# Neutrino Physics <br> <br> Introduction 

 <br> <br> Introduction}

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## The Nobel Prize in Physics 2015



Photo © Takaaki Kajita
Takaaki Kajita
Super-Kamiokande Collaboration


Photo: K. MacFarlane. Queen's University /SNOLAB

## Neutrino Oscillations $\sqrt{8}$ <br> Neutrinos have Mass

Arthur B. McDonald
Sudbury Neutrino Observatory Collaboration
The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass"

## Neutrino Physics Post-1998

1998: evidence for neutrino mass from Superk $\left(V_{\mu} \rightarrow V_{\tau}\right)$
first solid evidence of beyond the Standard Model Physics

Massive Neutrinos

2002: evidence for neutrino mass from $\mathrm{SNO}\left(\mathrm{V}_{e} \rightarrow \mathrm{~V}_{\mu, \tau}\right)$
2003: KamLand confirmed Large Mixing Angle solution to solar v problem
2011: hints for non-zero $\theta_{13}$ from T2K, MINOS, and Double Chooz
2012: evidences of non-zero $\theta_{13}$ from Daya Bay and RENO
for some parameters: discovery phase into precision phase; and yet, many great discoveries to come

## Neutrino Oscillation $\Rightarrow$ Massive Neutrinos

- Neutrino Masses are non-degenerate (at least two are non-zero)
- mass eigenstates $\neq$ weak eigenstates
- Accidental symmetries in SM
- Broken lepton flavor numbers: $L_{e}, L_{\mu}, L_{\tau}$
- Processes cross family lines in lepton sector now possible
- As a result
- neutrino oscillation
- lepton flavor violation decays?


ARE NEUTRINOS

- total lepton number? $L ? L_{e}+L_{\mu}+L_{\tau}$
 THEIR OWN'?
ANTIPARTICLES?


## What if Neutrinos Have Mass?

- Similar to the quark sector, there can be a mismatch between mass eigenstates and weak eigenstates
- weak interactions eigenstates: $V_{e}, V_{\mu} V_{\tau}$

- mass eigenstates: $\mathrm{V}_{1}, \mathrm{~V}_{2}, \mathrm{~V}_{3}$
- Pontecorvo-Maki-Nakagawa-Sakata (PMNS) Matrix

Maki, Nakagawa, Sakata, I962; Pontecorvo, I967

$$
\left(\begin{array}{c}
v_{e} \\
v_{\mu} \\
v_{\tau}
\end{array}\right)=\left(\begin{array}{ccc}
U_{e 1} & U_{e 2} & U_{e 3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{array}\right)\left(\begin{array}{c}
v_{1} \\
v_{2} \\
v_{3}
\end{array}\right) \quad \begin{gathered}
3 \text { mixing angles } \\
+1 \text { (3) phase(s) for } \\
\text { Dirac (Majorana) }
\end{gathered}
$$

## Leptonic Mixing Matrix

- Pontecorvo-Maki-Nakagawa-Sakata (PMNS) Matrix

$$
U_{\text {MNS }}=\underbrace{\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & c_{a} & s_{a} \\
0 & -s_{a} & c_{a}
\end{array}\right]}_{\text {PMNs }} \underset{\text { reactor }}{\left[\begin{array}{ccc}
c_{x} & 0 & s_{x} e^{-i \delta} \\
o & 1 & 0 \\
-s_{x} e^{i \delta} & 0 & c_{x}
\end{array}\right]} \underset{\text { solar }}{\left[\begin{array}{ccc}
c_{S} & s_{S} & 0 \\
-s_{S} & c_{s} & 0 \\
0 & 0 & 1
\end{array}\right]} \underset{\text { Majorana phases }}{\left[\begin{array}{ccc}
1 & o & 0 \\
0 & e^{i\left(\frac{1}{2} \phi_{12}\right)} & 0 \\
0 & 0 & e^{i\left(\frac{1}{2} \phi_{13}+\delta\right)}
\end{array}\right]}
$$

- Three mass eigenvalues $m_{1}, m_{2}, m_{3} \Rightarrow$ two $\Delta m_{i j}^{2} \equiv m_{i}^{2}-m_{j}^{2}$
- three mixing angles: $c_{a}, c_{s}, c_{x}$
- three CP phases (if Majorana): $\delta, \phi_{12}, \phi_{13}$
- 1 CP phase (if Dirac): $\delta$
- Oscillation experiments: sensitive only to $\delta$
- Neutrinoless double beta decay: sensitive only to Majorana phases: $\phi_{12}, \phi_{13}$


## Neutrino Oscillation: Macroscopic Quantum Mechanics

- production: neutrinos of a definite flavor produced by weak interaction
- propagation: neutrinos evolve according to their masses
- detection: neutrinos of a different flavor composition detected


$$
P\left[v_{\mu} \rightarrow v_{e}\right]=\sin ^{2} 2 \theta \sin ^{2}\left[1.27 \Delta m_{32}^{2}\left(\frac{(\mathrm{eV})^{2}}{c^{2}}\right) \frac{L(\mathrm{~km})}{E(\mathrm{GeV})}\right]
$$

## Classes of Experiments

Oscillation Experiments:
Atmospheric, solar, reactor, accelerator neutrinos

- mass ordering, CP phases, precision measurements
- Searches for BSM physics


## Weak Decay Kinematics:

- Absolute mass scale
- Precision cosmology

Neutrino cross sections, CELNS:

- Interpretation of data
- BSM

Neutrinoless Double Beta Decay:

- Majorana vs Dirac

Astrophysical Neutrinos:
SN, GRBs, AGNs, mergers

- Possible BSM physics


## Grand Unified Neutrino Spectrum at Earth

Edoardo Vitagliano, Irene Tamborra, Georg Raffelt. Oct 25, 2019. 54 pp. MPP-2019-205
e-Print: arXiv:1910.11878 [astro-ph.HE] I PDF

[Slide Curtesy: Kate Scholberg, Snowmass CSS 2022]


Neutrinos as messengers

IceCube: Talks by Jessie Micalleł, Qinrui Liu

## KM3NeT-ORCA: Talk by Bouke Jung

## Where Do We Stand?

- Latest 3 neutrino global analysis:

|  |  | Normal Ordering (Best Fit) |  | Inverted Ordering ( $\Delta \chi^{2}=7.0$ ) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | bfp $\pm 1 \sigma$ | $3 \sigma$ range | bfp $\pm 1 \sigma$ | $3 \sigma$ range |
|  | $\sin ^{2} \theta_{12}$ | $0.304_{-0.012}^{+0.012}$ | $0.269 \rightarrow 0.343$ | $0.304_{-0.012}^{+0.013}$ | $0.269 \rightarrow 0.343$ |
|  | $\theta_{12} /{ }^{\circ}$ | $33.45{ }_{-0.75}^{+0.77}$ | $31.27 \rightarrow 35.87$ | $33.45{ }_{-0.75}^{+0.78}$ | $31.27 \rightarrow 35.87$ |
|  | $\sin ^{2} \theta_{23}$ | $0.450_{-0.016}^{+0.019}$ | $0.408 \rightarrow 0.603$ | $0.570_{-0.022}^{+0.016}$ | $0.410 \rightarrow 0.613$ |
|  | $\theta_{23} /{ }^{\circ}$ | $42.1_{-0.9}^{+1.1}$ | $39.7 \rightarrow 50.9$ | $49.0_{-1.3}^{+0.9}$ | $39.8 \rightarrow 51.6$ |
|  | $\sin ^{2} \theta_{13}$ | $0.02246_{-0.00062}^{+0.0062}$ | $0.02060 \rightarrow 0.02435$ | $0.02241_{-0.00062}^{+0.00074}$ | $0.02055 \rightarrow 0.02457$ |
|  | $\theta_{13} /{ }^{\circ}$ | $8.62_{-0.12}^{+0.12}$ | $8.25 \rightarrow 8.98$ | $8.61{ }_{-0.12}^{+0.14}$ | $8.24 \rightarrow 9.02$ |
|  | $\delta_{\mathrm{CP}} /{ }^{\circ}$ | $230_{-25}^{+36}$ | $144 \rightarrow 350$ | $278{ }_{-30}^{+22}$ | $194 \rightarrow 345$ |
|  | $\frac{\Delta m_{21}^{2}}{10^{-5} \mathrm{eV}^{2}}$ | $7.42_{-0.20}^{+0.21}$ | $6.82 \rightarrow 8.04$ | $7.42_{-0.20}^{+0.21}$ | $6.82 \rightarrow 8.04$ |
|  | $\frac{\Delta m_{3 \ell}^{2}}{10^{-3} \mathrm{eV}^{2}}$ | $+2.510_{-0.027}^{+0.027}$ | $+2.430 \rightarrow+2.593$ | $-2.490_{-0.028}^{+0.026}$ | $-2.574 \rightarrow-2.410$ |

$\Rightarrow$ hints of $\theta_{23} \neq \pi / 4$

- expectation of Dirac CP phase $\delta$
- slight preference for normal mass ordering


## Neutrino Mass Measurements

- search for absolute mass scale:
- end point kinematic of tritium beta decays Tritium $\rightarrow H e^{3}+e^{-}+\bar{\nu}_{e}$

```
KATRIN: current limit ~0.8 eV
    Future sensitivity ~0.2 eV
```

Katrin: Talk by Bjoern Lehnert

Other ideas: Project 8 (Talk by Arina Telles), ...

- neutrinoless double beta decay
current bound: $|\langle m\rangle| \equiv\left|\sum_{i=1,2,3} m_{i} U_{i e}^{2}\right|<(0.061-0.165) \mathrm{eV}$ (Kamland-Zen, 2016)
CUORE (Talk by Daniel Mayer) nEXO (Talk by Soud AI Kharusi) CUPID (Talk by Krystal Alfonso) LEGEND (Talk by Danielle Schaper) AMoRE (Talk by Hanbeom Kim)
$N_{\text {eff }}=2.99 \pm 0.17$ [Planck 2018]
$\Rightarrow$ fully thermalized sterile neutrino disfavored


## How are masses ordered?

The known knowns:
normal hierarchy:


## The Known Knowns



NuFIT (2022)
$\left[\theta^{\text {lep }}{ }_{23} \sim 42^{\circ}\right]$
$\left[\theta^{\text {lep }}{ }_{12} \sim 33^{\circ}\right]$
$\left[\theta^{\mathrm{lep}_{13}} \sim 9^{\circ}\right]$

## Open Questions - Neutrino Properties

CP violation in lepton sector?
Mass ordering: sign of $\left(\Delta m_{13}{ }^{2}\right)$ ?
Precision: $\theta_{23}>\pi / 4, \theta_{23}<\pi / 4, \theta_{23}=\pi / 4$ ?

## CP Violation in Neutrino Oscillation

- With leptonic Dirac CP phase $\delta \neq 0 \rightarrow$ leptonic CP violation
- Predict different transition probabilities for neutrinos and antineutrinos

$$
P\left(\nu_{\alpha} \rightarrow \nu_{\beta}\right) \neq P\left(\overline{\nu_{\alpha}} \rightarrow \overline{\nu_{\beta}}\right)
$$

- One of the major scientific goals at current and planned neutrino experiments


DUNE


## Experimental Precision: Oscillation Parameters



## Neutrino Interactions




CC RES


CC DIS


CONUS: Talk by Nicola Ackermann
(TeV) Neutrinos at LHC

MINERvA: Talks by Tejin Cai, David Robert Last

FASERNu: Talk by Shih-Chieh Hsu
oscillation data oscillation data
@GeV: Needed


## Some Anomalies are

 more anomalous than others.
## Neutrino Anomalies

## Neutrinos Travel Faster Than Light, According to One Experiment

Others doubt the mind-boggling claim, which would overturn Einstein's theory of special relativity

22 SEP 2011 - BY ADRIAN CHO (Science)


## Multiple Lorentz Groups - A . domain <br> Neutpurio vi....s and DAMA annual modulation

Nell min al neutrin.


## Common oriain

## Ne Once Again, Physicists Debunk Faster-Than-Light Neutrinos <br> Multipl

Five different groups agree that the elusive particles obey Einstein's speed limit after all

## superlum $\Rightarrow$ tperiment

superluminal Neutrinos in the Minimal standard Model Extension

## Neutrino Anomalies

Measurements at < km disagree with state-of-the-art neutrino predictions


## Neutrino Anomalies



## Are there sterile neutrinos?

# New neutrino mass states (eV)? 

## Sterile neutrinos

MicroBooNE: Talk by Jay Hyun Jo

BeEST: Talk by Annika Lennarz

IceCube: Talks by Jessie Micalleł, Qinrui Liu

## (NoVA: Talk by

Alexander booth)

## Are Neutrinos their Own Antiparticles?

Two-neutrino double- $\beta$ decay

## LN conserved



Maria Goeppert-Mayer, 1935

$$
(A, Z) \rightarrow(A, Z+2)+e^{-}+e^{-}+\bar{\nu}_{e}+\bar{\nu}_{e}
$$

First observed in 1987

Neutrinoless double- $\beta$ decay


Wendell Furry, 1939

$$
\Delta L=2
$$

$$
(A, Z) \rightarrow(A, Z+2)+e^{-}+e^{-}
$$

Required massive Majorana neutrinos;


Not yet observed

## Neutrinoless Double Beta Decay ${ }^{\left|m_{s s}\right|}\left|\sum_{k=1}^{4} u_{\dot{\alpha}} m_{k}\right|$

## $3 \nu$ : 10 fully covered by 2035

## $4 \nu$ : NO can be probed




## Neutrinoless Double Beta Decay


[From Snowmass White Paper 2212.11099]

## Open Questions - Neutrino Properties

Majorana vs Dirac?

CP violation in lepton sector?
Absolute mass scale of neutrinos?
Mass ordering: sign of $\left(\Delta m_{13}{ }^{2}\right)$ ?

To understand some of these properties
$\Rightarrow$ BSM Physics

Sterile neutrino(s)?
Precision: $\theta_{23}>\pi / 4, \theta_{23}<\pi / 4, \theta_{23}=\pi / 4$ ?
Additional Neutrino Interactions?

## a suite of current and upcoming <br> experiments to address these puzzles

## Open Questions - Theoretical

Smallness of neutrino mass:
$m_{v} \ll m_{e, u, d}$


## Open Questions - Theoretical

Flavor structure:
weak interaction eigenstates

quark mixing

leptonic mixing

## Open Questions - Theoretical

Smallness of neutrino mass:

$$
m_{v} \ll m_{e, u, d}
$$



Fermion mass and hierarchy
problem $" \rightarrow$ Many free parameters in the Yukawa sector of SM

Flavor structure:

leptonic mixing

quark mixing

## Why Should We Care?

- Understanding a wealth of data, fundamentally
- SM flavor sector: no understanding of significant fraction (22/28) of SM parameters; (c.f. SM gauge sector)
- Neutrinos as window into BSM physics
- neutrino mass generation unknown (suppression mechanism, scale)
- Uniqueness of neutrino masses $\rightarrow$ connections w/ NP frameworks
- Neutrinos affords opportunities for new explorations
- New Tools
- May address other puzzles in particle physics
- Window into early Universe
- UV connection


## Smallness of neutrino masses

What is the operator for neutrino mass generation?

- Majorana vs Dirac
- scale of the operator
- suppression mechanism


## Neutrino Mass beyond the SM

- SM: effective low energy theory

$$
\mathcal{L}=\mathcal{L}_{\mathrm{SM}}+\frac{\mathcal{O}_{5 D}}{M}+\frac{\mathcal{O}_{6 D}}{M^{2}}+\ldots
$$

## new physics effects

- only one dim-5 operator: most sensitive to high scale physics

$$
\frac{\lambda_{i j}}{M} H H L_{i} L_{j} \quad \Rightarrow \quad m_{\nu}=\lambda_{i j} \frac{v^{2}}{M}
$$

Weinberg, I 979
$\bullet m_{v} \sim\left(\Delta m^{2}{ }_{\text {atm }}\right)^{1 / 2} \sim 0.1 \mathrm{eV}$ with $v \sim 100 \mathrm{GeV}, \lambda \sim O(1) \Rightarrow M \sim 10^{14} \mathrm{GeV}$

- Lepton number violation $\Delta L=2 \hookrightarrow$ Majorana fermions


## Neutrino Mass beyond the SM



Type-I seesaw

$N_{R}: S U(3)_{c} \times S U(2)_{w} x U(1) Y \sim(1,1,0)$
Minkowski, 1977; Yanagida, 1979; Glashow, 1979; Gell-mann, Ramond, Slansky,1979; Mohapatra, Senjanovic, 1979;

Type-II seesaw

$\Delta: S U(3)_{c} \times \operatorname{SU}(2)_{w} \times U(1) y \sim(1,3,2)$
Lazarides, 1980; Mohapatra, Senjanovic, 1980

## 3 possible portals

Type-III seesaw


$$
\Sigma=\left(\Sigma^{+}, \Sigma^{0}, \Sigma^{-}\right)
$$

$\Sigma_{\mathrm{R}}: \operatorname{SU}(3)_{\mathrm{c}} \times \mathrm{SU}(2)_{\mathrm{w}} \times \mathrm{U}(1)_{\mathrm{Y}} \sim(1,3,0)$ Foot, Lew, He, Joshi, 1989; Ma, 1998

## Why are neutrinos light? (Type-I) Seesaw Mechanism

- Adding the right-handed neutrinos:

$$
\begin{gathered}
\left(\begin{array}{cc}
v_{L} & v_{R}
\end{array}\right)\left(\begin{array}{cc}
0 & m_{D} \\
m_{D} & M_{R}
\end{array}\right)\binom{v_{L}}{v_{R}} \\
m_{v} \sim m_{\text {light }} \sim \frac{m_{D}^{2}}{M_{R}} \ll m_{D} \\
m_{\text {heavy }} \sim M_{R} \\
\text { For } \quad m_{v_{3}} \sim \sqrt{\Delta m_{\text {atm }}^{2}} \\
\text { If } \quad m_{D} \sim m_{t} \sim 180 \mathrm{GeV} \\
\quad \Longrightarrow \mathrm{M}_{\mathrm{R}} \sim 10^{15} \mathrm{GeV}(\mathrm{GUT}!!)
\end{gathered}
$$



## Grand Unification Naturally Accommodates Seesaw


origin of the heavy scale $\Rightarrow U(1)_{B-L}$
exotic mediators $\Rightarrow$ predicted in many GUT theories, e.g. $\mathrm{SO}(10)$

$$
\begin{aligned}
16 & =(3,2,1 / 6) \sim\left[\begin{array}{lll}
u & u & u \\
d & d & d
\end{array}\right] \\
& +\left(3^{*}, 1,-2 / 3\right) \sim\left(u^{c} u^{c} u^{c}\right) \\
& +\left(3^{*}, 1,1 / 3\right) \sim\left(d^{c} d^{c} d^{c}\right) \\
& +(1,2,-1 / 2) \sim\left[\begin{array}{l}
v \\
e
\end{array}\right] \\
& +(1,1,1) \sim e^{c} \\
& +(1,1,0) \sim v^{c}
\end{aligned}
$$

## Neutrino Mass beyond the SM



Type-I seesaw

$N_{R}: S U(3)_{c} \times S U(2)_{w} x U(1) Y \sim(1,1,0)$
Minkowski, 1977; Yanagida, 1979; Glashow, 1979; Gell-mann, Ramond, Slansky,1979; Mohapatra, Senjanovic, 1979;

## 3 possible portals

Type-III seesaw


$$
\Sigma=\left(\Sigma^{+}, \Sigma^{0}, \Sigma^{-}\right)
$$

$\Sigma_{\mathrm{R}}: \operatorname{SU}(3)_{\mathrm{c}} \times \mathrm{SU}(2)_{\mathrm{w}} \times \mathrm{U}(1)_{\mathrm{Y}} \sim(1,3,0)$ Foot, Lew, He, Joshi, 1989; Ma, 1998

## Low Scale Seesaws <br> $$
\begin{aligned} & m_{v} \sim\left(\Delta \mathrm{~m}^{2} \mathrm{~atm}\right)^{1 / 2} \sim 0.1 \mathrm{eV} \text { with } v \sim 100 \mathrm{GeV}, \lambda \sim 10^{-6} \\ & \Rightarrow M \sim 10^{2} \mathrm{GeV} \end{aligned}
$$

- New particles:
- Type I seesaw: generally decouple from collider experiments
- Type II seesaw: $\Delta^{++} \rightarrow e^{+} e^{+}, \mu^{+} \mu^{+}, \tau^{+} \tau^{+}$
- Type III seesaw: observable displaced vertex, dark matter candidate
- inverse seesaw: non-unitarity effects
- radiative mass generation: model dependent - singly/doubly charged SU(2) singlet, even colored scalars in loops, dark matter candidate
- New interactions:
- LR symmetric model: $W_{R}$
- R parity violation: $\tan ^{2} \theta_{\mathrm{atm}} \simeq \frac{B R\left(\tilde{\chi}_{1}^{0} \rightarrow \mu^{ \pm} W^{\mp}\right)}{B R\left(\tilde{\chi}_{1}^{0} \rightarrow \tau^{ \pm} W^{\mp}\right)}$


## Cautions!!! Is it really the $v_{R}$ in Type I seesaw?



Expanded view of the region:
$40 \mathrm{GeV}<\mathrm{m}_{\mathrm{N}}<250 \mathrm{GeV}$

RH neutrino production thru active-sterile mixing:


$$
\propto V=\frac{m_{D}}{M_{R}} \sim \frac{10^{-4} \mathrm{GeV}}{100 \mathrm{GeV}}=10^{-6}
$$

RH neutrino relevant for $v$ mass generation

$$
\Rightarrow\left|V_{\mu N}\right|^{2}=10^{-12}
$$

unless extremely fine-tuned

## Higher Dimensional Neutrino Masses



$$
m_{\nu} \propto \epsilon \cdot\left(\frac{1}{16 \pi^{2}}\right)^{n} \cdot\left(\frac{v}{\Lambda}\right)^{d-5} \cdot \frac{v^{2}}{\Lambda}
$$

Babu, Leung (2001); de Gouvea, Jenkins (2007);
e.g. at dim-7, 1-loop

$$
O_{1}^{\prime}=L L H H\left(H^{\dagger} H\right)
$$



For an excellent review on Radiative Neutrino Mass Generation: Cai, Herrero-García, Schmidt, Vincente,Volkas, I706.08524

## Higher Dimensional Neutrino Masses



Anamiati, Castillo-Felisola, Fonseca, Helo, Hirsch (2019)

$$
m_{\nu} \propto \epsilon \cdot\left(\frac{1}{16 \pi^{2}}\right)^{n} \cdot\left(\frac{v}{\Lambda}\right)^{d-5} \cdot \frac{v^{2}}{\Lambda}
$$

Babu, Leung (200I); de Gouvea, Jenkins (2007);
e.g. at dim-7, 1-loop

$$
O_{1}^{\prime}=L L H H\left(H^{\dagger} H\right)
$$



For an excellent review on Radiative Neutrino Mass Generation: Cai, Herrero-García, Schmidt, Vincente,Volkas, I706.08524

Need a lot of work to have realistic mixing

# What if neutrinos <br> are Dirac? 

## Small Masses - Dirac Neutrinos

## Randall-Sundrum <br> warped extra dimensions



SM (us)
Grossman, Neubert (2000); Huber, Shafi (200I)


Cheng, Li (1978); .....

## Clockwork Seesaw Mechanism

S.C. Park, C.S. Shin (2017); Hong, Kurup, Perelstein (2019); Babu, Saad (2020) ...


Figure from Babu, Saad (2020)

## SUSY Breaking

Arkani-Hamed, Hall, Murayama, Tucker-Smith, Weiner (200I)

Figure from Babu, He (1988);
For clarifications of radiative Dirac neutrino mass generation: see e.g. Farzan, Pascoli, Schmidt (20I2)


$$
Y_{\nu} \sim \frac{m 3 / 2}{M_{\mathrm{P}}} \sim \frac{\mu}{M_{\mathrm{P}}}
$$



## Flavor Structure - Anarchy

- there are no parametrically small
numbers

Hall, Murayama, Weiner (2000); de Gouvea, Murayama (2003);

- large mixing angle, near mass degeneracy statistically preferred
- UV theory prediction can resemble anarchy
- warped extra dimensions
- heterotic string models: O(100) RH neutrinos

Buchmüller, Hamaguchi, Lebedev, Ramos-Sánchez, Ratz (2007)

Feldstein, Klemm (2012)




$$
m_{v} \sim \frac{v^{2}}{M_{*}}>M_{*} \sim \frac{M_{\mathrm{GUT}}}{10 \ldots 100}
$$



## Flavor Structure from Symmetries

## Grand Unified Theories: GUT symmetry

## Quarks - Leptons

Family Symmetry:

[Figure Credit: arXiv:1301.1340]

$$
\text { e-family }- \text { muon-family } \rightarrow \text { tau-family }
$$

## Symmetry Relations

Symmetry $\Rightarrow$ relations among parameters $\Rightarrow$ reduction in number of fundamental parameters

## Symmetry Relations

# Symmetry $\Rightarrow$ relations among parameters $\Rightarrow$ reduction in number of fundamental parameters 

Symmetry $\Rightarrow$ experimentally testable correlations among physical observables

## Testing Symmetry Relations $\Rightarrow$ Precision

Symmetry $\Rightarrow$ experimentally testable correlations among physical observables

## mass hierarchy

CP phase

$$
0 \nu \beta \beta
$$

cLFV

## mixing angles

Testing correlations $\Rightarrow$ Precision

## Non-Abelian Discrete Flavor Symmetries

- Large neutrino mixing motivates discrete flavor symmetries
- $\mathrm{A}_{4}$ (tetrahedron)
- $T^{\prime}$ (double tetrahedron)
- $\mathrm{S}_{3}$ (equilateral triangle)
- $\mathrm{S}_{4}$ (octahedron, cube)
- $\mathrm{A}_{5}$ (icosahedron, dodecahedron)
- $\Delta_{27}$
- Q6
-.....

[Eligio Lisi for NOW2008 ]



## Tri-bimaximal Neutrino Mixing

- Latest Global Fit (3б)

$$
\begin{gathered}
\sin ^{2} \theta_{23}=0.437(0.374-0.626) \\
\sin ^{2} \theta_{12}=0.308(0.259-0.359) \\
\sin ^{2} \theta_{13}=0.0234(0.0176-0.0295)
\end{gathered}
$$

$$
\begin{aligned}
& {\left[\theta^{\mathrm{lep}} 23 \sim 49.2^{\circ}\right]} \\
& {\left[\theta^{\mathrm{lep}} 12 \sim 33.4^{\circ}\right]} \\
& {\left[\theta^{\mathrm{le}}{ }_{13} \sim 8.57^{\circ}\right]}
\end{aligned}
$$

- Tri-bimaximal Mixing Pattern

Harrison, Perkins, Scott (1999)

$$
U_{T B M}=\left(\begin{array}{ccc}
\sqrt{2 / 3} & \sqrt{1 / 3} & 0 \\
-\sqrt{1 / 6} & \sqrt{1 / 3} & -\sqrt{1 / 2} \\
-\sqrt{1 / 6} & \sqrt{1 / 3} & \sqrt{1 / 2}
\end{array}\right) \quad \begin{array}{ll}
\sin ^{2} \theta_{\mathrm{atm}, \mathrm{TBM}}=1 / 2 & \sin ^{2} \theta_{\odot, \mathrm{TBM}}=1 / 3 \\
\sin \theta_{13, \mathrm{TBM}}=0 .
\end{array}
$$

## Neutrino Mass Matrix from A4

- Imposing A4 flavor symmetry on the Lagrangian
- A4 spontaneously broken by flavon fields


## 2 free parameters

$$
M_{\nu}=\frac{\lambda v^{2}}{M_{x}}\left(\begin{array}{ccc}
2 \xi_{0}+u & -\xi_{0} & -\xi_{0} \\
-\xi_{0} & 2 \xi_{0} & u-\xi_{0} \\
-\xi_{0} & u-\xi_{0} & 2 \xi_{0}
\end{array}\right)
$$

## relative strengths $\Rightarrow$ CG's

- always diagonalized by TBM matrix, independent of the two free parameters

$$
U_{\mathrm{TBM}}=\left(\begin{array}{ccc}
\sqrt{2 / 3} & 1 / \sqrt{3} & 0 \\
-\sqrt{1 / 6} & 1 / \sqrt{3} & -1 / \sqrt{2} \\
-\sqrt{1 / 6} & 1 / \sqrt{3} & 1 / \sqrt{2}
\end{array}\right)
$$

Neutrino Mixing
Angles from Group
Theory

## Modular Flavor Symmetries

- Extra dimensional origin of non-Abelian discrete symmetries
- Modular symmetries Atarelli, Feruglio (2005); Feruglio (2017), .....
- Inspired by string theories
- Imposing modular invariance $Y=Y(\tau)$
- Highly predictive models



## A Toy Modular $\mathrm{A}_{4}$ Model

- Weinberg Operator $\quad \mathscr{W}_{v}=\frac{1}{\Lambda}\left[\left(H_{u} \cdot L\right) Y\left(H_{u} \cdot L\right)\right]_{1}$
- Traditional A4 Flavor Symmetry
- Yukawa Coupling $Y \rightarrow$ Flavon VEVs (A4 triplet, 6 real parameters)

$$
Y \rightarrow\langle\phi\rangle=\left(\begin{array}{c}
a \\
b \\
c
\end{array}\right) \Rightarrow m_{v}=\frac{v_{u}^{2}}{\Lambda}\left(\begin{array}{ccc}
2 a & -c & -b \\
-c & 2 b & -a \\
-b & -a & 2 c
\end{array}\right)
$$

- Modular A4 Flavor Symmetry
- Yukawa Coupling $Y \rightarrow$ Modular Forms (A4 triplet, 2 real parameters)

$$
Y \rightarrow\left(\begin{array}{l}
Y_{1}(\tau) \\
Y_{2}(\tau) \\
Y_{3}(\tau)
\end{array}\right) \quad \Rightarrow \quad m_{v}=\frac{v_{u}^{2}}{\Lambda}\left(\begin{array}{lll}
2 Y_{1}(\tau) & -Y_{3}(\tau) & -Y_{2}(\tau) \\
-Y_{3}(\tau) & 2 Y_{2}(\tau) & -Y_{1}(\tau) \\
-Y_{2}(\tau) & -Y_{1}(\tau) & 2 Y_{3}(\tau)
\end{array}\right)
$$

## A Toy Modular $A_{4}$ Model

- Input Parameters:

$$
\tau=0.0111+0.9946 i
$$

$$
v_{u}^{2} / \Lambda
$$

- Predictions:

$$
\begin{array}{lll}
\frac{\Delta m_{\text {sol }}^{2}}{\left|\Delta m_{\text {atm }}^{2}\right|}=0.0292 & & \\
\sin ^{2} \theta_{12}=0.295 & \sin ^{2} \theta_{13}=0.0447 & \sin ^{2} \theta_{23}=0.651 \\
\frac{\delta_{C P}}{\pi}=1.55 & \frac{\alpha_{21}}{\pi}=0.22 & \frac{\alpha_{31}}{\pi}=1.80 .
\end{array}
$$

$$
m_{1}=4.998 \times 10^{-2} \mathrm{eV} \quad m_{2}=5.071 \times 10^{-2} \mathrm{eV} \quad m_{3}=7.338 \times 10^{-4} \mathrm{eV}
$$

## Modular Invariance Beyond Neutrino Flavor



## CP Violation

## Origin of CP Violation

- CP violation $\Leftrightarrow$ complex mass matrices
$\bar{U}_{R, i}\left(M_{u}\right)_{i j} Q_{L, j}+\bar{Q}_{L, j}\left(M_{u}^{\dagger}\right)_{j i} U_{R, i} \xrightarrow{\mathfrak{C P}} \bar{Q}_{L, j}\left(M_{u}\right)_{i j} U_{R, i}+\bar{U}_{R, i}\left(M_{u}\right)_{i j}^{*} Q_{L, j}$
- Conventionally, CPV arises in two ways:
- Explicit CP violation: complex Yukawa coupling constants Y
- Spontaneous CP violation: complex scalar VEVs <h>

- Complex CG coefficients in certain discrete groups $\Rightarrow$ explicit CP violation
- CPV in quark and lepton sectors purely from complex CG coefficients

CG coefficients in non-Abelian discrete symmetries $\Rightarrow$ relative strengths and phases in entries of Yukawa matrices
$\Rightarrow$ mixing angles and phases (and mass hierarchy)

## Group Theoretical Origin of CP Violation

## Basic idea

Discrete
symmetry G


## Group Theoretical Origin of CP Violation




## Outlook

## Acknowledgements



## History of the Universe




## Outlook

- Fundamental origin of fermion mass \& mixing patterns still unknown
- It took decades to understand the gauge sector of SM
- Uniqueness of Neutrino masses offers exciting opportunities to explore BSM Physics
- Many NP frameworks; addressing other puzzles
- Early Universe (baryogengesis thru leptogenesis, non-thermal relic neutrinos)
- New Tools/insights:
- Non-Abelian Discrete Flavor Symmetries $\Rightarrow$ origin of CP
- Deep connection between outer automorphisms and CP
- Modular Flavor Symmetries
- Enhanced predictivity of flavor models
- Possible connection to more fundamental physics




## Example: $\operatorname{SU(5)~Compatibility~} \Rightarrow \mathrm{T}^{\prime}$ Family Symmetry

- Double Tetrahedral Group T': double covering of A4
M.-C.C, K.T. Mahanthappa $(2007,2009)$
- Symmetries $\Rightarrow 10$ parameters in Yukawa sector $\Rightarrow 22$ physical observables
- Symmetries $\Rightarrow$ correlations among quark and lepton mixing parameters

$$
\theta_{13} \simeq \theta_{c} / 3 \sqrt{2} \leftarrow \begin{gathered}
c G^{\prime} \text { of } \\
\mathrm{sU}(5) \& T^{\prime}
\end{gathered}
$$

no free
parameters!


## Group Theoretical Origin of CP Violation

## complex CGs $\Rightarrow$ G and physical CP transformations do not commute



$$
\begin{aligned}
& \Phi(x) \stackrel{\widetilde{C_{P}}}{\longmapsto} U_{\mathrm{CP}} \Phi^{*}(\mathcal{P} x) \\
& \rho_{r_{i}}(u(g))=U_{r_{i}} \rho_{r_{i}}(g)^{*} U_{r_{i}}^{\dagger} \quad \forall g \in G \text { and } \forall i \\
& \begin{array}{l}
\text { un has to be a class-inverting, } \\
\quad \text { involutory automorphism of } \mathrm{G} \\
\Rightarrow \begin{array}{l}
\text { non-existence of such automorphism } \\
\quad \text { in certain groups }
\end{array} \\
\Rightarrow \text { calculable physical CP violation in } \\
\text { generic setting }
\end{array}
\end{aligned}
$$

examples: $\mathrm{T}_{7}, \Delta(27), \ldots .$.

