Rapid-Turn Multi-Field Inflation

Sonia Paban, TACOS—10/10/2022

Robbie Rosati

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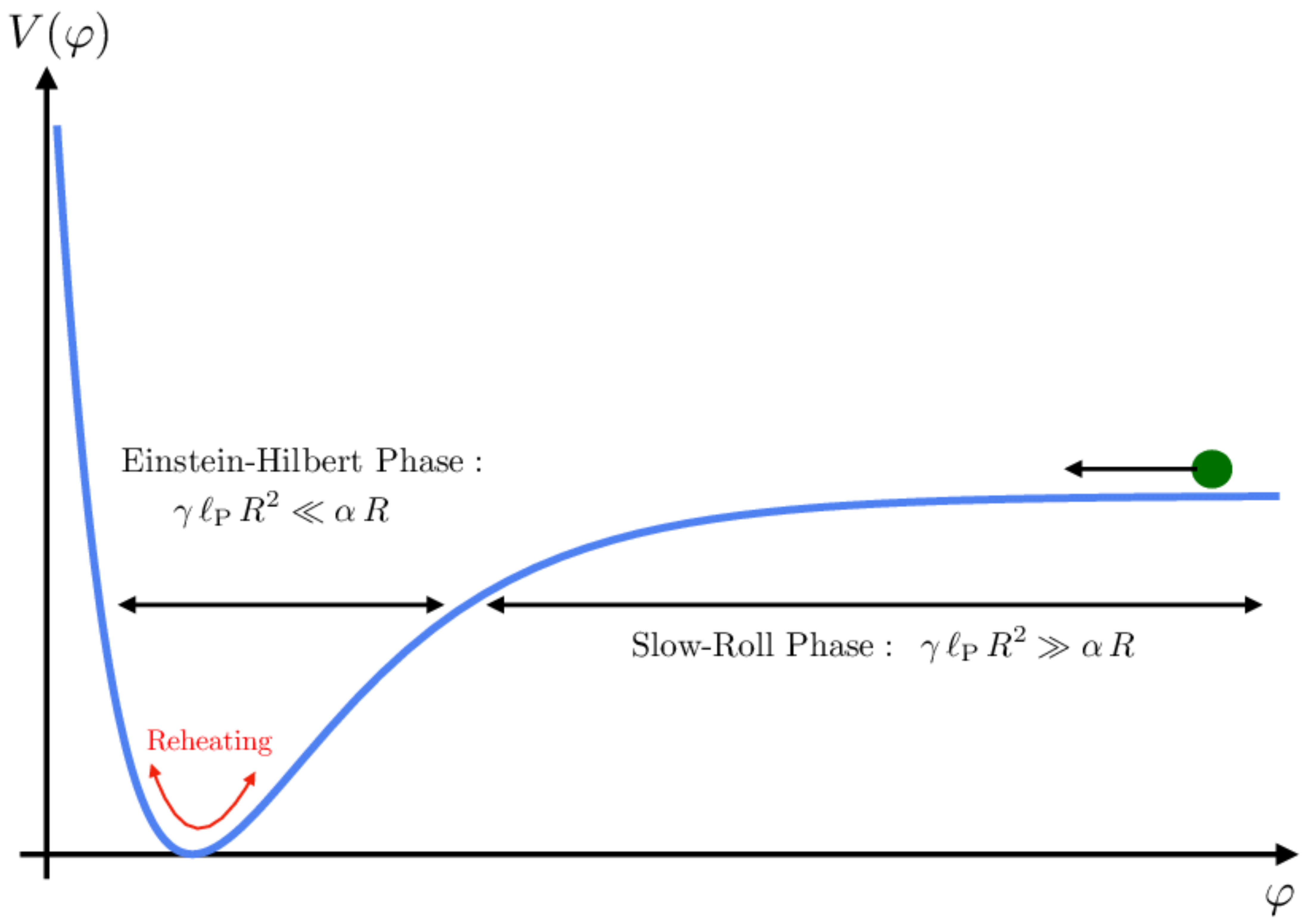
WHY: Multi-Field Inflation



$$H \sim \text{constant}$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1$$

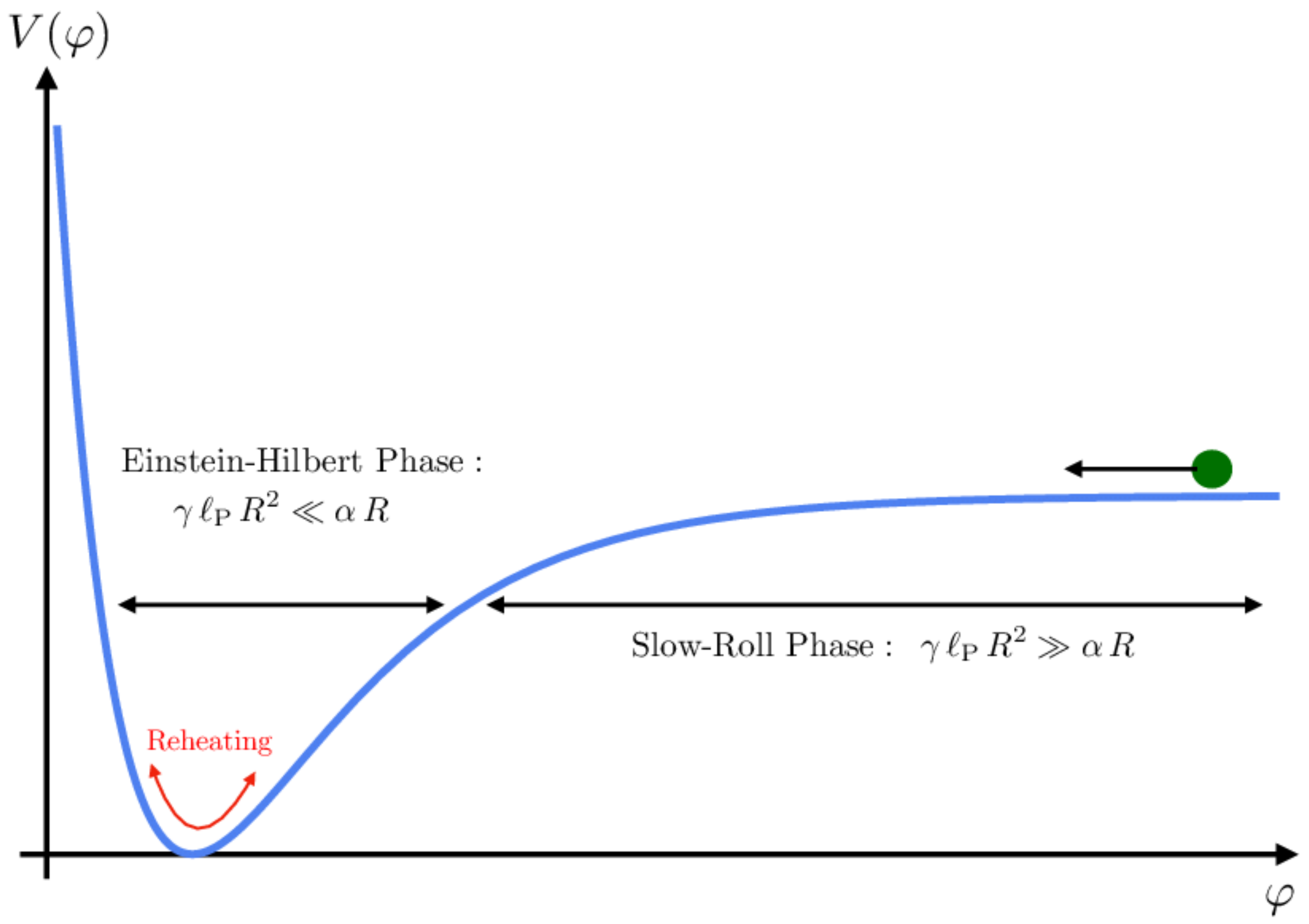
Inflation

$$\epsilon_V \equiv \frac{{M_P}^2}{2}\frac{V_{,\phi}^2}{V^2}$$

$$\epsilon_V \sim \epsilon$$

Single-Field Inflation

Single-Field Inflation



- Fits all known data : $n_s, f_{\mathrm{NL}}, \not{P}_{\mathrm{iso}}$

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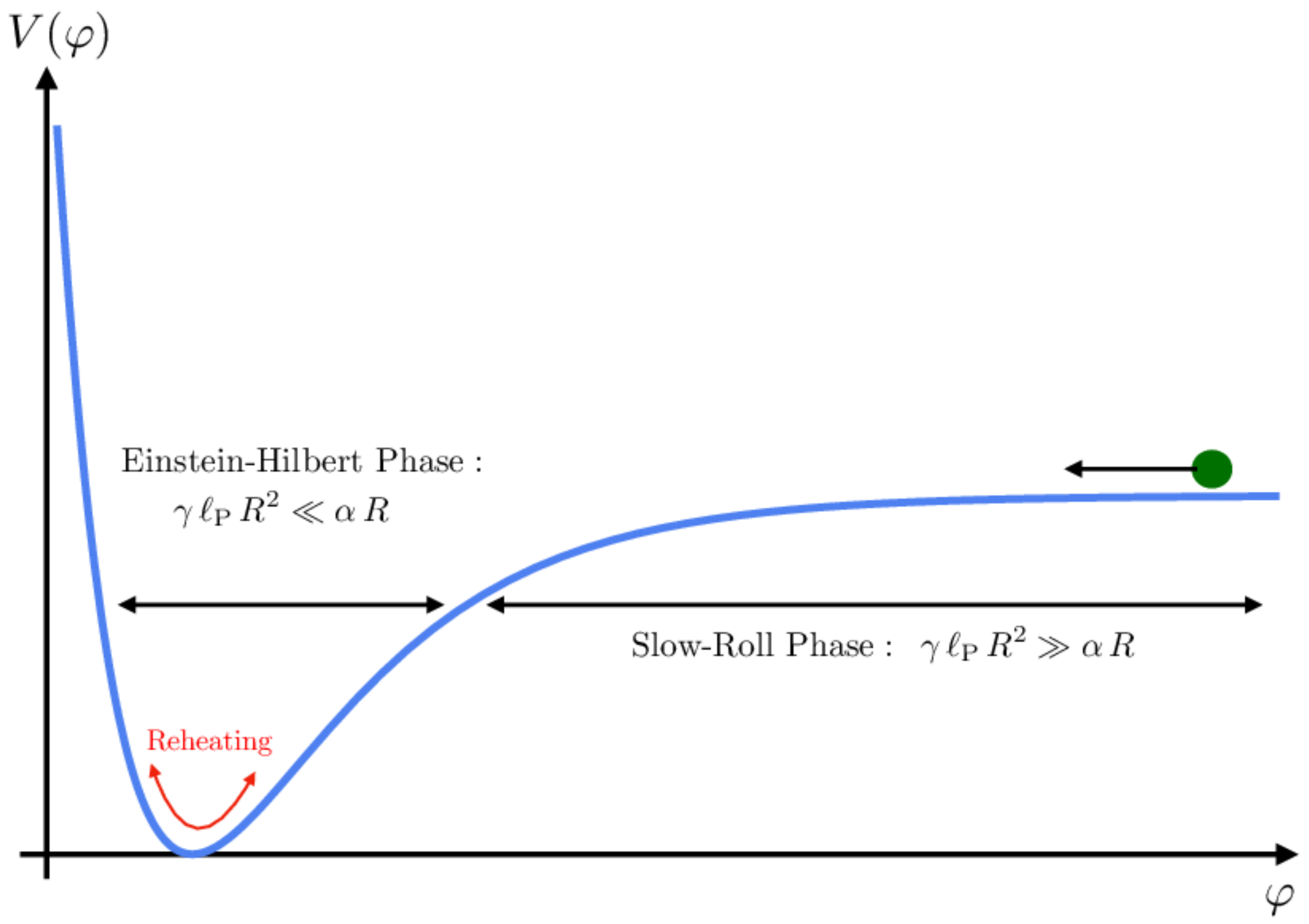
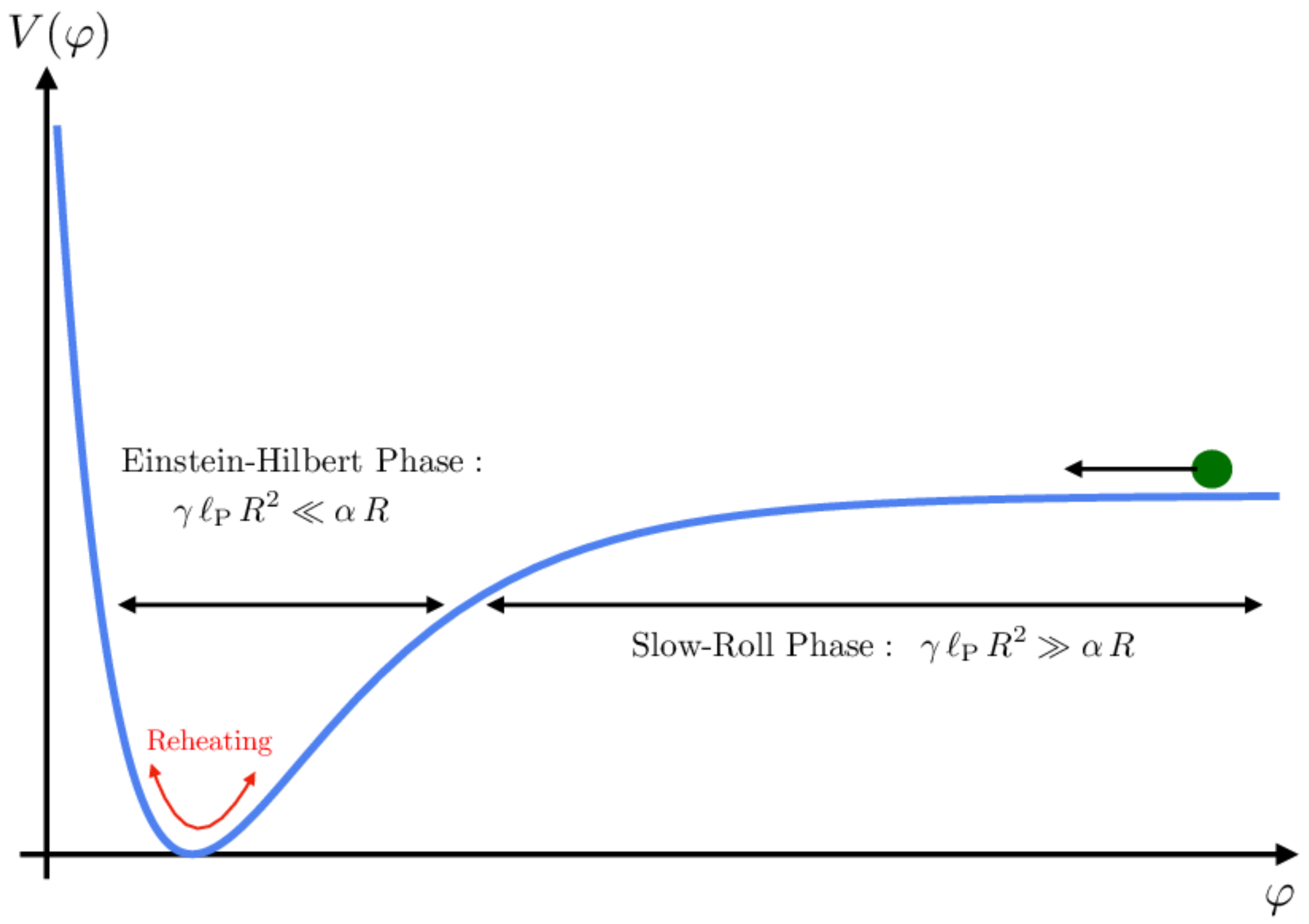


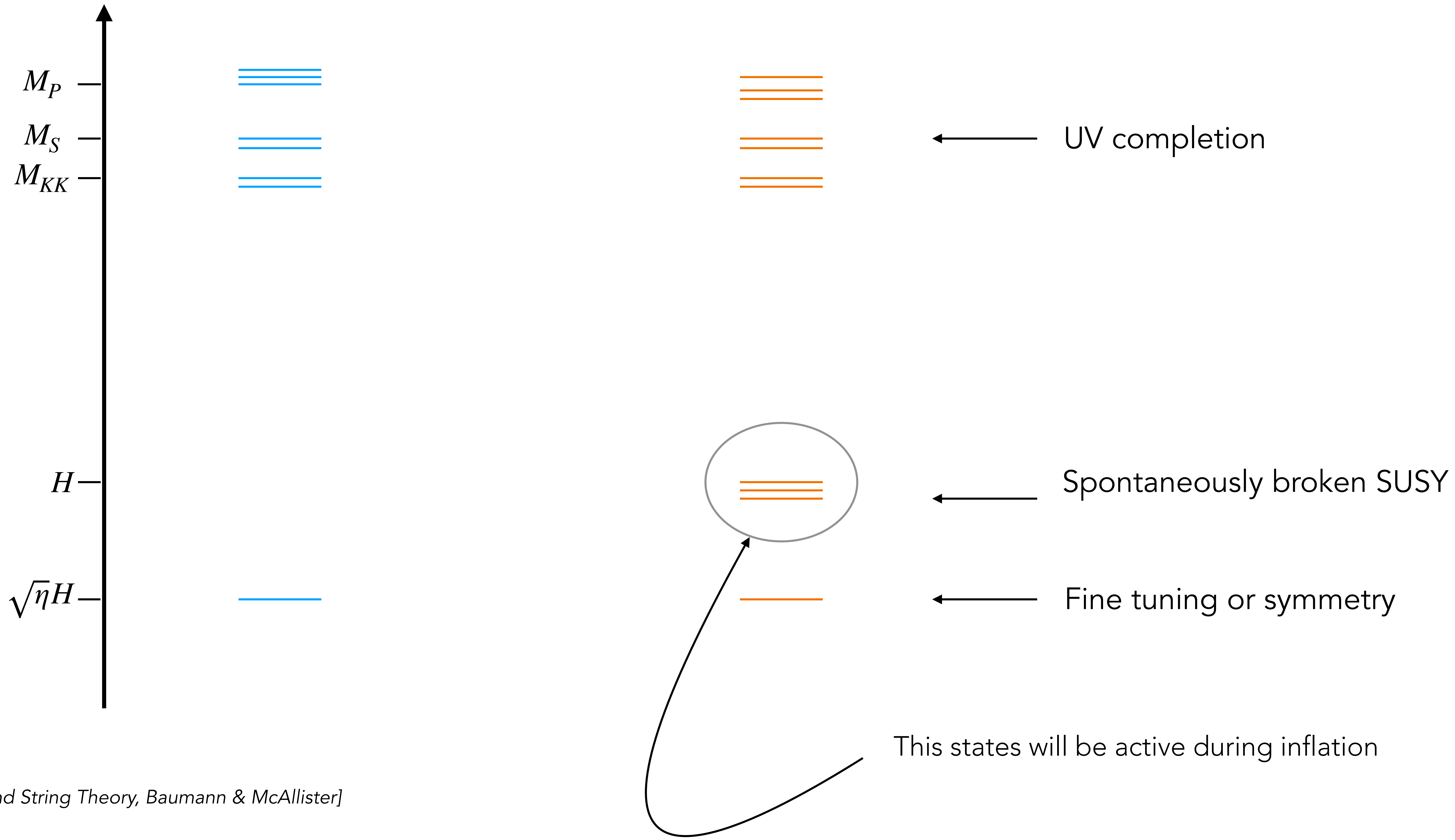
- Fits all known data : $n_s, f_{NL}, \not{P}_{iso}$

- There are many field theories that support inflationary phases. They can be understood as EFT valid at the inflationary energies.

Single-Field Inflation



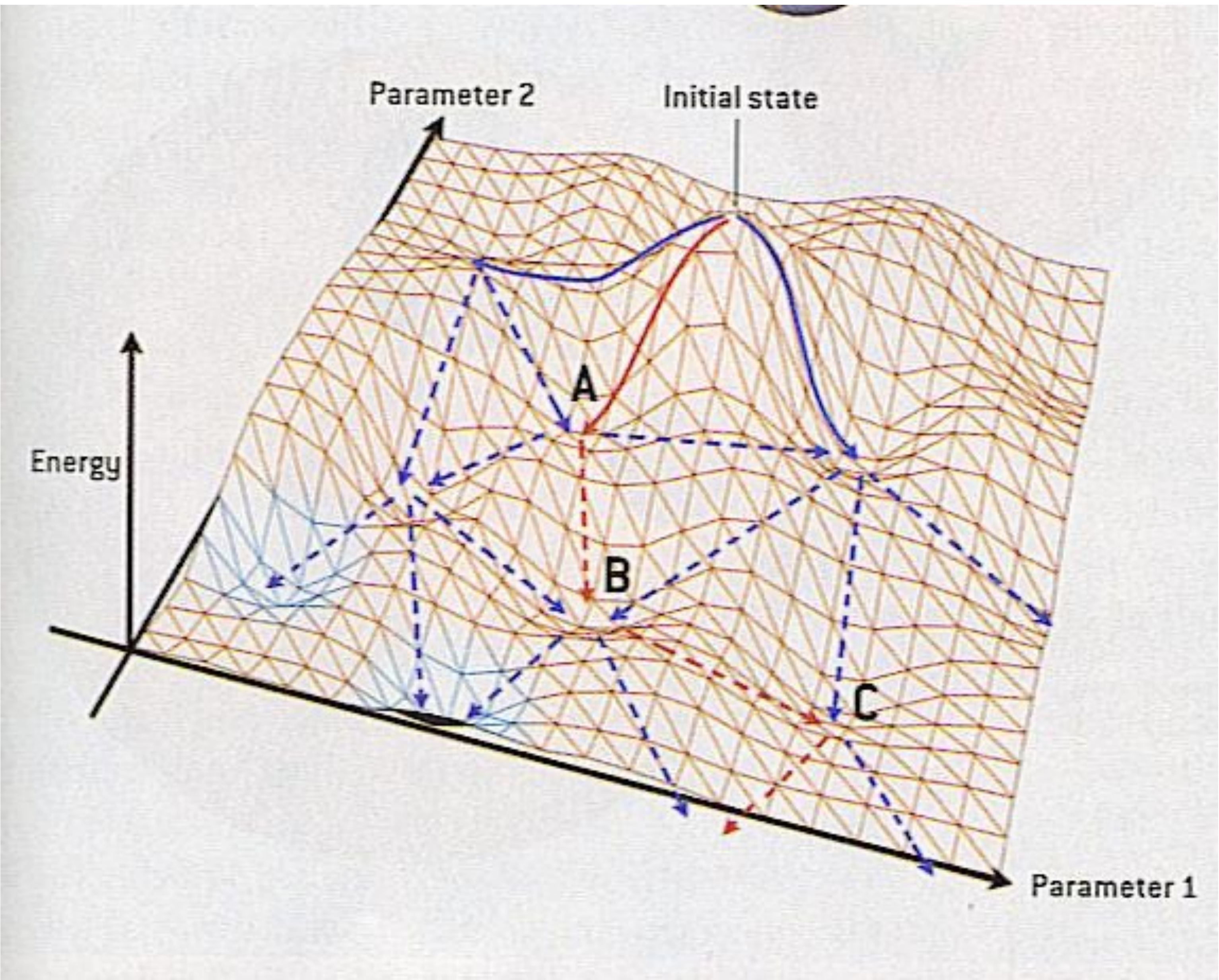
- Fits all known data : $n_s, f_{\rm NL}, \not{P}_{\rm iso}$
- There are many field theories that support inflationary phases. They can be understood as EFT valid at the inflationary energies.
- Fields with masses $m \lesssim H$ are classically and quantum mechanically active during inflation.

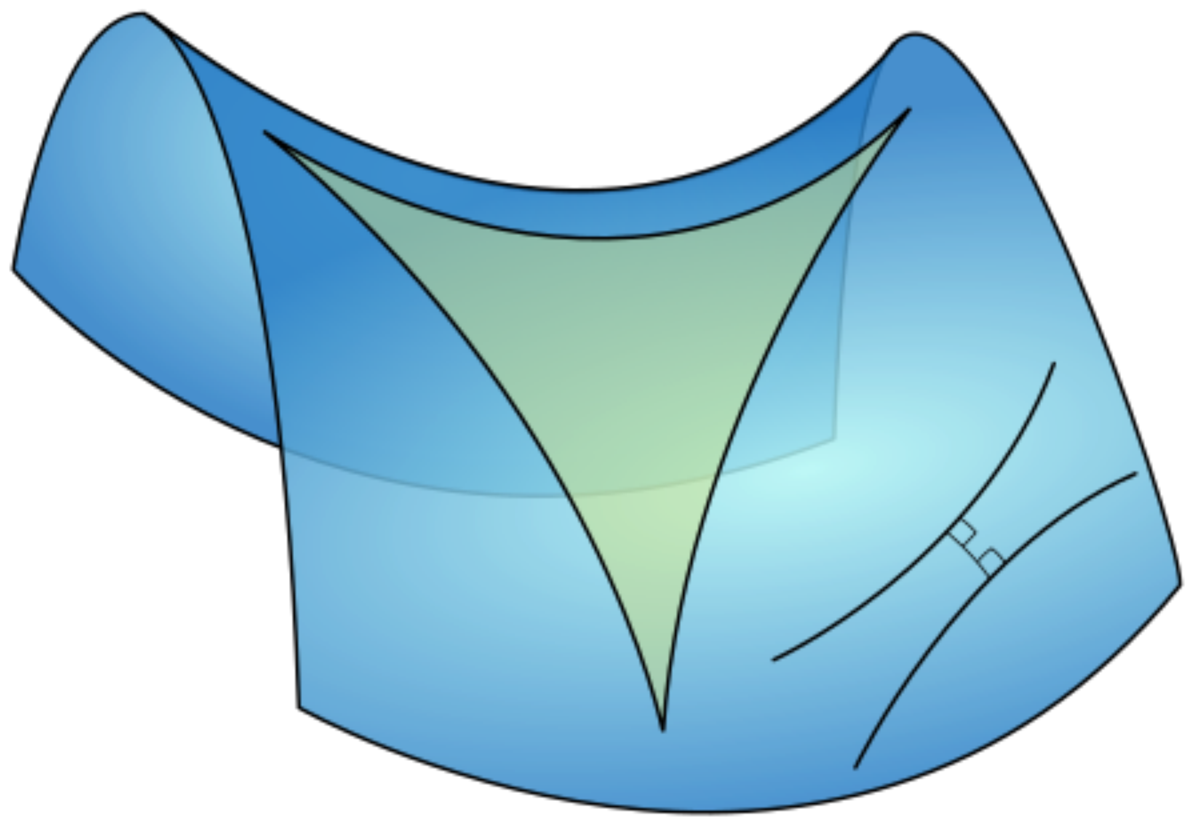


[Inflation and String Theory, Baumann & McAllister]

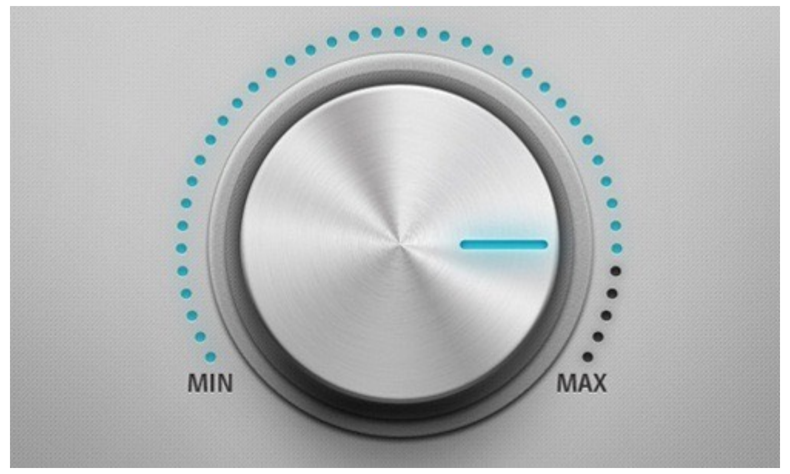
Motion in Multi-Field Space

Wands, astro-ph/0702187; Inflation and String Theory, Baumann & McAllister; Meyers, Tarrant, ArXiv:1311.3972;…





Potential: $V(\phi^I)$



Field Space Metric: $G_{IJ}(\phi^k)$

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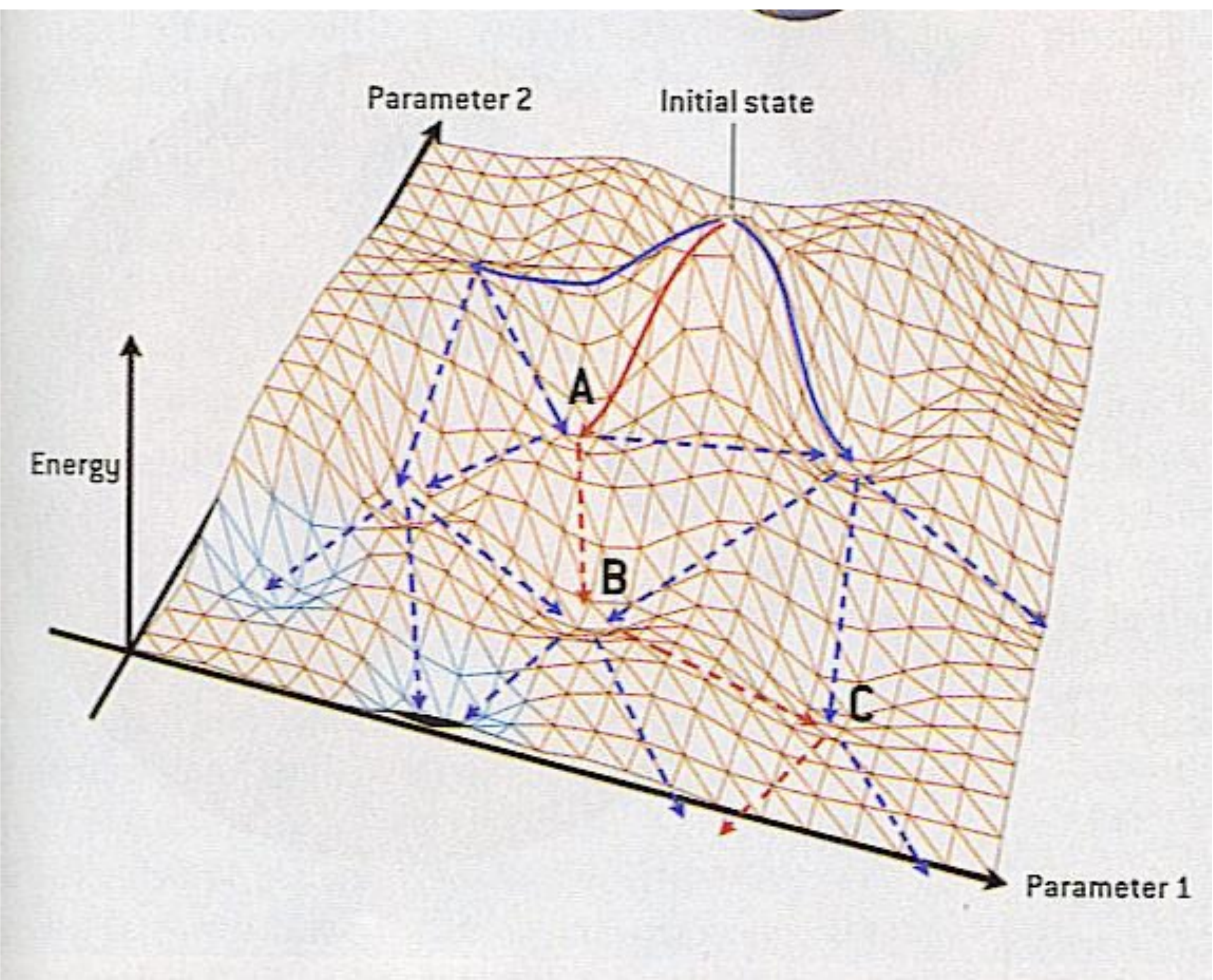
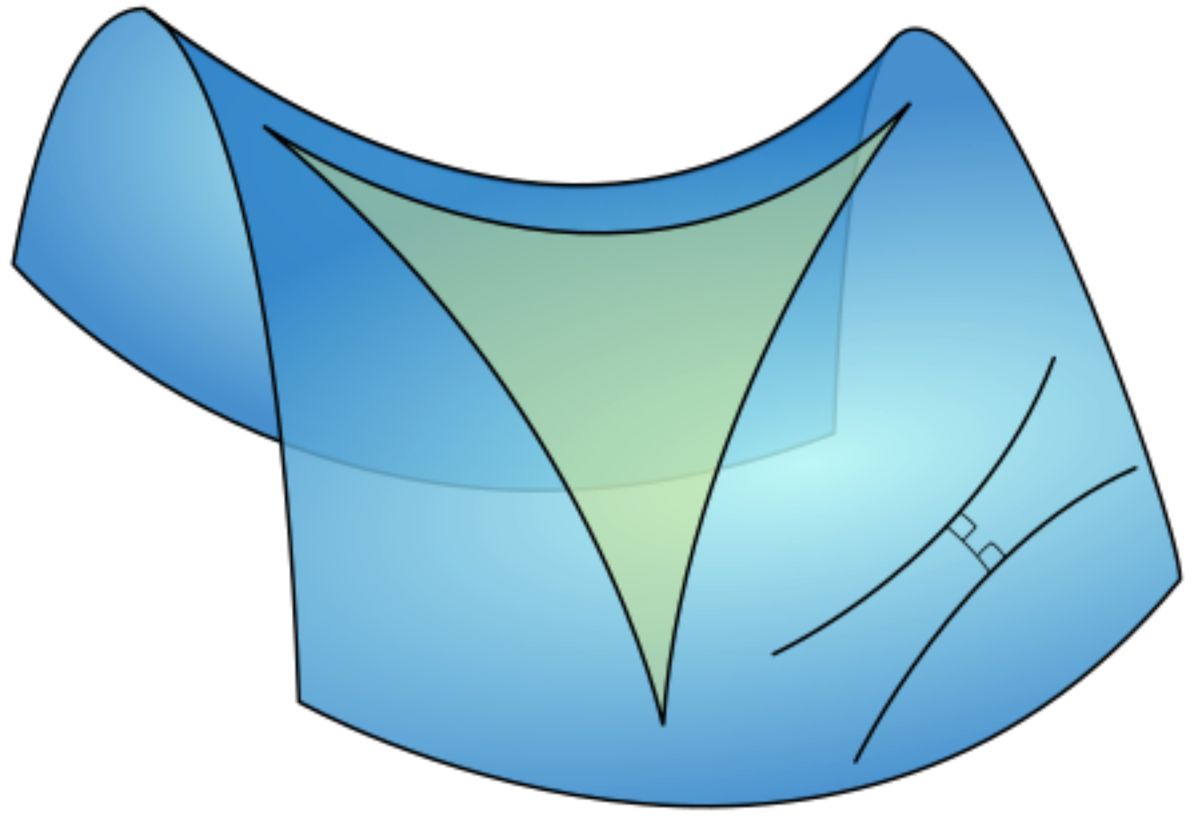


$$\ddot{\phi}^I + 3H\dot{\phi}^I + \Gamma^I_{JK}\dot{\phi}^I\dot{\phi}^J + \frac{\partial V}{\partial \phi_I} = 0$$



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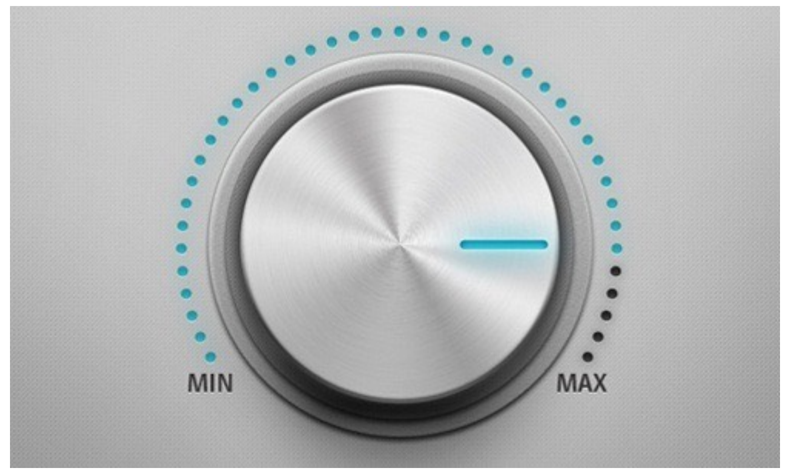


Field Space Metric: $G_{IJ}(\phi^k)$

- Multi-field effects quite generally shift the spectrum toward the red.

- For any single-field model of inflation, the signal in the squezed limit must satisfy:

$$\lim_{k_3 \to 0} \langle R_{\mathbf{k}_1} R_{\mathbf{k}_2} R_{\mathbf{k}_3} \rangle \propto f_{\mathrm{NL}}^{\mathrm{local}} \propto (n_s - 1)$$

Planck: $f_{\mathrm{NL}}^{\mathrm{local}} = -0.9 \pm 5.1$

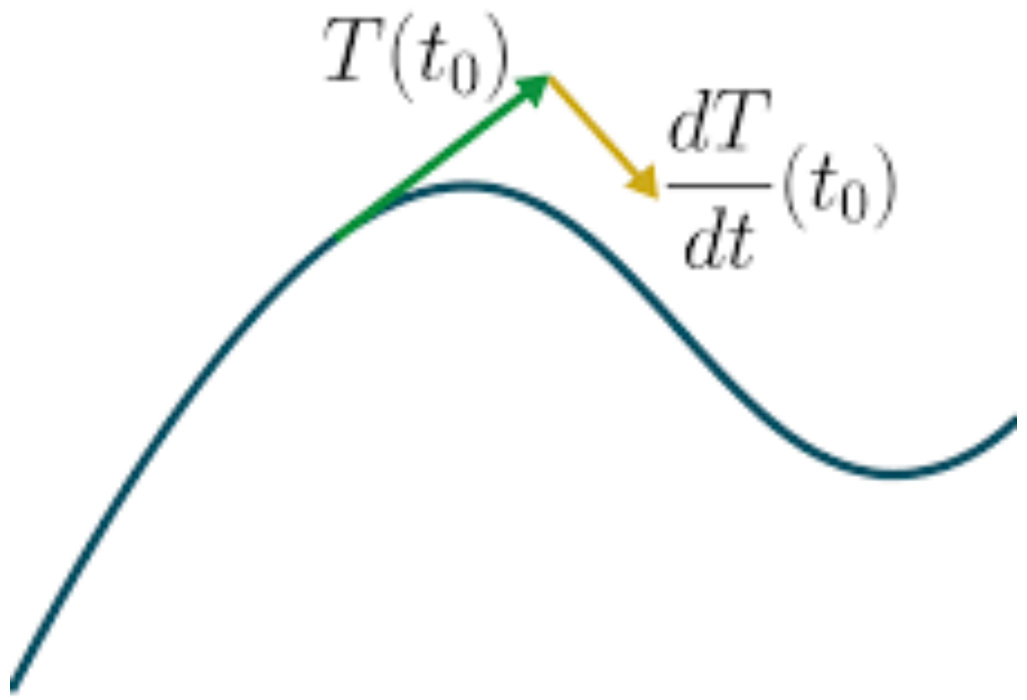
- Isocurvature perturbations have not been seen in the CMB temperature and polarization data so far.

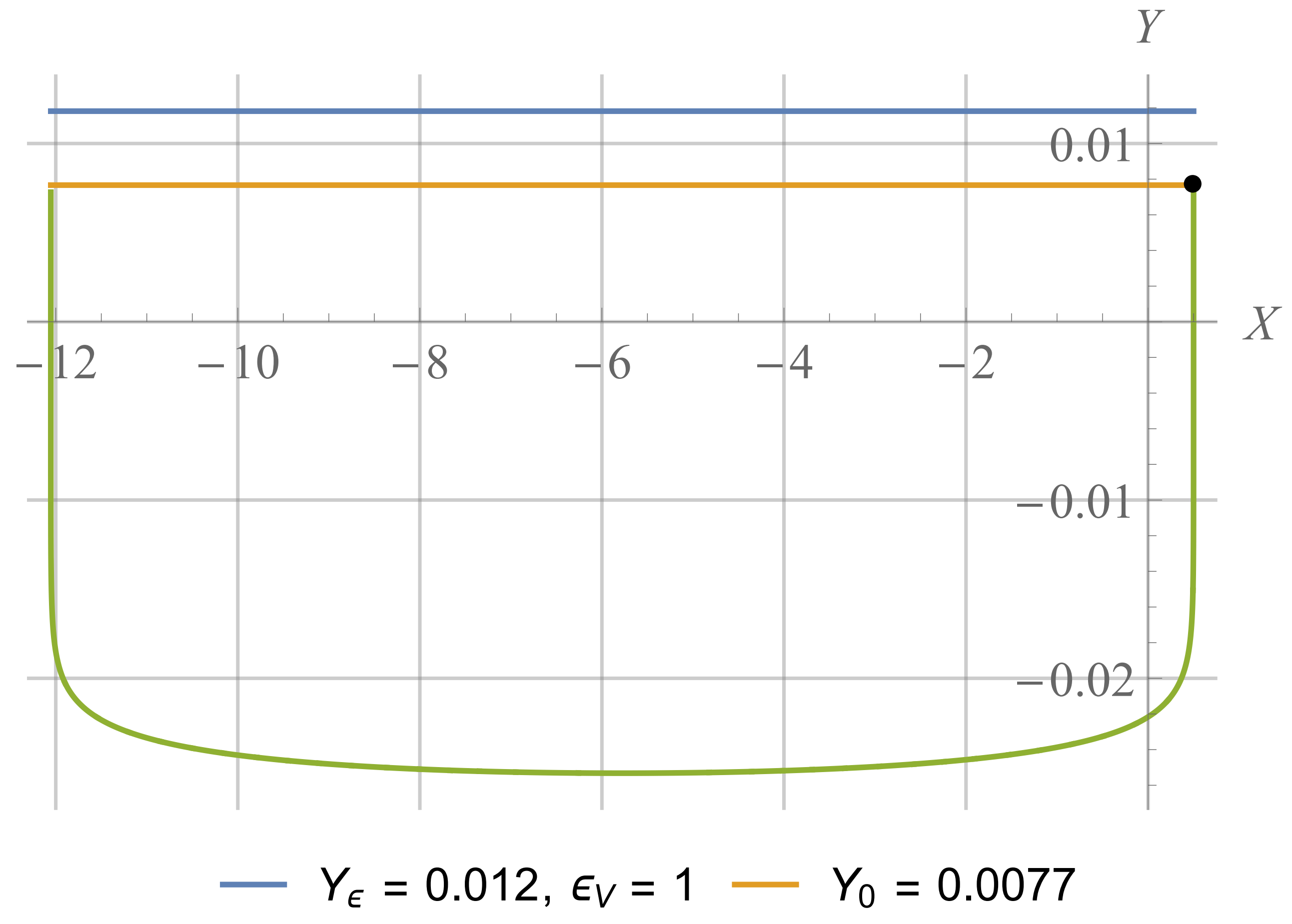
- Adiabatic perturbations are frozen on superhorizon scales, regardless of the uncertain physics of reheating. In contrast, the amplitude of primordial isocurvature perturbations is strongly model-dependent and sensitive to post-inflationary evolution.

WHY NOW: Rapid-Turn Multi-Field Inflation ?

$$\epsilon_{\rm V} = \epsilon \left\{ \left(1 + \frac{\eta}{2(3-\epsilon)}\right) + \frac{\omega^2}{9H^2}\frac{1}{(1-\epsilon/3)^2} \right\}$$

I-S Yang'12, Cespedes & Palma'13, Brown'17 , Achúcarro & Palma'18,





$$\epsilon \equiv -\frac{\dot H}{H^2}$$

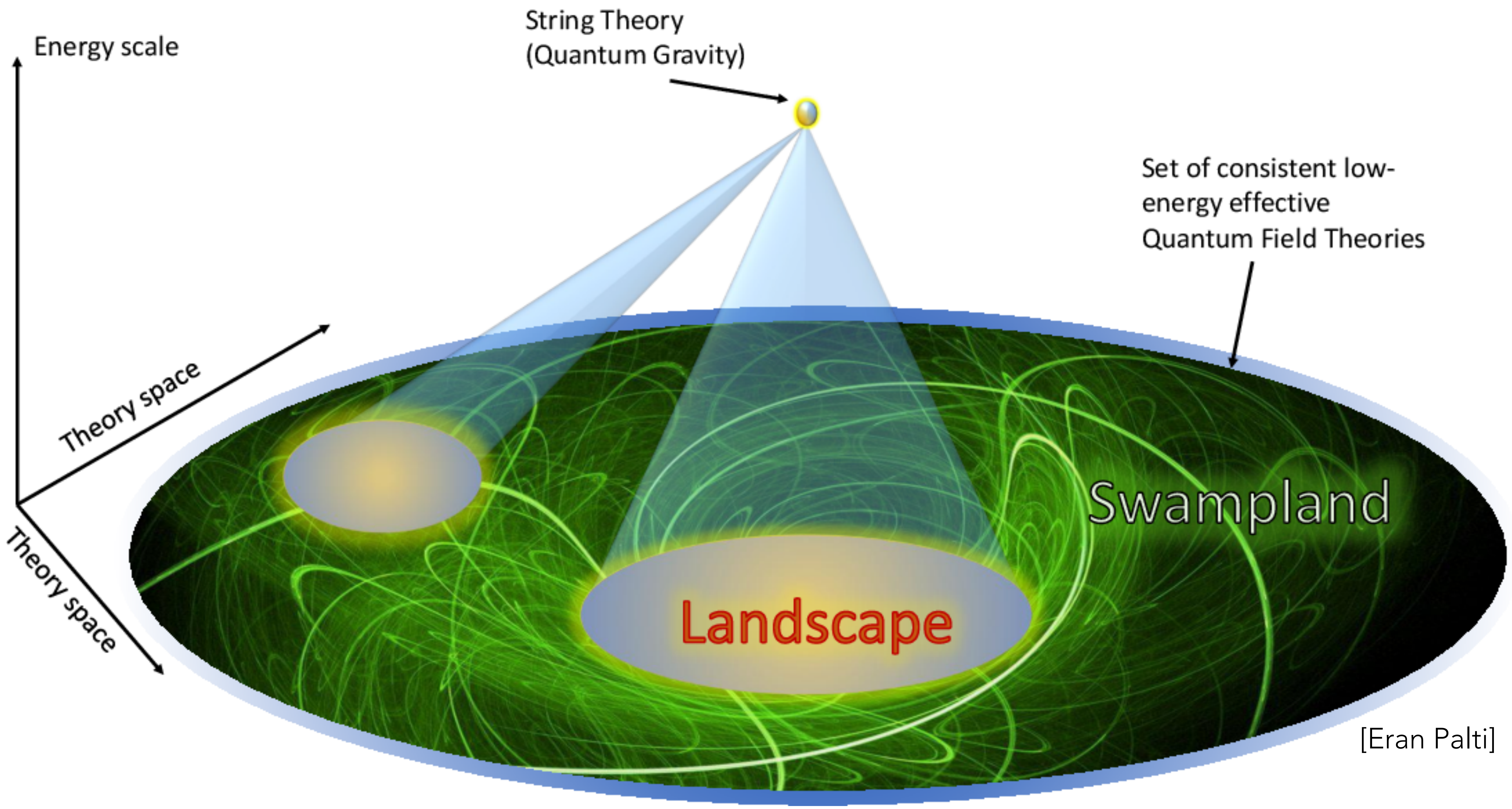
$$\eta \equiv \frac{\dot\epsilon}{H\epsilon}$$

$$\epsilon_V \equiv \frac{{M_P}^2}{2}\frac{|\nabla V|^2}{V^2}$$

$$\omega \equiv |D_t T^a|$$

WHY NOW: Rapid-Turn Multi-Field Inflation ?

String Theory is our motivation to look into multi-field models. What else does string theory imply?



[Ooguri, Vafa ’06; Obied, Ooguri, Spodyneiko, Vafa ’18; Ooguri, Palti, Shiu, Vafa ’18]

The Swampland Conjectures

[Ooguri, Vafa '06; Obied, Ooguri, Spodyneiko, Vafa '18; Ooguri, Palti, Shiu, Vafa '18; Androit, Roupec'18; Garg, Krishnan '18; Denef, Hebecker, Wrase '18]

- Asymptotic dS conjecture: Scalar field potentials arising from a consistent theory of quantum gravity satisfy that either

$$|\nabla V| > \frac{\mathcal{O}(1)}{M_{Pl}} V \qquad \text{or} \qquad \frac{\min(\mathrm{V}_{;\mathrm{IJ}})}{V} < -\frac{\mathcal{O}(1)}{M_{Pl}^2}$$

[Hertzberg, Tegmark, Kachru, Shelton, Ozcan '07; Flauger, SP, Robbins, Wrase '08]

- Distance conjecture: if a scalar field moves a distance $|\Delta\phi| \geq O(1)$ in Planck units, a tower of light states emerges that invalidates the EFT.

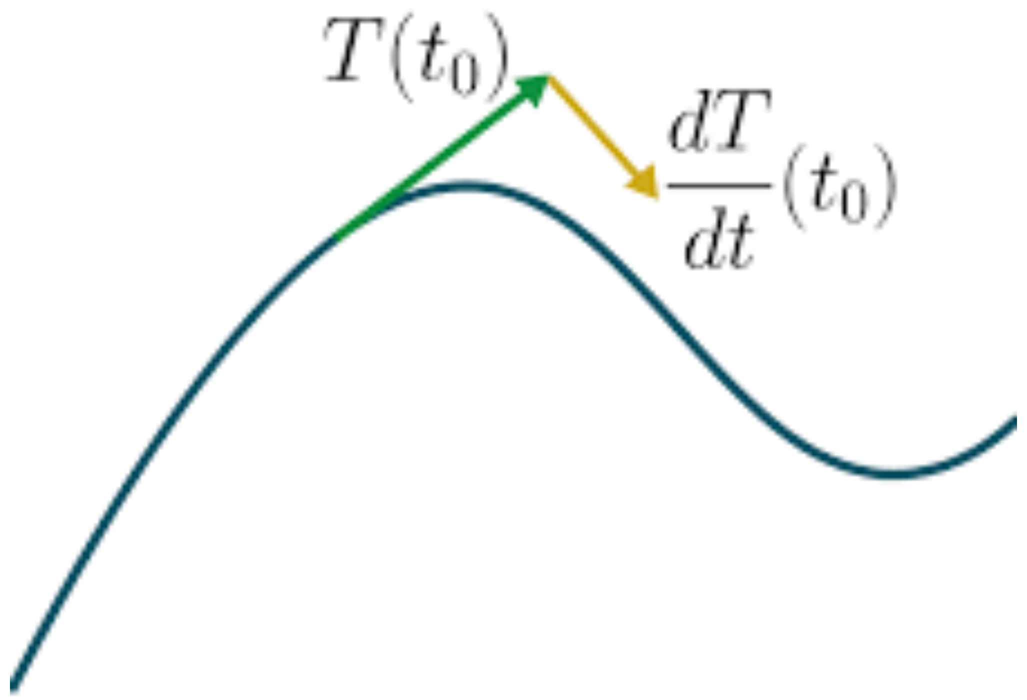
Disclaimer: We are agnostic about the Swampland conjectures. We want to understand their implications for inflation.

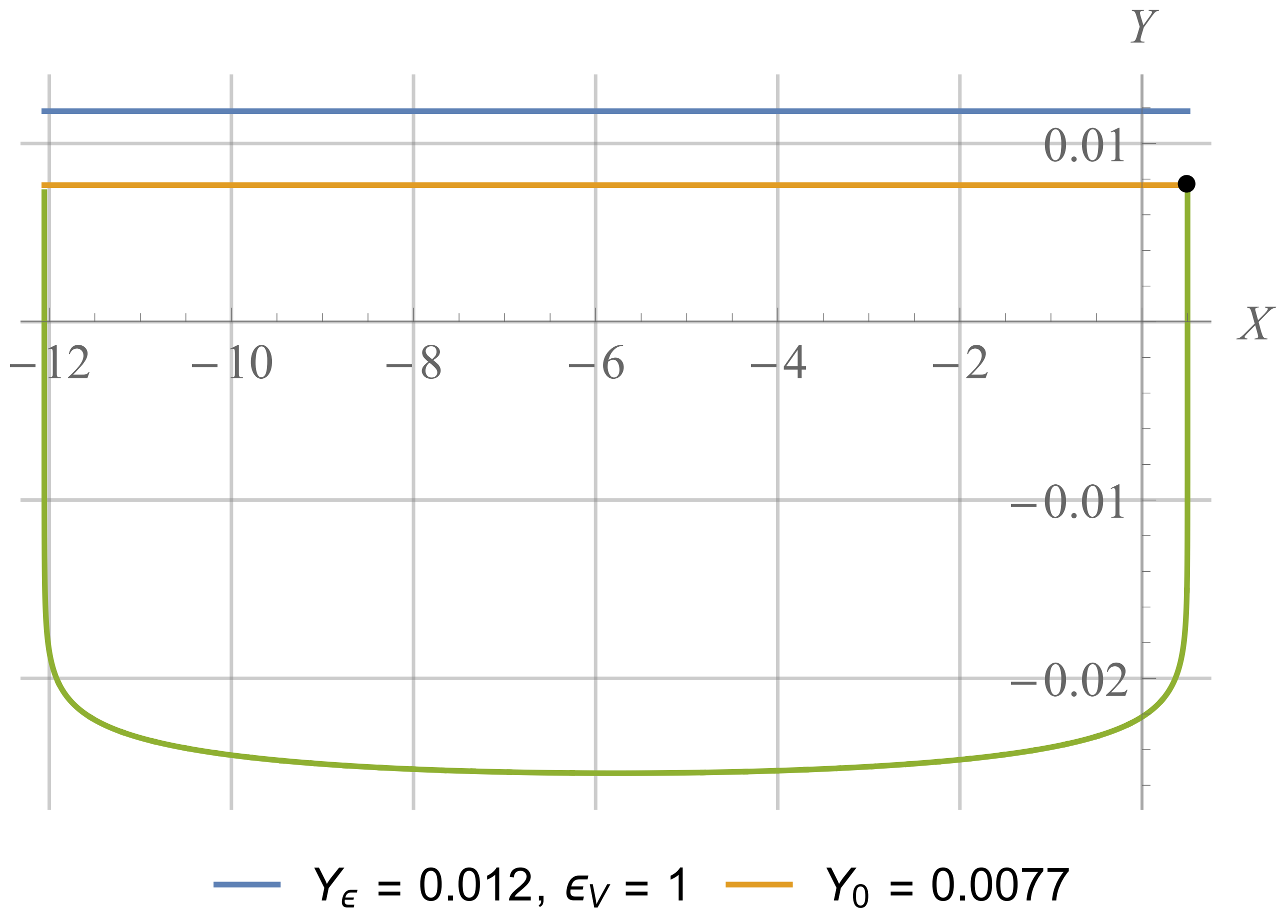
WHY NOW: Rapid-Turn Multi-Field Inflation ?

(Classical)

$$\epsilon_V = \epsilon \left\{ \left(1 + \frac{\eta}{2(3-\epsilon)}\right) + \frac{\omega^2}{9H^2} \frac{1}{(1-\epsilon/3)^2} \right\}$$

I-S Yang'12, Cespedes & Palma'13, Brown'17 , Achúcarro & Palma'18,





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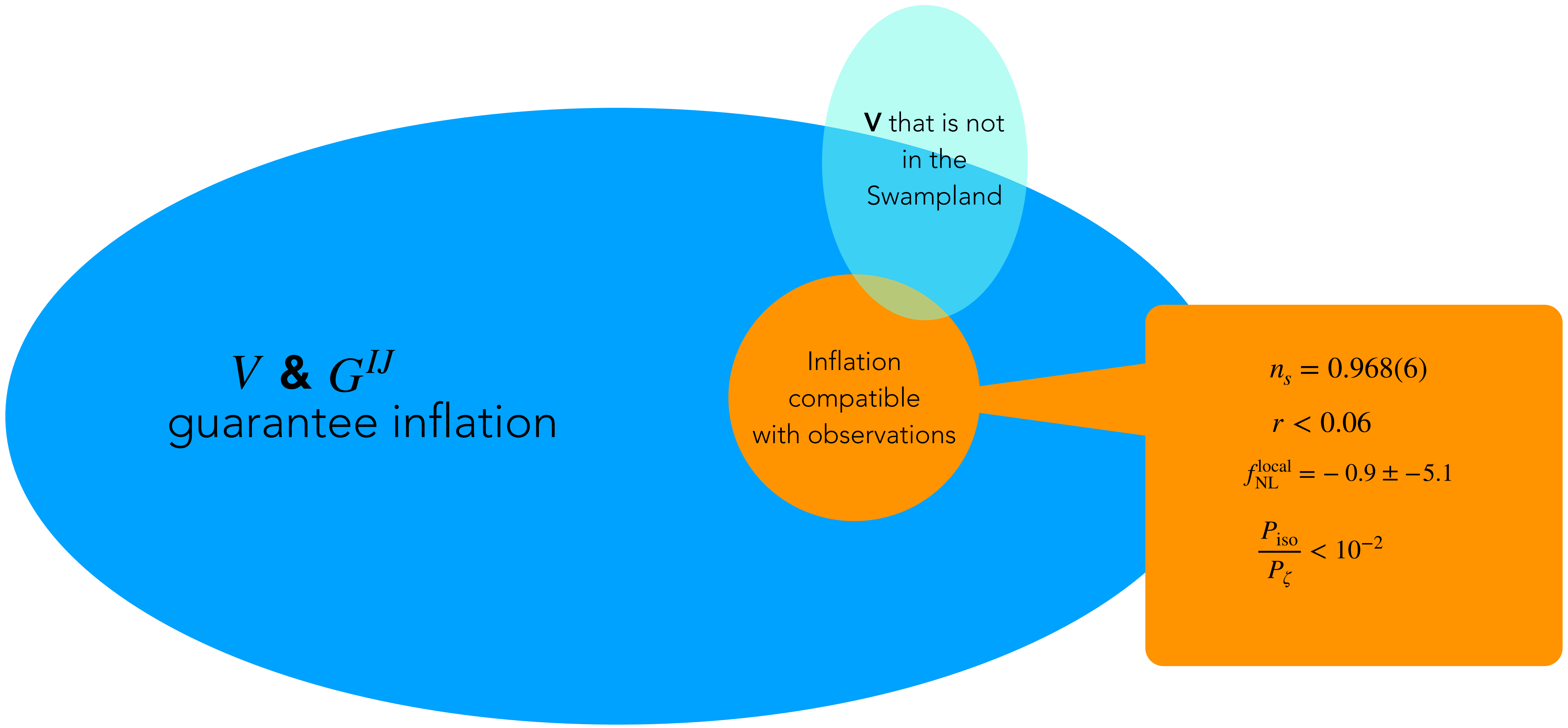
Multi-Field Inflation

Q: What conditions on V and G^{IJ} guarantee multi-field inflation?

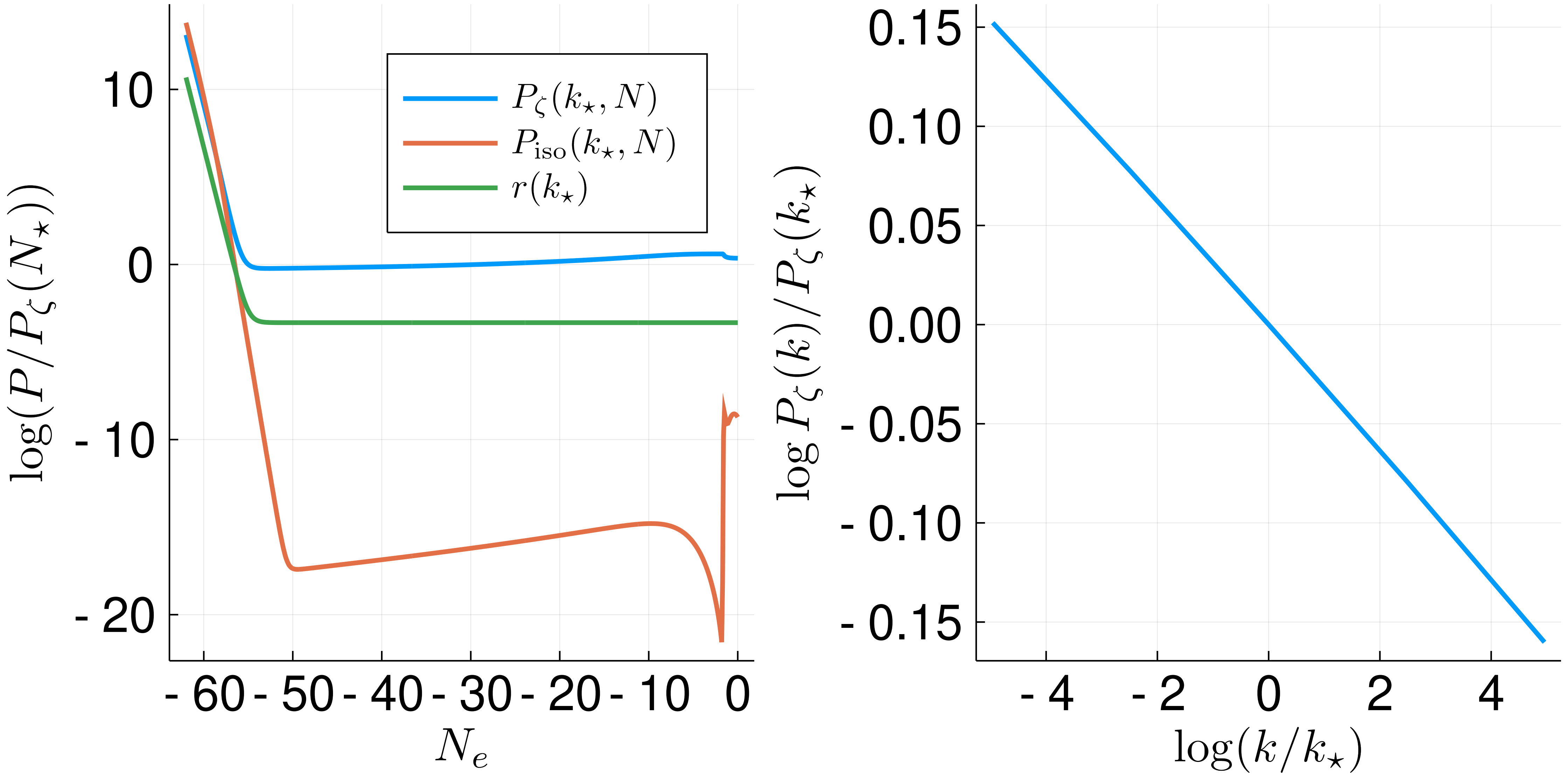


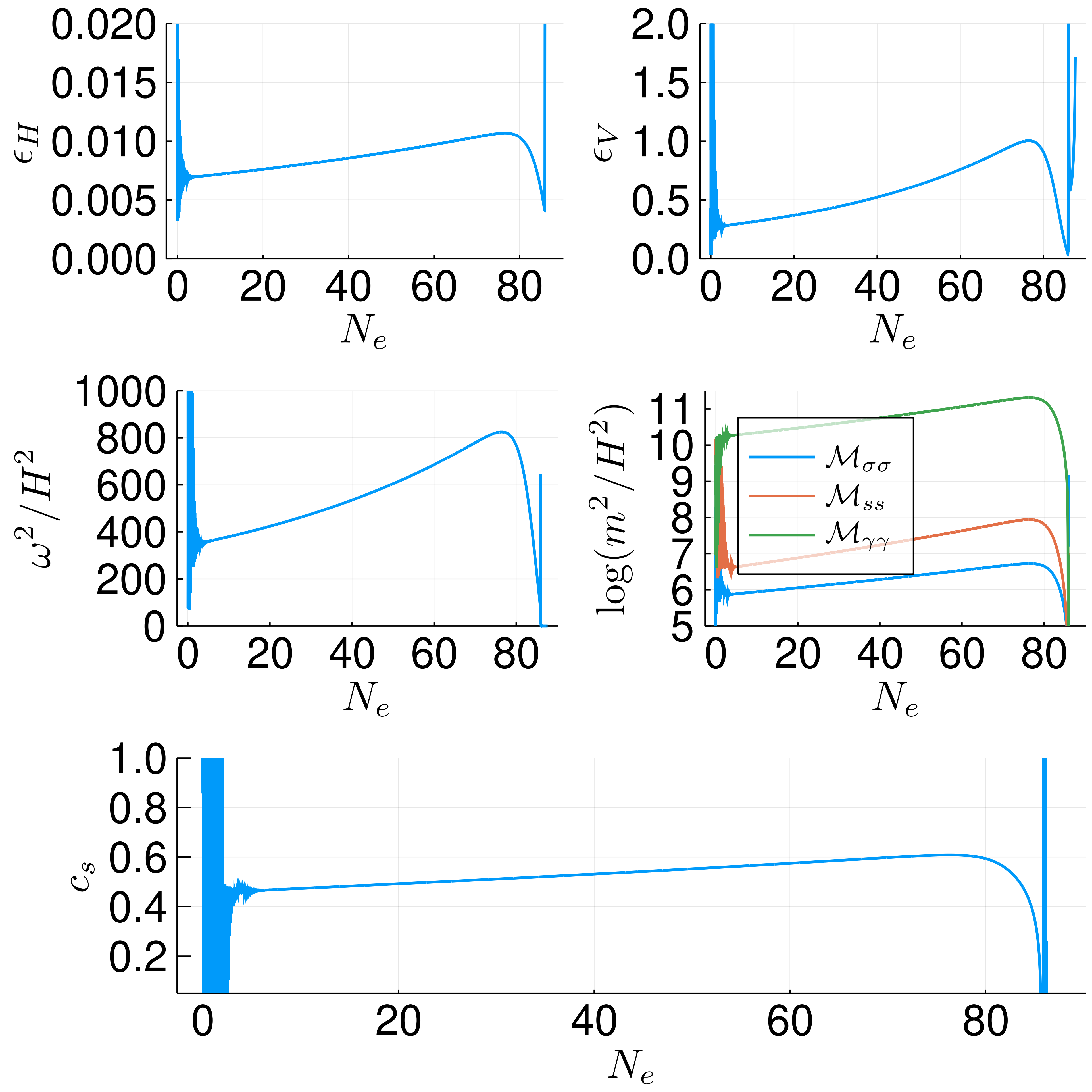
Two Examples of potentials with Rapid-turning and good phenomenology

Aragam, SP, Rosati: ArXiv:1905.07495

- Three fields, flat metric and potential

$$V = \Lambda^4 \left(e^{z/R} + \Delta \left(1 - \exp \left[\frac{-(x - A\cos z/f)^2 - (y - A \sin z/f)^2)}{2\sigma^2} \right] \right) \right)$$



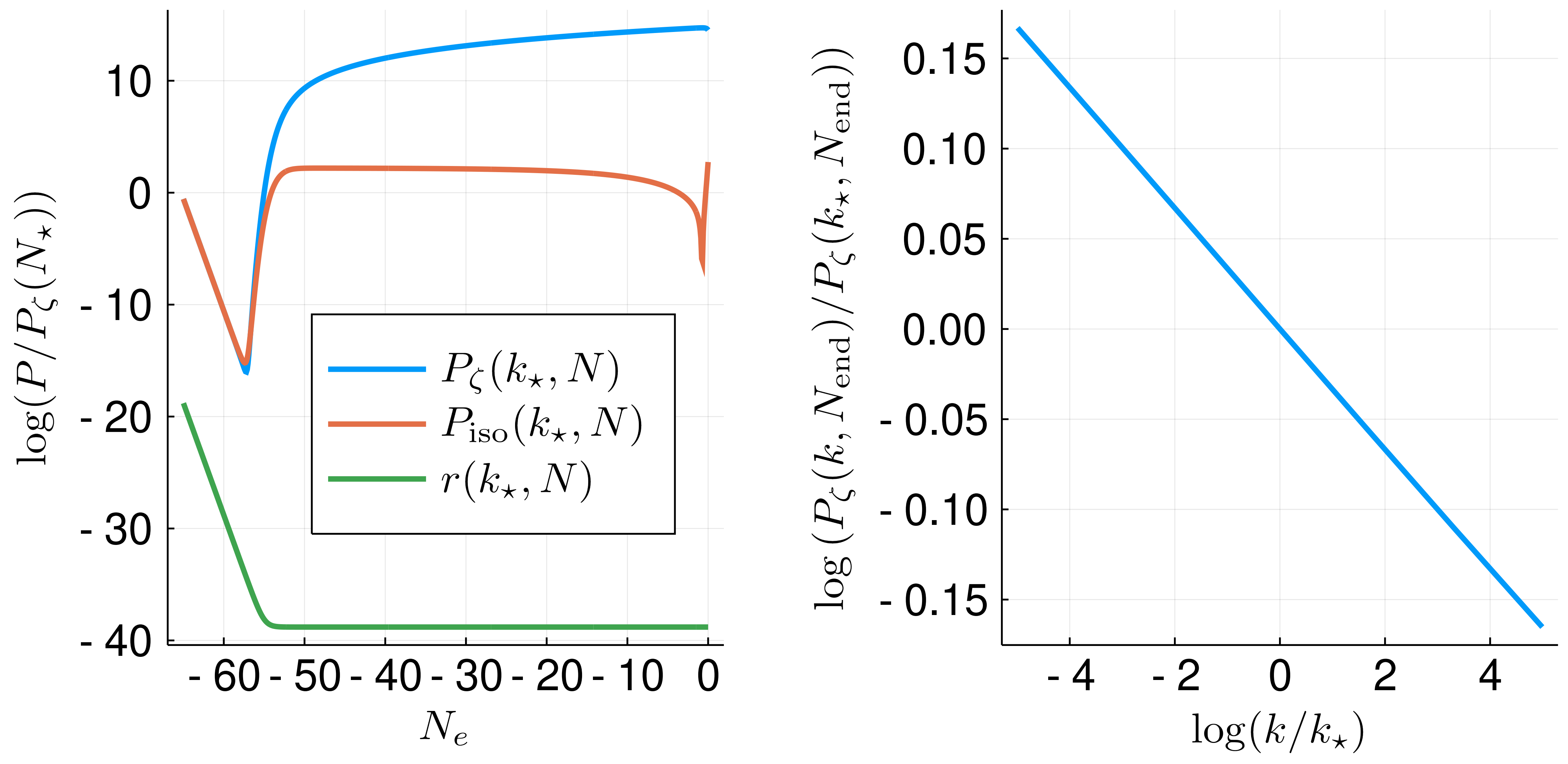


Two Examples of potentials with Rapid-turning and good phenomenology

[Aragam, SP, Rosati: ArXiv:1905.07495]

- Two fields, with metric and potential (Orbital Inflation) [Achúcarro, Welling 1907.02020]

$$G_{IJ} = \begin{pmatrix} e^{2Y/R_0} & 0 \\ 0 & 1 \end{pmatrix} \qquad V(X,Y) = 3W^2 - 2G^{IJ}W_{,I}W_{,J}\,. \qquad W(X) = Ae^{X/R_1}\left[\tanh\left(\frac{X}{R_2}\right)+1\right].$$



- Scalar: $n_s = 0.966$
- Didn't compute $f_{\rm NL}^{\rm local}$

[Bjorkmo, Ferreira, Marsh: ArXiv:1908.11316] showed that rapid-turn only produces mild NG.

- Notice that $r \sim 10^{-17}$

(Quantum Difference)

$$\mathscr{L}^{(2)} = a^3 \left[M_P^2 \epsilon \left(\dot{\zeta}^2 - \frac{(\partial\zeta)^2}{a^2} \right) + 2\dot{\phi}\, \omega\, \dot{\zeta} Q_s + \frac{1}{2} \left(\dot{Q_s}^2 - \frac{(\partial Q_s)^2}{a^2} - m_s^2 Q_s^2 \right) \right]$$

$$m_s^2 = V_{;ss} - H^2 \omega^2 + \epsilon H^2 M_P^2 R_{fs}$$

$$V_{;NN} \equiv N^I N^J V_{;IJ}; \quad T^I \equiv \frac{\dot{\phi}^I}{\dot{\phi}}; \quad D_t T^I = H\, \omega\, N^I$$

S. Renaux-Petel and K. Turzyński; 1510.01281, Brown, 1705.03023; Mizuno & Mukhoyama, 1707.05125

Take-away: There are Planck compatible multi-fields models of rapid-turn inflation.

Is there a criteria on the potential and field-space geometry for determining if slow-roll, rapid-turn, multi-field inflation is possible?

Aragam, SP, Rosati: ArXiv:2010.15933

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How generic are the potentials and field-space metrics in SUGRA?

Aragam, Chiovoloni, SP, Rosati, Zavala: ArXiv:2110.05516

Take-away: There are Planck compatible multi-fields models of rapid-turn inflation.

Do these models have other phenomenological consequences?

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How generic are the potentials and field-space metrics in SUGRA?

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Q: What are the criteria on the potential and field-space geometry for determining if slow-roll, rapid-turn, multi-field inflation is possible?

Bjorkmo, Marsh ArXiv:1901.08603; Bjorkmo: ArXiv:1902.10529; Aragam, SP, Rosati: ArXiv:2010.15933

$$\frac{\omega}{H} = \frac{3V_{vv}}{V_{v\perp}} + \mathcal{O}(\epsilon)$$

$$\frac{\omega^2}{H^2} = \frac{V_{\perp\perp}}{H^2} - \frac{V_{v\perp}^2}{V_{vv}H^2} - 9 + \mathcal{O}(\epsilon)$$

$$\perp^a \equiv \dot{\phi}_\perp^a / \dot{\phi}_\perp$$

$$v^I \equiv \frac{V^I}{|V^I|}$$

$$V_{\perp b} \equiv \perp^a V_{;ab}$$

- We quantify a limit, which we dub *extreme turning*, in which rapid-turn solutions may be found efficiently and develop methods to do so. In particular, simple results arise when the covariant Hessian of the potential has an eigenvector in close alignment with the gradient -- a situation we find to be common and we prove generic in two-field hyperbolic geometries.

- A sufficient condition is to have *fat inflation:* All fields heavier than H. It solves the η_V-problem.

Chakraborty, Chiovolini, Loaiza-Brito, Niz, Zavala: ArXiv:1908.09797

η_V**–Problem**: Quantum corrections tend to drive scalar masses to the cutoff scales, unless the fields are protected by symmetries.

$$\Delta m^2 \sim \Lambda^2_{\rm cutoff}$$

$$\eta_{\rm V} = \frac{M_P^2}{2}\frac{|V''|}{V}$$

$$\eta_V \equiv \Big|\min\ \ \text{eigenvalue}\{\mathbb{M}\}\Big| \qquad \mathbb{M} = \frac{M_P^2}{V}\begin{pmatrix} V_{TT} & V_{TN} \\ V_{NT} & V_{NN}\end{pmatrix}$$

How generic are the potentials and field-space metrics in SUGRA?

Aragam, Chiovoloni, SP, Rosati, Zavala: ArXiv:2110.05516

- Rapid-turn inflation in supergravity is rare and tachyonic
- Large turning rates can be generated in a wide class of models, at the cost of high field space curvature.

$$S = \int d^4x\sqrt{-g}\left[M_P^2\frac{R}{2} - K_{i\bar{j}}\partial_\mu\Phi^i\partial^\mu\bar{\Phi}^{\bar{j}} - V(\Phi^k,\bar{\Phi}^k)\right]$$

$$K = -3\,\alpha\,M_P^2\log[(\Phi+\bar{\Phi})/M_P] + S\bar{S}\,,$$

$$V = \frac{M_P^{3\alpha}\,|\,F\,|^2}{(\Phi+\bar{\Phi})^{3\alpha}}$$

$$\mathscr{R} = -4/(3\alpha)$$

$$\frac{\omega}{H} \simeq \frac{2\sqrt{\epsilon}}{\sqrt{3\,\alpha}}$$

In the 'large" regime (large volume, large complex structure, weak coupling) this coefficient is certainly $\alpha \sim O(1)$, but its value is unclear away from this limit
—A. Lukas

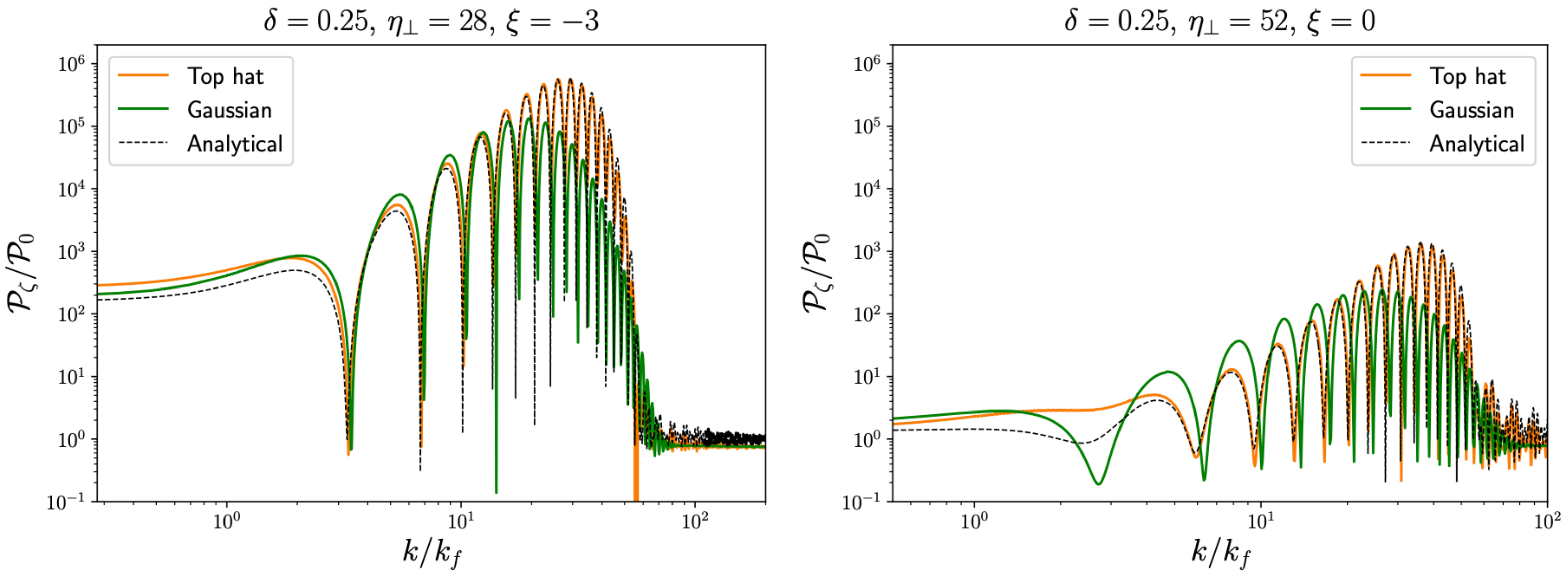
- Sustained turning is hard to achieve, but sporadic turning is pretty common. This, in turn, generates features in the spectrum.

- Observed scale invariance $10^{-4}\,\mathrm{Mpc}^{-1} \lesssim \mathrm{k} \lesssim 10^{-1}\,\mathrm{Mpc}^{-1}$

- PBHs if feauture with $k \gtrsim 10^{8}\mathrm{Mpc}^{-1}$ and amplification of 10^{7} larger than CMB. Palma, Sypsas, Zenteno ArXiv:2004.06106

- Generation of SGWB, possibly visible at LISA

Do these models have other phenomenological consequences?

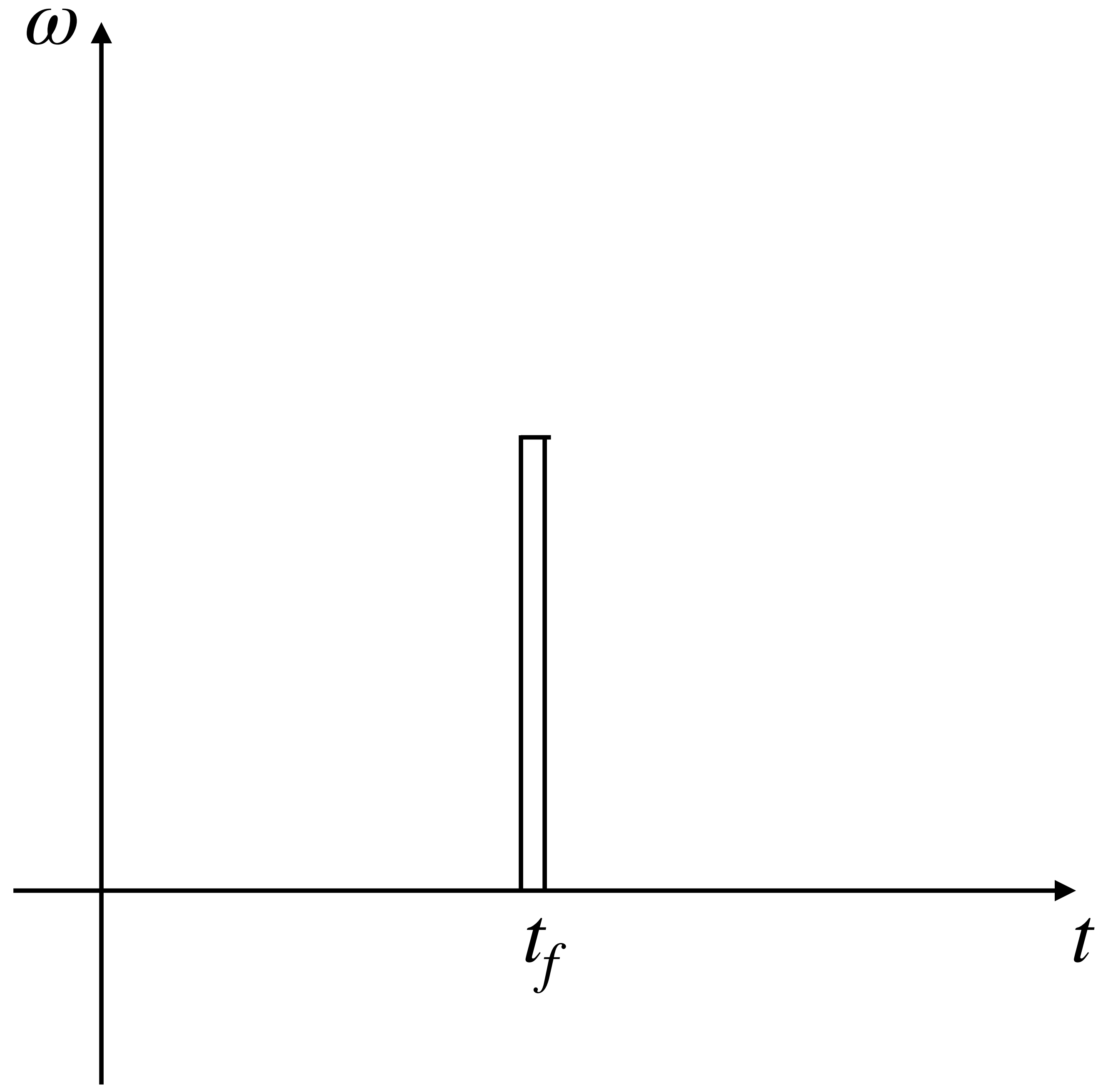


Fumagalli, Renaux-Petel, Witkowski: ArXiv:2012.02761

For a specific choice of masses,

$$D_t\dot{\zeta}_c + 3HD_t\zeta_c + \frac{k^2}{a^2}\zeta_c = 0$$

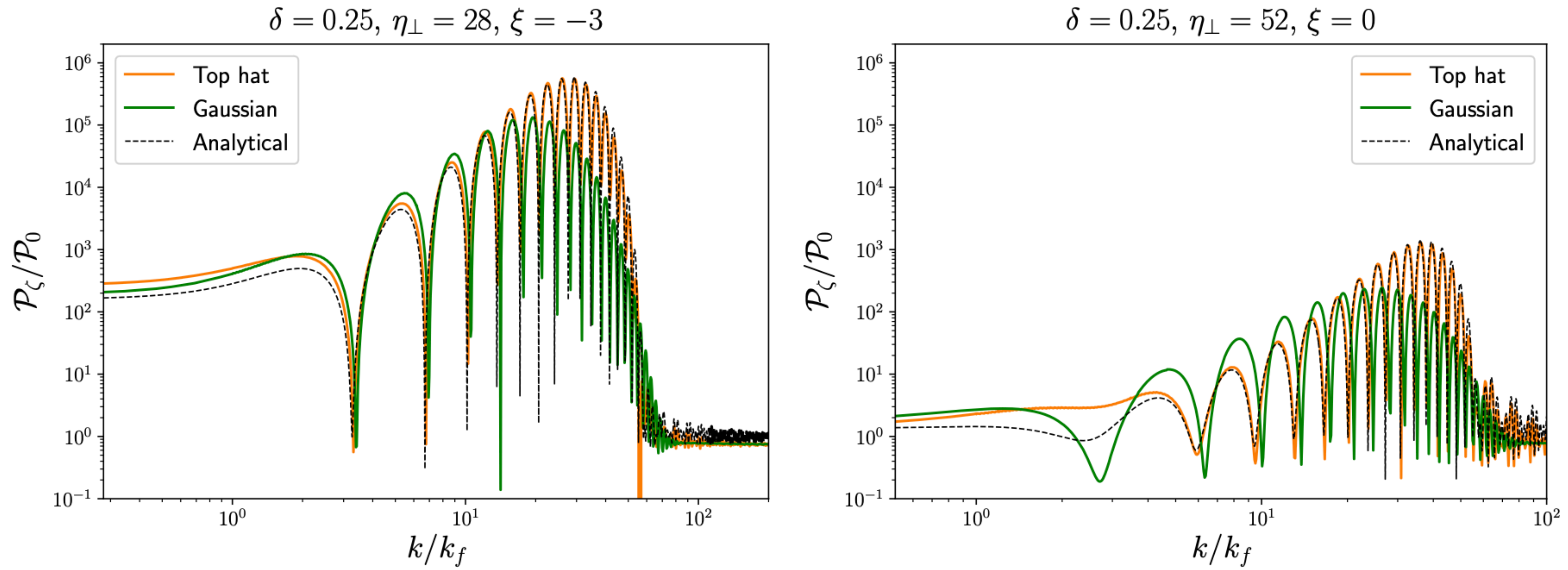
$$D_t\dot{Q}_s + 3HD_tQ_s + \frac{k^2}{a^2}Q_s = -\,2\omega D_t\zeta_c$$



During a top-hat feature:

$$\frac{q_\pm}{H} = \sqrt{\frac{k^2}{k_f^2} \pm 2\frac{\omega}{H}\frac{k}{k_f}}$$

$$k_f = Ha(t_f)$$



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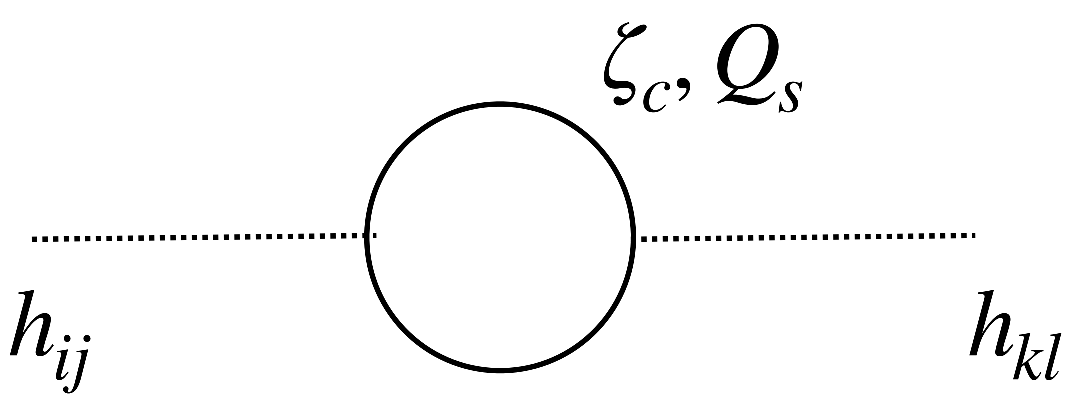
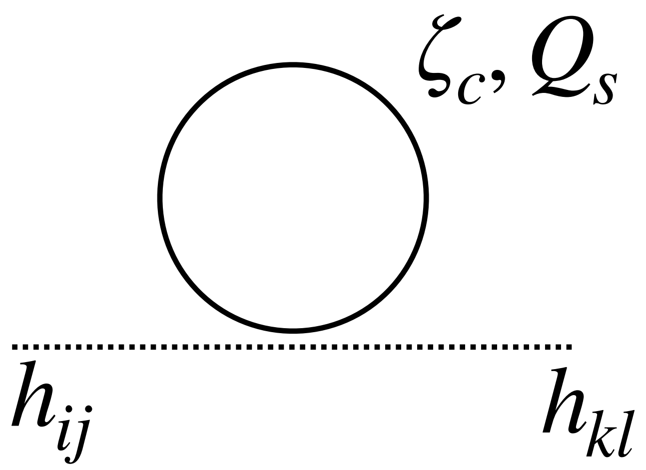
SGWB

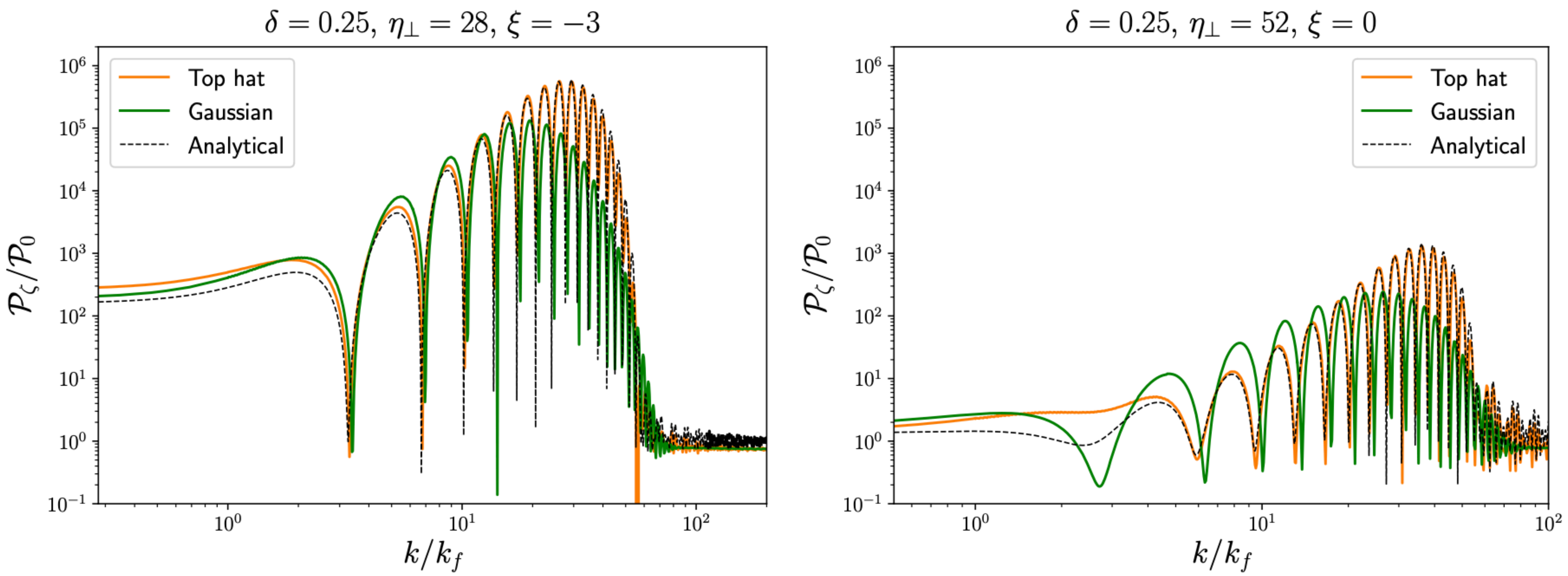
- Primordial scalar perturbations induce a gravitational wave spectrum.

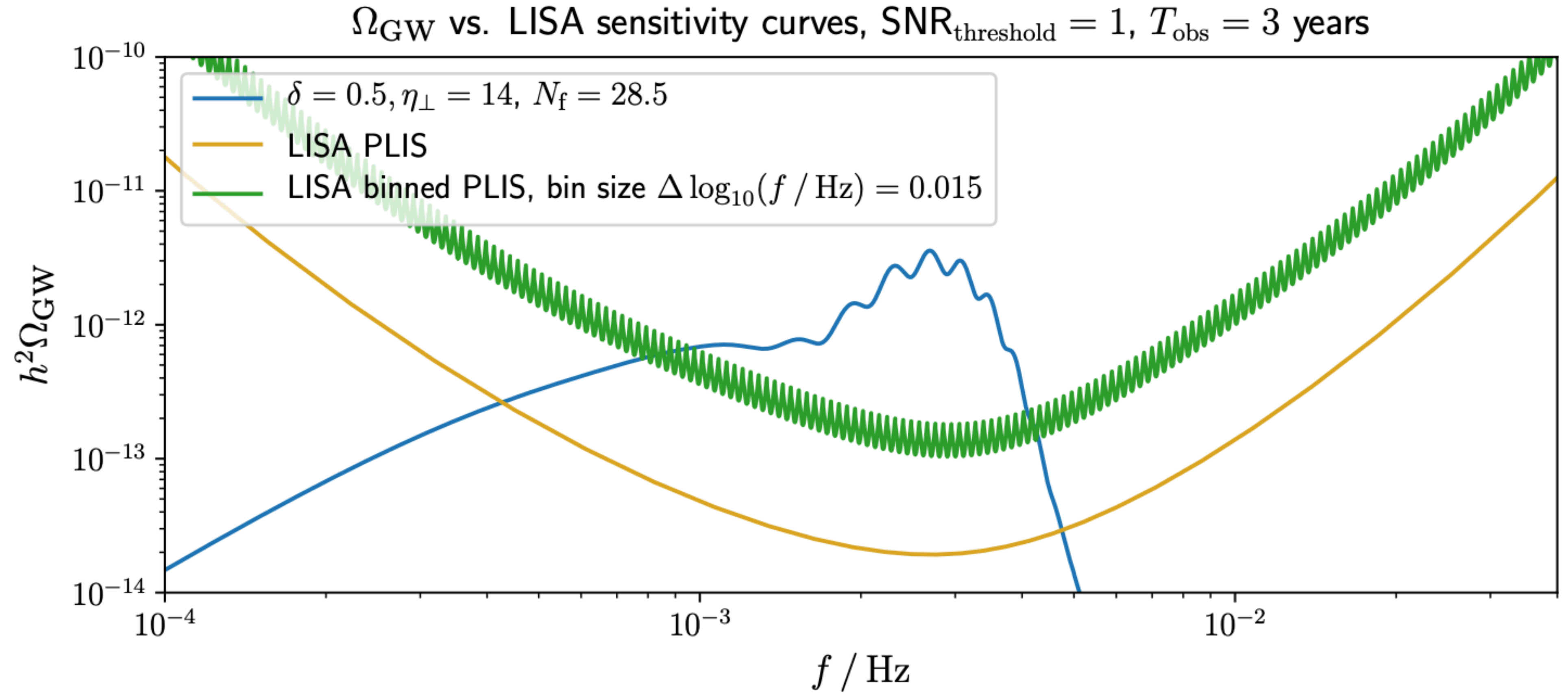
Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290





$$\Omega_{\rm GW} \sim 10^{-5} P_\zeta^2$$





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Summary

- Inflation models with rapid-turning fields can be realized in potentials far too steep for standard single-field slow-roll inflation.
- These models' predictions for $n_s, f_{NL}, \text{and } \mathrm{P}_{\text{iso}}$ are Planck compatible.
- If the turning rate is large enough, modes experience exponential growth close to horizon exit. This effect has two consequences:

** If the turning rate is constant, matching the measured amplitude for P_ζ, forces H^2 to be small, rendering r unobservable.

** For sporadic turning, it can seed PBH and SGWB

- In known SUGRA models, rapid-turn inflation models are rare and tachyonic.

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- Inflation models with rapid-turning fields can be realized in potentials far too steep for standard single-field slow-roll inflation.
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Thank you for your attention.

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  NOT A PROBLEM: Enough freedom in the potential to match experiment.

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Planck: $f_{\rm NL}^{\rm local} = -0.9 \pm 5.1$

[Bjorkmo, Ferreira, Marsh: ArXiv:1908.11316] prove that the overall growth is limited due to interference between decaying modes and growing modes. Not a problem

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- Isocurvature perturbations have not been seen in the CMB temperature and polarization data so far.

$$P_\zeta = \frac{H_*^2}{8\pi^2 \epsilon_*} e^{2x}, \quad x \sim \mathcal{O}(10)$$

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