Exotic Field Theories for Prosaic Natural Physics Solution

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Exotic approaches to Naturalness

Naturalness Terminology

A dimensionless numbers (or dimensionful number in units of the cut-off) in the action which is much less than one we will call this a
Dirac Fine Tuning

A Dirac fine tuning which is not radiatively stable we will call a t'Hooft fine tuning.

Dirac fine tunings of relevant operators which are not protected by symmetry are t'Hooft fine tuned

Some Examples:

Dirac	t'Hooft
Fermion masses in the SM, the Theta parameter	Higgs mass, CC

(This talk)

Resolutions to Naturalness Problems

``UV Solutions''

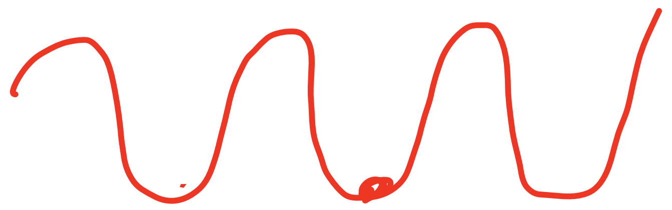
- Enhanced Symmetry (SUSY).
- Strong coupling dynamics shifts relevant to marginal.   (RS)
- No dimension 2 scalar operators (Technicolor).

``IR Solutions''

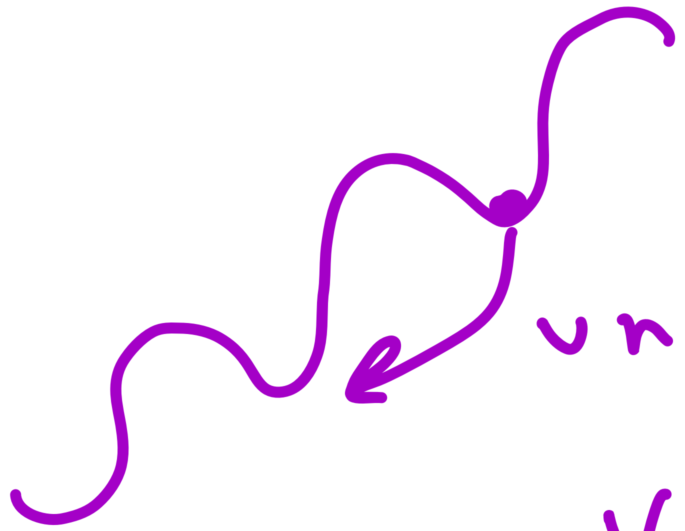
- Relaxation Mechanisms. PQ Mechanism mechanism (strong CP), Abbot (CC), Relaxion (EW Hierarchy)

Relaxation Mechanisms

$U(1)_A \to \emptyset$ $\qquad L = \phi F \wedge F$ $\qquad \langle \phi \rangle \equiv \theta$



Abbotts: Relaxation of c.c



$$\frac{V}{M_{pl}^2} > \Lambda^?$$

$\Lambda \equiv$ phantom sector strong scale

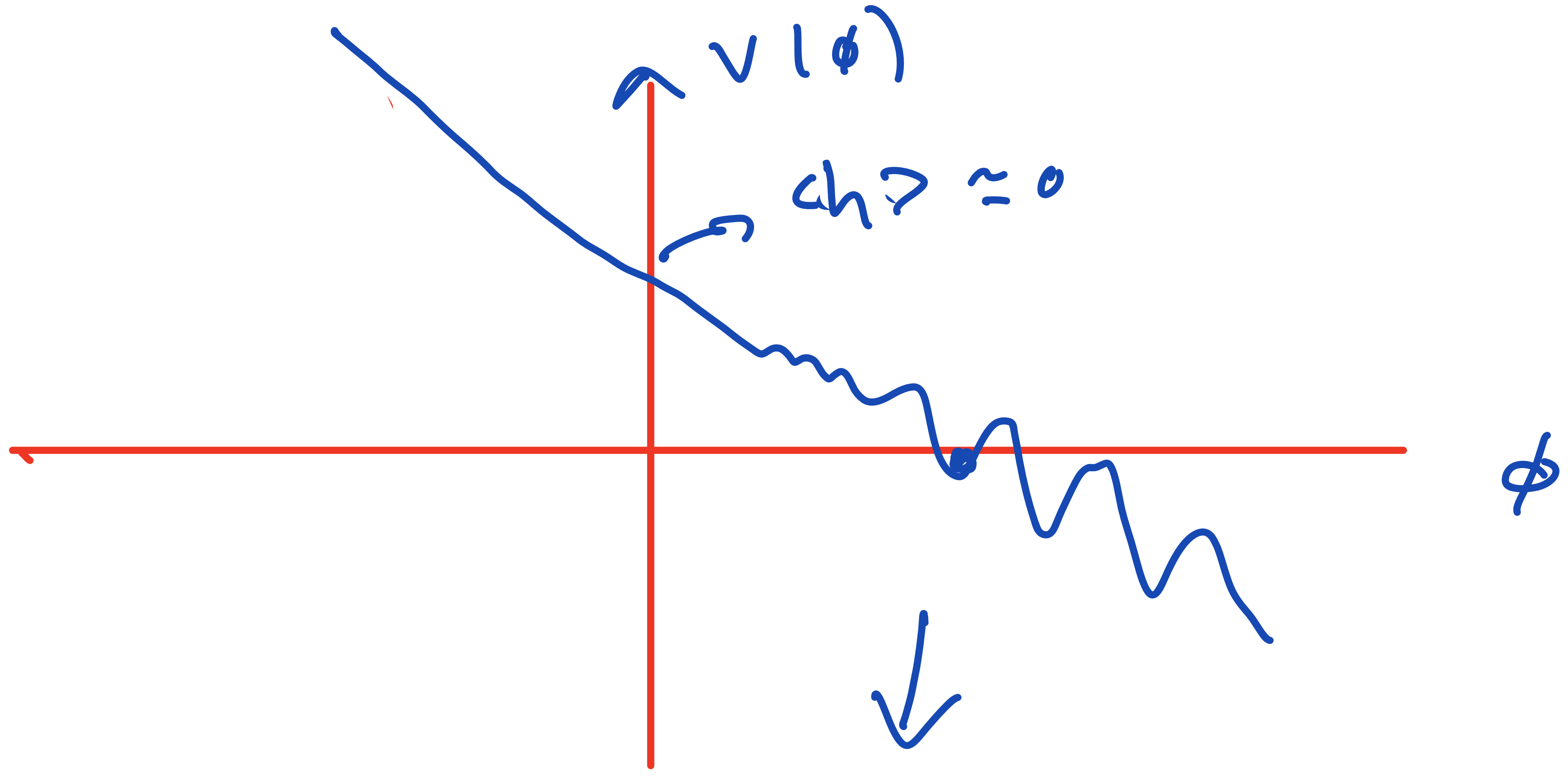
$$V = V_0 + \frac{\epsilon B}{f_B} - \Lambda^4 \cos(B/f_B)$$

Relaxion

Graham
Kaplan
Rajendran

$$V \sim g\Lambda^3\phi - (M^2 - g\Lambda\phi)|h|^2$$
$$+ v(h)\cos(\phi/f)$$

$g \sim 10^{-36}$



Slide stops @ EW scale

These paradigms are compelling (though they still suffer from Dirac fine-tunings) especially since one need not have any new physics beyond the weak scale (testability?)
But they dont seem very generic

However, perhaps the problem is that we are thinking in to narrow a space of QFT's

Consider any macroscopic object

Λ Inter-actomic spacing

$$R \sim N\Lambda \qquad N \sim 10^{23}$$

To determine if this system his fine tuned we need to place it in a field theoretical context. Perhaps we can learn about field theories which look finely tuned but are not.

Consider the following quantum field theories

$$S = \int d^2x (\frac{1}{2}\dot{\phi}_I^2 + C_1(\nabla^2\phi_I)^2 + V(\nabla\phi_I))$$

[redacted]

$$S = \int d^4x (\dot{\vec{\pi}}^2 - (\vec{\nabla}\cdot\vec{\pi})^2 - \partial_{(i}\pi_{j)}\partial_{(i}\pi_{j)}) + V(\partial_i\pi_j)$$

[redacted]

The QFTs which describe these theories are distinguished from the class of theories we typically consider when looking for solutions to hierarchy problems:

- Spontaneously break space-time symmetries
- Target space have non vanishing boundaries

Effective Field Theory of Solids

Label the atoms by D fields $\phi^I(t,\vec{x})$ $\quad I = 1 - D$

Lagrangian ``Co-moving coordinates’’

$$X^I(\phi, t)$$ Eulerian

$$\langle \phi^I \rangle = \alpha x^I$$ Ground state solution

Assumption: of homgeneity and isotropy on large scales

Broken spacte-time symmetries but leaves unbroken diagonal sub-groups

$$x^I \to x^I + a^I$$

$$\phi^I \to \phi^I - a^I$$

$$T_{ST} \otimes T_I \otimes SO(3)_I \otimes SO(3)_{ST} \to T_{I+ST} \otimes SO(3)_{T+ST}$$

$$S = \int G(B^{IJ})\, d^4x \ ,$$

Only three Goldstones

$$B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J \ ,$$

Inverse Higgs Constraints

Power Counting: $\partial\phi \sim 1$ $\quad \partial^2\phi \sim\ll 1$ $\quad \phi^I(x) = \alpha\big(x^I + \pi^I(x)\big)$

$$S = \int d^4x \left(G_0 + \frac{\partial G}{\partial B^{IJ}}\bigg|_0 s^{IJ} + \frac{1}{2}\frac{\partial G}{\partial B^{IJ}\partial B^{KL}}\bigg|_0 s^{IJ} s^{KL} + \dots \right)$$

$$s^{IK} = (\partial_\mu \phi^I \partial^\mu \phi^K - \alpha^2 \delta^{IK}) = \alpha^2\big(2\,\partial^{(I}\pi^{K)} + \partial_\mu \pi^I \partial^\mu \pi^K\big)$$

$$S = \int d^4x \left[\frac{1}{2} G_0\, \partial_\mu \pi^i \partial^\mu \pi^i + 2C_0\, (\vec{\nabla}\cdot\vec{\pi})^2 + 2D_0\, \partial_{(i}\pi_{j)}\partial_{(i}\pi_{j)}\right]$$

This is NOT what you will find in Landau+Lifshitz ! AS part should be absent.

Consider a “Beam” (string) Embedded

$d/L \ll 1$

d

L

$$S = \int d^2\sigma \sqrt{g}\; G(B) \qquad g_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$$

$$X^0 = \tau \equiv t\ , \qquad X^1 = \sigma \equiv x$$

Ground State $\quad X^{2,3} = 0, \qquad \phi = \alpha\, x,$

Action for Longitudinal mode: $\quad S = \int dt dx (\dot{\phi}^2 - (\partial_x \phi)^2 + \ldots$

Again NOT what is in L+L!

1-D Embedded Solid: Need to impose boundedness of target space

$$\mid \phi \mid < L$$

$$S_{\text{bar}} = \int d\tau d\sigma \, \theta(\phi - \phi^\star) \sqrt{g} \, G(B) \, , \qquad B \equiv g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi$$

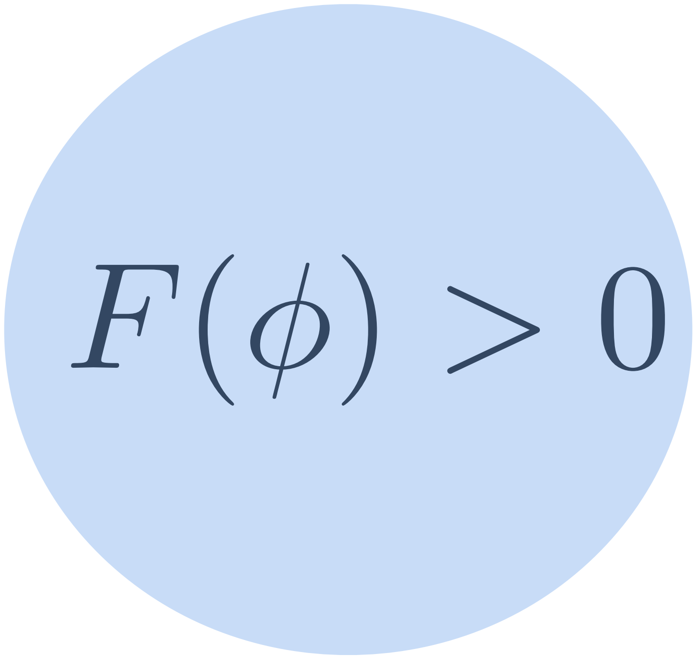
$$T_{\mu\nu} = \theta\big(F(\phi)\big) \tilde{T}_{\mu\nu} \qquad \tilde{T}_{\mu\nu} = 2 \frac{\partial(\sqrt{g}\, G)}{\partial g^{\mu\nu}}$$

(On shell)

$$\partial_i T^{i\nu} = \delta\big(F(\phi)\big) \frac{\partial F}{\partial \phi^K} (\partial_i \phi^K) \tilde{T}^{i\nu} + \theta\big(F(\phi)\big) \cancel{\partial_i \tilde{T}^{i\nu}} = 0$$

$$n_i(\phi^\star) \, \tilde{T}^{i\nu}(\phi^\star) = 0 \quad \longrightarrow \quad \boxed{\tilde{T}^{ij} = 0}$$

3-D Solid Need to impose boundedness of target space



$$e.g.\ F(\phi^I) = R^2 - \phi^I \phi^I$$

Implement at the level of the action

$$S = \int \theta\big(F(\phi)\big)\, G(B^{IJ}) d^4x$$

Equations of motion:

$$T_{\mu\nu} = \theta\big(F(\phi)\big)\tilde{T}_{\mu\nu}$$

Static Configuration:

$$\partial_i T^{i\nu} = \delta\big(F(\phi)\big)\, \partial_i F(\phi)\, \tilde{T}^{i\nu} + \theta\big(F(\phi)\big)\, \partial_i \tilde{T}^{i\nu} = 0$$

$$n_i(\phi^\star)\,\tilde{T}^{i\nu}(\phi^\star) = 0$$

$\phi^\star$ (Boundary value)

Homogeneity of ground state:

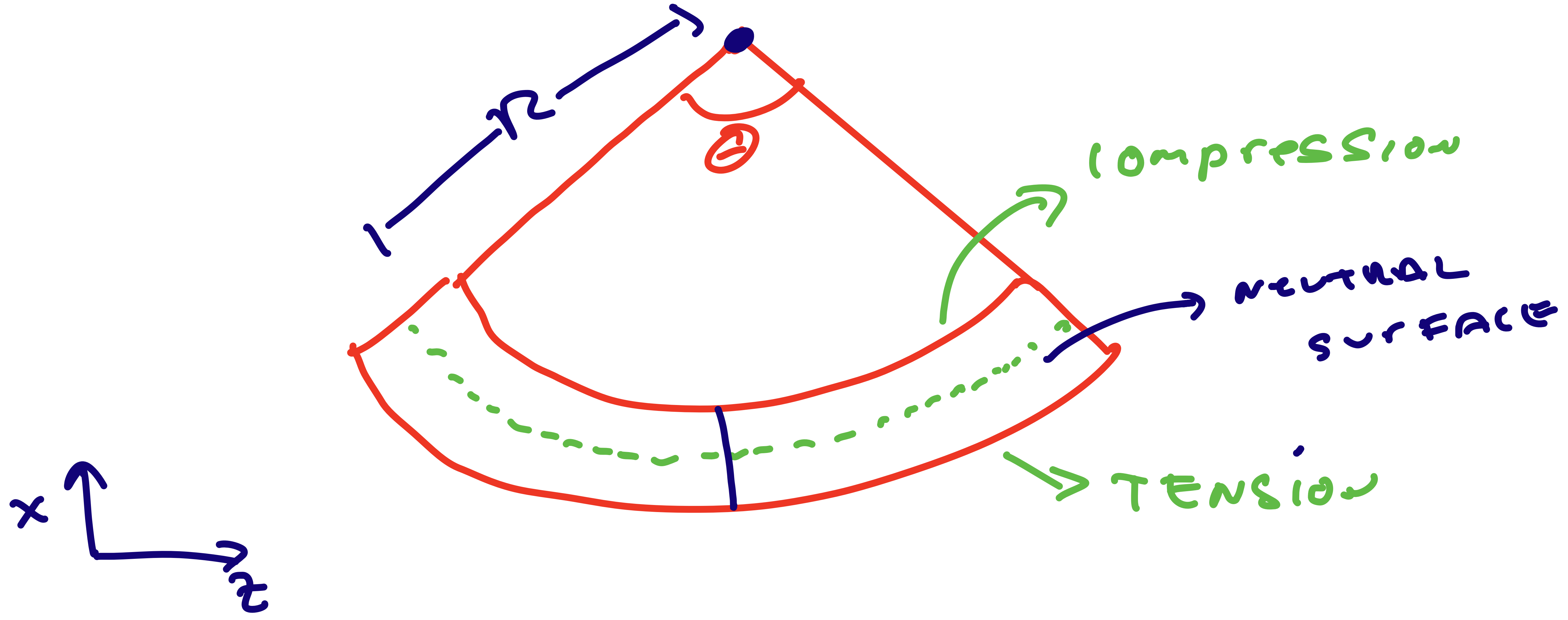
$$\tilde{T}^{ij} = 0 \quad \text{everywhere}$$

Generates a constraint on Wilson coefficients of the action

Most Striking for the case of the string

$$S_{\text{bar}} \supset \frac{1}{2}\int d^4x\, \tilde{T}^{\alpha\beta}\, \partial_\alpha\vec{\varphi}\cdot\partial_\beta\vec{\varphi} \quad = 0$$

U.V completion



Free - Energy Density : pure finite size effect.

$$E = \frac{1}{2}\sigma_{ij}u_{ij}$$

u_{ij} = strain tensor

$\partial_{(i}\Delta x_{j)}$

$$\sigma_{ij} = \varepsilon u_{ij}$$

$\Delta x_j \equiv$ Displacement from EQ.

$u_{ij} \approx 0$ Except u_{zz}

$$u_{zz} = \frac{x}{R}$$

$R \equiv$ radius of curvature.

Follows from "Beam" Assumption. $\ell / L \to 0$.

$$E \sim \int (u_{zz})^2 \, dx \sim \frac{1}{R^2} \sim \left(\frac{\partial^2 X}{\partial z^2} \right)^2$$

origin on neutral axis

position of neutral axis is space fixed frame.

From the points of view of RG flow this is quite remarkable

Consider dimensional reduction: $(\nabla\phi)^2$ IRRELEVENT

$(\nabla^2\phi)^2$ RELEVENT

IN FREE FIELD THEORY

Highly non-trivial RG flow as a consequence of boundedness of target space

Can see it from explicit calculation

$$S = \frac{1}{2}\int_{\mathcal{R}} dxdydt\left(\dot{\vec{\pi}}^{\,2} - a\big(\vec{\nabla}\cdot\vec{\pi}\big)^2 - b(\partial_i\pi_j)(\partial_j\pi_i) - c(\partial_i\pi_j)(\partial_i\pi_j)\right)\;,$$

Full Theory (2+1) Propagator:

$$G^{ij}(x) \equiv \int dtdydy'\,\langle T\pi^i(x,y,t)\pi^j(0,y',0)\rangle$$

$$\tilde{G}^{11}(k) = -i\frac{\delta}{c_L^2\,k^2}\left[1 + \frac{(c_L^2 - 2c_T^2)^2}{(c_L^2 - c_T^2)c_T^2}\,f_+(k\delta)\right]$$

$$\tilde{G}^{22}(k) = -i\frac{\delta}{c_T^2\,k^2}\left[1 + \frac{c_L^2}{c_L^2 - c_T^2}\,f_-(k\delta)\right]\;,$$

δ Thickness

$$\tilde{G}^{11}(k \gg 1/\delta) \simeq -i\frac{\delta}{c_L^2\,k^2}\;, \qquad \tilde{G}^{22}(k \gg 1/\delta) \simeq -i\frac{\delta}{c_T^2\,k^2}$$

$$f_\pm(\xi) \equiv \frac{2\sinh^2(\xi/2)}{\big(\sinh\xi \pm \xi\big)\,\xi}\;.$$

$$\tilde{G}^{11}(k \ll 1/\delta) \simeq -i\frac{\delta}{\bar{c}_L^2\,k^2}\;, \qquad \tilde{G}^{22}(k \ll 1/\delta) \simeq -i\frac{\delta}{\bar{c}_L^2/c_L^2}\frac{1}{\delta^2 k^4}$$

Note that radiative stability is a consequence of the fact that we are working at the level of the effective action. Calculating at any given order simply shift the boundary

For this to be effective it was crucial that the action enjoy a shift symmetry, so radiative corrections preserver the local structure of the effective action

$$\frac{\partial V(\alpha)}{\partial \alpha} = 0.$$

Equivalent to

$$n \cdot T = 0.$$

This mechanism does not seem that useful for model building since it causes vanishing coefficients as opposed to hierarchally small coefficients

However, we have been too cavalier!!!!

When we opposed the boundary we broke the shift symmetry:

$$L \to L + \delta(F(\phi))L_B + \ldots.$$

Surface physics:

Replace theta function with finite thickness boundary

$$\theta_\ell(F(\phi)) = \frac{1}{2}\Big(1 + \tanh\frac{R - |\phi^I|}{\ell}\Big)$$

$$\theta_\ell(x) = \theta(x) + C_1\,\ell\,\delta(x) + \frac{1}{2!}C_2\,\ell^2\delta'(x) + \ldots$$

In our case:

$$\theta_\ell\big(F(\phi)\big) = \theta\big(F(\phi)\big) + C_1\,\ell\,\delta\big(F(\phi)\big)\,|\partial F| + \dots\ , \qquad C_1 = \mathcal{O}(1)$$

Ensures independence of F

$$S_{\text{bdy}} = \ell \int d^4x\,\delta(F(\phi))\,|\partial F|\,\mathcal{L}_{\text{bdy}}\big(B^{IJ},\phi^I\big)$$

For a static configurations:

$$\mathcal{L}_{bdy} \sim \rho$$

$$\tilde{T}_{ij} \sim \frac{\ell}{R}\rho$$

R Radius of curvature

More Concretely:

Suppose: $$\mathcal{L}_{\rm bdy}\big(B^{IJ},\phi^I\big) = \text{const} \equiv -\Lambda$$

$$T_{\mu\nu} = \theta\big(F(\phi)\big)\tilde{T}_{\mu\nu} - \ell\Lambda\,\delta(F(\phi))|\partial F|\,h_{\mu\nu}$$

Induced metric: $$h_{\mu\nu} = \eta_{\mu\nu} - n_\mu n_\nu\ , \qquad n_\mu \equiv -\partial_\mu F/|\partial F|$$

$$\partial_i T^{i\nu} = \delta\big(F(\phi)\big)|\partial F|\left[-\,n_i\,\tilde{T}^{i\nu} - \ell\Lambda\,\frac{1}{|\partial F|}\partial_i\big(|\partial F|h^{i\nu}\big)\right] + \theta\big(F(\phi)\big)\,\partial_i\tilde{T}^{i\nu} = 0$$

$\nu = j$ $$\boxed{n_i\,\tilde{T}^{ij} = \ell\Lambda\,n^j K}$$

We see that the vanishing of the Wilson coefficient Is a conequence of the boundedness of the target space. This is a dynamical relaxation mechanism, though we have not had to do ``engineering''.

How can we generalize this mechanism?

-Space-time symmetry breaking

-Boundedness of Target Space

How can we choose our vanishing Wilson coefficients?

Consider a Superfluid

Shift Symmetry: $\phi \to \phi + a$

$$S = \int d^d x P(X) \qquad X = \partial_\mu \phi \partial^\mu \phi$$

$$J^\mu = 2P' \partial_\mu \phi \qquad < Q > \neq 0, < J^i > = 0$$

$$\langle \phi \rangle \sim t$$

Time versions of solid vev

$$\langle \phi^I \rangle \sim x^I$$

Analogy: ``time solid”

Bound target space $\theta(F(\phi)) = \theta(\phi - \phi_\star)$

Time Relaxation Constraint

$$\tilde{T}_{\mu\nu} = 2P'(X)\partial_\mu\phi\partial_\nu\phi + \eta_{\mu\nu}P(X)$$

$$n^\mu = \delta^\mu_0$$

$$n_\mu \tilde{T}^{\mu\nu}(\phi^\star) = 0$$

$$\tilde{T}^{00}(x) = 2\mu^2 P'(\mu^2) - P(\mu^2) = 0$$

Everywhere

We can go a step further: Consider a Super Solid:

$$T_{ST} \otimes T_I \otimes SO(3)_I \otimes SO(3)_{ST} \to T_{I+ST} \otimes SO(3)_{T+ST}$$

$$U(1) \to \emptyset$$

$$\phi = \mu t \ , \qquad \phi^I = \alpha x^I$$

Bound in both space and time:

$$\theta\Big(F(\phi, \phi^I)\Big)$$

$$T_{\mu\nu} = 0.$$

So far we have only set the vacuum energy due to one field (The GB) to zero.

Generalize: Use GB as ``control field”:

$$L = \theta(F(\phi))L_0(\phi')$$

Automatically sets $T = 0$ for all fields