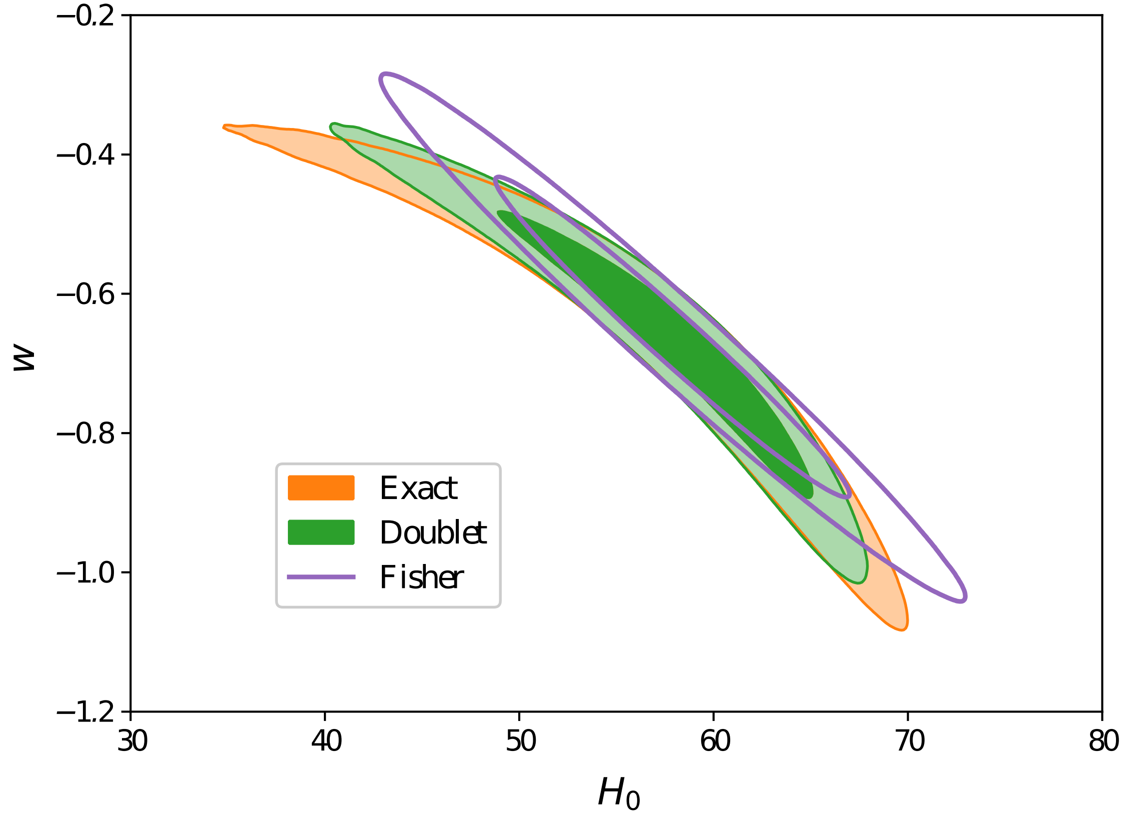
Beyond Fisher Forecasting for Cosmology



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Based on work in progress being conducted with Joel Meyers, Brandon Stevenson, and Cynthia Trendafilova (paper currently in preparation).

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Why forecasting?

» Upcoming cosmological surveys are expected to produce vast quantities of data.

» It is important to be able to accurately forecast the constraints those data can place on cosmological models, so that instrumental and computational time and resources can be used most effectively.

Fisher forecasting

» Forecasts of the parameter constraints on a given model often assume:

$$P = N \exp\left[-\frac{1}{2} F_{ab} \Delta p^a \Delta p^b\right]$$

» Where P is the posterior probability of the model, F_{ab} is known as the “Fisher matrix”, and $\Delta p := p - p_{fid}$. p_{fid} is the fiducial value of the parameter p.

E. Sellentin, M. Quartin, and L. Amendola, “Breaking the spell of Gaussianity: forecasting with higher order Fisher matrices,” Mon. Not. Roy. Astron. Soc. 441 no. 2, (2014) 1831–1840, `arXiv:1401.6892 [astro-ph.CO]`.

Fisher forecasting

» Given a data covariance matrix $\overleftrightarrow{C}$ and a vector of model predictions $\vec{\mu}$, the Fisher matrix can be written in the form

$$F_{ab} := \vec{\mu}_{,a}\overleftrightarrow{M}\vec{\mu}_{,b}$$

» Where $\overleftrightarrow{M} = \overleftrightarrow{C}^{-1}$, the subscript "$,\mathrm{a}$" refers to a partial derivative taken with respect to the parameter p^a, and the model vectors are contracted with $\overleftrightarrow{M}$ in the data space (in Einstein notation, $\vec{\mu}_{,a}\overleftrightarrow{M}\vec{\mu}_{,b} = \mu^i_{,a}M_{ij}\mu^j_{,b}$ where i and j are indices in the data space).

E. Sellentin, M. Quartin, and L. Amendola, "Breaking the spell of Gaussianity: forecasting with higher order Fisher matrices," Mon. Not. Roy. Astron. Soc. 441 no. 2, (2014) 1831–1840, arXiv:1401.6892 [astro-ph.CO].

Fisher forecasting

» Advantages of Fisher forecasting: speed and computational simplicity.

» Disadvantage: assumes that the components of $\vec{\mu}$ are linear in the model parameters, and that data have a Gaussian distribution.

» Bottom line: Fisher forecasting can, in some situations, produce oversimplified constraints.

E. Sellentin, M. Quartin, and L. Amendola, "Breaking the spell of Gaussianity: forecasting with higher order Fisher matrices," Mon. Not. Roy. Astron. Soc. 441 no. 2, (2014) 1831–1840, `arXiv:1401.6892 [astro-ph.CO]`.

Beyond Fisher forecasting

» For an arbitrary posterior P, the Derivative Approximation for Likelihoods (DALI), to second order in the model vector derivatives, is

$$P = N \exp\left[-\frac{1}{2}F_{ab}\Delta p^a \Delta p^b - \frac{1}{2}G_{abc}\Delta p^a \Delta p^b \Delta p^c - \frac{1}{8}H_{abcd}\Delta p^a \Delta p^b \Delta p^c \Delta p^d\right],$$

» where $G_{abc} \coloneqq \vec{\mu}_{,ab}\overleftrightarrow{M}\vec{\mu}_{,c}$, $H_{abcd} \coloneqq \vec{\mu}_{,ab}\overleftrightarrow{M}\vec{\mu}_{,cd}$, and N is a normalization constant.

» Note: this order of approximation is known as “Doublet-DALI”, or “Doublet”, for short.

E. Sellentin, M. Quartin, and L. Amendola, “Breaking the spell of Gaussianity: forecasting with higher order Fisher matrices,” Mon. Not. Roy. Astron. Soc. 441 no. 2, (2014) 1831–1840, arXiv:1401.6892 [astro-ph.CO].

When can we use Fisher?

- If exact likelihood is concave in one or more directions, Fisher approximation will break down.
- Not easy to tell, a priori, when this will happen. Therefore, it’s not easy to tell, ahead of time, whether it’s appropriate to use Fisher.
- Can we quantify the difference between Fisher and DALI without sampling?

When can we use Fisher?

» Think of likelihood contours as (D-1)-dimensional hypersurfaces embedded D-dimensional parameter space:

$$\mathcal{L}(p^a) \coloneqq -\ln[P(p^a)] = C$$

» C is a constant, and $P(p^a)$ is the posterior probability as a function of the model parameters $\{p^a\}$. This is a constraint equation of the form

$$\Phi(p^a) = 0$$

When can we use Fisher?

» We can define a unit vector that is normal to the likelihood hypersurfaces:

$$n_a = \frac{\Phi_{,a}}{\sqrt{g^{ab}\Phi_{,a}\Phi_{,b}}}$$

» Where $g^{ab} = \delta^{ab}$. The divergence of the unit normal vector field is

$$K = n^a_{,a}$$

E. Poisson, *A Relativist's Toolkit: The Mathematics of Black-Hole Mechanics*. Cambridge University Press, Cambridge, UK, 2004

When can we use Fisher?

» The divergence $K = h^{AB}K_{AB}$ is the trace of the extrinsic curvature K_{AB}, where "A" and "B" refer to coordinates on the hypersurface, and h_{AB} is the metric on the hypersurface.

» If $K > 0$ at a given point, then the hypersurface is convex there. On the other hand, if $K < 0$ at a given point, then the hypersurface is concave there.

E. Poisson, *A Relativist's Toolkit: The Mathematics of Black-Hole Mechanics*. Cambridge University Press, Cambridge, UK, 2004

Extrinsic curvature test

- Step 1: compute K on grids covering every possible cross-section of the exact posterior.
  - Why cross-sections? K can be computed very quickly in two dimensions, with results that are easier to interpret compared to the results obtained in spaces with more than two dimensions.

- Step 2: Plot cross-sections on which negative (or zero) curvature has been detected, visually inspect plots to determine extent of deviation from Fisher.

Beyond Fisher forecasting at low redshift

» Flat ΛCDM, characterized by the Hubble parameter

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}}$$

» Flat wCDM, characterized by the Hubble parameter

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})(1+z)^{3(1+w_X)}}$$

Low-redshift data

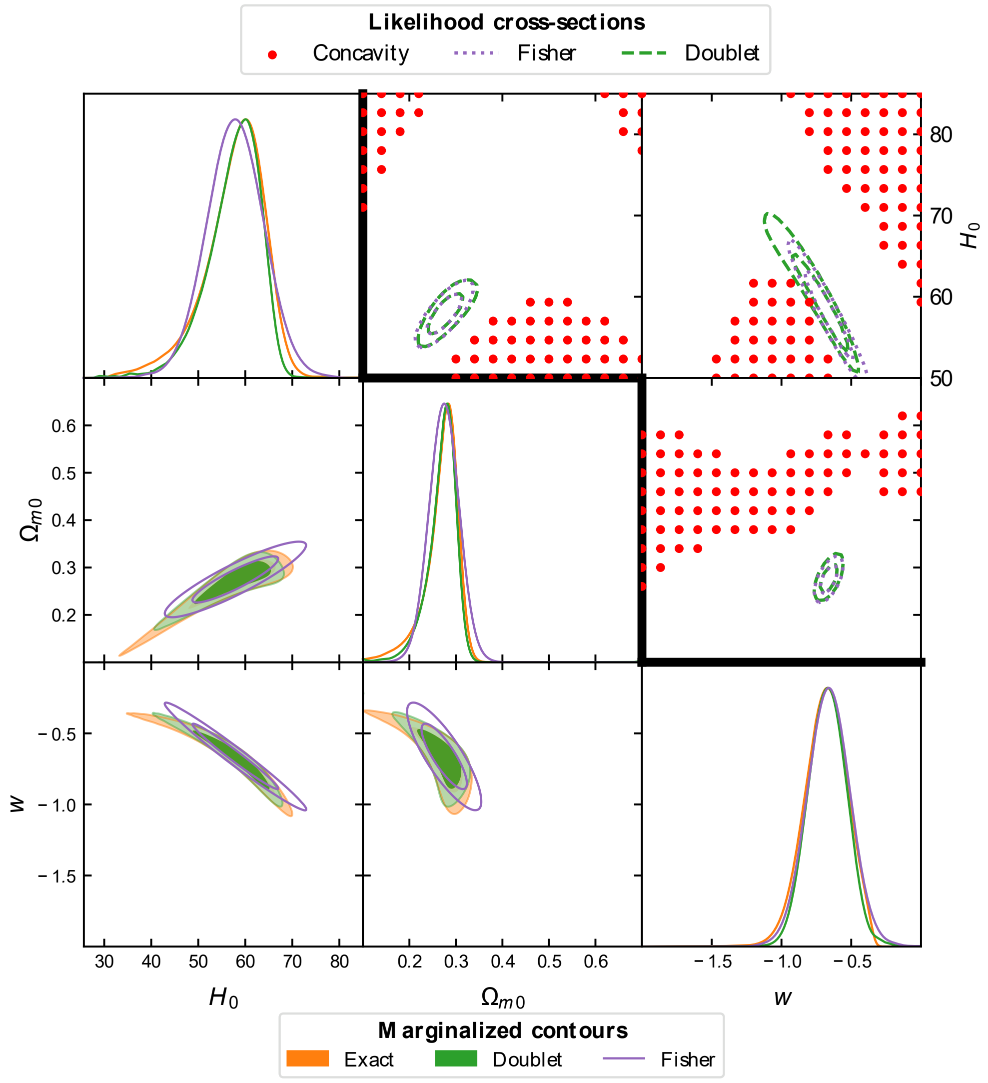
» 31 $H(z)$ measurements from cosmic chronometers.

» 6 comoving distance and $H(z)$ measurements from BAO.

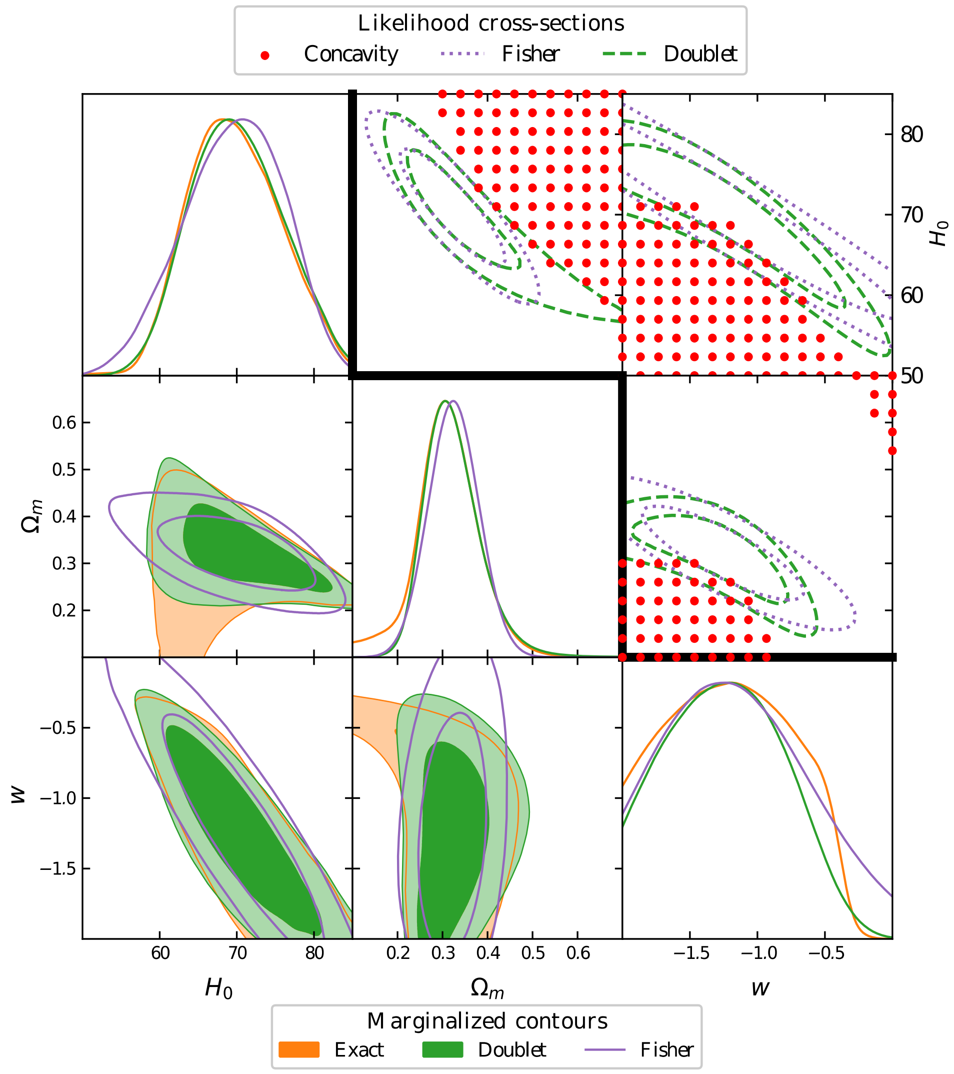
» 120 standard ruler measurements from QSOs.

See J. Ryan, Y. Chen, and B. Ratra, “Baryon acoustic oscillation, Hubble parameter, and angular size measurement constraints on the Hubble constant, dark energy dynamics, and spatial curvature,” MNRAS 488 no. 3, (Sept., 2019) 3844–3856, `arXiv:1902.03196[astro-ph.CO]`, and J. Ryan, S. Doshi, and B. Ratra, “Constraints on dark energy dynamics and spatial curvature from Hubble parameter and baryon acoustic oscillation data,” MNRAS 480 (Oct., 2018) 759–767, `astro-ph/1805.06408` for details.

Flat wCDM cross-sections and marginalized contours

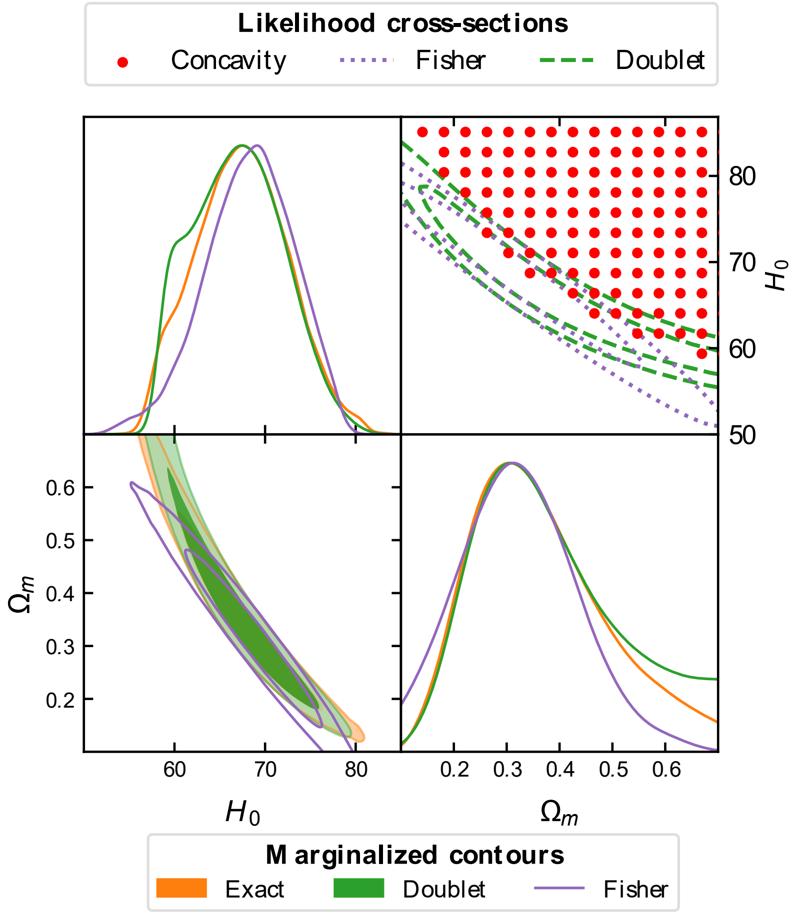


BAO data

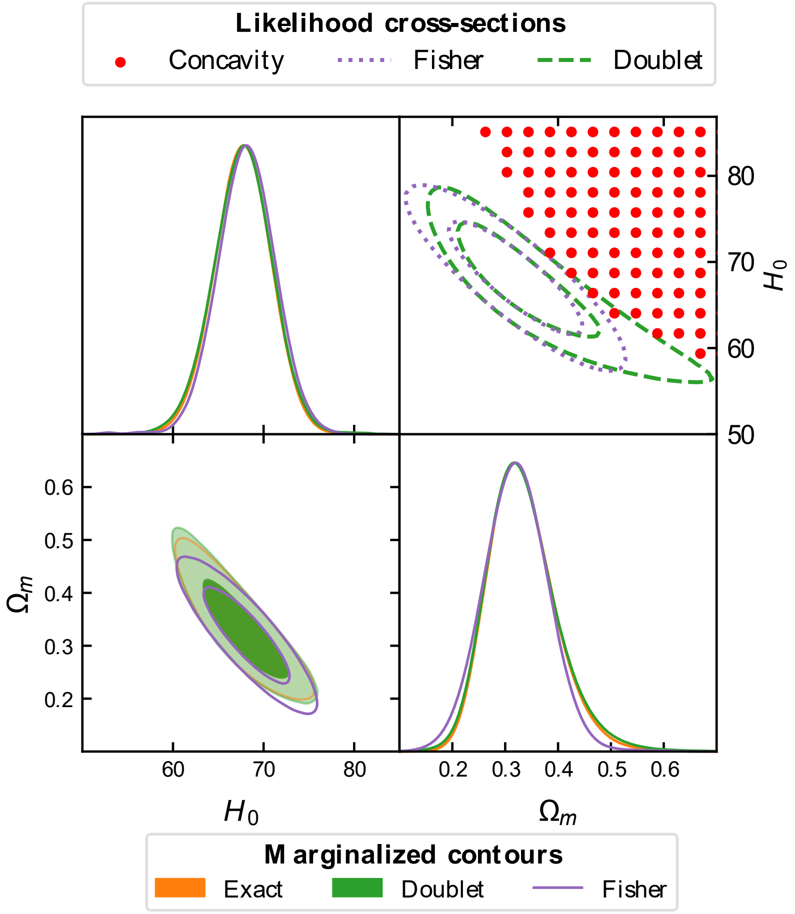


H(z) data

Flat ΛCDM cross-sections and marginalized contours

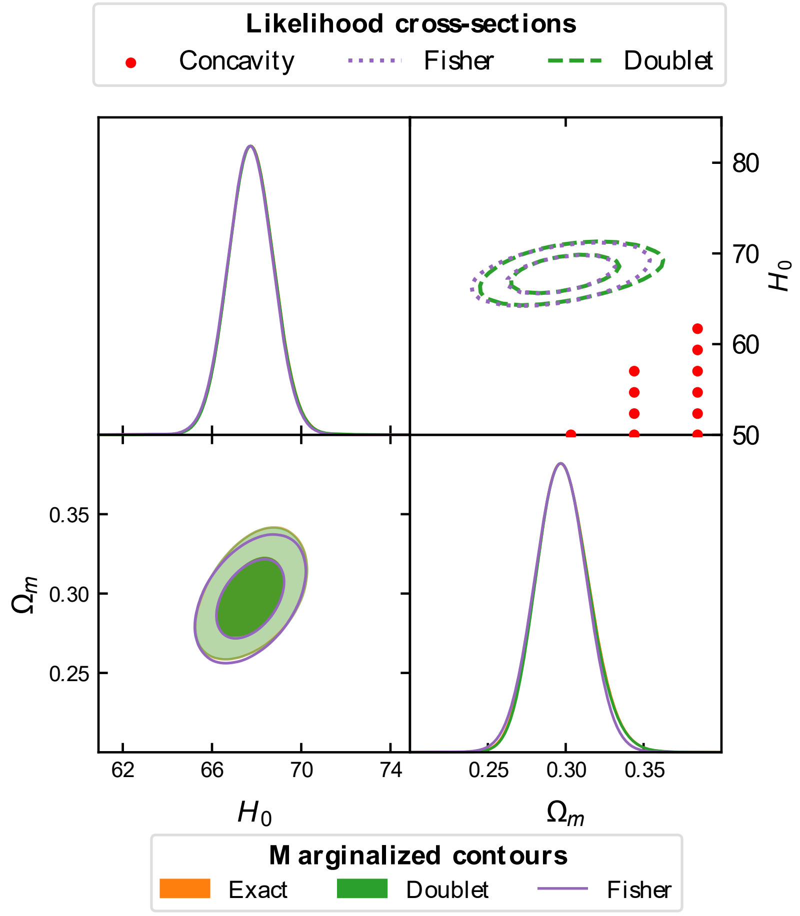


QSO data



H(z) data

Flat ΛCDM cross-sections and marginalized contours



BAO data

» Extrinsic curvature of likelihood cross-sections can be used as a proxy for curvature of marginalized contours.

» This allows us to check the range of validity of the Fisher approximation without needing to sample a large parameter space.

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Why don't we always use DALI?

» In some cases, the Doublet approximation can over-predict the variances of model parameters.

Parameters	Exact	Fisher	Doublet
H_0	1.18	1.16	1.22
Ω_m	0.00029	0.00028	0.00038

» Table shows the variances of the parameters of the flat ΛCDM model from BAO data, using the exact likelihood, the Fisher approximation, and the Doublet approximation.

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Conclusion

» We have discovered a simple way to leverage the DALI method to judge whether the Fisher approximation is appropriate for a given forecast.

» We caution against using the DALI method blindly, however, because it can give inaccurate parameter variances, leading to inaccurate summary statistics.

Acknowledgements

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