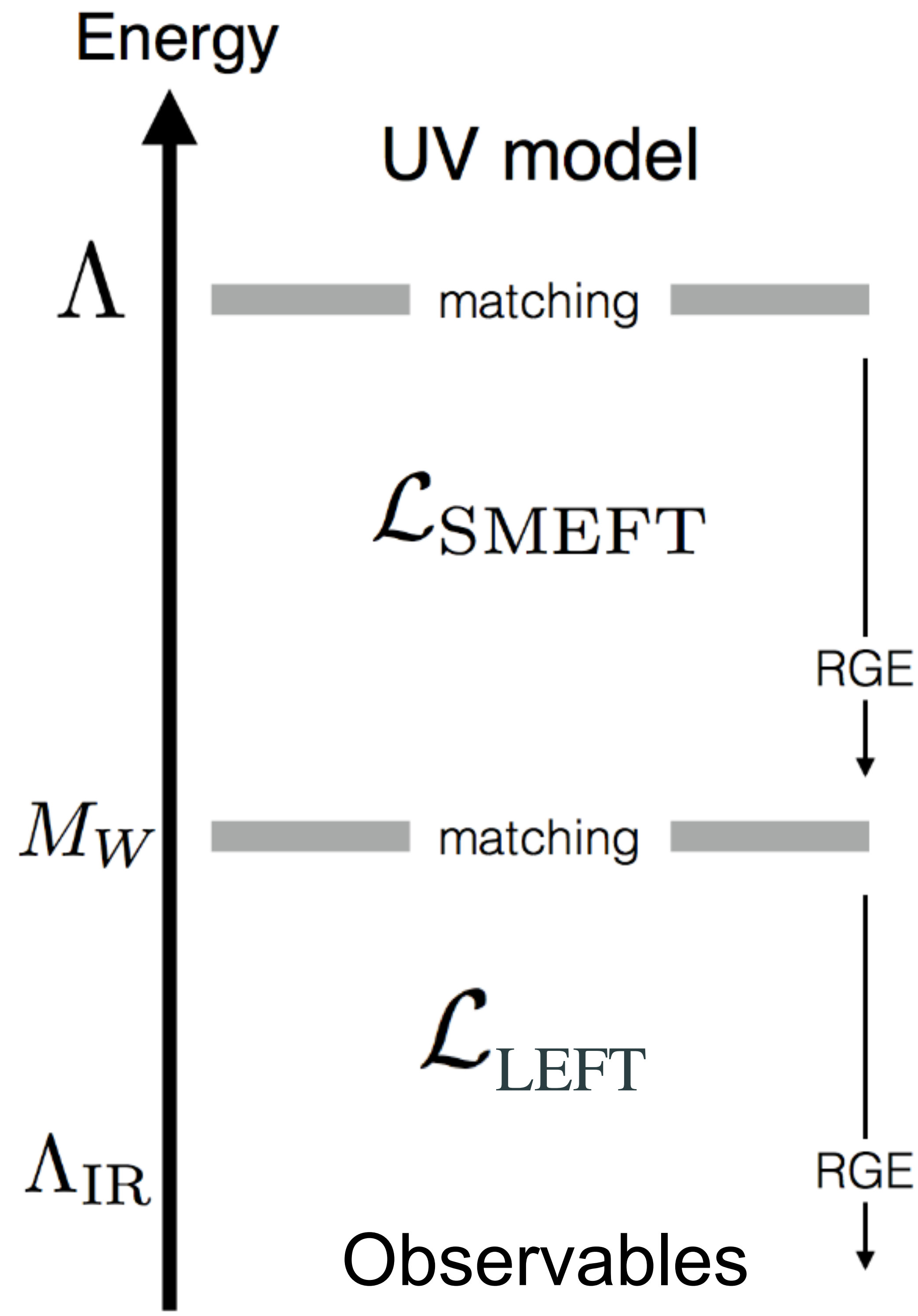
- RGE evolution in the SMEFT and LEFT, and one-loop matching of the SMEFT to the LEFT is also known

  [Jenkins, Manohar, Trott, 1308.2627, 1310.4838, Alonso et al., 1312.2014, Jenkins, Manohar, Stoffer, 1709.04486, 1711.05270; Dekens, Stoffer, 1908.05295]

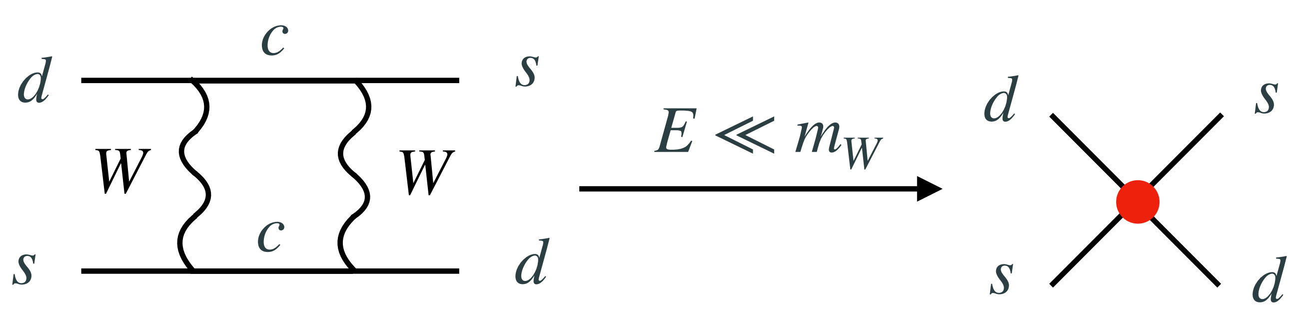
  DsixTools and Wilson:
  [JFM, Ruiz-Femenía, Vicente, Virto, 2010.16341; Aebischer, Kumar, Straub, 1804.05033]

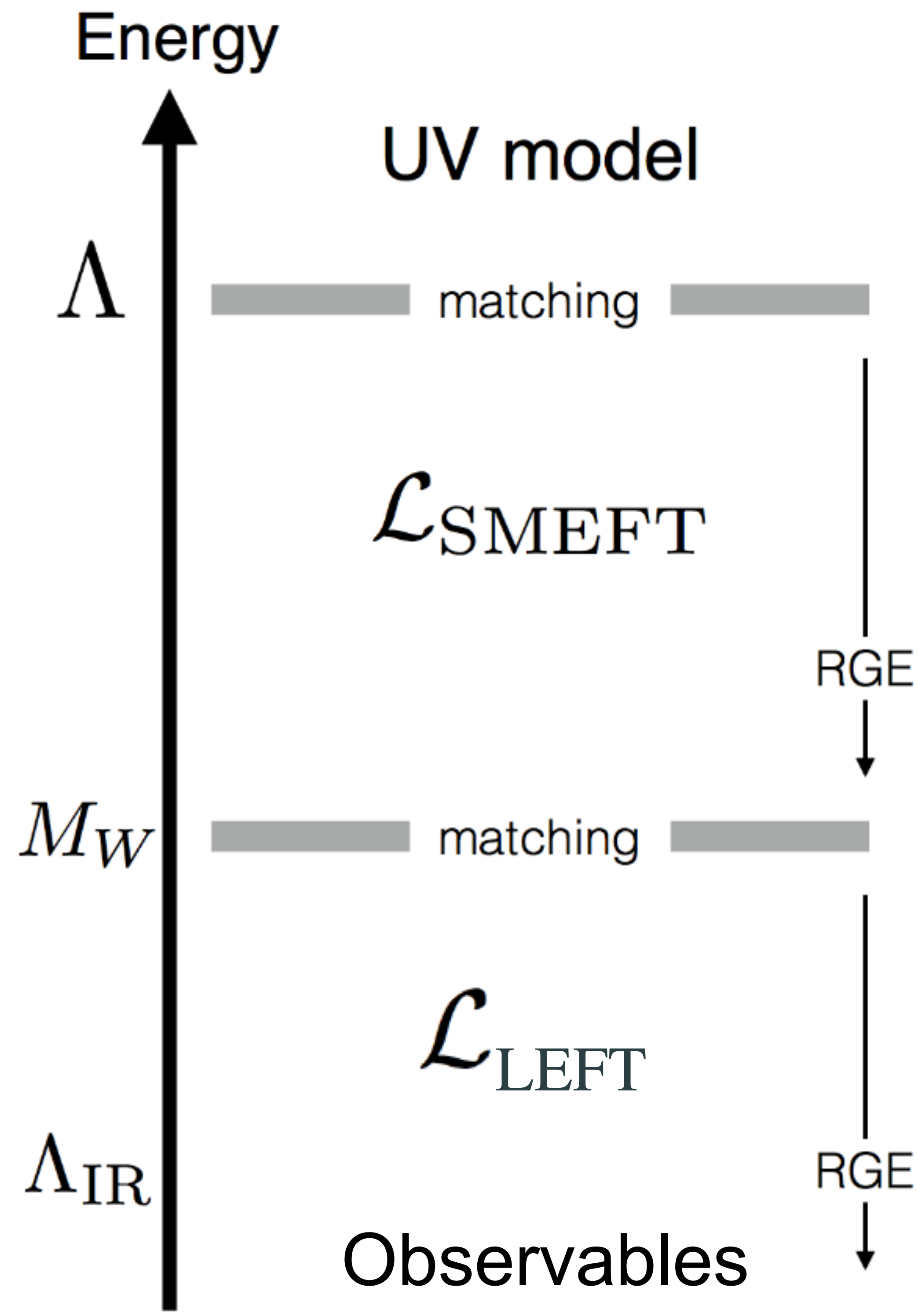


The EFT approach: the need to go beyond

However, we need to go beyond:

- One-loop can be the leading effect in important processes. E.g., in the SM





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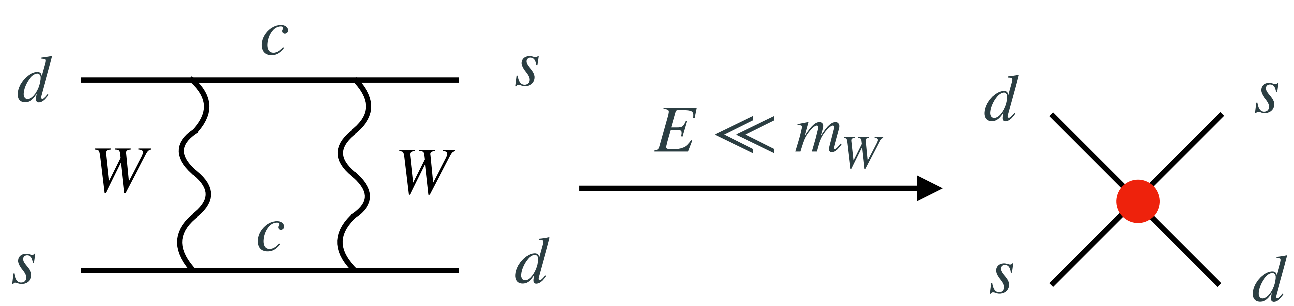
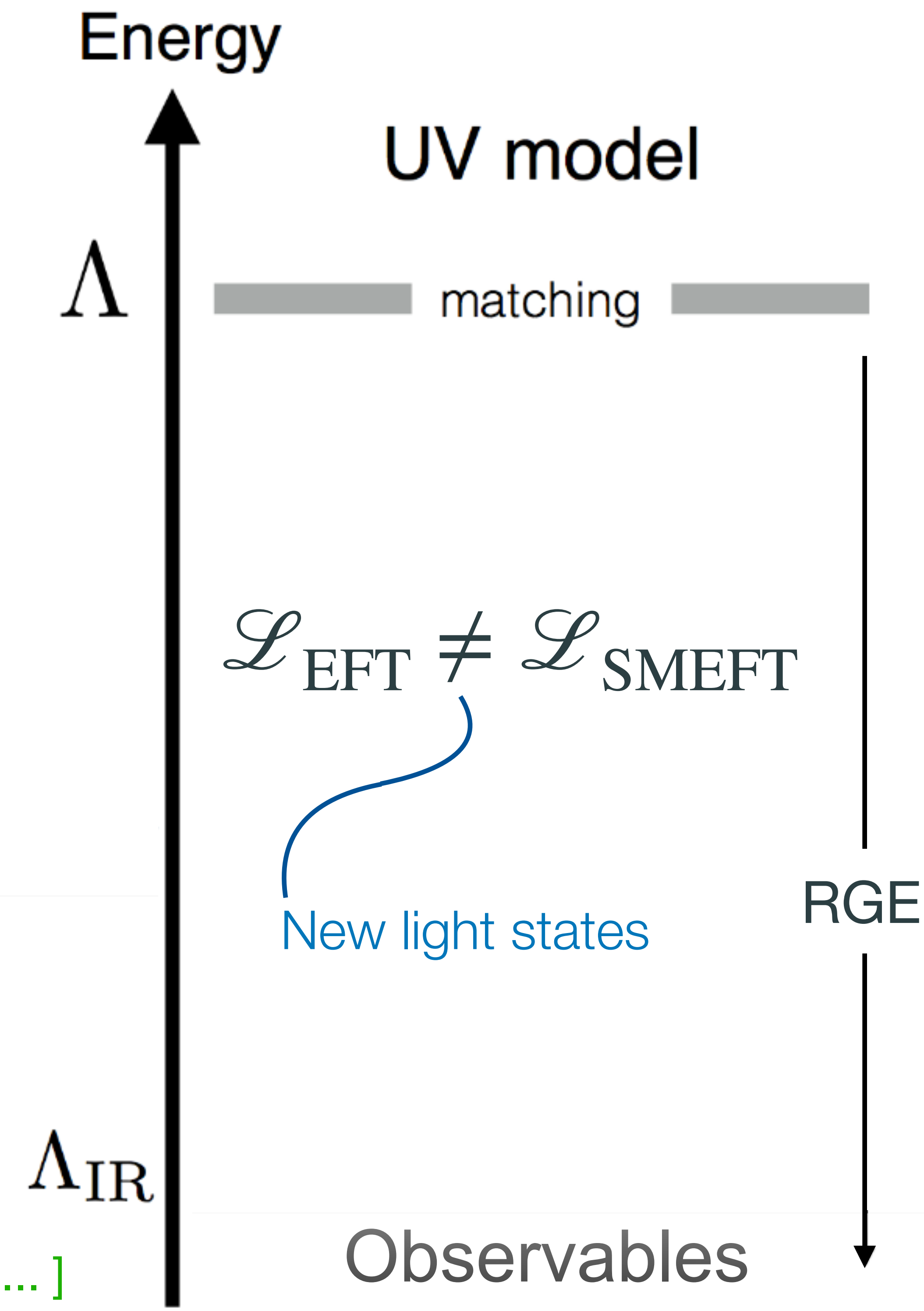


- Perhaps the relevant EFT below the NP scale is not the SMEFT, e.g.

  - SM + ALP EFT
    [Chala, Guedes, Ramos, Santiago, 2012.09017;
    Bauer et al., 2012.12272;
    Galda, Neubert, Renner, 2105.01078;...]

  - SM + DM EFT
    [Criado, Djouadi, Pérez-Victoria, Santiago, 2104.14443,...]

  - …



EFT matching

The path-integral approach in a nutshell

Matching weakly-coupled theories

$\mathscr{L}_{\rm EFT}$ has to reproduce the physics of $\mathscr{L}_{\rm UV}$ at low energies:

$$\mathscr{L}_{\rm UV}(\eta_H, \eta_L) \xrightarrow[E \ll \Lambda \sim m_H]{\text{Matching}} \mathscr{L}_{\rm EFT}(\eta_L)$$

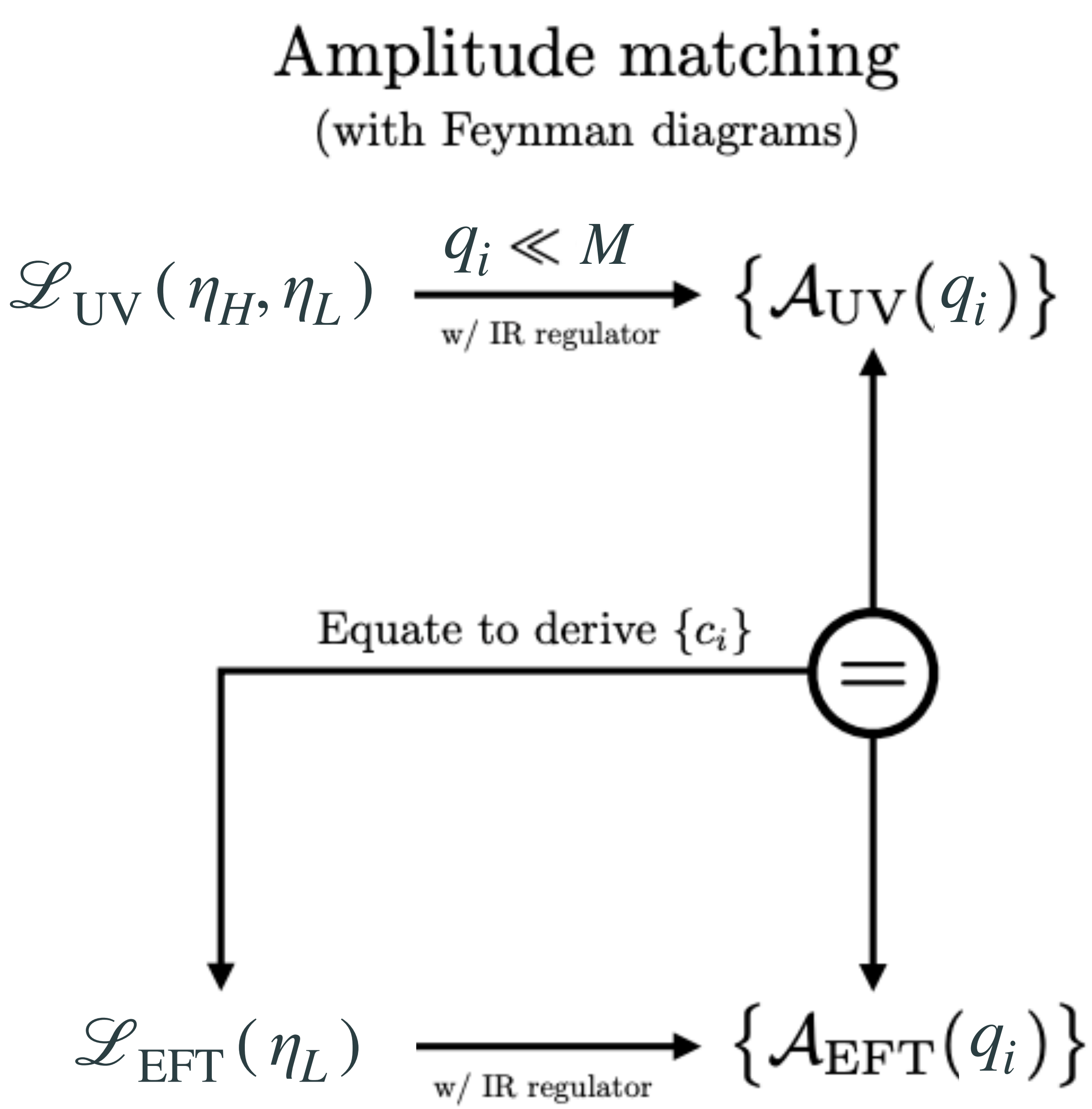
$$\mathscr{L}_{\rm EFT}(\eta_L) = \mathscr{L}_{\rm kin}(\eta_L) + \sum_{n=2}^{\infty} \sum_{l=0}^{\infty} \frac{C_{n,i}^{(l)}}{(16\pi^2)^l \Lambda^{n-4}} O_{n,i}(\eta_L)$$

Two standard EFT matching approaches:

i) **Diagrammatic matching**: Compare the amplitudes of both theories in the low-energy limit

ii) **Functional matching**: Integrate out the heavy modes directly from the path-integral

The tools of the trade: diagrammatic matching



[Figure from Cohen, Lu, Zhang, 2011.02484]

- Traditional, well-established procedure. Valid to any loop order
- Matching usually done off-shell: Additional redundancies but need to consider 1LPI diagrams only
- Need a priori knowledge of the EFT Lagrangian in off-shell basis and with redundancies (e.g. Fierz related ops.)

  [dim-6 SMEFT basis in Carmona, Lazopoulos, Olgoso, Santiago, 2112.10787; Gherardi, Marzocca, Venturini, 2003.12525]

Matchmakereft

https://ftae.ugr.es/matchmakereft/

[Carmona, Lazopoulos, Olgoso, Santiago, 2112.10787]

The tools of the trade: path-integral matching

Functional matching
(path-integral methods)

$$\mathscr{L}_{\rm UV}(\eta_H, \eta_L)$$

$$\Big\downarrow \text{ Using classical EOMs}$$

$$\Gamma_{\rm UV}[\,\hat{\eta}_H(\eta_L), \eta_L\,]$$

$$\Big\downarrow \text{ Method of regions}$$

$$\mathscr{L}_{\rm EFT}(\eta_L)$$

- Many aspects developed only recently.
  Currently well-established up to one loop only

  [Henning, Lu, Murayama, 1412.1837; del Aguila, Kunszt, Santiago, 1602.00126; JFM, Portolés, Ruiz-Femenía, 1607.02142; Henning, Lu, Murayama, 1604.01019; Zhang, 1610.00710; Cohen, Lu, Zhang, 2011.02484...]

- Some closed-form formulas are known

  [Drozd et al.,1512.03003; Ellis et al., 1604.02445; Ellis et al., 1706.07765; Summ, Voight, 1806.05171; Krämer, Summ, Voight, 1908.04798; Ellis et al., 2006.16260; Angelescu, Huang, 2006.16532]

- Manifestly gauge invariant by construction

  [Gaillard '86, Chan '86, Cheyette '88]

- The EFT Lagrangian comes out automatically.
  No knowledge of the EFT Lagrangian is required!

Evaluating the effective action

Given a general theory $\mathscr{L}_{\mathrm{UV}}(\eta)$, its effective action is given by

$$e^{i\,\Gamma_{\mathrm{UV}}[\hat{\eta}]} = \int \mathscr{D}\eta\ \exp\left(i\int \mathrm{d}^d x\ \mathscr{L}_{\mathrm{UV}}(\eta+\hat{\eta})\right)$$

η : Quantum fields (loop lines)

$\hat{\eta}$: Classical fields (tree lines)

where

Tree-level

One-loop

Higher loop orders

$$\mathscr{L}_{\mathrm{UV}}(\hat{\eta}+\eta) = \mathscr{L}_{\mathrm{UV}}(\hat{\eta}) + \frac{1}{2}\,\bar{\eta}_i \underbrace{\left.\frac{\delta^2\mathscr{L}_{\mathrm{UV}}}{\delta\eta_j\,\delta\bar{\eta}_i}\right|_{\eta=\hat{\eta}}}_{\delta_{ij}\,\Delta_i^{-1} - X_{ij}} \eta_j + \mathcal{O}(\eta^3)$$

X : Interaction term

Δ : Covariant propagator

e.g. $\Delta^{-1} = -D^2 - M^2$
for a heavy scalar

$$\mathscr{L}^{(0)}_{\mathrm{EFT}}(\hat{\eta}_L) \equiv \mathscr{L}_{\mathrm{UV}}(\hat{\eta}_L, \hat{\eta}_H(\hat{\eta}_L))$$

[usual removal of heavy fields via EOMs]

Performing the *gaussian* path integral over the second piece

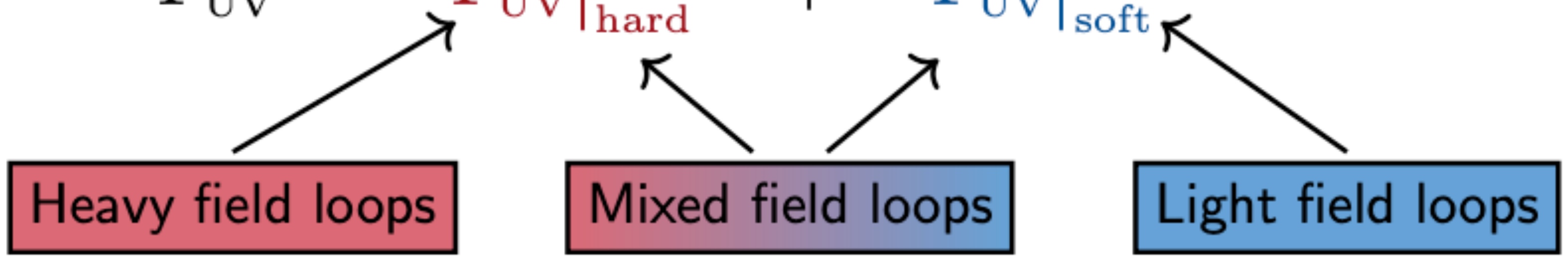
Contains loop integration

$$\Gamma^{(1)}_{\mathrm{UV}} = -\,i\ln\left[\left(\mathrm{SDet}\,(\Delta^{-1}+X)\right)^{-\frac{1}{2}}\right] = \frac{i}{2}\,\mathrm{STr}\ \ln\ \Delta^{-1} + \frac{i}{2}\,\mathrm{STr}\ \ln\,(1-\Delta X)$$

Taking the hard part

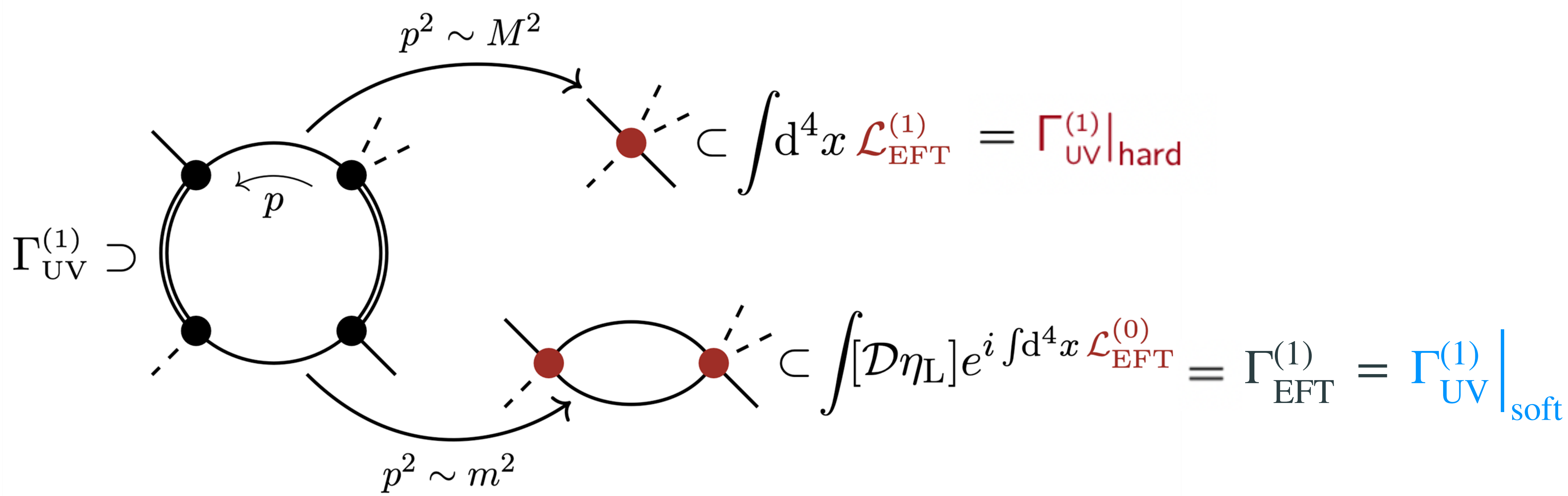
We can separate $\Gamma^{(1)}_{\rm UV}$ in two regions (for $q^2, m^2 \ll M^2$): **hard** ($p^2 \sim M^2$) & **soft** ($p^2 \sim m^2$)

$$\Gamma^{(1)}_{\rm UV} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm hard} + \Gamma^{(1)}_{\rm UV}\Big|_{\rm soft}$$



No need to subtract non-local EFT contributions if only the <u>hard part of the loop</u> is considered

[JFM, Portolés, Ruiz-Femenía, <u>1607.02142</u>]



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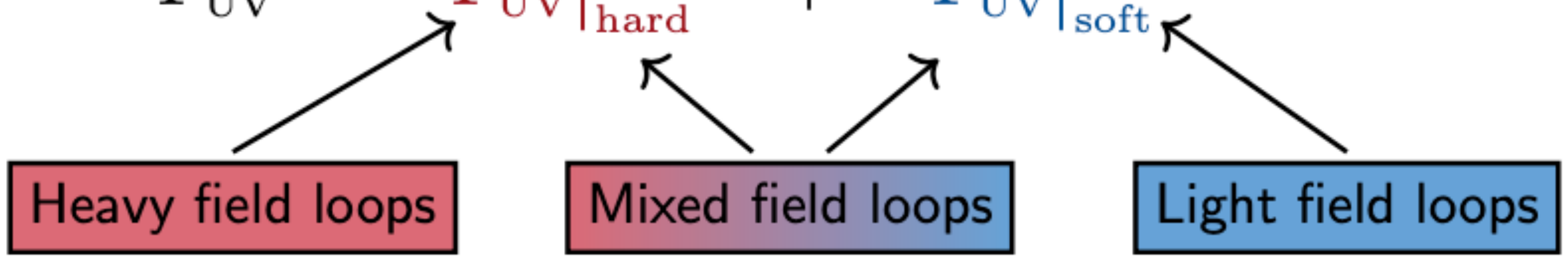

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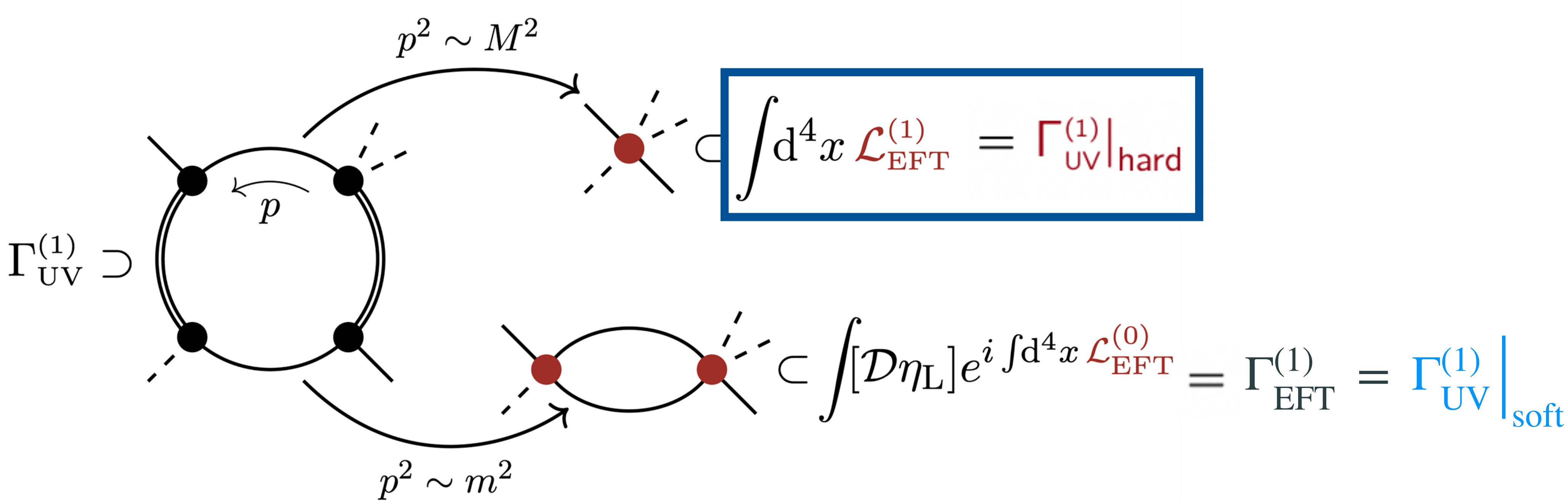


Master formula for one-loop functional matching

$$\int \mathrm{d}^d x \, \mathscr{L}^{(1)}_{\mathrm{EFT}} = \Gamma^{(1)}_{\mathrm{UV}}\Big|_{\mathrm{hard}} = \frac{i}{2}\,\mathrm{STr}\,\ln\,\Delta^{-1}\Big|_{\mathrm{hard}} + \frac{i}{2}\,\mathrm{STr}\,\ln\,(1-\Delta X)\Big|_{\mathrm{hard}}$$

Since ΔX is at most $\mathscr{O}(M^{-1})$, we can expand the second logarithm and get

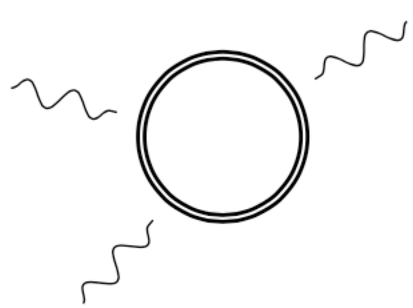
[Cohen, Lu, Zhang, 2011.02484]

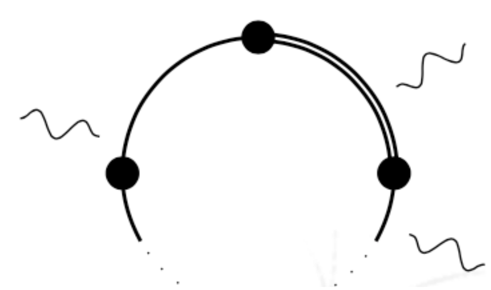
$$\int \mathrm{d}^d x \, \mathscr{L}^{(1)}_{\mathrm{EFT}} = \boxed{\frac{i}{2}\,\mathrm{STr}\,\ln\,\Delta^{-1}\Big|_{\mathrm{hard}}} - \boxed{\frac{i}{2}\sum_{n=1}^{\infty}\mathrm{STr}\left[(\Delta X)^n\right]\Big|_{\mathrm{hard}}}$$

Supertraces evaluated covariantly using the CDE “trick”

log-type supertrace
(universal)

power-type supertrace
(model-dependent interactions)





STrEAM

[Cohen, Lu, Zhang, 2012.07851]

SUPER TRACER

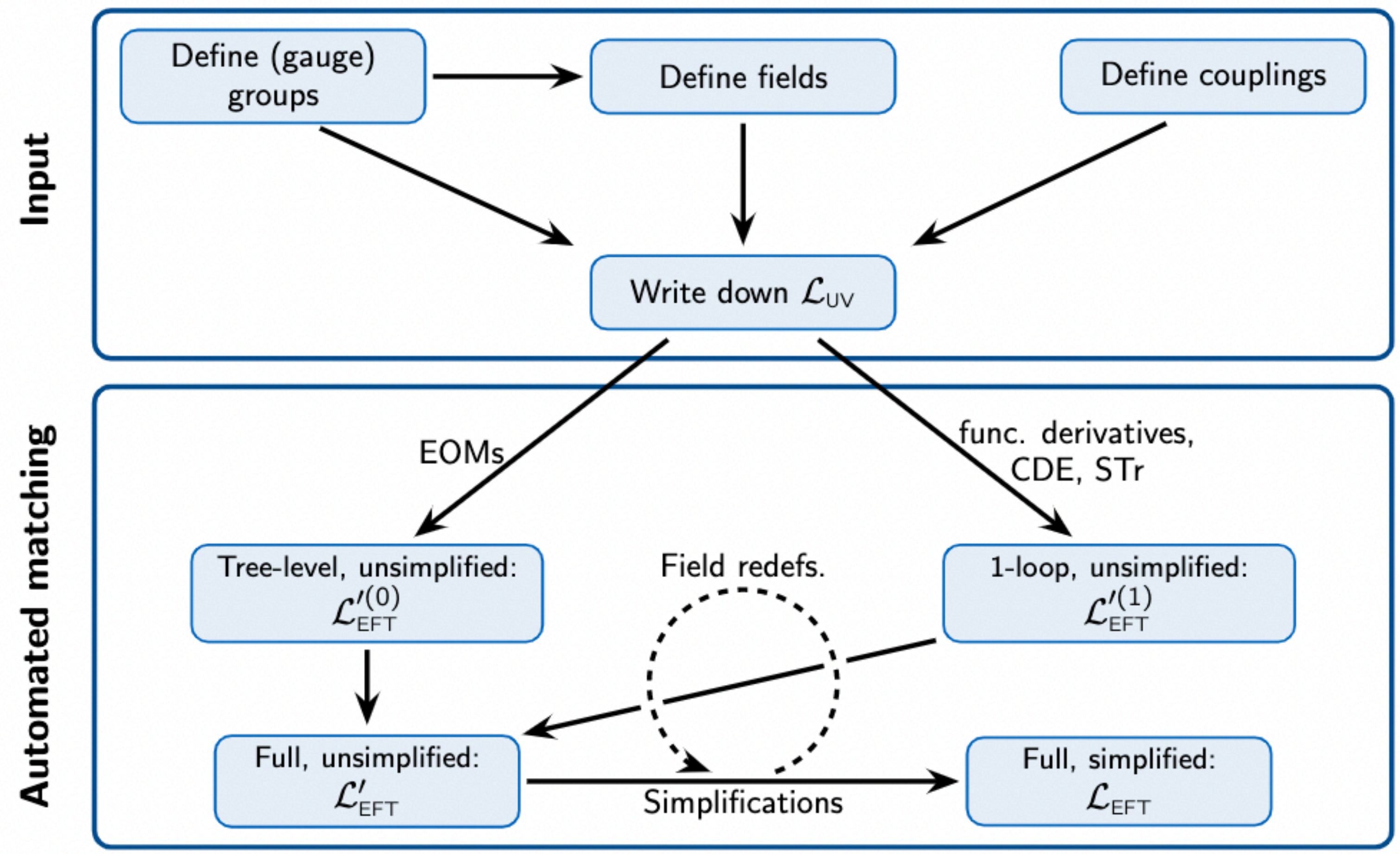
[JFM, König, Pagès, Thomsen, Wilsch, 2012.08506]

MATCHETE

To make your way through the BSM jungle

Automated EFT matching

MATCHETE is a Mathematica package aimed at fully automating one-loop EFT matching and RG running of arbitrary weakly-coupled UV theories using functional methods



[JFM, König, Pagès, Thomsen, Wilsch, 2212.04510]

Proof of concept version (Matchete v0.1) is now publicly available:

- Works with *any* model with heavy scalars and/or fermions
- Simple and intuitive input/output
- Handles *arbitrary* group theory
- Partial simplifications of the resulting EFT Lagrangian (IBP, field redef.,…)
- Heavy vectors not yet supported

  [w.i.p with Olgoso, Santiago, Thomsen]
- RG computations not yet available

Simplifications and basis reduction

The resulting EFT Lagrangian is typically redundant

$$\mathscr{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{C_2}{\Lambda^2}\phi^3\partial^2\phi + \frac{C_3}{\Lambda^2}\phi^2(\partial_\mu\phi)^2$$

Exact simplifications (linear): IBP, Dirac and group identities, commutation relations…

$$\mathscr{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{3C_2 - C_3}{3\Lambda^2}\phi^3\partial^2\phi$$

On-shell equivalence (non-linear): Field redefinitions (*sometimes* equivalent to using of EOMs)

[Criado, Pérez-Victoria, 1811.09413]

$$\phi \to \phi + \frac{3\,C_2 - C_3}{3\,\Lambda^2}\phi^3 \qquad \left[\partial^2\phi = -\,m^2\,\phi - \frac{\lambda}{3!}\phi^3\right]$$

$$\mathscr{L} \to \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \left(\frac{\lambda}{4!} + \frac{m^2(3C_2 - C_3)}{3\,\Lambda^2}\right)\phi^4 + \frac{18\,C_1 - \lambda\,(3C_2 - C_3)}{18\,\Lambda^2}\phi^6$$

MATCHETE routines implement both kinds of simplifications (no Fierz identities at the moment)

Two BSM matching examples

SM extension with a scalar SM-singlet

$$\mathscr{L}_{\rm UV} = \mathscr{L}_{\rm SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - \frac{\mu_S}{3!} S^3 - \frac{\lambda_S}{4!} S^4 - \frac{\lambda_{SH}}{2} (H^\dagger H) S^2 - \kappa (H^\dagger H) S$$

with $M, \kappa, \mu_S \gg v_{\rm EW}$

Less than a minute to compute the one-loop matching (which was correctly computed only after several iterations in the literature)

[Henning, Lu, Murayama 1412.1837;
Ellis, Quevillon, You, Zhang 1706.07765;
Jiang, Craig, Li, Sutherland 1811.08878;
Haisch, Ruhdorfer, Salvioni, Venturini, Weiler 2003.05936]

SM extension with a vector-like lepton ($E \sim (\mathbf{1}, \mathbf{1})_{-1}$)

$$\mathscr{L}_{\rm UV} = \mathscr{L}_{\rm SM} + i\,(\bar{E}\gamma_\mu D^\mu E) - m_E \bar{E} E - \left(y_E \ell_L H E_R + \mathrm{h.c.}\right)$$ with $M_E \gg v_{\rm EW}$

About a ~1 minute to compute the one-loop matching plus ~3 minutes to simplify (result validated against **matchmakereft**)

Let’s see how it works!

An example of Matchete in action

Matchete in action

Next is a live demonstration, please see the attached Mathematica notebook

So, in a nutshell

plus medium- and long-term goals

Outlook

- (Automated) EFT matching is crucial to BSM phenomenology

- Functional matching is ideal for automation (also useful for pen-and-paper computations!)

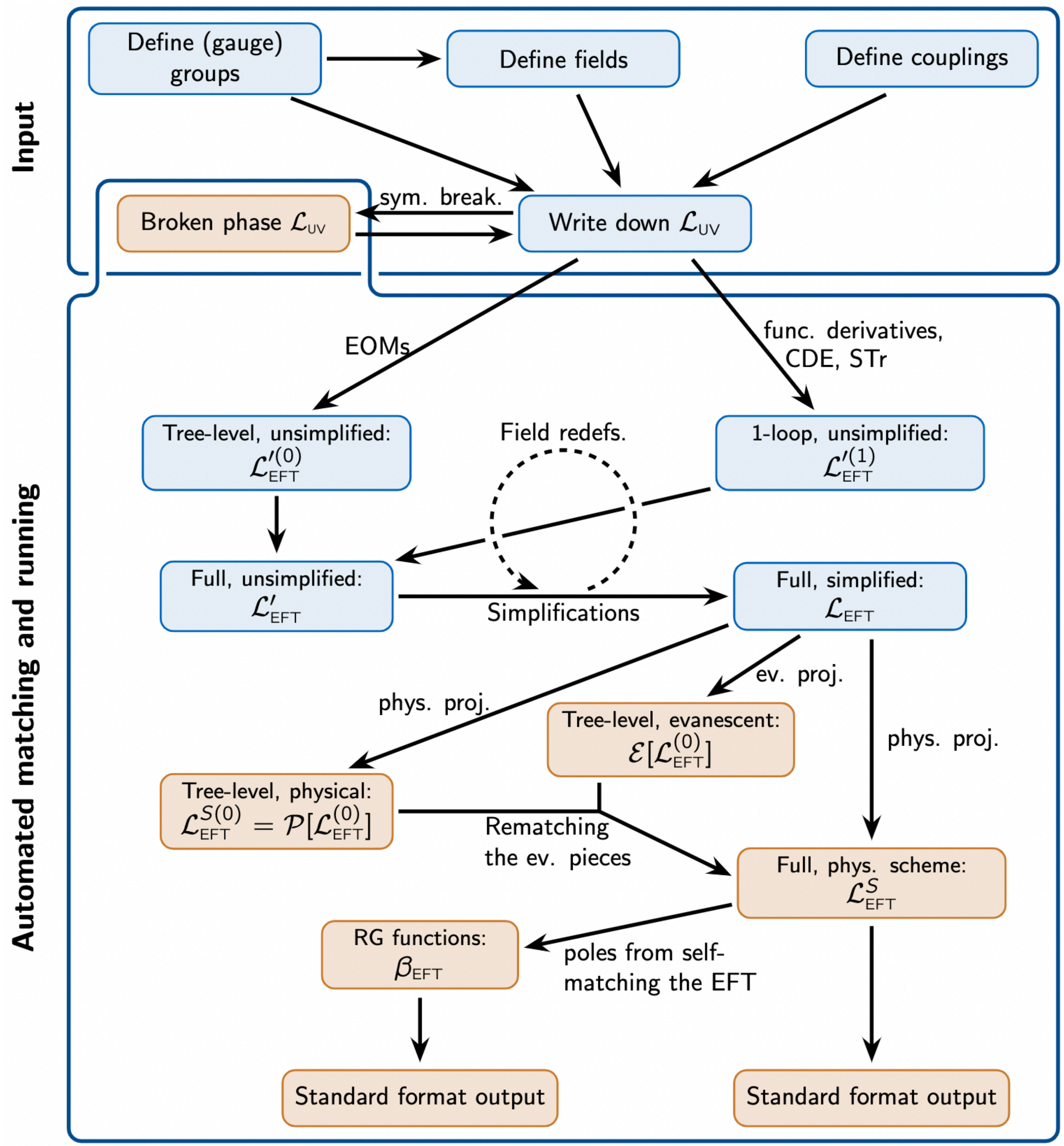
- Complete automation: Lagrangian in, Lagrangian/WCs out not yet available
  - Ongoing progress with MATCHETE

- The ultimate goal is a code (or chain of codes) that fully automates
  - One-loop matching
  - RG evolution

  Multi-step matching

  - Connection to observables / fit to data **Interface with other EFT pheno codes**

Future plans



Proof of concept already available at:

https://gitlab.com/matchete/matchete

Expected functionalities include:

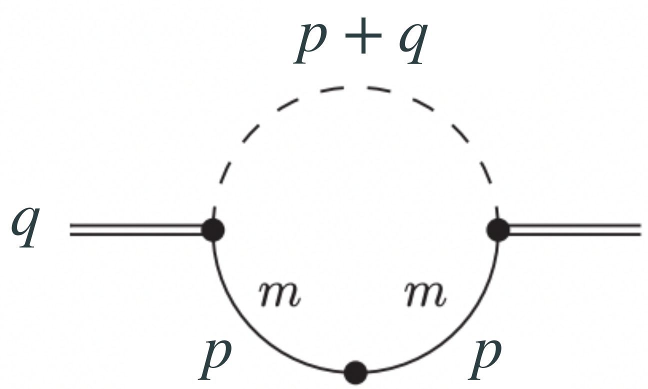
- Complete basis reduction
- Handling of evanescent contributions
- Other γ_5 and regularization/ renormalization schemes
- EFT basis identification
- Heavy vectors and symmetry breaking
- One-loop RG computations
- Interface with other EFT tools (UFO / WCxf outputs)

Thank you

Matching models is about to become easy!

Backup

Method of regions



$$I = \int Dp \, \frac{1}{(p+q)^2\,(p^2-m^2)^2} \qquad Dp \equiv (\mu^2\, e^{\gamma_E})^{\epsilon}\, \frac{d^d p}{i\pi^{\frac{d}{2}}}$$

Say $m^2 \ll q^2$ $\longrightarrow$ hard: $p^2 \sim q^2$ & soft: $p^2 \sim m^2$

Expansion by regions provides a method for scale separation in dimensional regularization:

[Beneke, Smirnov, hep-ph/9711391; Jantzen, 1111.2589]

$$I_{\text{hard}} = \int Dp \, \frac{1}{(p+q)^2\, p^4} + \mathcal{O}(m^2) = \frac{1}{q^2}\left[-\frac{1}{\epsilon} + \ln\frac{-q^2}{\mu^2}\right] + \mathcal{O}(m^2,\epsilon)$$

$$I_{\text{soft}} = \int Dp \, \frac{1}{q^2\,(p-m^2)^2} + \mathcal{O}(m^2) = \frac{1}{q^2}\left[\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\right] + \mathcal{O}(m^2,\epsilon)$$

Spurious IR/UV divergences

$$I = I_{\text{hard}} + I_{\text{soft}} = \frac{1}{q^2}\ln\frac{-q^2}{m^2} + \mathcal{O}(m^2,\epsilon)$$

Manifest covariance of the supertraces

Supertraces are not automatically manifestly covariant due to “open” covariant derivatives ($D_\mu \mathbb{1}$)

$$\mathrm{STr}[Q(iD_\mu, U_k)] = \pm\int \frac{\mathrm{d}^d p}{(2\pi)^d}\,\langle p\,|\,\mathrm{tr}\,Q(iD_\mu, U_k)\,|\,p\rangle = \pm\int \mathrm{d}^d x \int \frac{\mathrm{d}^d p}{(2\pi)^d}\,\mathrm{tr}\left[Q\big(iD_\mu + p_\mu, U_k(x)\big)\right]\,\mathbb{1}$$

Covariant Derivative Expansion (CDE)

Path integral transformation sandwiching the trace between $e^{-iD\cdot\partial_p}$ and $e^{iD\cdot\partial_p}$

When passing $e^{-i\partial_p\cdot D}$ through Q to cancel against $e^{i\partial_p\cdot D}$ it has the desired effect of putting all covariant derivatives into commutators

$$e^{-iD\cdot\partial_p}\,(iD_\mu + p_\mu)\,e^{iD\cdot\partial_p} = p_\mu + i\sum_{n=0}^{\infty}\frac{(-i)^n}{(n+2)n!}\,(D_{\alpha_1,\ldots\alpha_n}G_{\mu\nu})\,\partial_p^{\alpha_1}\cdots\partial_p^{\alpha_n}\,\partial_p^{\nu}$$

$$e^{-P\cdot\partial_p}\,U_k\,e^{P\cdot\partial_p} = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\,(D_{\alpha_1,\ldots\alpha_n}U_k)\,\partial_p^{\alpha_1}\cdots\partial_p^{\alpha_n}$$

➡ This renders the all supertraces manifestly covariant!

Evanescent operators when changing basis in the EFT

In $d = 4$, we can use Fierz identities so that $\mathscr{L}_{\rm EFT} = \mathscr{L}'_{\rm EFT}$

$$\mathscr{L}_{\rm EFT} \supset C_{eW}^{pr}\, Q_{eW}^{pr} + C_{\ell e}^{prst}\, R_{\ell e}^{prst} \qquad R_{\ell e}^{prst} = (\bar{\ell}_p\, e_r)(\bar{e}_s\, \ell_t)$$

$$\mathscr{L}'_{\rm EFT} \supset C_{eW}^{pr}\, Q_{eW}^{pr} - \frac{1}{2}\, C_{\ell e}^{prst}\, Q_{\ell e}^{prst} \qquad Q_{\ell e}^{prst} = (\bar{\ell}_p \gamma_\mu\, \ell_t)(\bar{e}_s \gamma^\mu\, e_r)$$

but their one-loop amplitudes (evaluated in $d = 4 - 2\epsilon$) are different!

$$i\left(A_{eH\to\ell W} - A'_{eH\to\ell W}\right) = \frac{g_L}{64\pi^2}\,[C_{\ell e}]^{prst} y_e^{ts}\left(\bar{u}\,\tau^I \sigma_{\mu\nu} P_R u\right) q^\mu \epsilon^{*\,\nu}$$

In $d = 4 - 2\epsilon$, there is an evanescent operator

$$R_{\ell e}^{prst} = -\frac{1}{2}\, Q_{\ell e}^{prst} + E_{\ell e}^{prst} \qquad E_{\ell e}^{prst} \xrightarrow{\epsilon \to 0} 0$$

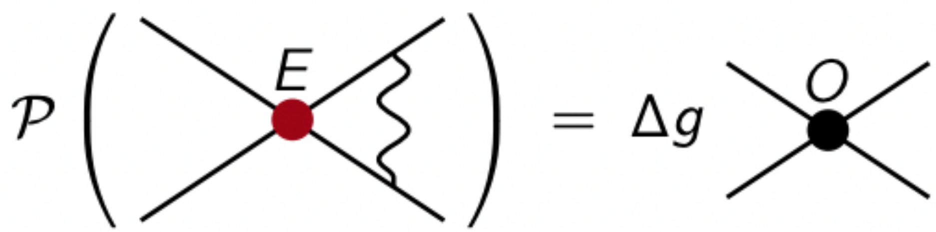
Evanescent operators when changing basis in the EFT

An evanescent operator E is an operator that satisfies

$$E = \text{rank}(\epsilon) \xrightarrow{d \to 4} 0$$

Evanescent contributions have long been accounted for in the LEFT (Weak Effective Hamiltonian). Not so much in BSM context [Buras, Weisz '90; Dugan, Grinstein, '91; Herrlich, Nierste, hep-ph/9412375;…]

The physical contributions from evanescent operators are finite and local



e.g., in the previous example

$$E_{\ell e}^{pr} \to -\frac{g_L\, y_e^{ts}}{128\pi^2}\, Q_{eW}^{pr} + \text{[many other contributions]}$$