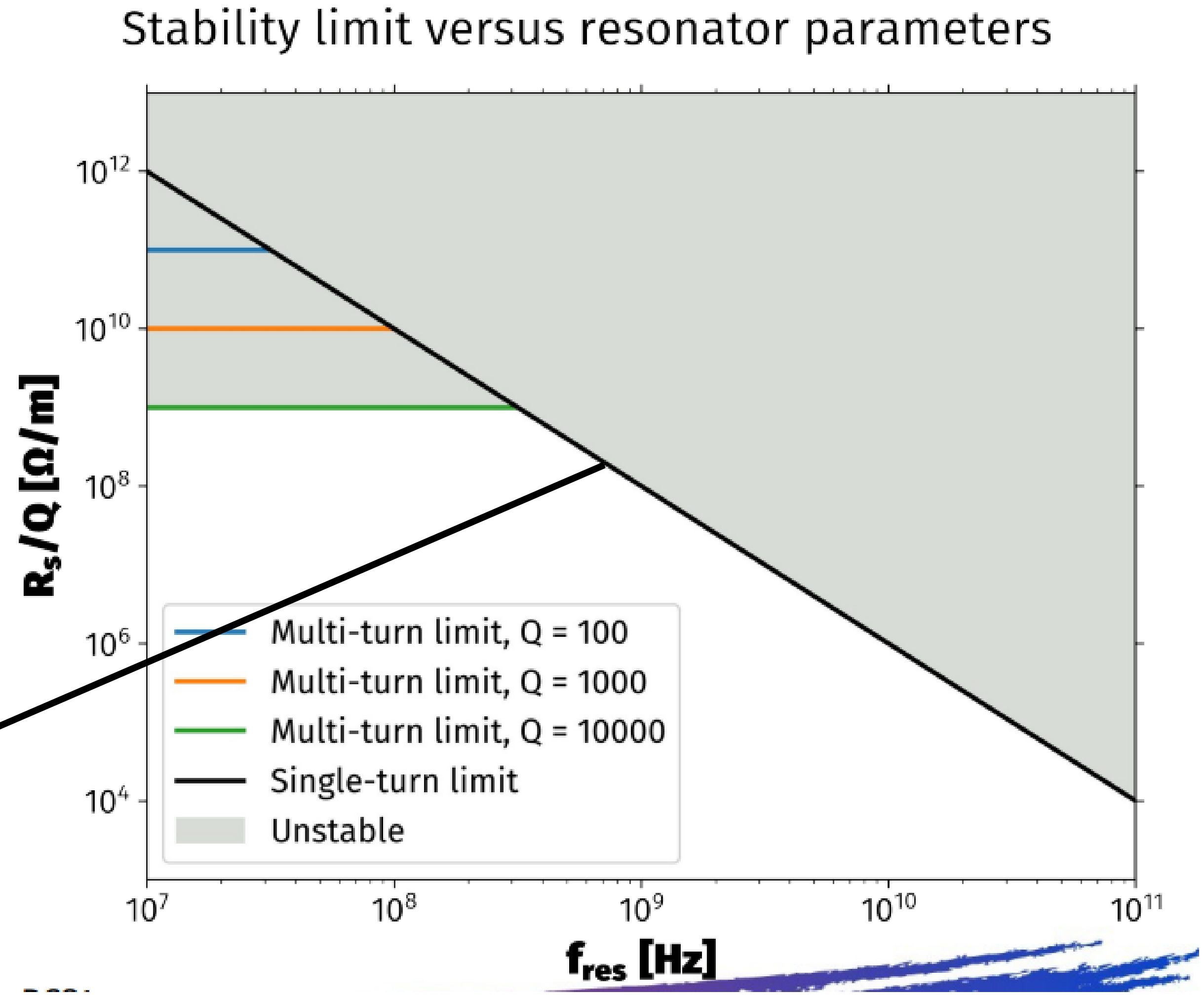
Transverse stability in RCS and TESLA cavity
Preliminary considerations

Alexej Grudiev

4/10/2022

Single-turn stability limit in MC RCS from David Amorim. 2022-09-06_HEMAC_RCS1_impedance_resonator_scan_v2.pdf (cern.ch)

- Some RCS parameters:
  - f_RF = 1.3GHz
  - C = 5990m
  - Einj = 63.1GeV/c
  - dE = 14.2 GeV/c per turn
  - N_turns = 20
  - V_RF = 20.1 GV
  - Sigma_z = 25mm
- Single turn stability limit: **Rs/Q*f^2 = 100 [MΩ/m*GHz^2]**



ILC cavity parameters, reminder from:R/Q in linacΩ

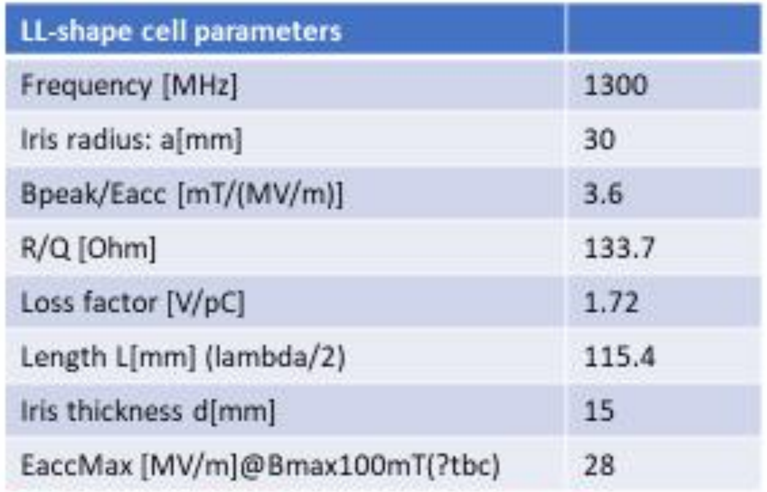
[SRF cavity parameter model for HEC RF system design (cern.ch)](https://cern.ch)

An example: 1.3 GHz SRF cavity for the ILC

Proceedings of 2005 Particle Accelerator Conference, Knoxville, Tennessee

DESIGN OF A LOW LOSS SRF CAVITY FOR THE ILC

J. Sekutowicz, DESY, 22607 Hamburg, Germany
K. Ko, L. Ge, L. Lee, Z. Li, C. Ng, G. Schussman, L. Xiao, SLAC, Menlo Park, 94025 California, USA
I. Gonin, T. Khabibouline, N. Solyak, FNAL, Batavia, 60510-0500 Illinois, USA
Y. Morozumi, K. Saito, KEK, Tsukuba, 305-0801, Japan
P. Kneisel, TJNAF (JLAB), Newport News, 23606 Virginia, USA



LL-shape cell parameters	
Frequency [MHz]	1300
Iris radius: a[mm]	30
Bpeak/Eacc [mT/(MV/m)]	3.6
R/Q [Ohm]	133.7
Loss factor [V/pC]	1.72
Length L[mm] (lambda/2)	115.4
Iris thickness d[mm]	15
EaccMax [MV/m]@Bmax100mT(?tbc)	28

Do we need short range wake?

Do we need anything else?

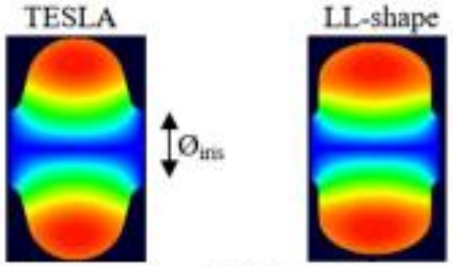
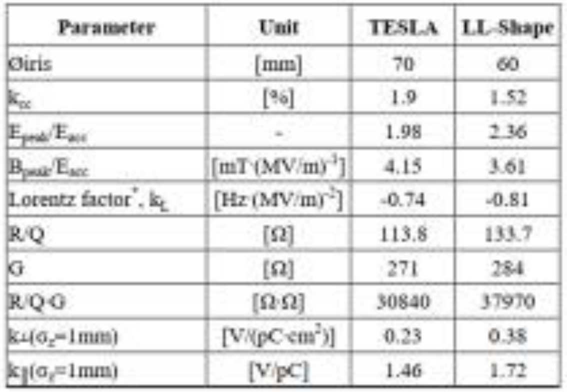
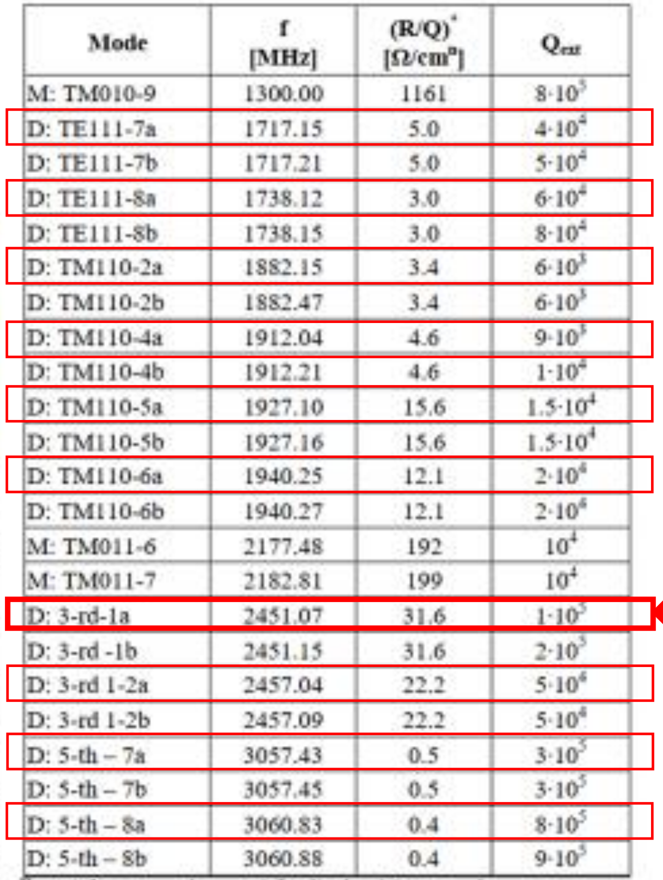


Table 1. RF parameters of both inner cells.



Parameter	Unit	TESLA	LL-Shape
Øiris	[mm]	70	60
k_{cc}	[%]	1.9	1.52
E_{peak}/E_{acc}	-	1.98	2.36
B_{peak}/E_{acc}	$[mT\cdot(MV/m)^{-1}]$	4.15	3.61
Lorentz factor*, k_L	$[Hz\cdot(MV/m)^{-2}]$	-0.74	-0.81
R/Q	[Ω]	113.8	133.7
G	[Ω]	271	284
R/Q·G	[Ω·Ω]	30840	37970
$k_\perp(\sigma_z=1mm)$	$[V/(pC\cdot cm^2)]$	0.23	0.38
$k_\parallel(\sigma_z=1mm)$	[V/pC]	1.46	1.72

Table 3: FM and HOM data.



Mode	f [MHz]	$(R/Q)^*$ $[\Omega/cm^n]$	Q_{ext}
M: TM010-9	1300.00	1161	$8\cdot10^5$
D: TE111-7a	1717.15	5.0	$4\cdot10^4$
D: TE111-7b	1717.21	5.0	$5\cdot10^4$
D: TE111-8a	1738.12	3.0	$6\cdot10^4$
D: TE111-8b	1738.15	3.0	$8\cdot10^4$
D: TM110-2a	1882.15	3.4	$6\cdot10^3$
D: TM110-2b	1882.47	3.4	$6\cdot10^3$
D: TM110-4a	1912.04	4.6	$9\cdot10^3$
D: TM110-4b	1912.21	4.6	$1\cdot10^4$
D: TM110-5a	1927.10	15.6	$1.5\cdot10^4$
D: TM110-5b	1927.16	15.6	$1.5\cdot10^4$
D: TM110-6a	1940.25	12.1	$2\cdot10^4$
D: TM110-6b	1940.27	12.1	$2\cdot10^4$
M: TM011-6	2177.48	192	10^4
M: TM011-7	2182.81	199	10^4
D: 3-rd-1a	2451.07	31.6	$1\cdot10^5$
D: 3-rd -1b	2451.15	31.6	$2\cdot10^5$
D: 3-rd 1-2a	2457.04	22.2	$5\cdot10^4$
D: 3-rd 1-2b	2457.09	22.2	$5\cdot10^4$
D: 5-th – 7a	3057.43	0.5	$3\cdot10^5$
D: 5-th – 7b	3057.45	0.5	$3\cdot10^5$
D: 5-th – 8a	3060.83	0.4	$8\cdot10^5$
D: 5-th – 8b	3060.88	0.4	$9\cdot10^5$

[*] n = 0 for monopoles, n = 2 for dipoles; M-monopole, D-dipole; a, b indicate polarizations.

Max R/Q

One HOM from LL ILC cavity and S-turn stability

- Some RF cavity parameters:
    - Active Acc. Gradient: 30 MV/m
    - Cavity length: L_cav~1m
    - V_cav: 30MV
    - N_cav = 20100/30 = **670**
- Max R/Q HOM: R/Q = 32 linacΩ/cm^2 => Rs/Q = R/Q/2*c/ω = **3.1 kΩ/m** per cavity
- For 670 cavities: Rs/Q = **2.1 MΩ/m**
- f = **2.45 GHz,**
- Ncav*Rs/Q*f^2 = **12.6 [MΩ/m*GHz^2] < 100,** Single turn stability limit from David
- **It is below stability limit by about factor 8 for one HOM.**
- In fact, all HOMs must be taken into account for S-turn stability calculation

Stability for several HOMs

- Wake of single mode expressed as

$$W(t) = \frac{2\pi f_{res} R_s}{Q\sqrt{1-\frac{1}{4Q^2}}} \exp\left(-\frac{2\pi f_{res}}{2Q}t\right) \sin\left(2\pi f_{res}\sqrt{1-\frac{1}{4Q^2}}t\right)$$

- In SRF $Q \ll 1$, and for short bunch t_max << 1/fres

$$W(t) = \omega_{res}\frac{R_s}{Q}\omega_{res}t$$

- Stability limit: Rs/Q*fres^2 = 100 =>

$$\frac{dW(t)}{dt} = \omega_{res}\frac{R_s}{Q}\omega_{res} = 4\pi^2 100\ [\frac{M\Omega}{m}GHz^2]$$

- In fact, Stability limit == $\boldsymbol{\frac{dW(t)}{dt} = const}$
- In the case of N modes:

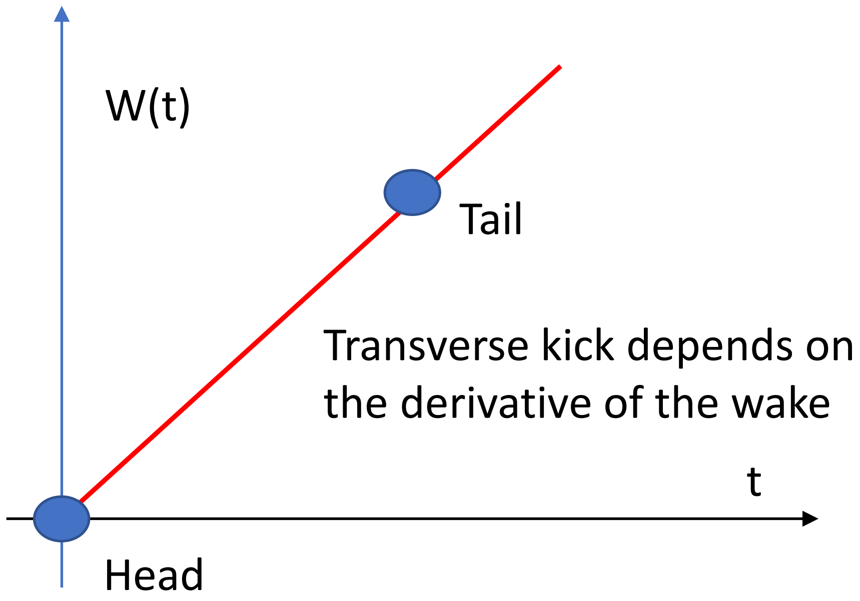
$$W(t) = \sum_i W_i\,(\mathrm{t}) \qquad \frac{dW(t)}{dt} = \sum_i \frac{dW_i(t)}{dt}$$

- Stability limit for total S-turn wake: $\sum_i \frac{dW_i(t)}{dt} = \sum_i \left[\frac{R_s}{Q}\omega_{res}^2\right]_i < 4\pi^2 100\ [\frac{M\Omega}{m}GHz^2]$

- Taking into account the bunch spectrum limit the number of relevant HOMs in the sum

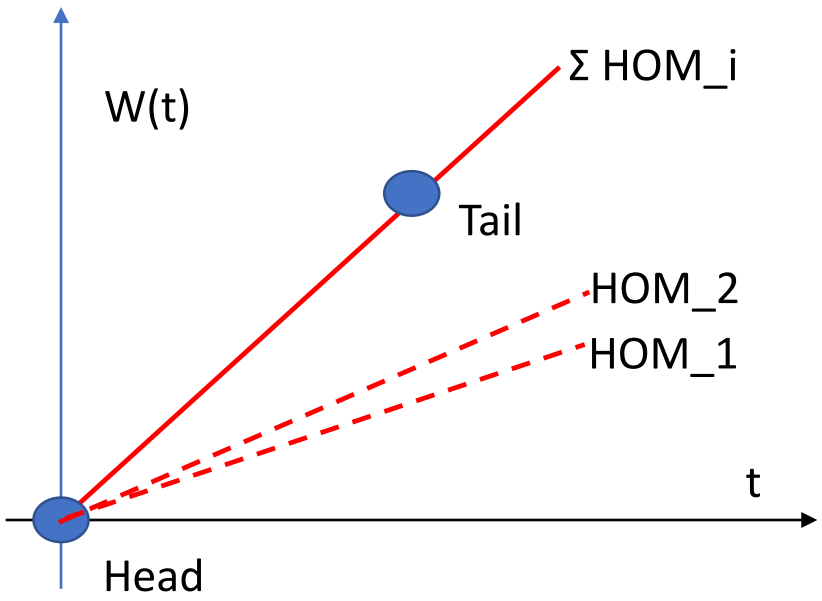
Physical interpretation of stability limit

$$W(t) = \omega_{res}\frac{R_s}{Q}\omega_{res}t$$



Transverse kick depends on the derivative of the wake

$$W(t) = \sum_i W_i\,(\mathrm{t})$$



Stability for several HOMs up to 3 GHz

mode	f[GHz]	R/Q[linacOhm/cm^2]	Rs/Q[kohm/m]	Ncav*Rs/Q[Mohm/m]	Rs/Q*f^2[MOhm/m*GHz^2]
D:TE111-7	1.717	5	0.695202139	0.465785433	1.373176912
D:TE111-8	1.738	3	0.412081268	0.27609445	0.833983043
D:TE110-2	1.882	3.4	0.431291291	0.288965165	1.023492653
D:TE110-4	1.912	4.6	0.574356228	0.384818673	1.406798553
D:TE110-5	1.927	15.6	1.932654732	1.29487867	4.808310518
D:TE110-6	1.94	12.1	1.489001143	0.997630766	3.75468315
D:3rd-1	**2.451**	**31.6**	**3.077904652**	**2.062196117**	**12.38843902**
D:3rd-2	2.457	22.2	2.157045016	1.445220161	8.724575883
D:5th-7	3.057	0.5	0.039046846	0.026161387	0.244484672
D:5th-8	3.06	0.4	0.031206852	0.020908591	0.195779679
					34.75372408

This is closer to the stability limits calculated by David: Rs/Q*f^2 = 100

BUT it is still factor 3 lower. More detailed studies needed

Closed expression of total Transverse wake.

- Karl Bane (2003, SLAC-PUB-9663)
- Wake per meter length

$$W_x(s) = \frac{4Z_0 c s_0}{\pi a^4}\phi(s)\left[1-\left(1+\sqrt{\frac{s}{s_0}}\right)\exp\left(-\sqrt{\frac{s}{s_0}}\right)\right] , \qquad (15)$$

$$s_0 = 0.169\frac{a^{1.79}g^{0.38}}{L^{1.17}} . \qquad (16)$$

s0=1.6mm !

- Derivative of the wake

$$\frac{dW_x(s)}{ds} = \frac{4Z_0 c}{\pi a^4} e^{-\sqrt{s/s_0}}$$

- per meter length :

$$\frac{dW_x(0)}{ds} = \frac{4Z_0 c}{\pi a^4}$$

$$\boldsymbol{\frac{dW_x(0)}{dt} = \frac{4Z_0 c^2}{\pi a^4}}$$

- Total wake of 670 cavities:

$$\frac{dW(0)}{dt}L = \frac{4Z_0 c^2 L}{\pi a^4}$$

$$L = L_{cav}\, N_{cav}$$

$$L = 670m$$
$$a = 30mm$$

$$\frac{4Z_0 c^2 L}{\pi a^4} = \boldsymbol{36000 \frac{M\Omega}{m} GHz^2}$$

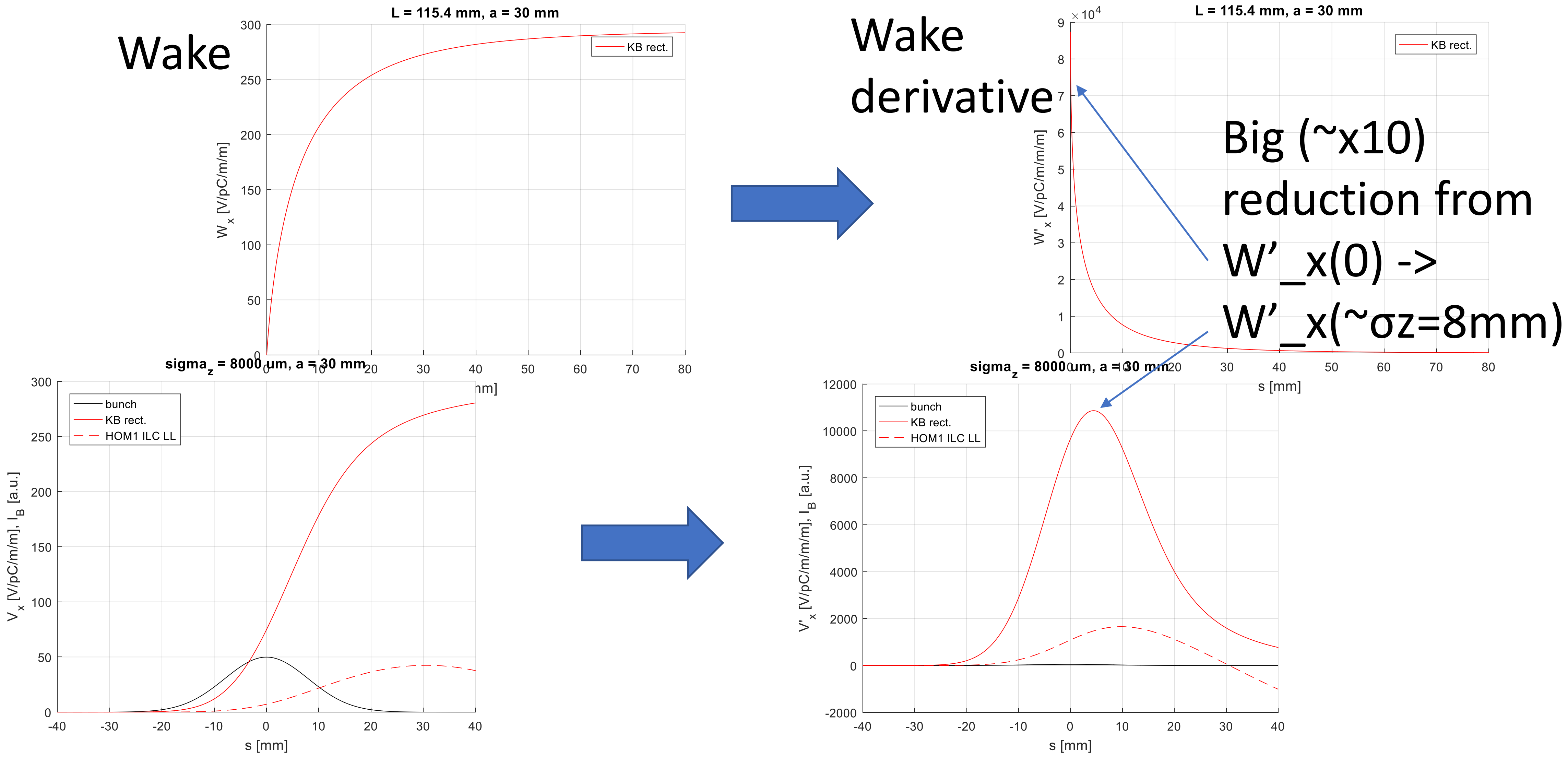
- Stability limit:

$$4\pi^2 100\left[\frac{M\Omega}{m}GHz^2\right] = \boldsymbol{3950\left[\frac{M\Omega}{m}GHz^2\right]}$$

The wake derivative is about **factor 9** larger than stability limit. Hm!?

- Big discrepancy between KB model and HOM sum. Probably due to KB model applicability for s << s0=1.6mm, where derivative is very high and for s > s0 it goes down -> less kick on the witness particle
- **Nevertheless KB model can be used for beam dynamic simulations where Wx will be convoluted with realistic bunch distribution**

KB wake and discrepancy with HOM estimate



Wake

Wake derivative

Big (~x10) reduction from W’_x(0) -> W’_x(~σz=8mm)