Wilson Coefficients and Natural Zeros from the On-Shell Viewpoint

Benedict von Harling

based on 2201.10572 with
Luigi Delle Rose and Alex Pomarol

Introduction

Muon g-2 experiment at Fermilab:

$$\Delta a_\mu = \frac{(g-2)_{\rm exp} - (g-2)_{\rm SM}}{2} = (2.51 \pm 0.59) \times 10^{-9}$$

Plethora of beyond-SM models to explain this. One example:

Consider two vector-like fermions, a singlet S and an SU(2) doublet L with couplings

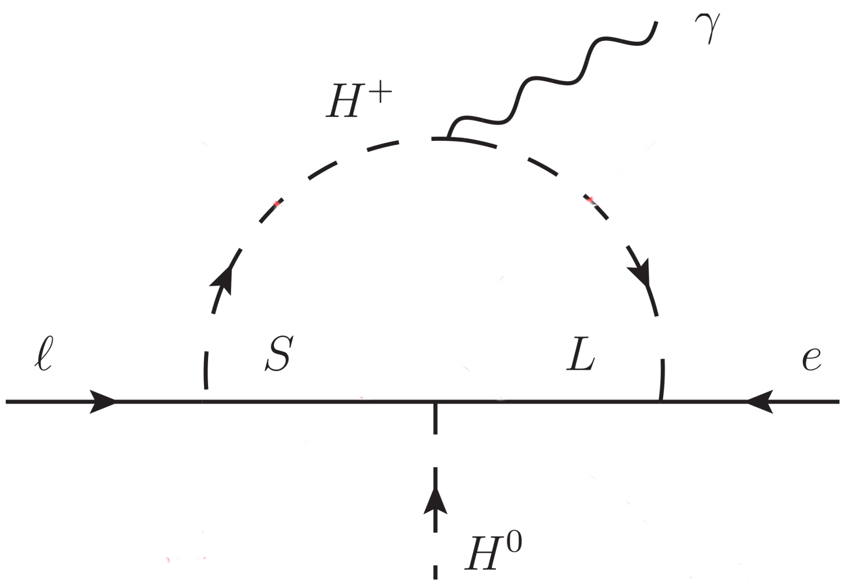
$$\mathcal{L} = -Y_L \underset{\substack{\uparrow \\ \text{lepton doublet}}}{\ell} S\tilde{H} - Y_R L \underset{\substack{\uparrow \\ \text{right-handed muon}}}{e} H - Y_V \tilde{H}^\dagger L^c S^c - Y_V' L S \underset{\substack{\uparrow \\ \text{Higgs}}}{\tilde{H}} - M_S\, S S^c - M_L\, L L^c + h.c.$$

K. Kannike, M. Raidal, D. M. Straub, and A. Strumia [arXiv:1111.2551]

A. Freitas, J. Lykken, S. Kell, and S. Westhof [arXiv:1402.7065]

Introduction

Expect that leading contribution g-2 of muon arises from (for $Y_V = 0$)



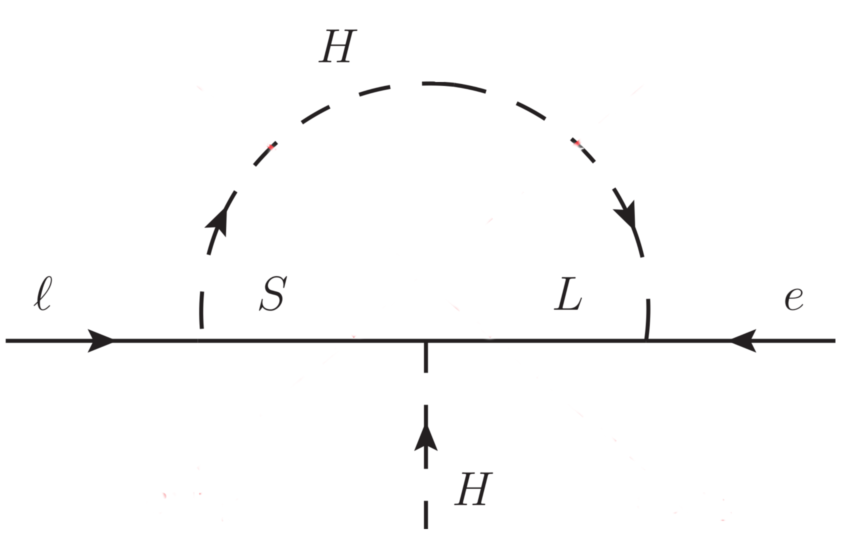
$\Rightarrow \Delta a_\mu = 0!$

$$\Rightarrow \mathcal{L}_{\text{eff}} \supset \frac{C_\gamma}{M^2} H \ell \sigma^{\mu\nu} e^c F_{\mu\nu}$$

with $C_\gamma = 0!$

N. Arkani-Hamed and K. Harigaya [arXiv:2106.01373]

But muon Yukawa coupling is generated (none at tree-level). Again for $Y_V = 0$:



$\Rightarrow \Delta Y_\ell \neq 0$

Introduction

N. Arkani-Hamed and K. Harigaya [arXiv:2106.01373]

Why is this interesting ? Apparent UV/IR conspiracy!

Consider physicist at energy scales $\ll M_S, M_L$. Integrating out S and L gives:

$$\mathcal{L}_{\rm eff} \supset \frac{Y_L Y_R Y_V'}{M_S^2 M_L^2} (\ell H^\dagger)\, H \not{D}^2 (H e^c)$$

Closing Higgs loop (and attaching photon for g-2), generate ΔY_ℓ and Δa_μ from quartically and quadratically divergent diagrams, respectively.
The former is generated but the latter vanishes!

At intermediate energy scales $\ll M_L, \gtrsim M_S$ integrating out only L gives

$$\mathcal{L}_{\rm eff} \supset \frac{Y_R Y_V'}{M_L^2} S H \not{D} (H e^c)$$

Closing Higgs and S loop calculable IR contribution $\Delta a_\mu \neq 0$ is generated.
But now there is additional UV matching contribution that makes total $\Delta a_\mu = 0$!

Introduction

$\Delta a_\mu^{(\mathrm{lo})} = 0$ can be understood from integrand being total derivative (but depends on labelling of loop momenta).

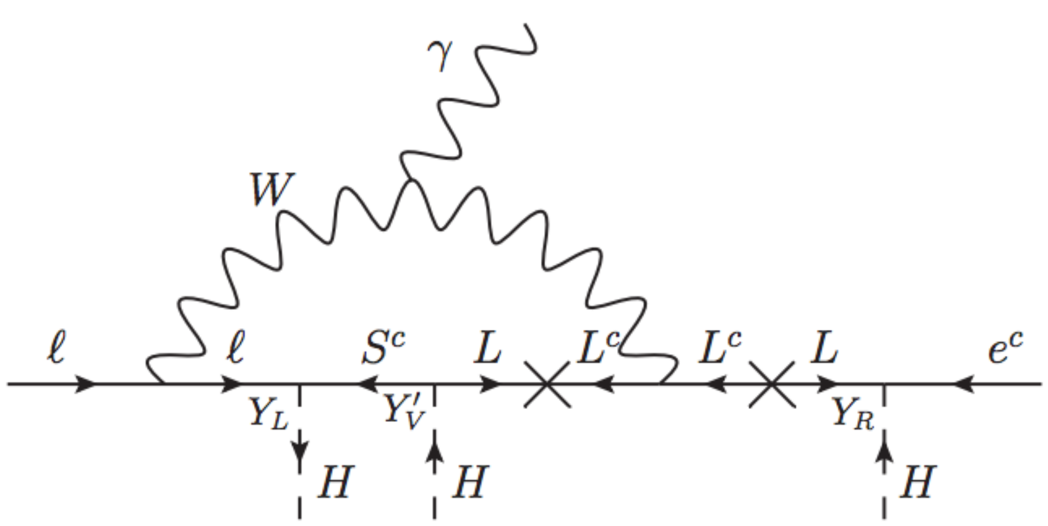
N. Arkani-Hamed and K. Harigaya [arXiv:2106.01373]

Can also be understood from an exchange symmetry.

N. Craig, I. G. Garcia, A. Vainshtein and Z. Zhang [arXiv:2112.05770]

Here will explain also with exchange symmetry using amplitude methods.

Note that contribution to g-2 arises at higher order in M_S, M_L :



$$\Rightarrow \mathcal{L}_{\mathrm{eff}} \supset \frac{C_\gamma'}{M^4} |H|^2 H \ell \sigma^{\mu\nu} e^c F_{\mu\nu}$$

with $C_\gamma' \neq 0$

$$\Rightarrow \Delta a_\mu \neq 0$$

Outline

- Amplitudes, spinor-helicity variables
- Loops from amplitudes
- g-2 from vector-like fermions S and L
- g-2 from vector-like fermions E and L
- hγγ from vector-like fermions E and L
- Conclusions

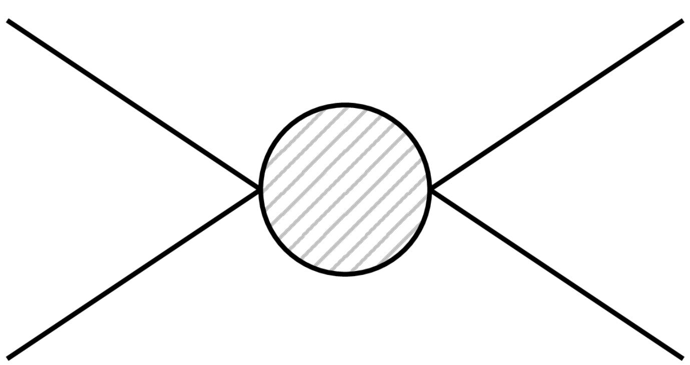
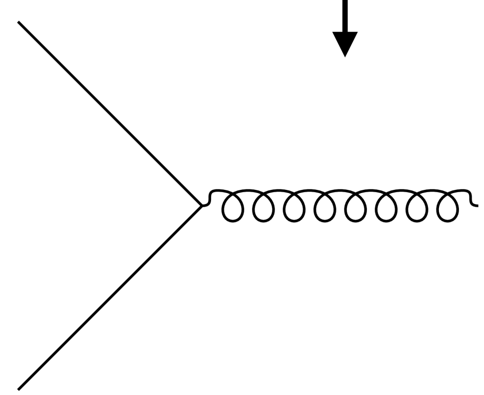
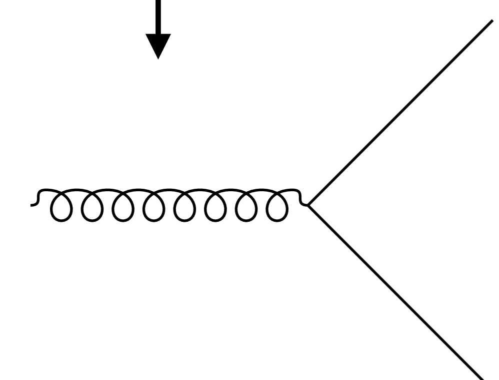
Amplitude methods

Instead of Lagrangian, define theory by particle content and certain on-shell building-blocks amplitudes.

The other tree-level amplitudes can be constructed from the building-blocks by requiring proper factorisation.

For example:

building-block amplitudes

$\lim_{s\to 0} s \times$  $=$  $\times$ 

Spinor-helicity variables

Write momenta as $$p_{\alpha\dot{\alpha}} = p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$$

For massless on-shell particles: $\det p = p_\mu p^\mu = 0$

$\Rightarrow$ Can write: $p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} = |p\rangle[p|$

For real momenta: $$\lambda_\alpha = (\tilde{\lambda}_{\dot{\alpha}})^* = \begin{pmatrix} \sqrt{p_0 + p_3} \\ \frac{p_1 + i\,p_2}{\sqrt{p_0 + p_3}} \end{pmatrix}$$

But useful to keep momenta complex! $\Rightarrow \lambda_\alpha \neq (\tilde{\lambda}_{\dot{\alpha}})^*$

Little group ISO(2) acts as simple rescaling

$$|p\rangle \to t\,|p\rangle \qquad\qquad |p] \to t^{-1}|p]$$

Amplitudes expressed in terms of building blocks:

$$\lambda_{i\alpha}\lambda_{j\beta}\,\epsilon^{\alpha\beta} \equiv \langle ij\rangle \qquad\qquad \tilde{\lambda}_{i\dot{\alpha}}\tilde{\lambda}_{j\dot{\beta}}\,\epsilon^{\dot{\alpha}\dot{\beta}} \equiv [ij]$$

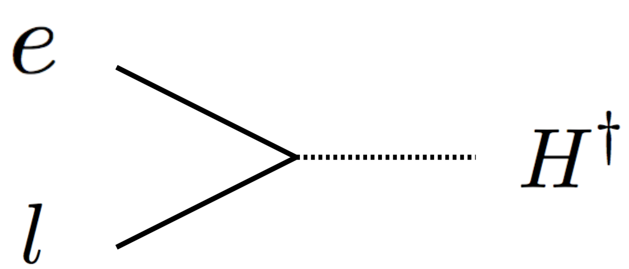
Scattering amplitude transforms as

$$\mathcal{A}(1^{h_1}\dots n^{h_n}) = \epsilon^{h_1}_{\mu_1}\dots\epsilon^{h_n}_{\mu_n}\,\mathcal{A}^{\mu_1\dots\mu_n} \to \prod_i t_i^{-2h_i}\mathcal{A}(1^{h_1}\dots n^{h_n})$$

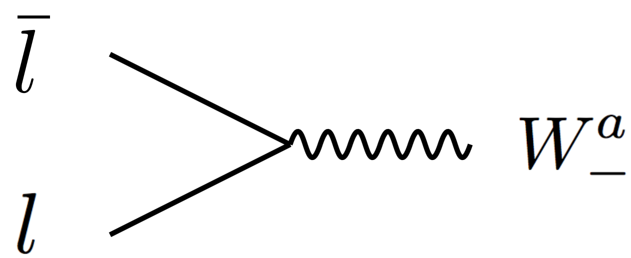
polarisation vectors

with helicity $h_i = \begin{cases} \pm\frac{1}{2} & \text{for massless fermions,} \\ \pm 1 & \text{for massless vectors,} \\ \pm 2 & \text{for gravitons.} \end{cases}$

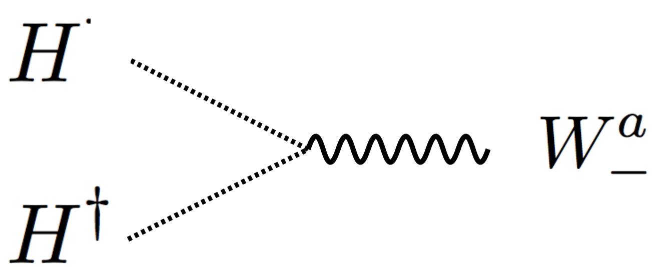
This scaling fixes all 3-point amplitudes. For example for SM:



$$\mathcal{A}(1_e, 2_{l_i}, 3_{H_i^\dagger}) = y_e \langle 12\rangle$$

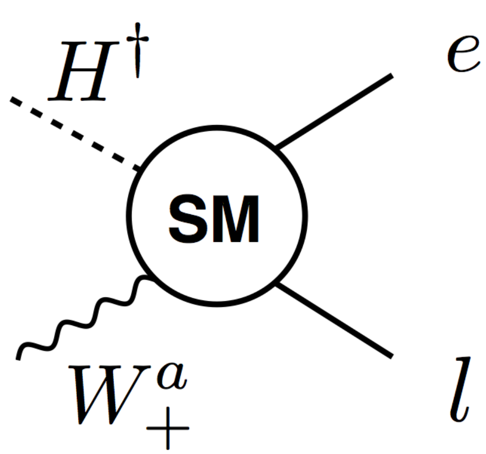


$$\mathcal{A}(1_{l_j}, 2_{\bar{l}_i}, 3_{W_-^a}) = g_2 \frac{\langle 13\rangle^2}{\langle 12\rangle}(T^a)_{ij}$$



$$\mathcal{A}(1_{H_j}, 2_{H_i^\dagger}, 3_{W_-^a}) = g_2 \frac{\langle 13\rangle\langle 23\rangle}{\langle 21\rangle}(T^a)_{ij}$$

Requiring proper factorisation one for example finds:



$$\mathcal{A}(1_{H_i^\dagger}, 2_{W_+^a}, 3_{l_j}, 4_e) = y_e\, g_2\, (T^a)_{ij} \frac{\langle 43\rangle\langle 13\rangle}{\langle 21\rangle\langle 23\rangle}$$

Spinor-helicity variables for massive particles

For massive particles: N. Arkani-Hamed, T.-C. Huang, Y.-t. Huang [1709.04891]

$$p_{\alpha\dot\alpha} \;=\; \epsilon_{IJ}|p\rangle_\alpha^I[p|_{\dot\alpha}^J \;=\; |p\rangle_\alpha^I[p|_{\dot\alpha\, I}$$

$I, J = 1, 2$ indices under little group SU(2).

Satisfy Dirac equation:

$$p|p]^I \;=\; M|p\rangle^I, \qquad p|p\rangle^I \;=\; M|p]^I$$

For internal particles, SU(2) indices contracted. Useful identities e.g.:

$$|p\rangle_\alpha^I[-p|_{\dot\alpha\, I} \;=\; p_{\alpha\dot\alpha} \qquad |p\rangle_\alpha^I\,\langle -p|_I^\beta \;=\; M\delta_\alpha^\beta$$

Often suppress SU(2) indices and use bold variables for massive particles:

$$|p\rangle^I = |\mathbf{p}\rangle \qquad |p]^I = |\mathbf{p}]$$

Loops from amplitudes

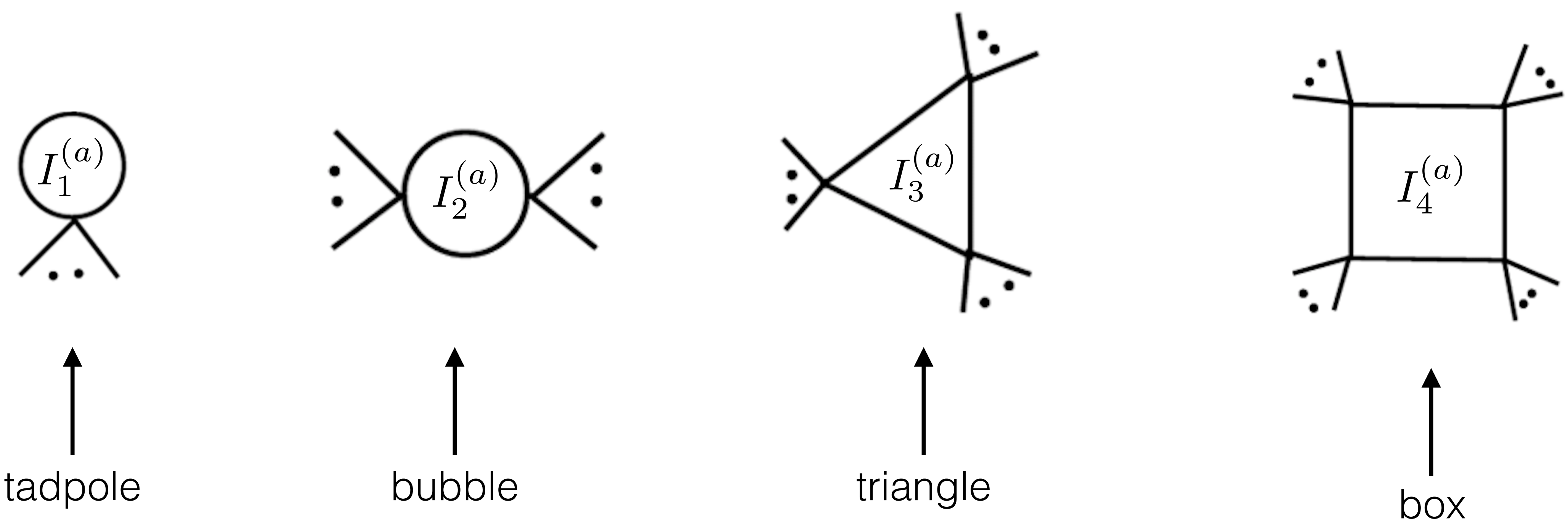
Starting point: Passarino-Veltman decomposition

$$\mathcal{A}_{\text{loop}} = \sum_a C_1^{(a)} I_1^{(a)} + \sum_b C_2^{(b)} I_2^{(b)} + \sum_c C_3^{(c)} I_3^{(c)} + \sum_d C_4^{(d)} I_4^{(d)} + R$$

rational term

with master integrals given by

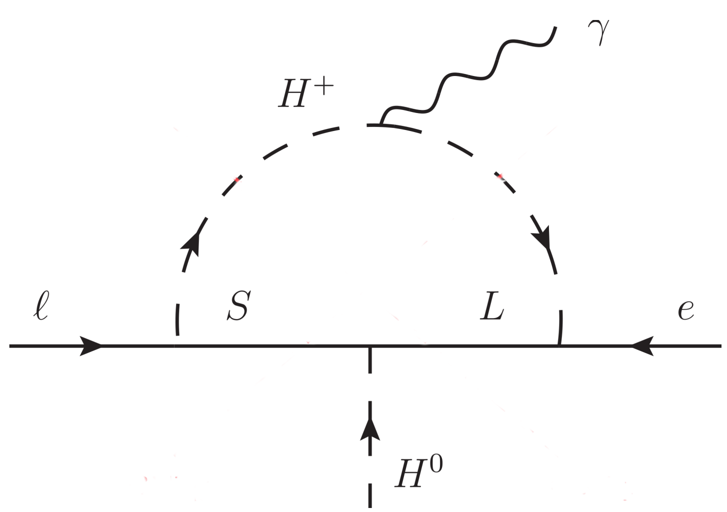
$$I_n = (-1)^n \mu^{4-D} \int \frac{d^D \ell}{i(2\pi)^D} \frac{1}{(\ell^2 - M_0^2)\,((\ell - P_1)^2 - M_1^2)\,((\ell - P_1 - P_2)^2 - M_2^2) \cdots}$$



Coefficients C_n can be obtained by performing (generalized) unitarity cuts on both sides of above relation.

R. Britto, F. Cachazo, B. Feng [arXiv:hep-th/0412103]
D. Forde [arXiv:0704.1835]
and others

Each cut puts internal particle on-shell. E.g. 4-cut:



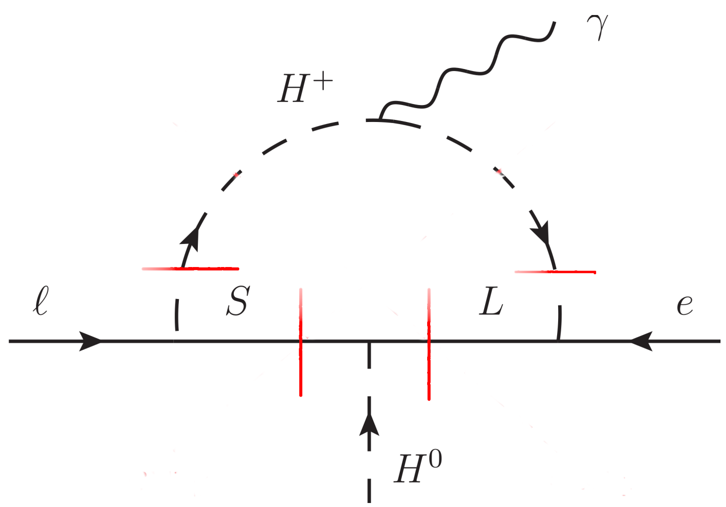
$$\sim \sum_a C_1^{(a)} I_1^{(a)} + \sum_b C_2^{(b)} I_2^{(b)} + \sum_c C_3^{(c)} I_3^{(c)} + \sum_d C_4^{(d)} I_4^{(d)} + R$$

Similarly, $C_2^{(b)}$ and $C_3^{(c)}$ obtained from 2- and 3-cuts (after subtracting contributions from triangles and boxes).

Coefficients C_n can be obtained by performing (generalized) unitarity cuts on both sides of above relation.

R. Britto, F. Cachazo, B. Feng [arXiv:hep-th/0412103]
D. Forde [arXiv:0704.1835]
and others

Each cut puts internal particle on-shell. E.g. 4-cut:



$$\sim \mathcal{A}_1 \cdot \mathcal{A}_2 \cdot \mathcal{A}_3 \cdot \mathcal{A}_4$$

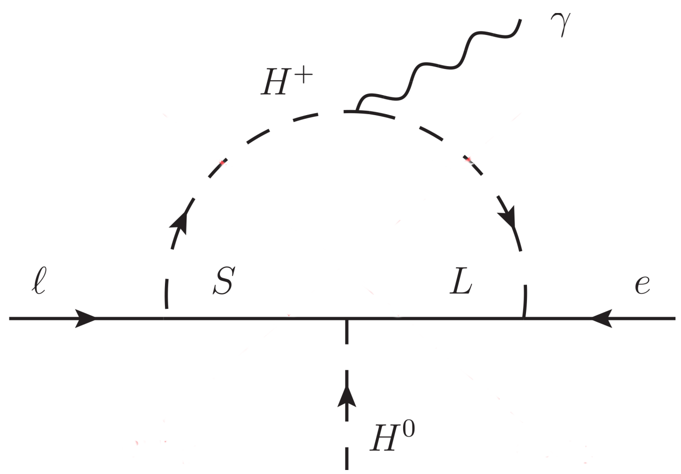
$$\sim \sum_a \cancel{C_1^{(a)}} I_1^{(a)} + \sum_b \cancel{C_2^{(b)}} I_2^{(b)} + \sum_c \cancel{C_3^{(c)}} I_3^{(c)} + \cancel{\sum_d} C_4^{(d)} \cancel{I_4^{(d)}} + \cancel{R}$$

projects to one $C_4^{(d)}$

Similarly, $C_2^{(b)}$ and $C_3^{(c)}$ obtained from 2- and 3-cuts (after subtracting contributions from triangles and boxes).

g-2 from vector-like fermions S and L

Operator in EFT for $M_S, M_L \gg v_{\rm EW}$:



$$\mathcal{L}_{\rm eff} \supset \frac{C_\gamma}{M^2}\, q_e\, \ell_\alpha e_\beta H F^{\alpha\beta}$$

Gives rise to amplitude

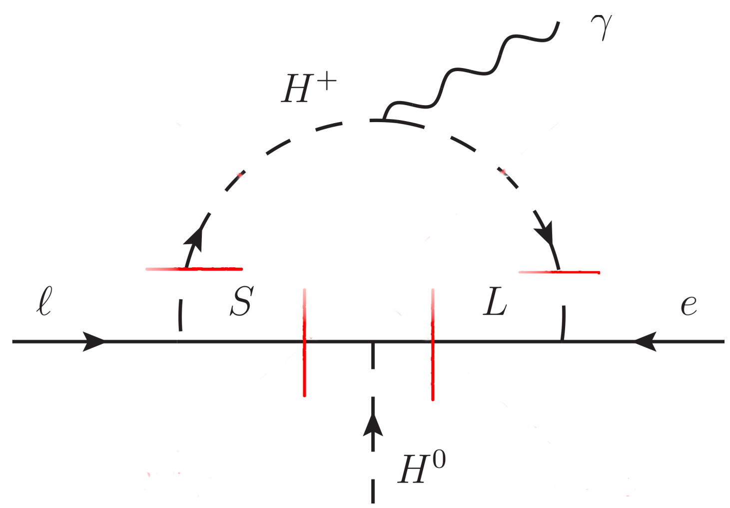
$$\frac{C_\gamma}{M^2}\mathcal{A}_D(1_\ell, 2_e, 3_{\gamma_-}, 4_{H^0}) = \frac{C_\gamma}{M^2}\, q_e\, \langle 13\rangle\langle 23\rangle$$

Obtain Wilson coefficient C_γ via

$$C_\gamma = \frac{1}{\mathcal{A}_{\mathcal{O}_D}} \lim_{\frac{P_i}{M}\to 0} M^2 \left(\sum_a C_1^{(a)} I_1^{(a)} + \sum_b C_2^{(b)} I_2^{(b)} + \sum_c C_3^{(c)} I_3^{(c)} + \sum_d C_4^{(d)} I_4^{(d)} + R \right)$$

Important simplification: Set $p_H = 0.$ $\qquad \mathcal{A}_D(1_\ell, 2_e, 3_{\gamma_-}, 4_{H^0}) = q_e \langle 13\rangle\langle 23\rangle$

$$\Rightarrow C_4 \equiv 0 \quad \text{(no boxes)}$$



$$= 0 \quad \text{for } M_S \neq M_L$$

Only possible triangles arise from cutting one massive and two massless internal lines. Absence of IR divergencies

$$\Rightarrow C_3 \equiv 0 \quad \text{(no triangles)}$$

P. Baratella, C. Fernandez, A. Pomarol [2005.07129]

$$I_2(p^2; M^2, 0) = \frac{1}{\epsilon} + 1 + \log\frac{\mu^2}{M^2} + \frac{2p^2}{M^2} + \ldots \qquad I_1(M^2) = M^2\left(\frac{1}{\epsilon} + 1 + \log\frac{\mu^2}{M^2}\right) + \ldots$$

Absence of UV divergence $\Rightarrow$ tadpoles cancel!

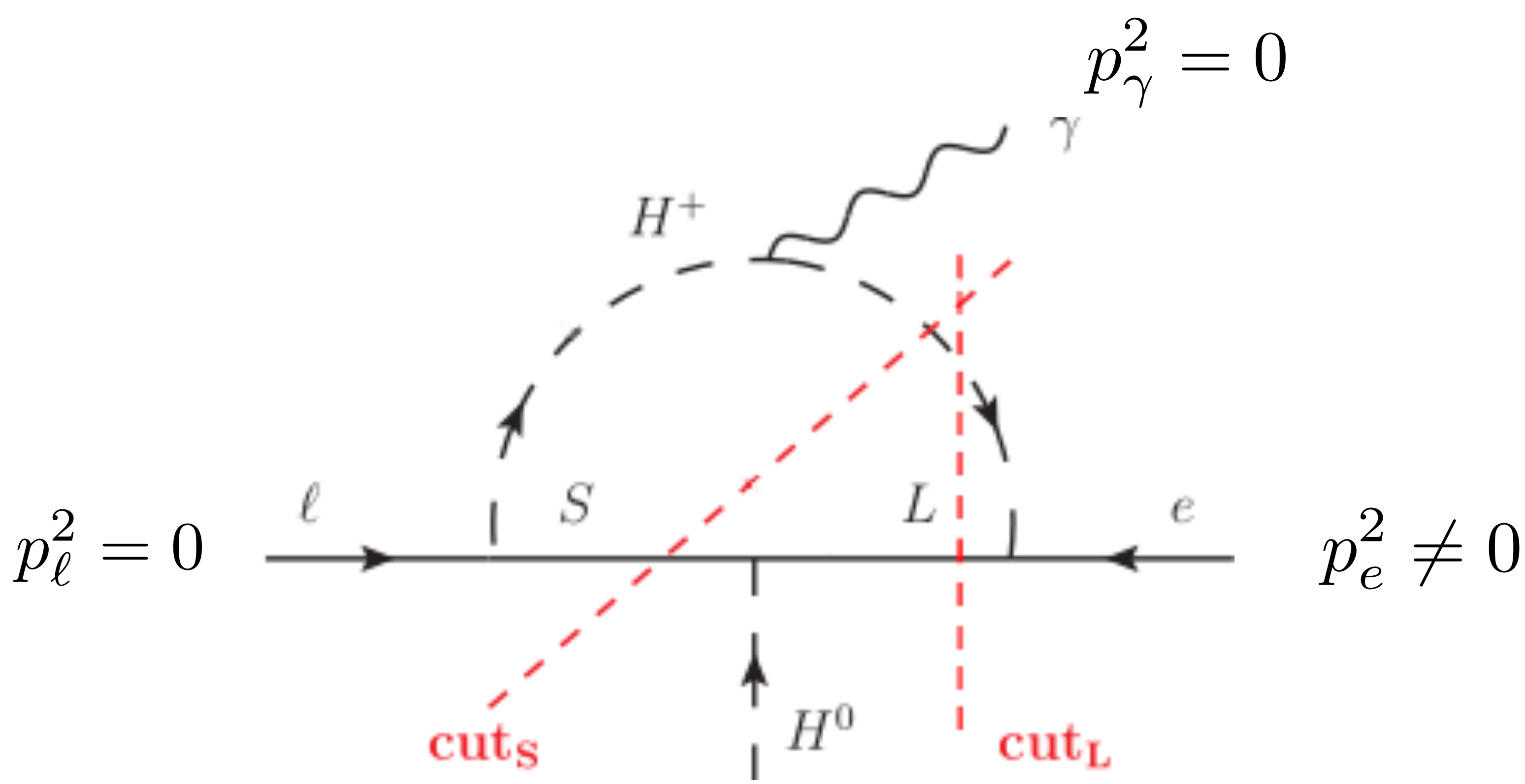
Similarly can show that rational terms do not contribute.

$$\Rightarrow C_\gamma = \frac{1}{\mathcal{A}_{\mathcal{O}_D}} \lim_{\frac{P_i}{M}\to 0} M^2 \sum_a C_2^{(a)} I_2^{(a)}$$

Kinematics: $p_H = 0 \Rightarrow p_\ell + p_\gamma + p_e = 0$

E.g. keep ℓ and γ massless. $\Rightarrow p_e^2 = (p_\ell + p_\gamma)^2 = 2p_\ell p_\gamma \equiv s \neq 0$

Possible 2-cuts:



cut_S and cut_L related via

$$M_S \leftrightarrow M_L$$

Amplitudes from cut_S :

$$\mathcal{A}(1_\ell, 3_{\gamma_-}, 1'_S, 3'_{H^+}) = q_e Y_L M_S \frac{[3'\mathbf{1}']}{[3'3][13]} \qquad \mathcal{A}(3'_{\bar{H}^+}, 1'_{\bar{S}}, 2_e, 4_{H^0}) = Y_R Y'_V \frac{[-\mathbf{1}'|p_{1'}|2\rangle}{M_S^2 - M_L^2}$$

Corresponding bubble coefficient:

$$C_2^{(S)} = -\int d\mathrm{LIPS}\, \mathcal{A}(1_\ell, 3_{\gamma_-}, 1'_S, 3'_{H^+}) \times \mathcal{A}(3'_{\bar{H}^+}, 1'_{\bar{S}}, 2_e, 4_{H^0})$$

Lorentz-invariant phase space

$$\begin{aligned}\mathcal{A}(1_\ell, 3_{\gamma_-}, 1'^I_S, 3'_{H^+})\,\epsilon_{IJ}\,\mathcal{A}(3'_{\bar{H}^+}, 1'^J_{\bar{S}}, 2_e, 4_{H^0}) &= -q_e Y_L Y_R Y'_V \frac{M_S^2}{M_S^2 - M_L^2} \frac{[3'|(p_3+p_1)|2\rangle}{[3'3][13]} \\ &= -q_e Y_L Y_R Y'_V \frac{M_S^2}{M_S^2 - M_L^2} \left(\frac{\langle 32\rangle}{[13]} + \frac{[3'1]\langle 12\rangle}{[3'3][13]} \right)\end{aligned}$$

Looking for amplitude $\sim \langle 13\rangle\langle 23\rangle \;\Rightarrow$ Only first term contributes.

dLIPS integral trivial

$$\Rightarrow C_2^{(S)} = Y_L Y_R Y_V' \frac{M_S^2}{M_S^2 - M_L^2} \frac{1}{s_{13}} \mathcal{A}_D(1_l, 2_e, 3_{\gamma_-}, 4_{H^0})$$

Recall that

$$C_\gamma = \frac{1}{\mathcal{A}_{\mathcal{O}_D}} \lim_{\frac{P_i}{M}\to 0} M^2 \sum_{a=S,L} C_2^{(a)} I_2^{(a)}$$

Need to multiply $C_2^{(S)}$ with

$$I_2^{(S)}(s_{13}, M_S^2, 0) \simeq \frac{1}{16\pi^2}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{M_S^2} + 1 + \frac{s_{13}}{2M_S^2} + \cdots\right)$$

$$\Rightarrow \frac{\Delta C_\gamma}{M^2} = \frac{Y_L Y_R Y_V'}{32\pi^2} \frac{1}{M_S^2 - M_L^2}$$

Contribution from cut_L obtained via $M_S \leftrightarrow M_L$

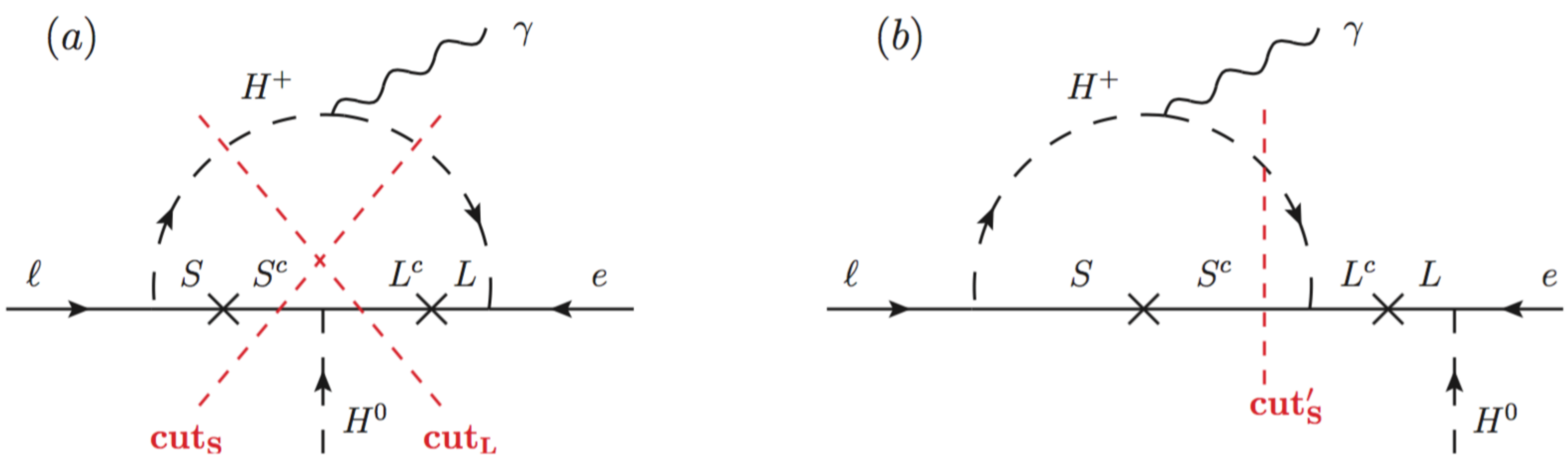
$$\Rightarrow C_\gamma = 0$$

Recall

$$\mathcal{L} = -Y_L \ell S \tilde{H} - Y_R L e H - Y_V \tilde{H}^\dagger L^c S^c - Y_V' L S \tilde{H} - M_S\, S S^c - M_L\, L L^c + h.c.$$

So far $Y_V = 0, Y_V' \neq 0.$ What about opposite case $Y_V \neq 0, Y_V' = 0?$

Diagrams in this case:



Relevant amplitudes for cut_S :

$$\mathcal{A}(1_\ell, 3_{\gamma_-}, 1'_S, 3'_{H^+}) = q_e Y_L M_S \frac{[3'\mathbf{1}']}{[3'3][13]} \qquad \mathcal{A}(3'_{\bar{H}^+}, 1'_{\bar{S}}, 2_e, 4_{H^0}) = Y_R Y_V \langle -\mathbf{1}'2\rangle \frac{M_L}{M_S^2 - M_L^2}$$

Same amplitudes as in other case, except for extra factor M_L/M_S in $\mathcal{A}(3'_{\bar{H}^+}, 1'_{\bar{S}}, 2_e, 4_{H^0})$

$$\Rightarrow \frac{\Delta C_\gamma}{M^2} = \frac{Y_L Y_R Y_V}{32\pi^2} \frac{M_L/M_S}{M_S^2 - M_L^2}$$

Contribution from cut_L again obtained via $M_S \leftrightarrow M_L$

$$\Rightarrow \frac{\Delta C_\gamma}{M^2} = \frac{Y_L Y_R Y_V}{32\pi^2} \frac{(M_L/M_S - M_S/M_L)}{M_S^2 - M_L^2} = -\frac{Y_L Y_R Y_V}{32\pi^2} \frac{1}{M_S M_L}$$

Finally contribution from cut'_S arises from extra term to amplitude

$$\mathcal{A}(3'_{\bar{H}^+}, 1'_{\bar{S}}, 2_e, 4_{H^0}) \; = \; -Y_R Y_V \langle -\mathbf{1}'2\rangle \frac{M_L}{p_2^2 - M_L^2} \; \simeq \; Y_R Y_V \langle -\mathbf{1}'2\rangle \frac{1}{M_L}$$

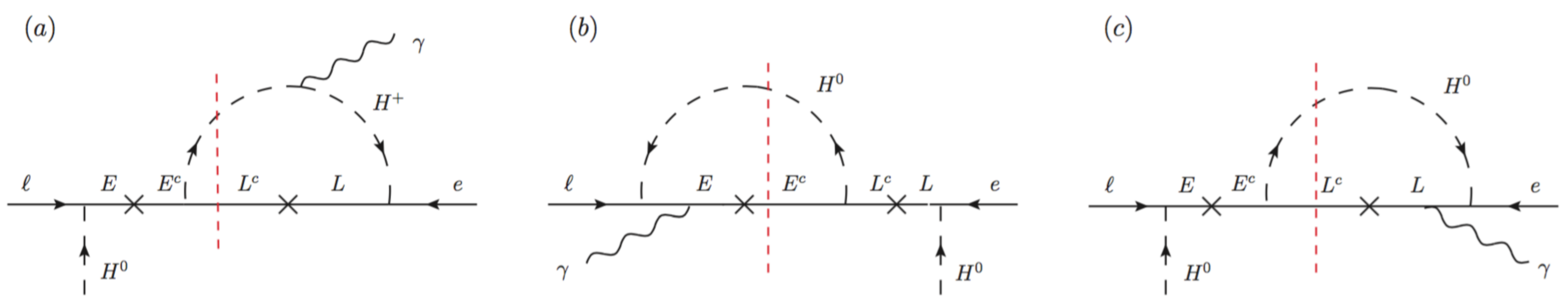
$$\Rightarrow C_\gamma = 0$$

g-2 from vector-like fermions E and L

Now consider vector-like fermions E and L with same quantum numbers as SM leptons:

$$\mathcal{L} = -Y_L \ell E H - Y_R L e H - Y_V H^\dagger L^c E^c - Y_V' L E H - M_E\, E E^c - M_L\, L L^c + h.c.$$

No Feynman diagram for $Y_V' \neq 0, Y_V = 0.$ $\Rightarrow$ study $Y_V' = 0, Y_V \neq 0.$



No boxes $\Longrightarrow$ No triangles.

No UV divergence. $\Longrightarrow$ Possible tadpoles cancelled.

$\Longrightarrow$ Again only bubbles!

Contribution from diagram (a) same as from diagram (b) in other case.

$$\Rightarrow \frac{\Delta C_\gamma}{M^2} = \frac{Y_L Y_R Y_V}{32\pi^2} \frac{1}{M_S M_L}$$

Amplitudes for cut in diagram (b):

$$\mathcal{A}(1_\ell, 3_{\gamma_-}, 1'_E, 3'_{H^0}) = q_e Y_L \frac{M_E}{2p_3 p_{1'}} \frac{\langle 33' \rangle [3' \mathbf{1}']}{[31]} \qquad \mathcal{A}(3'_{\bar{H}^0}, 1'_{\bar{E}}, 2_e, 4_{H^0}) = Y_R Y_V \frac{M_L \langle -\mathbf{1}' 2 \rangle}{p_2^2 - M_L^2}$$

$$\Rightarrow \mathcal{A}(1_\ell, 3_{\gamma_-}, 1'^I_E, 3'_{H^0})\, \epsilon_{IJ}\, \mathcal{A}(3'_{\bar{H}^0}, 1'^J_{\bar{E}}, 2_e, 4_{H^0}) \simeq -q_e Y_L Y_R Y_V \frac{M_E}{M_L} \frac{\langle 32 \rangle}{[31]} + \cdots$$

dLIPS integral again trivial. Multiplying with $I_2^{(S)}(s_{13}, M_S^2, 0)$ gives

$$\frac{\Delta C_\gamma}{M^2} = -\frac{Y_L Y_R Y_V}{32\pi^2} \frac{1}{M_S M_L}$$

Contribution from diagram (c) same with $M_E \leftrightarrow M_L$. $\Rightarrow$ Factor 2.

$$\Rightarrow \frac{C_\gamma}{M^2} = -\frac{Y_L Y_R Y_V}{32\pi^2} \frac{1}{M_S M_L}$$

Agrees with result from e.g.
A. Freitas et al. [1402.7065]

hγγ from vector-like fermions E and L

Another example of unexpected cancellation: Consider again model with vector-like fermions E and L (but with $Y_L = Y_R = 0$):

$$\mathcal{L} = -Y_V H^\dagger L^c E^c - Y_V' LEH - M_E\, EE^c - M_L\, LL^c + h.c.$$

Leading contribution to following operator vanishes

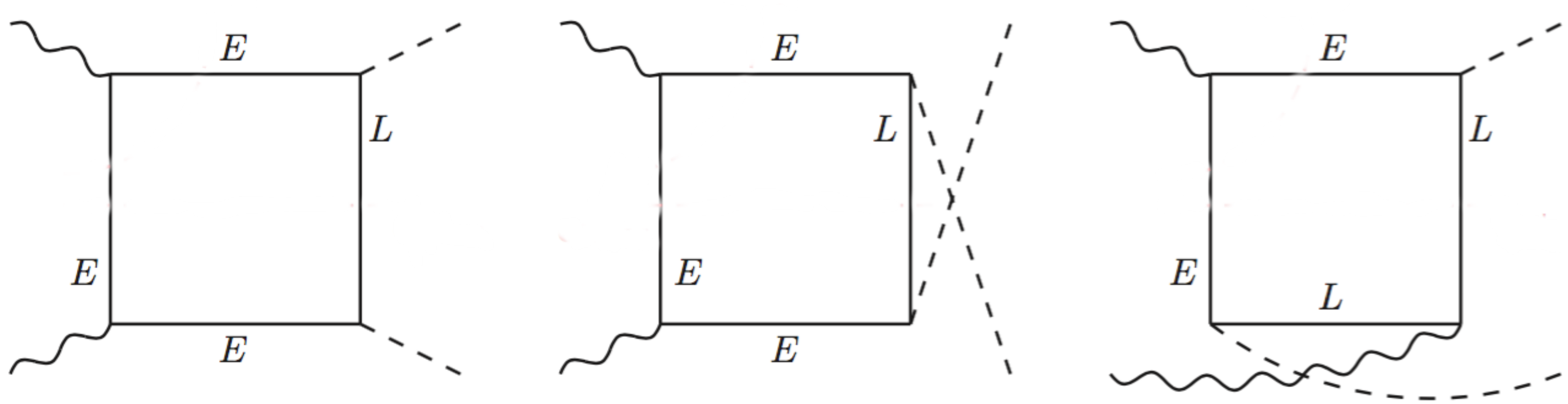
e.g. G. Panico, A. Pomarol, M. Riembau [1810.09413]

$$\mathcal{L}_{\rm eff} \supset \frac{C_{\gamma\gamma}}{M^2}\frac{q_e^2}{2}\,|H|^2 F_{\mu\nu}^2$$

Corresponds to amplitude:

$$\frac{C_{\gamma\gamma}}{M^2}\mathcal{A}_{H^2F^2}(1_{\gamma_-}, 2_{\gamma_-}, 3_{H^0}, 4_{H^0}) = -\frac{C_{\gamma\gamma}}{M^2}\, q_e^2\, \langle 12\rangle^2$$

Diagrams:

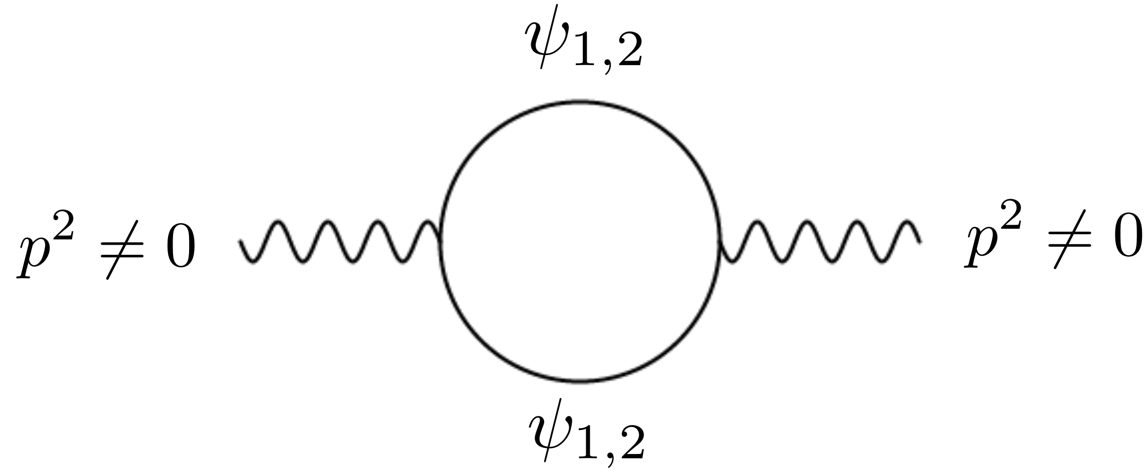


Choose $Y_V = 0, Y'_V \neq 0$ (discussion for $Y_V \neq 0, Y'_V = 0$ identical).

Set $v \equiv \langle H^0 \rangle \neq 0$. Mass matrix in basis $(E\, E^{c\dagger} L^- L^{c-})^T$:

$$\begin{pmatrix} 0 & M_E & vY'_V & 0 \\ M_E & 0 & 0 & 0 \\ vY'_V & 0 & 0 & M_L \\ 0 & 0 & M_L & 0 \end{pmatrix} \quad \Rightarrow \quad \text{Two mass eigenstates } \psi_{1,2}.$$

Diagram in mass eigenstate basis:



Now make photon massive, $m_\gamma^2 \equiv p^2 \neq 0$. Relevant vertex:

$$\mathcal{A}(1_\gamma, \ell_{\psi_i}, \ell'_{\psi_i}) = \frac{q_e}{p}\big(\langle \mathbf{1}\ell\rangle\,[\mathbf{1}\ell'] + [\mathbf{1}\ell]\langle \mathbf{1}\ell'\rangle\big)$$

$$\Rightarrow \mathcal{A}(1_\gamma, \ell^I_{\psi_i}, \ell'^K_{\psi_i})\,\epsilon_{IJ}\epsilon_{KL}\,\mathcal{A}(\ell'^L_{\bar\psi_i}, \ell^J_{\bar\psi_i}, 2_\gamma) \;=\; \frac{q_e^2}{p^2}\left[2\,M_i^2\langle \mathbf{12}\rangle[\mathbf{12}] + \big(\langle \mathbf{1}|\ell|\mathbf{2}]\langle \mathbf{2}|\ell'|\mathbf{1}] + (\ell \leftrightarrow \ell')\big)\right]$$

Choose particular basis for photon polarizations:

$$|1^{I=1}\rangle = \sqrt{p}\begin{pmatrix}1\\0\end{pmatrix}, \qquad |1^{I=2}\rangle = \sqrt{p}\begin{pmatrix}0\\1\end{pmatrix}$$

Next project onto particular component of SU(2) tensor:

$$\langle\mathbf{12}\rangle[\mathbf{12}] \;\rightarrow\; \langle 1^{I=1}2^{I=2}\rangle[1^{I=1}2^{I=2}] \;=\; p^2$$

$$\Rightarrow \int d\mathrm{LIPS}\,\mathcal{A}(1_\gamma,\ell_{\psi_i},\ell'_{\psi_i})\times\mathcal{A}(\ell'_{\bar\psi_i},\ell_{\bar\psi_i},2_\gamma) \;\rightarrow\; q_e^2\int_0^{2\pi}\frac{d\phi}{4\pi}\int_0^{\pi}d\theta\,s_\theta\Big[2\,M_i^2(1-c_\theta^2)+\frac{1}{2}p^2(1+c_\theta^2)\Big]$$

$$= \frac{2q_e^2}{3}\left(2M_i^2+p^2\right)$$

Furthermore, from the bubble integral:

$$I_2(p^2,M_1^2,M_1^2) = \frac{1}{16\pi^2}\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{M_1^2}+\ldots\right) = \frac{1}{16\pi^2}\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{M_E^2}-\frac{|Y_V'|^2v^2}{M_E^2-M_L^2}+\cdots\right)$$

expand to order v^2

and $M_E \leftrightarrow M_L$ for M_2. Match with $\langle 12\rangle^2 \rightarrow \langle\mathbf{12}\rangle^2 \rightarrow p^2$:

$$\Rightarrow \frac{C_{\gamma\gamma}}{M^2} = \frac{1}{16\pi^2}\frac{2}{3}|Y_V'|^2\left(\frac{1}{M_E^2-M_L^2}+\frac{1}{M_L^2-M_E^2}\right) = 0$$

Conclusions

- Leading contribution to g-2 and $h\gamma\gamma$ vanishes in certain models with vector-like fermions. Why?
- Amplitude methods useful to address this question.
- Setting Higgs momenta to zero, calculation of Wilson coefficients for g-2 and $h\gamma\gamma$ reduces to double cuts.
- Vanishing of Wilson coefficients can then be understood from exchange symmetry acting on amplitude level.
- Also useful to calculate Wilson coefficients in models where they do not vanish.

Backup slide: Absence of rational terms

Rational terms arise in Pasarino-Veltman decomposition from terms of the type ϵ/ϵ.

Accessible via generalized unitarity: Write

$$l^2 = l^2_{(4)} + l^2_{(-2\epsilon)} \equiv l^2_{(4)} - \mu^2$$

and interpret extra-dimensional momentum as 4D mass μ^2.

Rational term then given by

$$R \;=\; -\frac{1}{6}\sum_{i,j}\tilde{C}_2^{(ij)}(s_{ij} - 3(M_i^2 + M_j^2)) - \frac{1}{2}\sum_b \tilde{C}_3^{(b)} - \frac{1}{6}\sum_c \tilde{C}_4^{(c)}$$

To determine $\tilde{C}_2^{(ij)}, \tilde{C}_3^{(b)}$ and $\tilde{C}_4^{(c)}$ shift particle masses by μ^2 and take μ^2-term for $\tilde{C}_2^{(ij)}, \tilde{C}_3^{(b)}$ and μ^4-term for $\tilde{C}_4^{(c)}$ in large μ^2-limit.

We match to EFT at order $1/M^2$. However,

$$\lim_{\mu^2\to\infty}\frac{1}{s_{ij} - M^2 - \mu^2} \;=\; -\frac{1}{\mu^2} + \frac{M^2 - s_{ij}}{\mu^4} + \ldots$$

$\Rightarrow$ No rational terms at order $1/M^2$.