

KIRILL MELNIKOV

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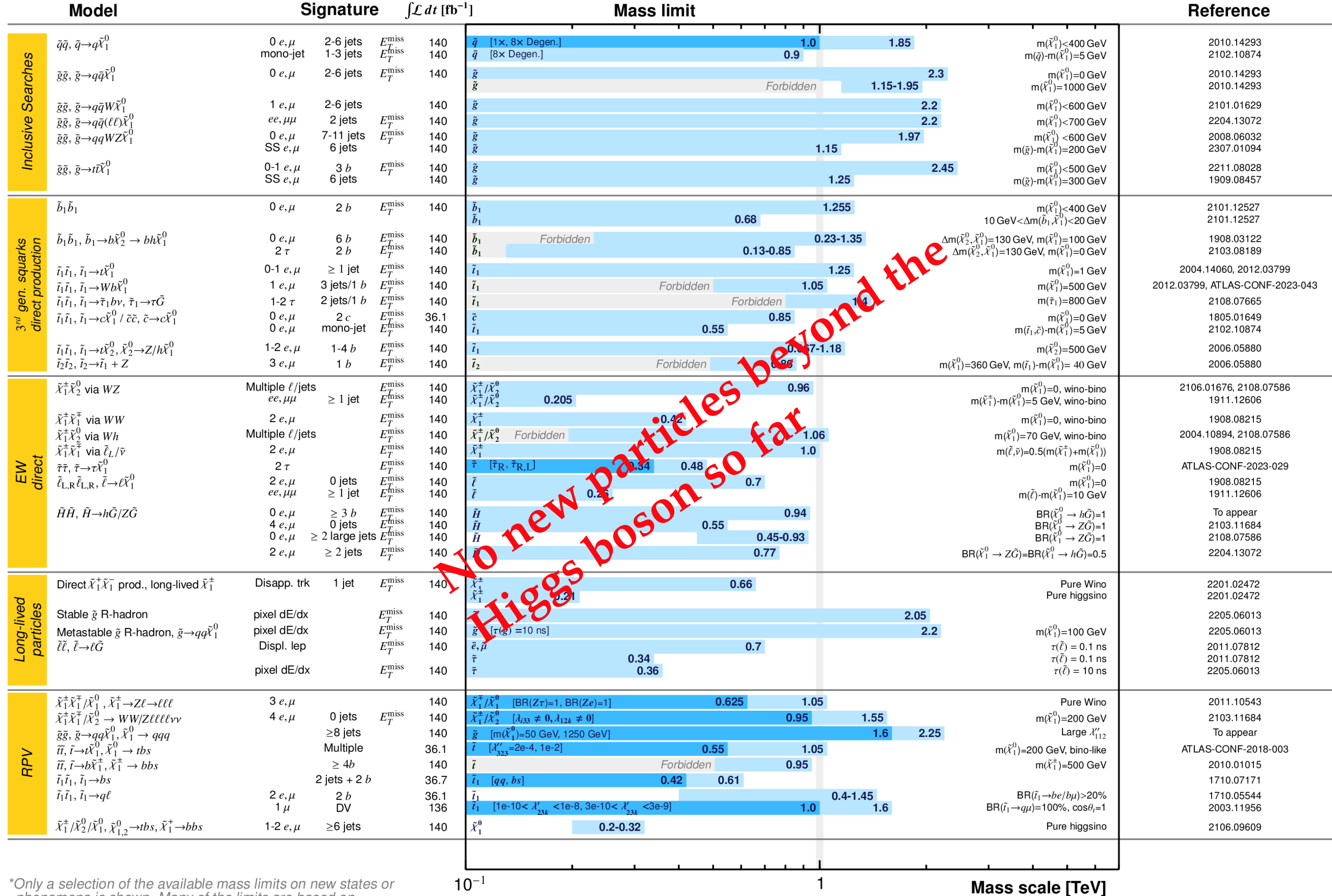
# PRECISION PHYSICS AT THE LHC

Lake Louise Winter Institute 2024

# A NIGHTMARE SCENARIO FOR THE LHC?

ATLAS SUSY Searches\* - 95% CL Lower Limits  
August 2023

ATLAS Preliminary  
 $\sqrt{s} = 13$  TeV



\*Only a selection of the available mass limits on new states or phenomena is shown. Many of the limits are based on simplified models, c.f. refs. for the assumptions made.

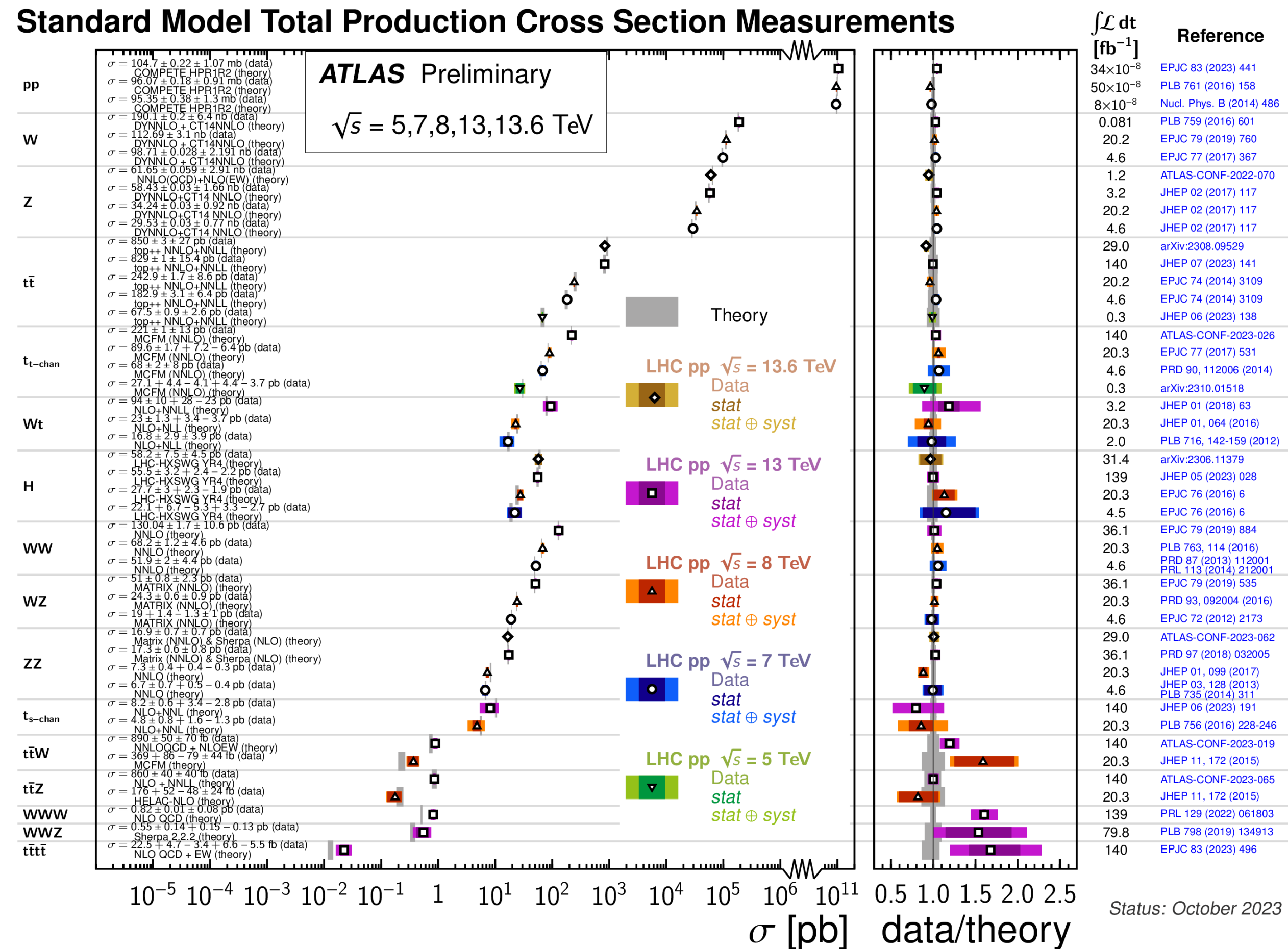
10<sup>-1</sup> 1 Mass scale [TeV]

No new particles beyond the Higgs boson so far

# AN OPPORTUNITY TO TRY SOMETHING NEW

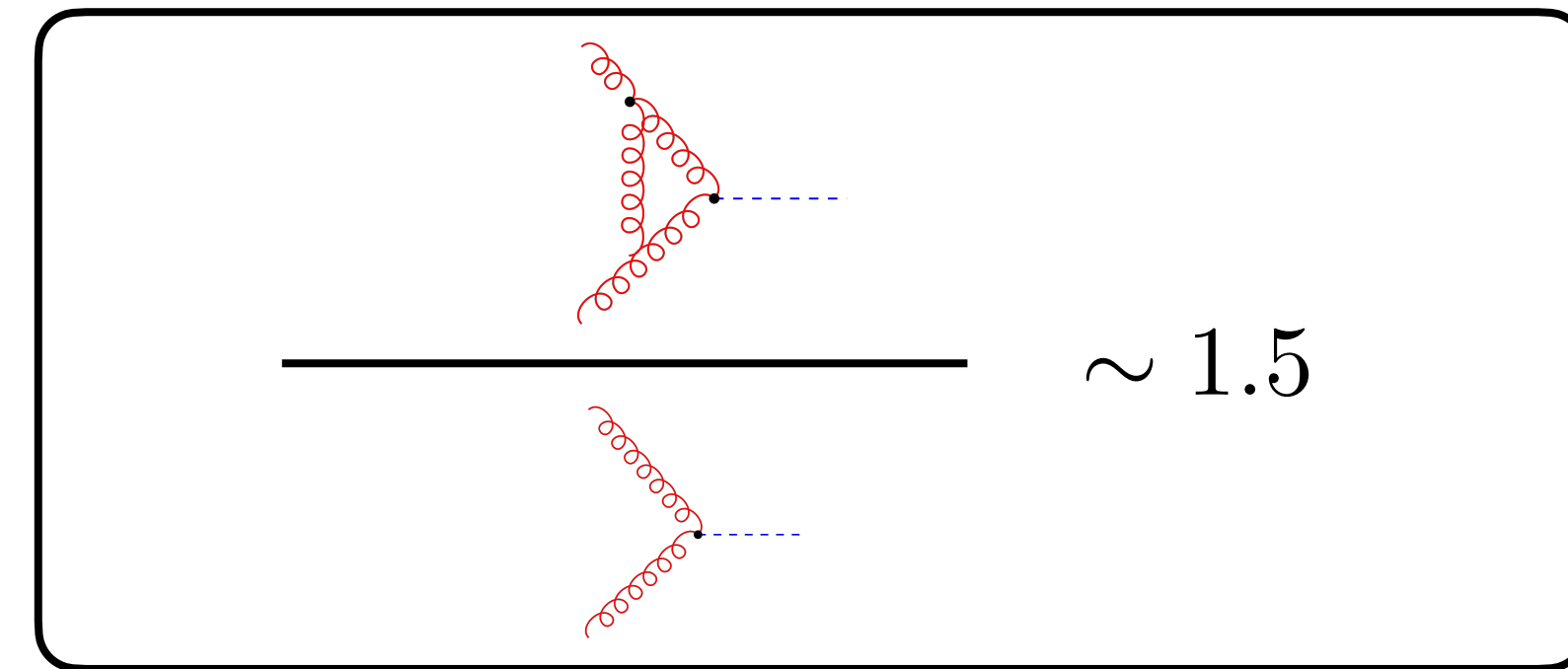
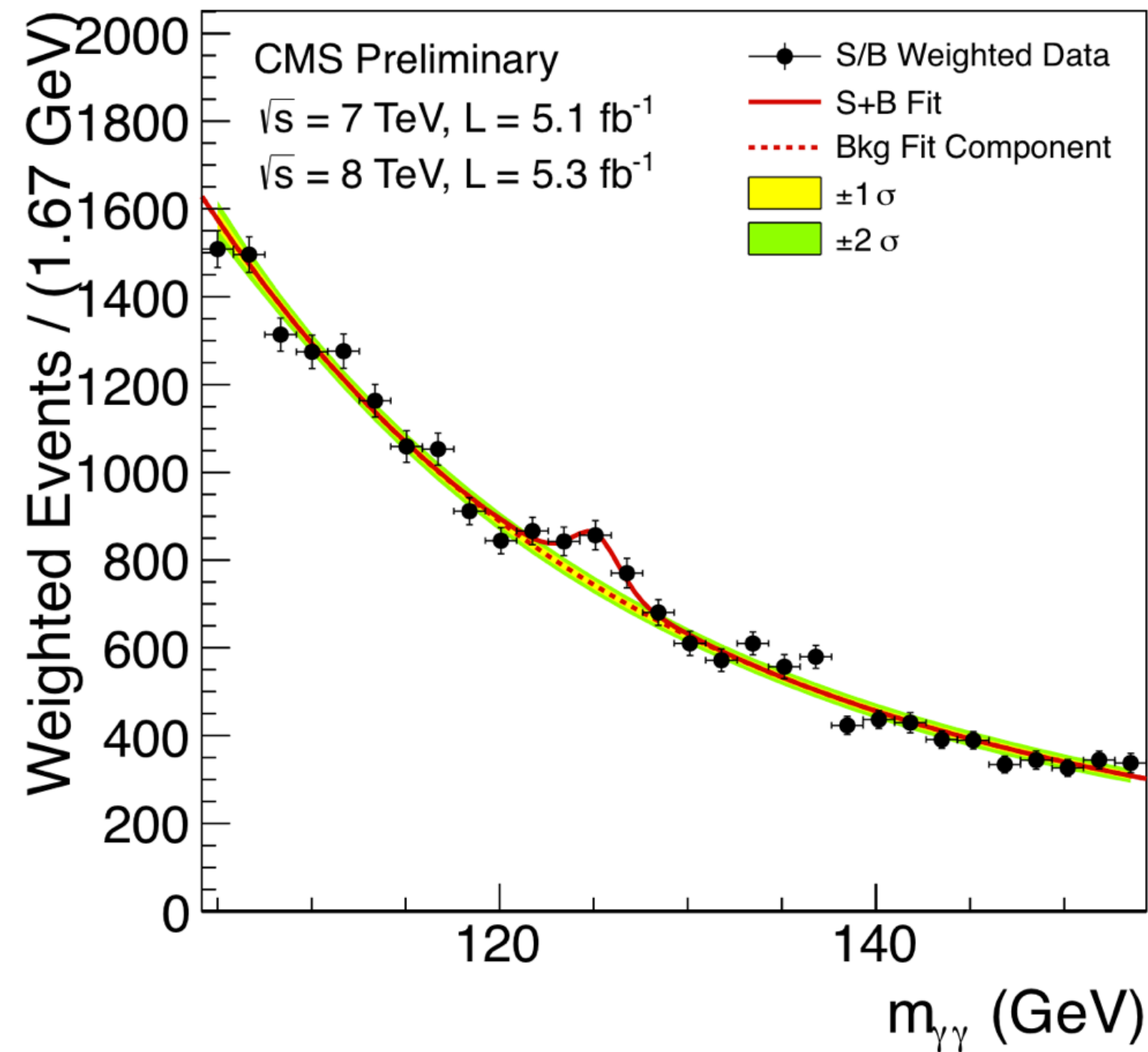
Absence of clear signal of physics beyond the Standard Model (SM) puts an added value on precision studies of SM processes and suggests to focus on a systematic discovery-through-precision program at the LHC.

If we have a solid theory that allows us to describe with confidence the expected outcomes of hard LHC collisions, differences between predictions and measurements can be attributed to effects of physics beyond the SM.



# AN OPPORTUNITY TO TRY SOMETHING NEW

One can not emphasize enough that the actual usefulness of this statement depends on the quality of the theory that one uses. If the Higgs boson production rate in hadron collisions would have been measured in the early 1990s before QCD radiative corrections to Higgs production in gluon fusion were computed, it would have been claimed that we observe physics beyond the Standard Model ...



48.58 pb =	16.00 pb	(+32.9%)	(LO, rEFT)
	+ 20.84 pb	(+42.9%)	(NLO, rEFT)
	- 2.05 pb	(-4.2%)	(( <i>t, b, c</i> ), exact NLO)
	+ 9.56 pb	(+19.7%)	(NNLO, rEFT)
	+ 0.34 pb	(+0.7%)	(NNLO, $1/m_t$ )
	+ 2.40 pb	(+4.9%)	(EW, QCD-EW)
	+ 1.49 pb	(+3.1%)	(N <sup>3</sup> LO, rEFT)

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger

The good news is that we are definitely passed comparisons within “factors of two”. Nowadays, when we say “precision LHC physics”, we talk about [systematic comparisons between predictions and measurements at a few percent level...](#)

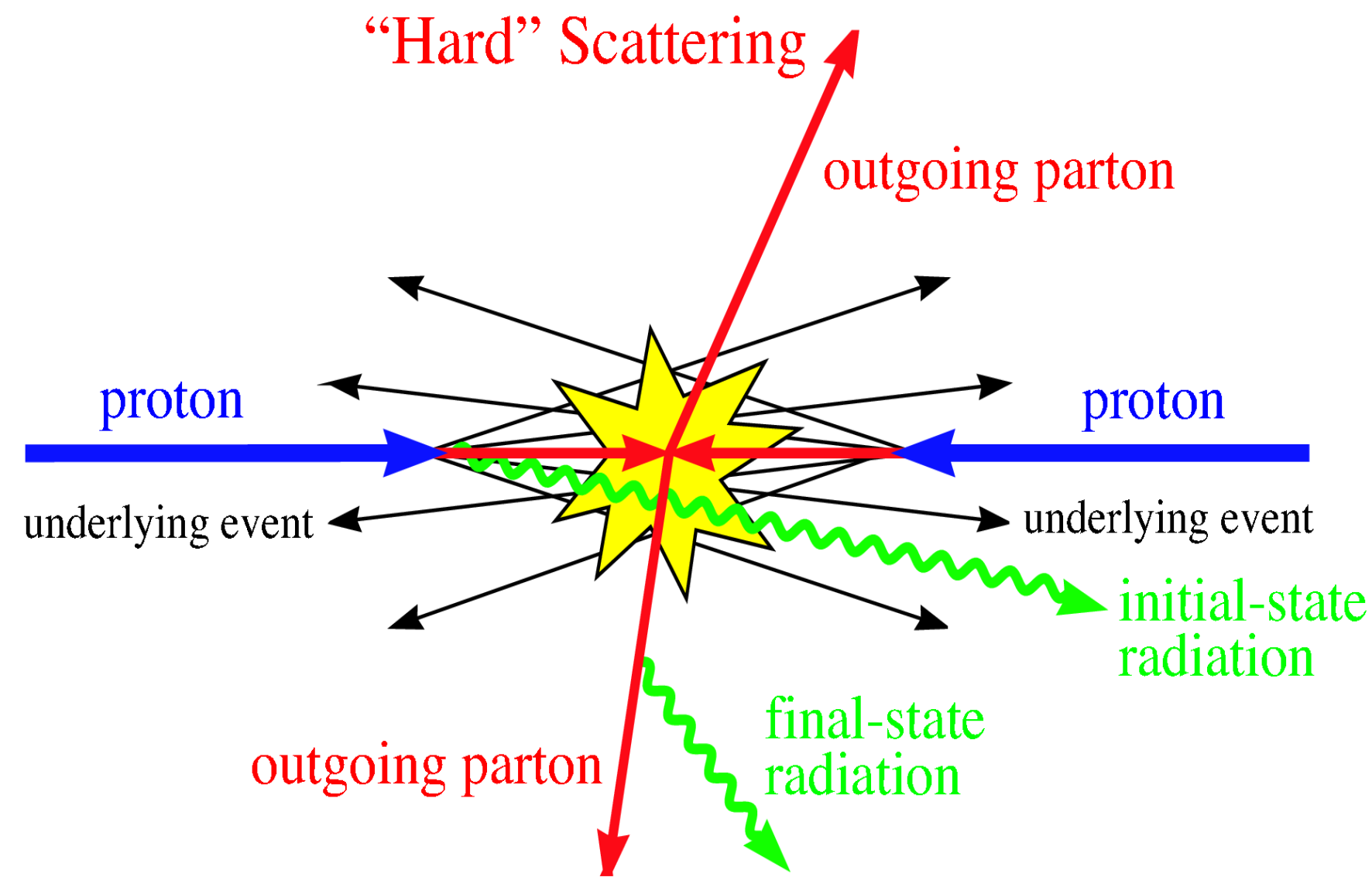
# PROTONS VS QUARKS AND GLUONS?

If we want to do precision physics at the LHC, we have to explain why it is possible to describe collisions of **protons** starting from a Lagrangian that contains quarks and gluons. The explanation rests on 3 pillars.

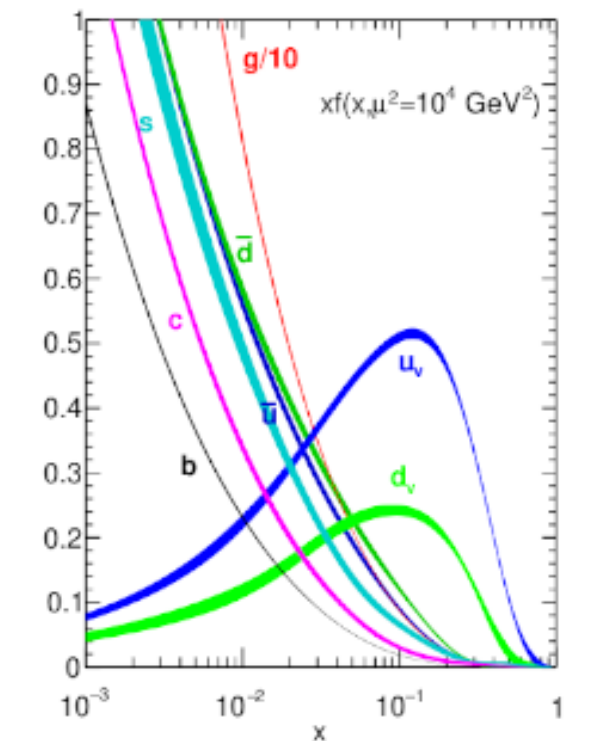
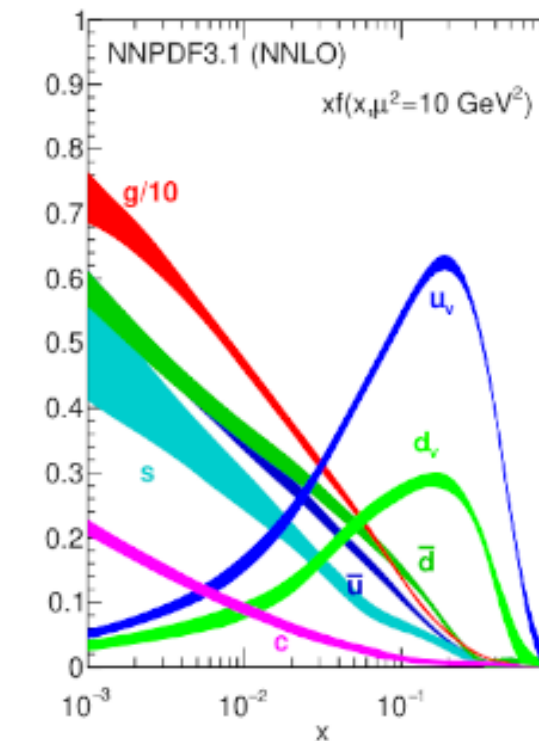
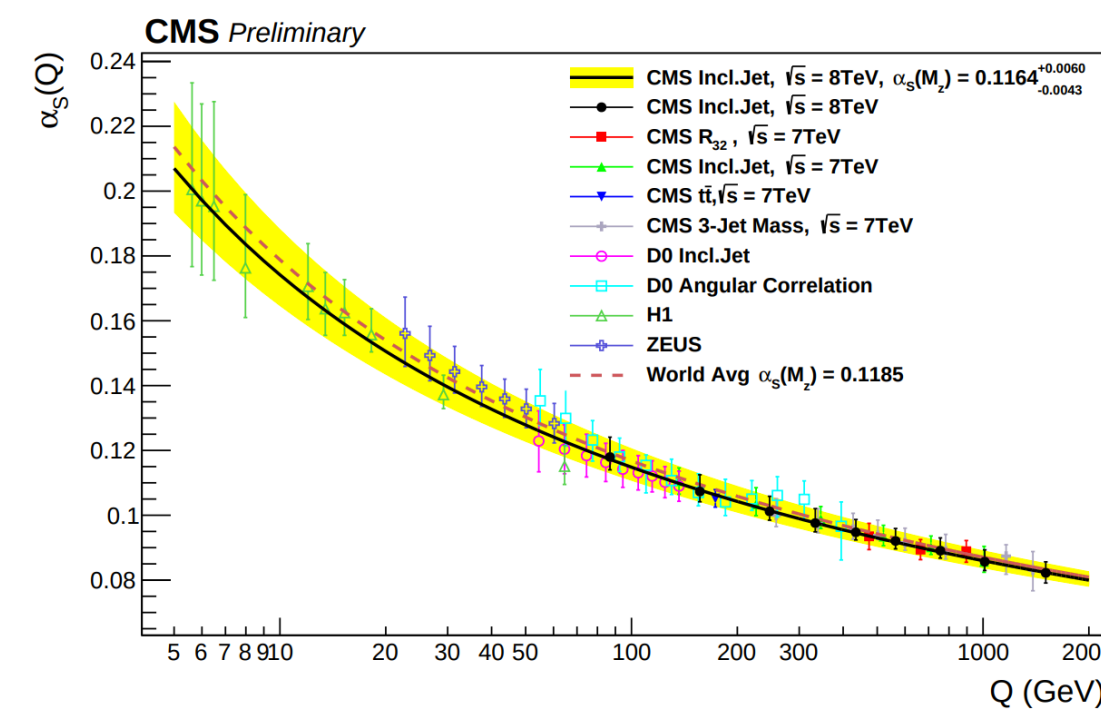
1) **Asymptotic freedom** which implies that the QCD coupling constant is small at short distances;

2) **Collinear factorization** which implies that only quark/gluon momenta distributions in a proton are relevant for the description of hard collisions;

3) **“Soft fragmentation of quarks and gluons into observable hadrons”** — allows to define cross sections in terms of energy flows (jets) associated with primary quarks and gluons.



$$\mathcal{L}_{\text{QCD}} = \sum \bar{q}_j \left( i\hat{D} - m_j \right) q_j - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}$$



$$d\sigma_{\text{hard}} = \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{ij}(x_1, x_2, \{p_{\text{fin}}\}) O_J(\{p_{\text{fin}}\})$$

## FIXED ORDERS VS POWER CORRECTIONS

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The above considerations work only up to power corrections which are proportional to the ratio of the non-perturbative parameter of QCD  $\Lambda_{\text{QCD}}$  and a typical hard scale in a given process.

$$d\sigma_{\text{hard}} = \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{ij}(x_1, x_2, \{p_{\text{fin}}\}) O_J(\{p_{\text{fin}}\}) \left(1 + \mathcal{O}(\Lambda_{\text{QCD}}^n/Q^n)\right), \quad n \geq 1$$

We can use this formula to estimate how far we can go with the perturbative expansion in QCD before we have to start paying attention to non-perturbative effects.

$$d\sigma_{ij} = d\sigma_{ij,\text{LO}} \left(1 + \alpha_s \Delta_{ij,\text{NLO}} + \alpha_s^2 \Delta_{ij,\text{NNLO}} + \dots\right)$$

$$N_c \left(\frac{\alpha_s}{\pi}\right)^2 \sim \frac{\Lambda_{\text{QCD}}}{Q}, \quad \Lambda_{\text{QCD}} \sim 0.3 \text{ GeV}, \quad Q \sim 30 \text{ GeV}$$

We find that [two or three orders in the perturbative expansion in QCD](#) (NNLO and/or N3LO) can be studied without worrying about non-perturbative effects; however, it does not make sense to continue with even higher order pQCD computations without addressing generic non-perturbative corrections (provided that  $n=1$ ) in the factorization formula.

[Conversely, this discussion implies that on the theory side it should be possible to reach a \(few\) percent precision for generic LHC observables. This is the goal of the LHC precision program. Achieving this requires solid NNLO/N3LO theory of hard parton collisions in the context of perturbative QCD, supplemented with reliable knowledge of parton distribution functions.](#)

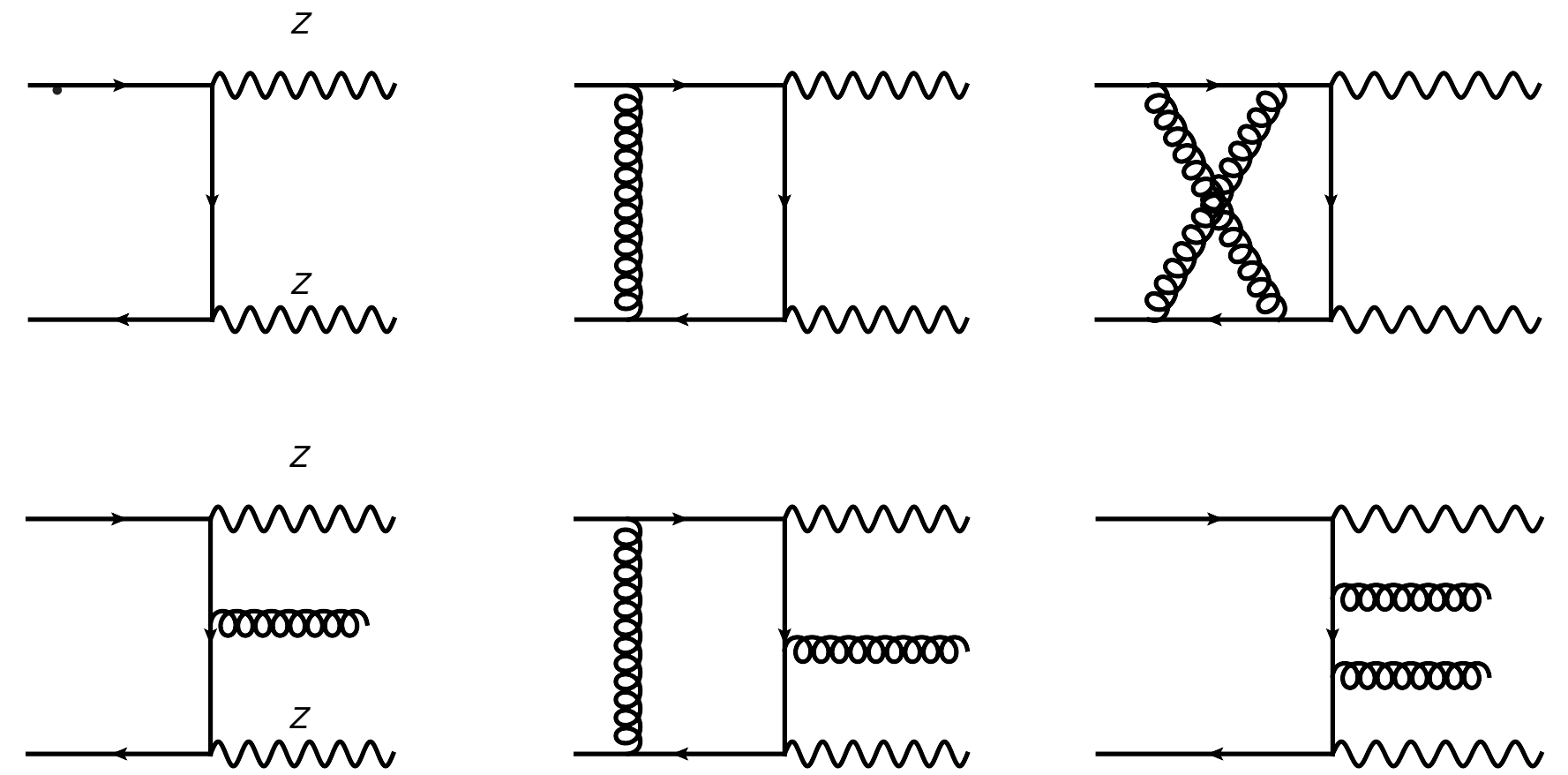
# FIXED ORDER

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To compute  $\mathcal{O}(\alpha_s^k)$  correction to a process with N partons at leading order we require

$$d\sigma_{ij} = d\sigma_{ij,\text{LO}} \left( 1 + \alpha_s \Delta_{ij,\text{NLO}} + \alpha_s^2 \Delta_{ij,\text{NNLO}} + \dots \right),$$

- 1) a k-loop correction to N-parton scattering amplitude;
- 2) a (k-1) -loop correction to (N+1)-parton scattering amplitude;
- 3) a (k-2) -loop correction to (N+2)-parton scattering amplitude;
- .....
- 4) a 0 -loop (tree) (N+k)-parton scattering amplitude.



Once amplitude are known, they have to be integrated over phase spaces subject to kinematic constraints.

These integrations are complex since, when taken separately, the individual contributions exhibit singularities (soft and collinear) which only disappear once the different contributions are combined. This requires the development of the so-called subtraction or slicing schemes that allow one to compute partonic cross sections.

$$\int d\Phi_n V + \int d\Phi_{n+g} R = \text{finite}$$

# PRECISION PHYSICS REQUIRES HIGHLY-DEVELOPED THEORY

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An opportunity to perform high-precision physics studies at the LHC, with an idea of using gains in precision for a potential discovery of New Physics, provides a strong motivation for continuous improvements in the theory of hard hadron collisions. As the result, during the past decade, we have witnessed a very impressive progress in that direction.

Modern NLO computations describe realistic final states and include quantum interferences. They incorporate electroweak corrections and are often matched to parton showers allowing one to simulate realistic events.

NNLO QCD computations have become available for many interesting processes. Results are obtained for realistic fiducial cross sections. Typically, the agreement between theory and experiment is improved once NNLO QCD theory is employed.

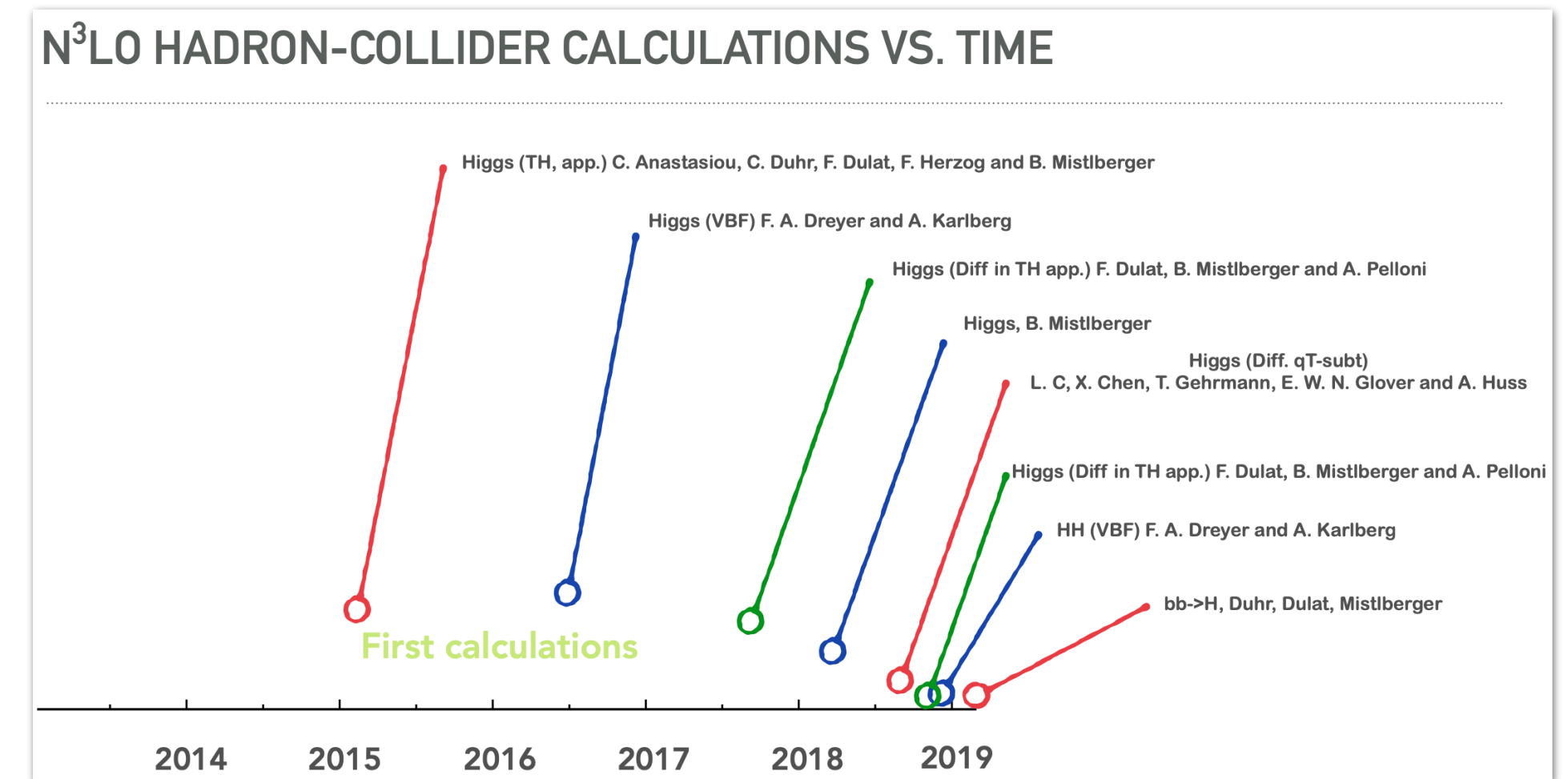
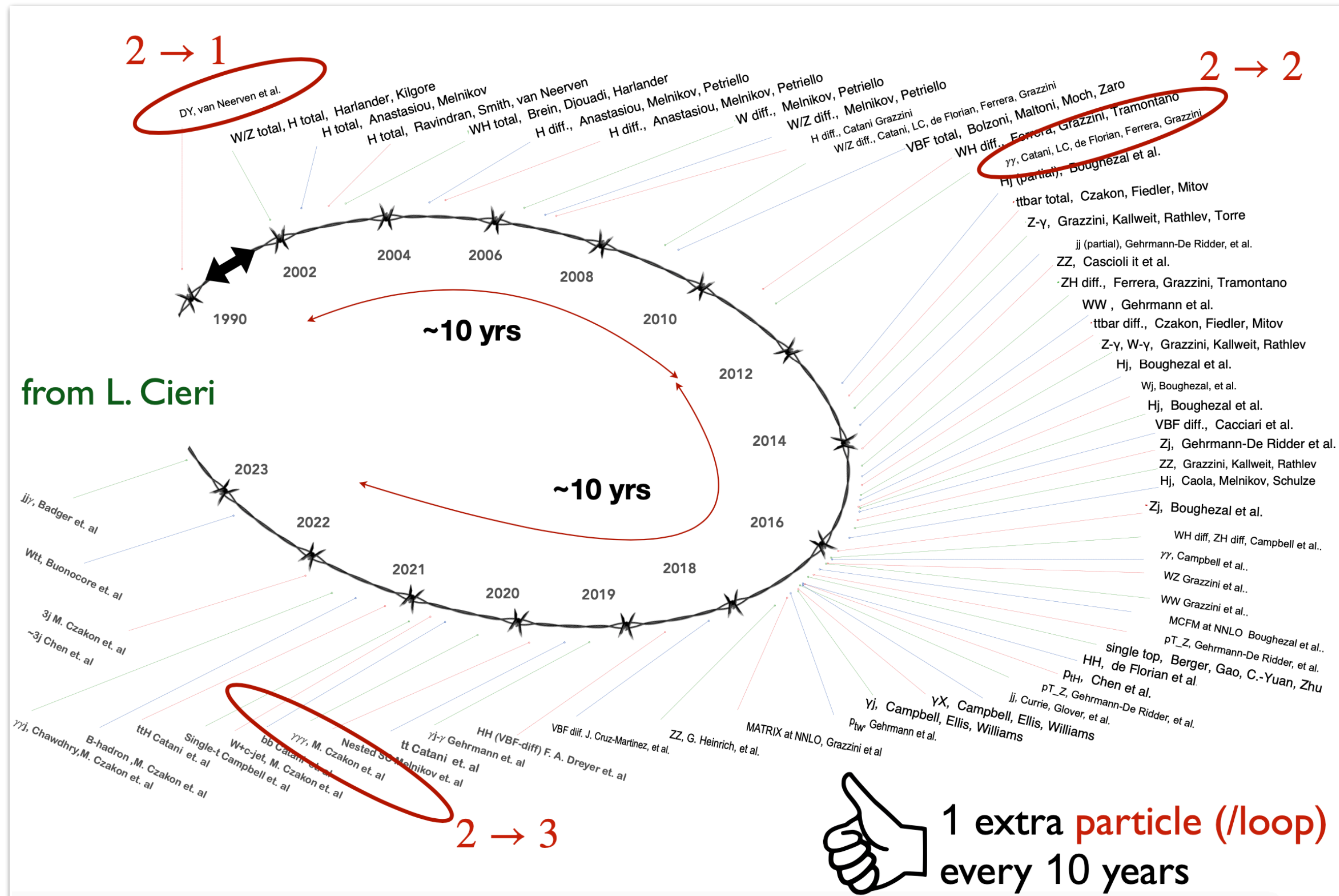
First N3LO QCD computations appeared (Higgs cross section and rapidity distribution in gluon fusion, Drell-Yan cross section and rapidity distributions). These results stress-test applications of perturbative QCD for hadron collisions.



# STEADY PROGRESS WITH PERTURBATIVE COMPUTATIONS

A graphic illustration of what I just said shows very impressive progress with NNLO and N3LO computations, as well as the current multiplicity frontiers ( $2 \rightarrow 4$  for NNLO and  $2 \rightarrow 1$  for N3LO computations).

These “frontiers” occur for different reasons: lack of understanding of how to efficiently compute loops with large number of external partons (NNLO) and lack of understanding how subtractions of infra-red singularities should be organized (N3LO).



# ULTRA-HIGH PRECISION MEASUREMENTS AT THE LHC

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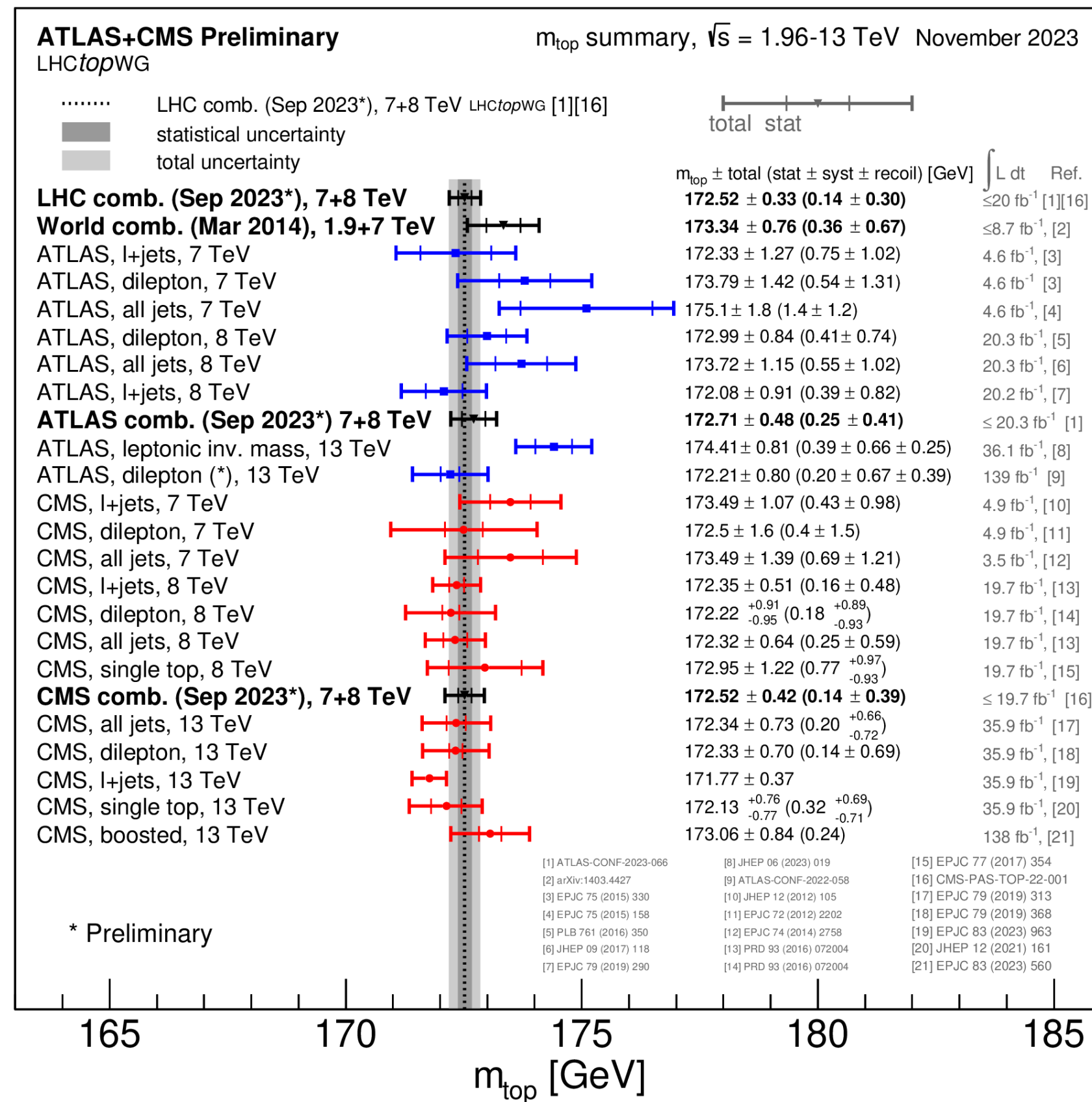
Overcoming these technical challenges is very important to ensure further advances in the precision physics program at the LHC. We have seen in the past that such advances often force us to develop a deeper understanding of theoretical foundations of perturbative quantum field theories in general and of the Standard Model and QCD in particular.

Unfortunately, explaining this would have meant giving a very technical talk which is not ideal. Instead, [I decided to talk about poster children of high-precision physics program at the LHC namely measurements of the top quark mass, the W boson mass and the strong coupling constant, that one can call “ultra-precise”.](#)

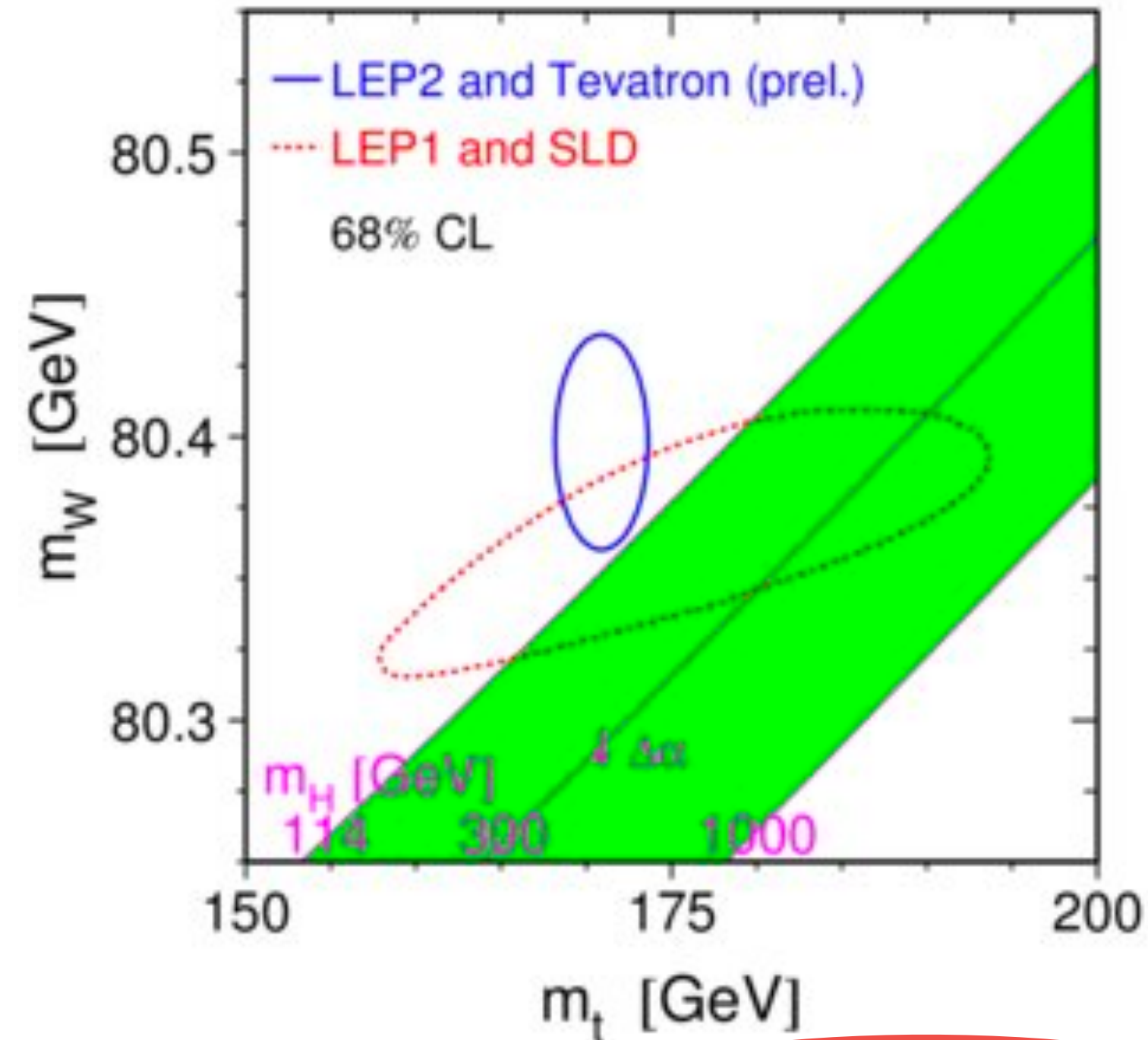
These are not the only measurements (results) that the precision physics program has delivered — many things that we now know, from parton distribution functions, to Higgs properties and its couplings to gluons, photons and quarks, to top quark physics, are directly related to our ability to properly describe partonic cross sections and simulate events in a self-consistent way. This should not be taken for granted since even a decade ago the situation was quite different.

The three observables that I want to talk about are very special; their measurements are driven by physics interests (important quantities!) and superb experimental capabilities. However, because of extremely high precision, they are very challenging from a theoretical point of view (I am sure an experimentalist would say the same ...), so that thinking about them reveals important subtleties and emphasizes required extensions of the precision physics program that I outlined earlier.

The top quark mass is being measured with higher and higher precision. But: can we use the measured value of the top-quark mass measurements to argue about physics, for example **the consistency of the SM and or the stability of electroweak vacuum?**



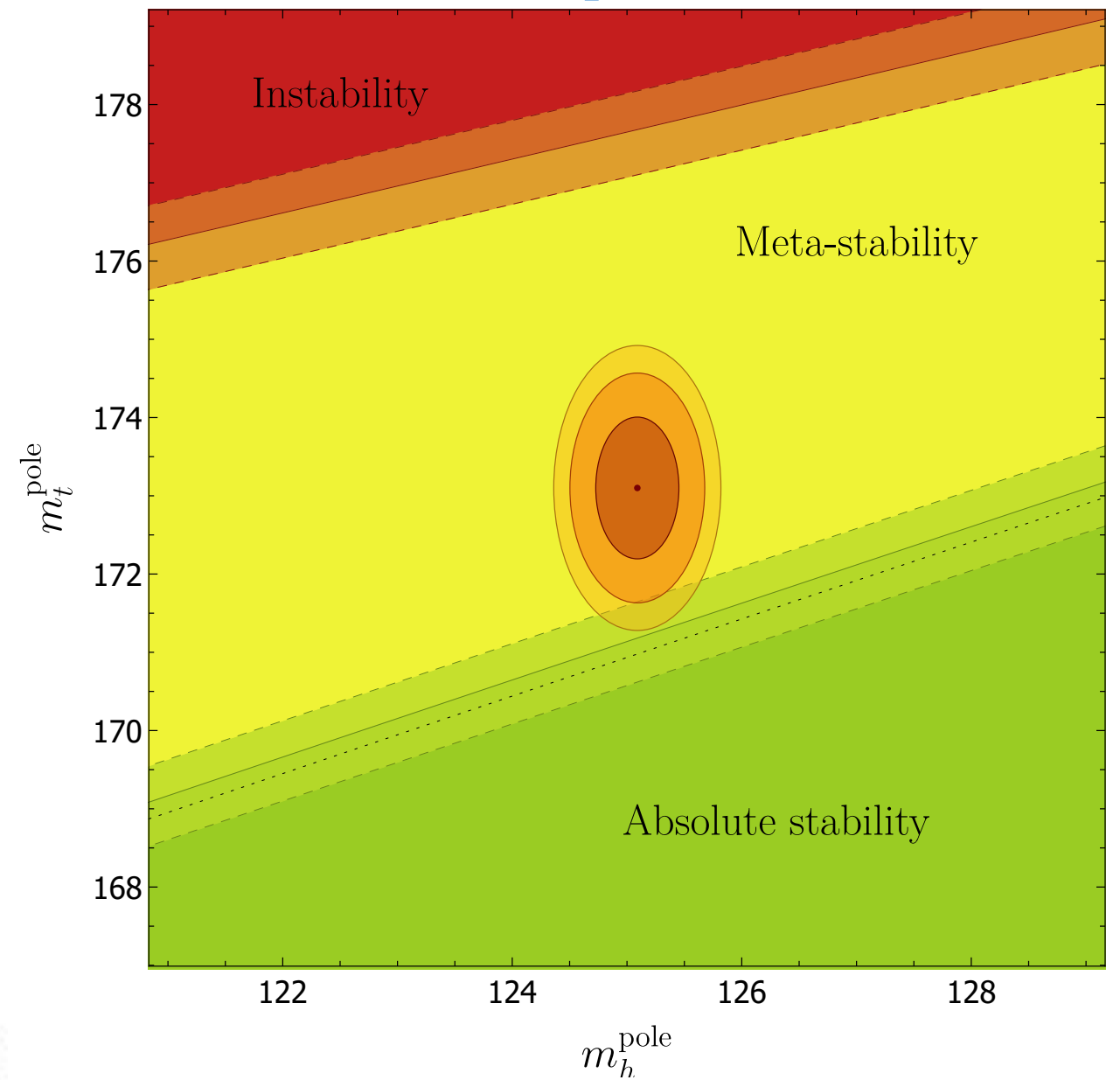
$$m_t = 172.52 \pm 0.33 \text{ GeV}$$



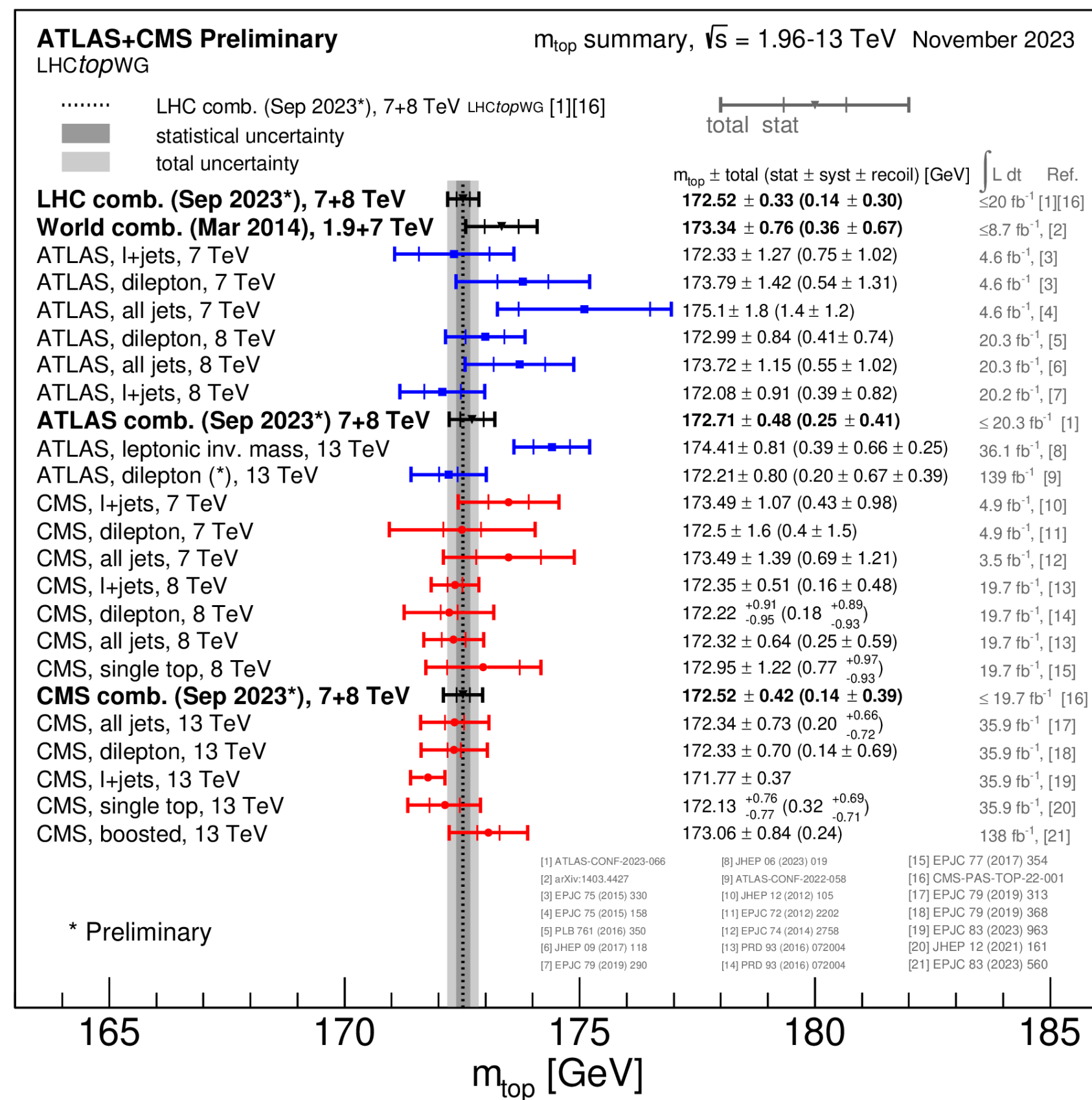
$$\tau_{SM} = \left( \frac{\Gamma}{V} \right)^{-1/4} = 10^{139+102-51} \text{ years}$$

Uncertainty equal parts  $m_t$ ,  $\alpha_s$ , threshold corrections

Stable EW vacuum requires  $m_t < 171 \text{ GeV}$



The top quark mass is being measured with higher and higher precision. But: can we use the measured value of the top-quark mass measurements to argue about physics, for example [the consistency of the SM](#) and or [the stability of electroweak vacuum](#)?



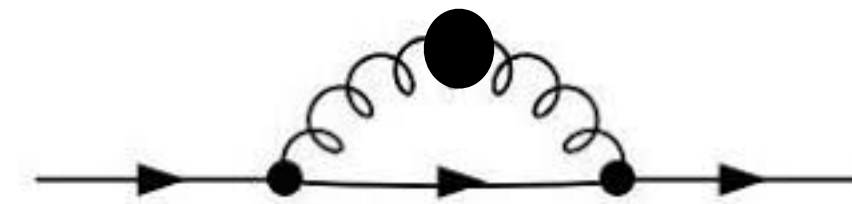
To make the discussion more structured, this question can be split into three separate questions:

- 1) is the top quark mass a well-defined parameter ?
- 2) do we model physics of hadron collisions — that we know and understand — well enough, so that we do not bias the top quark mass extraction?
- 3) do we pay enough attention to subtle details and to [poorly understood aspects](#) of hadron collisions, so that we do not compromise extractions of the top quark mass from experimental data?

$$m_t = 172.52 \pm 0.33 \text{ GeV}$$

In quantum field theory, particle masses are inferred from poles of propagators. This does not work for quarks beyond fixed-order perturbation theory (confinement). Obscure issue for the top quark since it is very heavy and very unstable (top is the only quasi-free quark, as we often say).

$$G(p, m) \sim \frac{1}{p^2 - m^2} \quad \text{pole at} \quad \Rightarrow \quad p^2 = m^2$$



$$m_{\text{pole}} = m_{\text{bare}} + \frac{4}{3} \int_0^\infty \frac{d^3 \vec{k}}{4\pi^2} \frac{\alpha_s(|\vec{k}|)}{k^2} w(|\vec{k}|, m) \quad w(|\vec{k}|, m) \approx 1, \quad |\vec{k}| \leq m \quad \alpha_s(|\vec{k}|) \approx \frac{\Lambda_{\text{QCD}}^2}{k^2 - \Lambda_{\text{QCD}}^2}$$

The pole mass of a top quark **can not be determined** with the precision better than  $\delta m_{\text{pole}} \sim \Lambda_{\text{QCD}} \sim 300 \text{ MeV}$

We think about the top quark mass as a parameter of the Lagrangian and define it according to a particular renormalization “scheme”. Depending on the choice of scale and the exact definition, we get different mass parameters from MS-bar to low-scale short-distance masses (kinetic, potential-subtracted, 1S etc.). **These short-distance masses can be determined with a much higher precision than the pole mass.**

$$m(\mu) = m_{\text{bare}} + \frac{4}{3} \int_\mu^\infty \frac{d^3 \vec{k}}{4\pi^2} \frac{\alpha_s(|\vec{k}|)}{k^2} w(|\vec{k}|, m) \quad \Lambda_{\text{QCD}} \ll \mu \quad m_{\text{pole}} = m(\mu) + \frac{4}{3} \int_0^\mu \frac{d^3 \vec{k}}{4\pi^2} \frac{\alpha_s(|\vec{k}|)}{k^2}$$

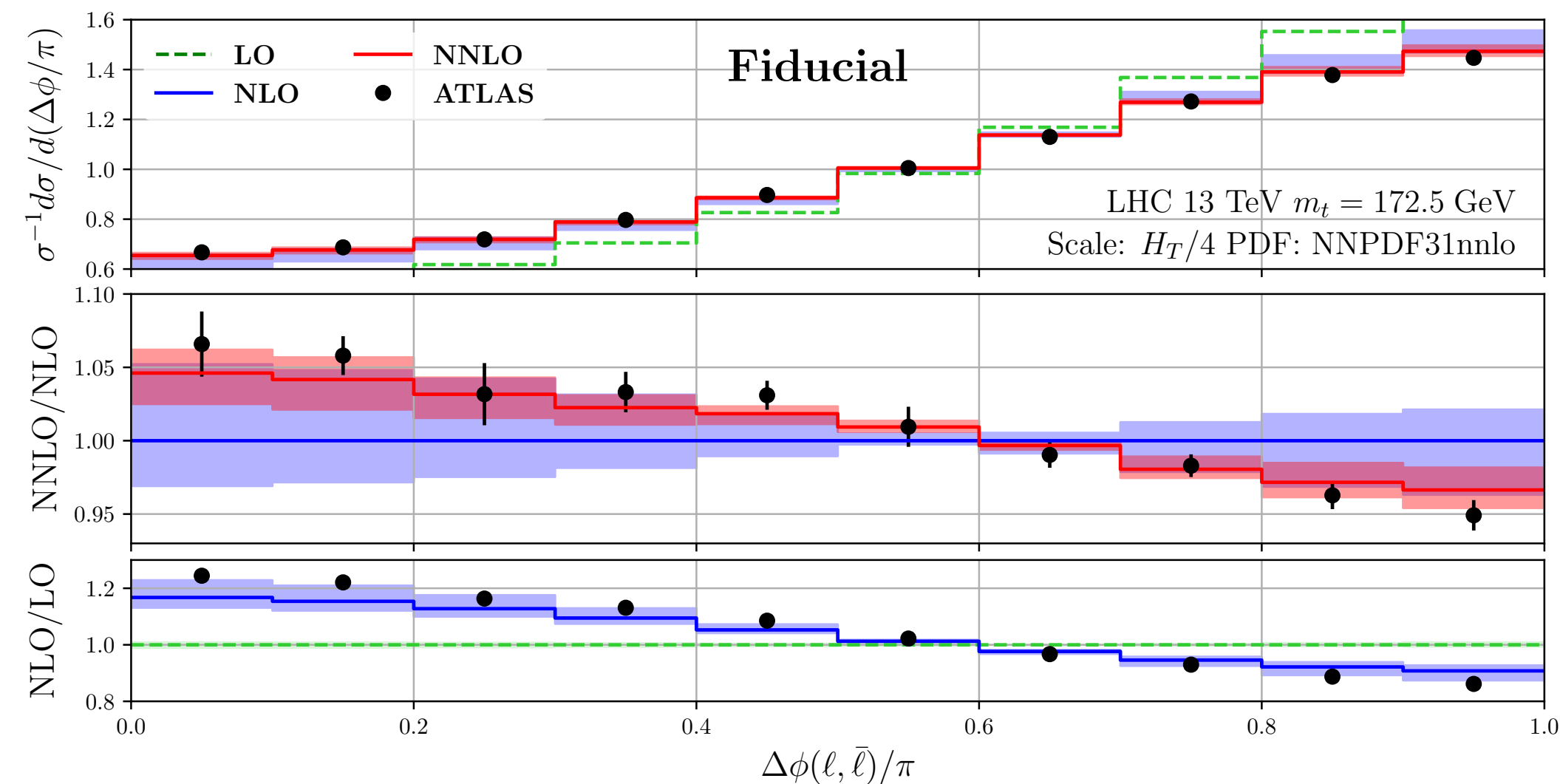
$$m_{\text{pole}} = m(\mu) + \frac{4}{3} \alpha_s(\mu) \mu$$

# MODELLING TOP QUARK PRODUCTION AND DECAY

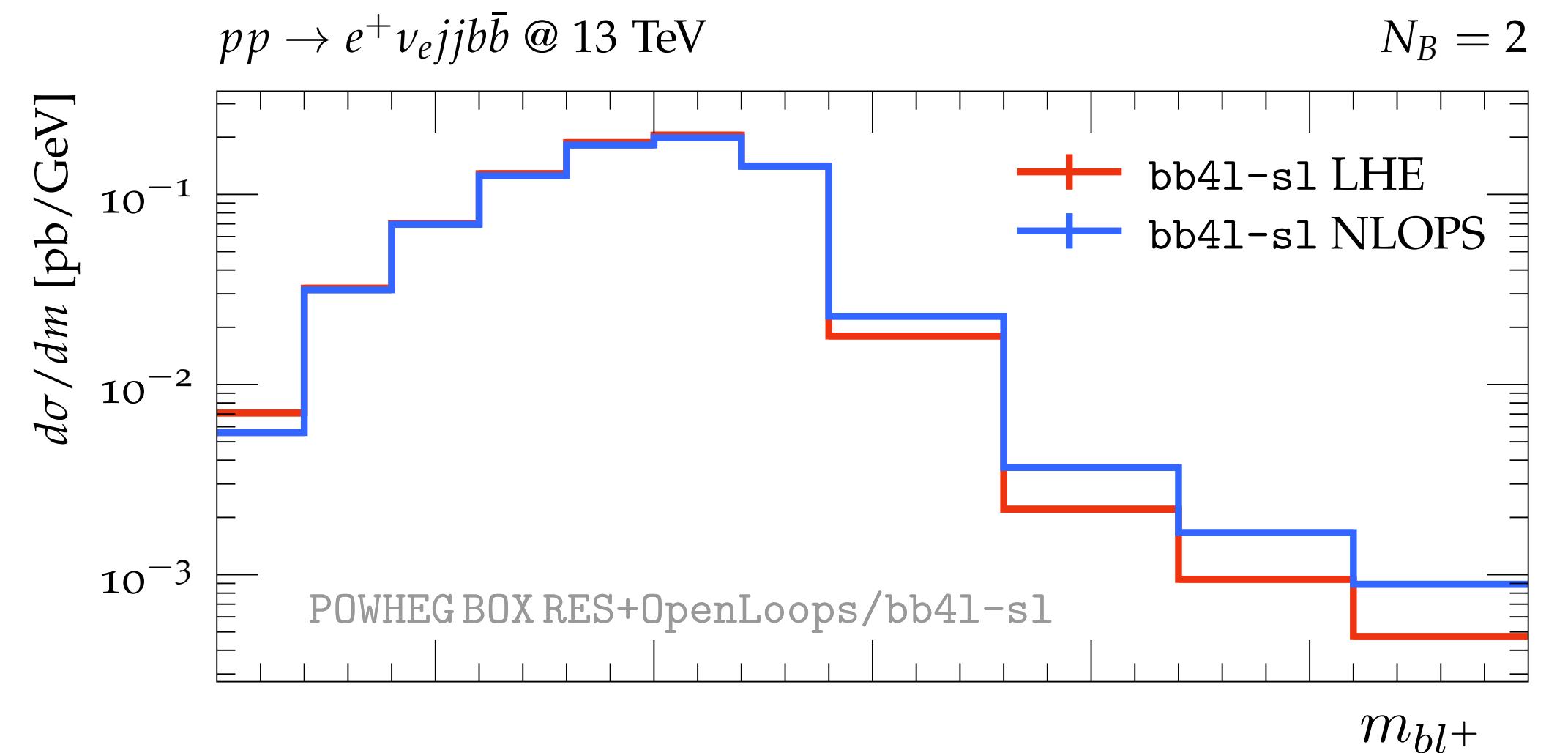
Values of the top quark mass are extracted from two types of observables at the LHC: “inclusive quantities” (i.e. total cross sections) and exclusive distributions of particles from top decays whose kinematic properties are related to the top quark mass.

Theoretical modelling involves NNLO QCD predictions for top-quark production processes (few percent precision) and NLO QCD predictions for top-like final states matched to parton showers (O(10) percent precision).

Such a precision for generic observables is not high enough to determine the top quark mass with a O(100) MeV precision. Need to look for observables with the strong dependence on  $m_t$ .



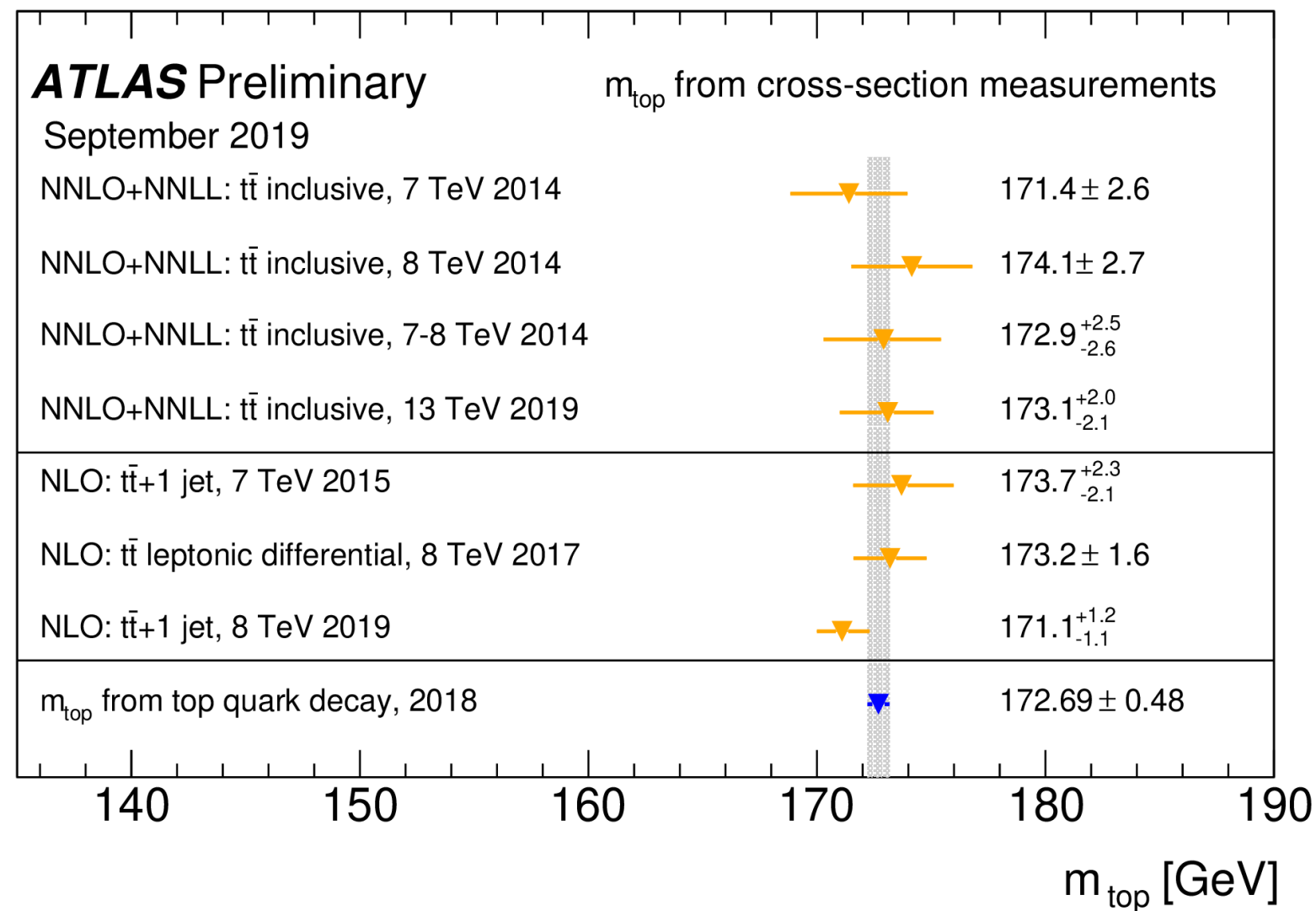
Behring, Czakon, Mitov, Papanastasiou, Poncelet



Jezo, Lindner, Pozzorini

Top quark mass extractions from cross sections rely on its strong sensitivity on  $m_t$  and (high-order) perturbative predictions for the top-pair production cross section. They reach an impressive precision of about 700 MeV.

$$d\sigma_{\text{hard}} = \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{ij}(x_1, x_2, \{p_{\text{fin}}\}) O_J(\{p_{\text{fin}}\}) (1 + \mathcal{O}(\Lambda_{\text{QCD}}^n/Q^n))$$



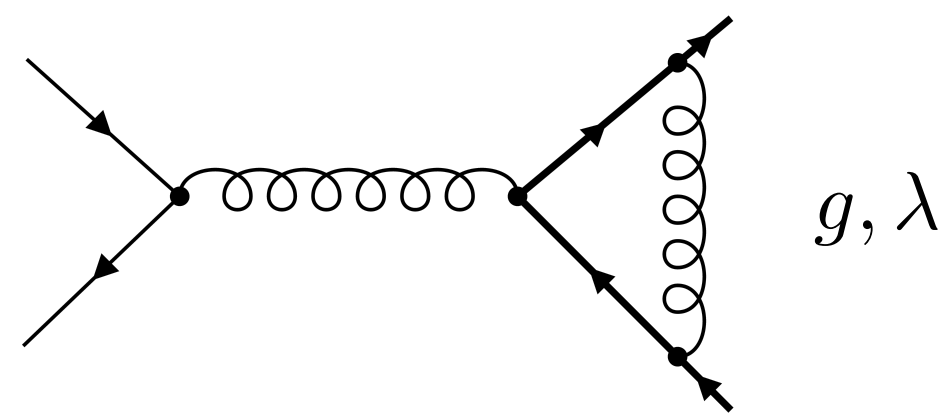
However, if there are linear power corrections to cross sections, the extraction of the top quark mass becomes biased. It is important to realize that, at this point, we cannot deduce from first principles whether such linear power corrections exist..

$$\sigma_{t\bar{t}} = \sigma_0 \left( \frac{m_0}{m_t} \right)^5 \left[ 1 + c_{\text{np}} \frac{\Lambda_{\text{QCD}}}{m_t} + \dots \right]$$

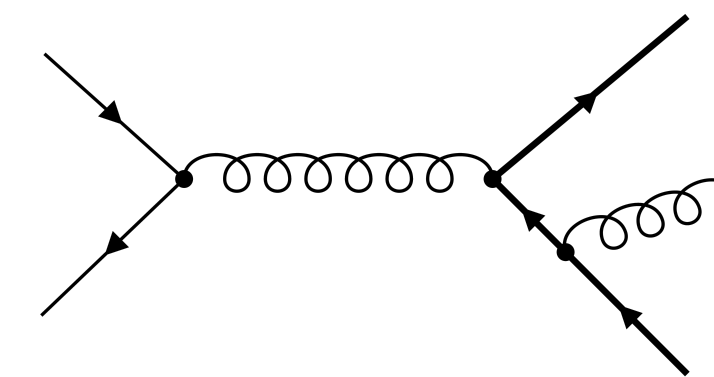
$$m_t \rightarrow m_t + \frac{c_{\text{np}}}{5} \Lambda_{\text{QCD}}$$

The best that we can do currently is to [estimate the sensitivity of various observables to infra-red physics](#), including those that play a role in the extraction of the top quark mass.

Perturbative computations in QFTs with massless particles (QCD, QED) cannot be performed for final states [with fixed multiplicities](#) because individual contributions exhibit logarithmic sensitivity to infra-red physics. We can make this explicit by exploring how perturbative predictions depend on the fictitious photon or gluon mass.



$$d\sigma_{q\bar{q}\rightarrow t\bar{t}} \sim d\sigma_0 (1 - c_1 \alpha_s \ln^2 \lambda \dots)$$



$$d\sigma_{q\bar{q}\rightarrow t\bar{t}+g} \sim d\sigma_0 c_1 \alpha_s \ln^2 \lambda \dots$$

However, in the proper combination of loop and real-emission corrections, infra-red divergencies [disappear](#) allowing us to claim that properly defined perturbative cross sections are not sensitive to infra-red physics. This is the essence of Kinoshita-Lee-Nauenberg theorem.

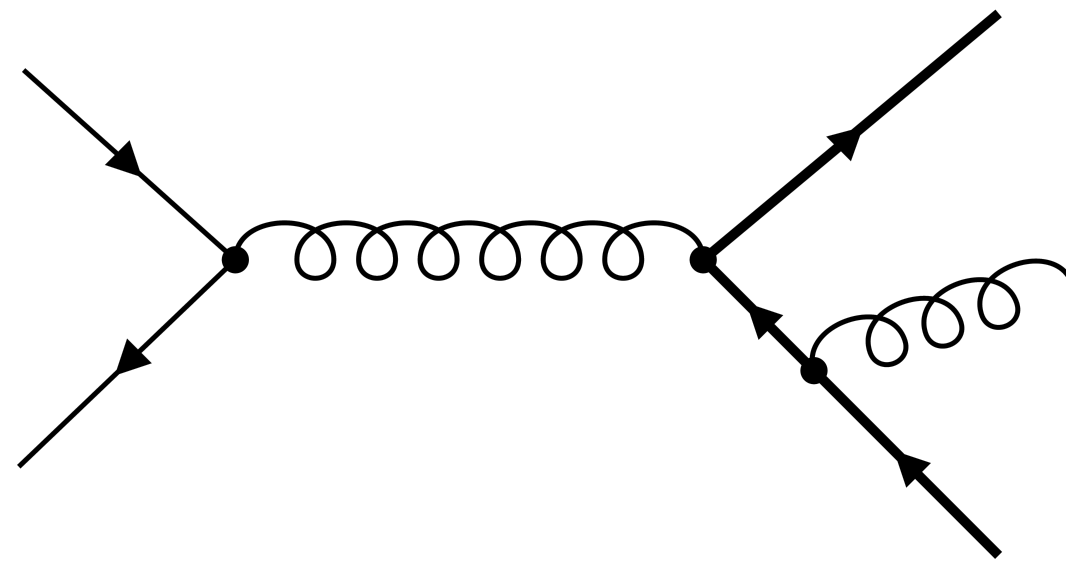
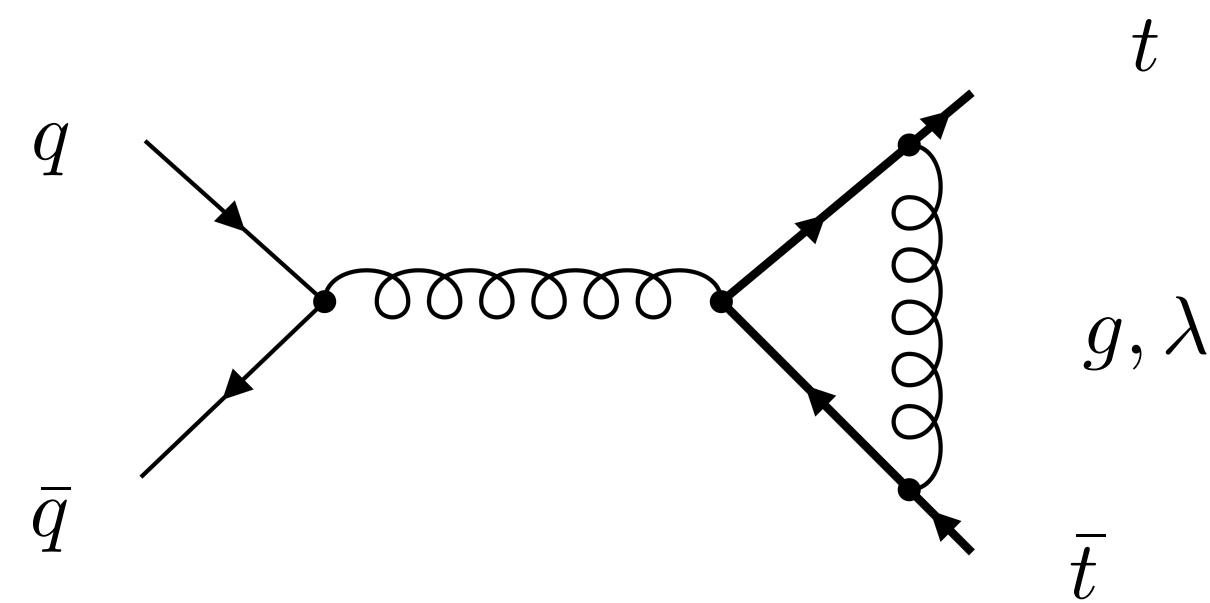
$$d\sigma_{q\bar{q}\rightarrow t\bar{t}} = d\sigma_{q\bar{q}\rightarrow t\bar{t}} + d\sigma_{q\bar{q}\rightarrow t\bar{t}+g} \sim \mathcal{O}(\lambda^0)$$

We can use fictitious gluon mass to probe the sensitivity of observables related to top quark physics — including the total cross sections — to [infra-red effects](#). This may give us an idea about the robustness of the top quark mass determinations from cross sections.



The fictitious gluon-mass dependence can be related to  $\mathcal{O}(\Lambda_{\text{QCD}})$  **corrections** computed within the renormalon model where perturbatively-induced Landau singularity in the running QCD coupling constant is the **only** source of non-perturbative effects.

$$\int dk k^{p-1} \alpha_s(\mu) F(k) \Rightarrow \int dk k^{p-1} \alpha_s(k) F(k) \quad \alpha_s(k^2) \approx \frac{1}{2\beta_0} \frac{\Lambda_{\text{QCD}}^2}{k^2 - \Lambda_{\text{QCD}}^2}, \quad k^2 \approx \Lambda_{\text{QCD}}^2$$



$$\sigma_{t\bar{t}} = \sigma_0 \left[ 1 + c \frac{\lambda}{m_t} + \mathcal{O}(\lambda^2) \right]$$

$$\sigma_{t\bar{t}} = \sigma_0 \left[ 1 + c \frac{\Lambda_{\text{QCD}}}{m_t} + \dots \right]$$

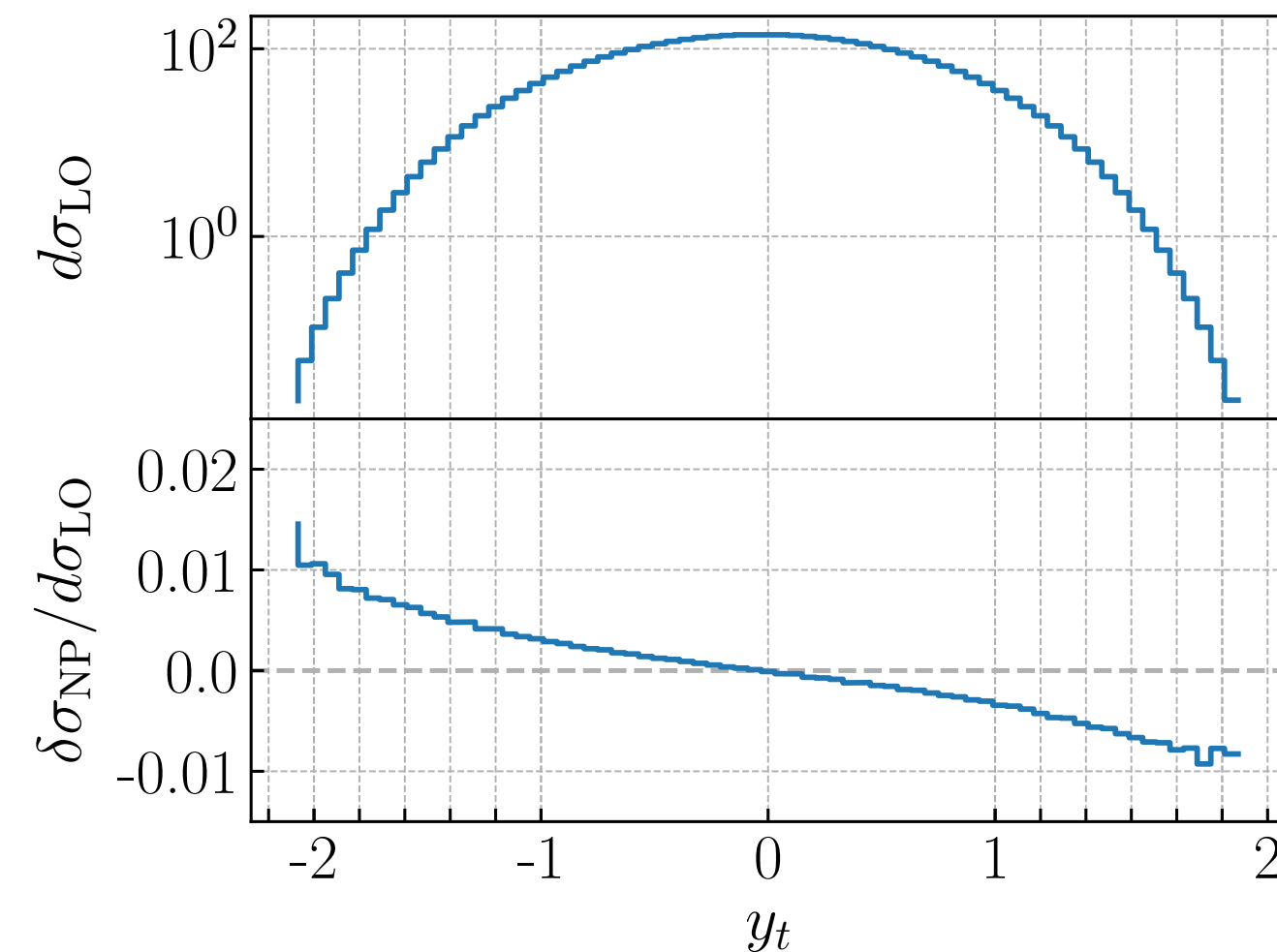
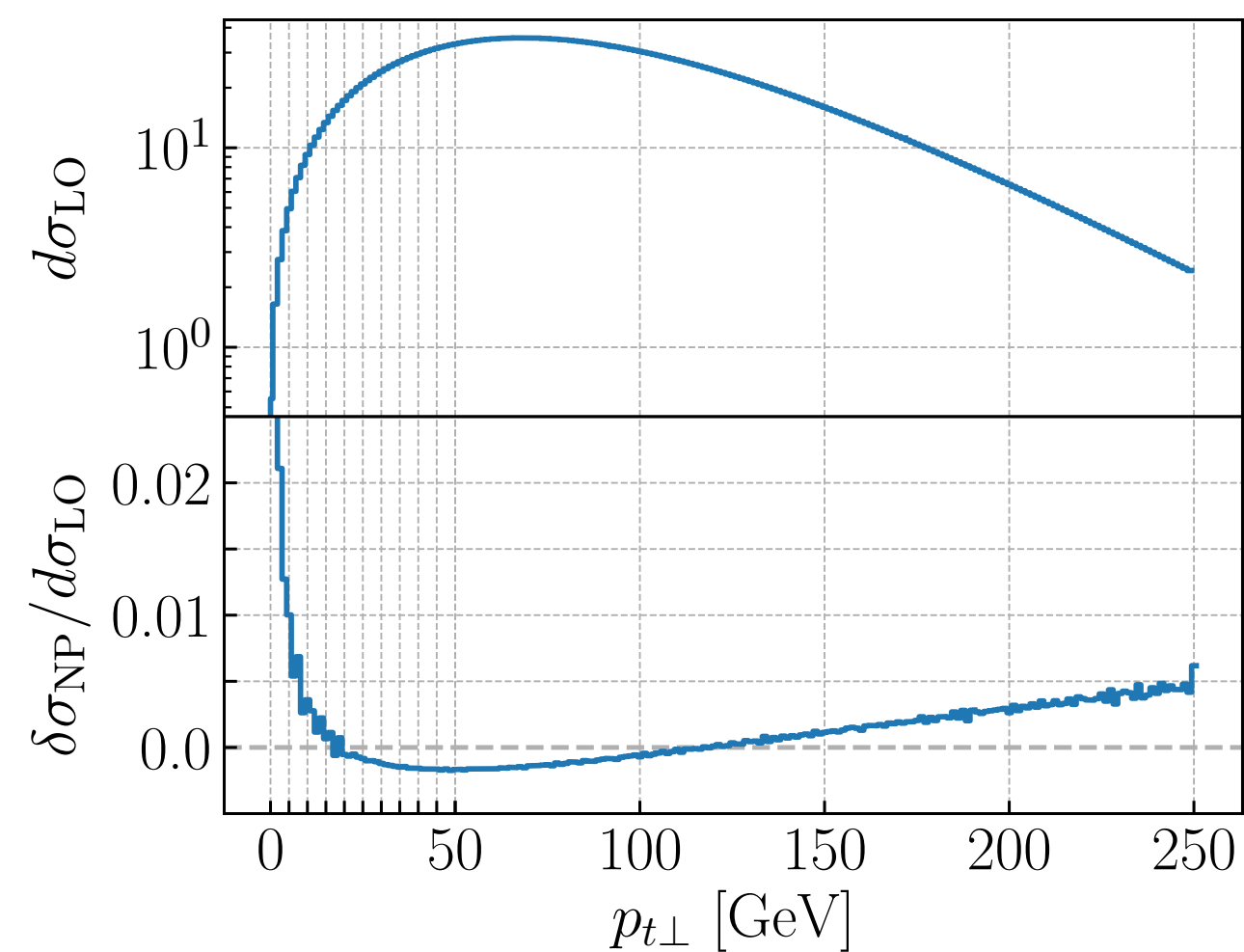
Within this model, one needs to compute NLO QCD corrections to a process of interest (top quark pair production) in a theory with **massive gluons**, assume that the gluon mass is the smallest parameter in the problem and expand the cross section or an observable to linear terms in the gluon mass. Such linear terms are then easily translated to  $\mathcal{O}(\Lambda_{\text{QCD}})$  corrections.

**A caveat:** this set up has been extensively used for processes which **do not contain gluons at leading order**, i.e. non-abelian vertices are not allowed through NLO QCD. Its modification can be applied to top quark pair production in quark collisions where this requirement can be lifted; processes with on-shell gluons at tree level cannot be studied using these methods.

One finds that there are no linear power corrections to top quark pair production cross section provided that it is expressed through one of the short distance masses; linear corrections are present if the cross section is written in terms of the pole mass.

Linear power corrections do exist in kinematic distributions. In general, shifts are not large but they become enhanced and reach a few percent close to edges of the allowed kinematic regions. Linear power corrections are not universal and exhibit non-trivial dependencies on kinematic variables even in a such a simple case as top quark pair production.

$$\frac{\delta_{\text{NP}} [p_{t\perp}]}{p_{t\perp}} = \frac{\alpha_s \pi \lambda}{2\pi m_t} \frac{(2C_F - C_A \tau)}{2(1 - \tau)} \quad \delta_{\text{NP}} [y_t] = \frac{\alpha_s \pi \lambda}{2\pi m_t} \left[ (3C_A - 8C_F) \tau \cosh^2 y_t - (C_A - 2C_F) \frac{\tau(2 - \tau)}{4(1 - \tau)} \sinh(2y_t) \right]$$



$$\tau = 4m_t^2/s_{t\bar{t}}$$

$$\alpha_s \lambda = \frac{0.4 \text{ GeV}}{C_F} = 0.3 \text{ GeV}$$

**Makarov, K.M., Nason, Oczelik**

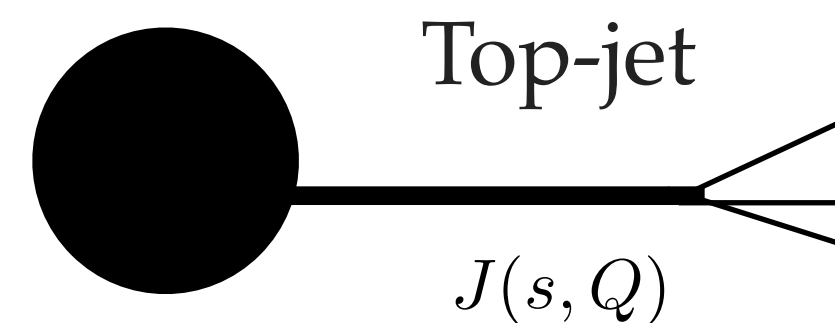
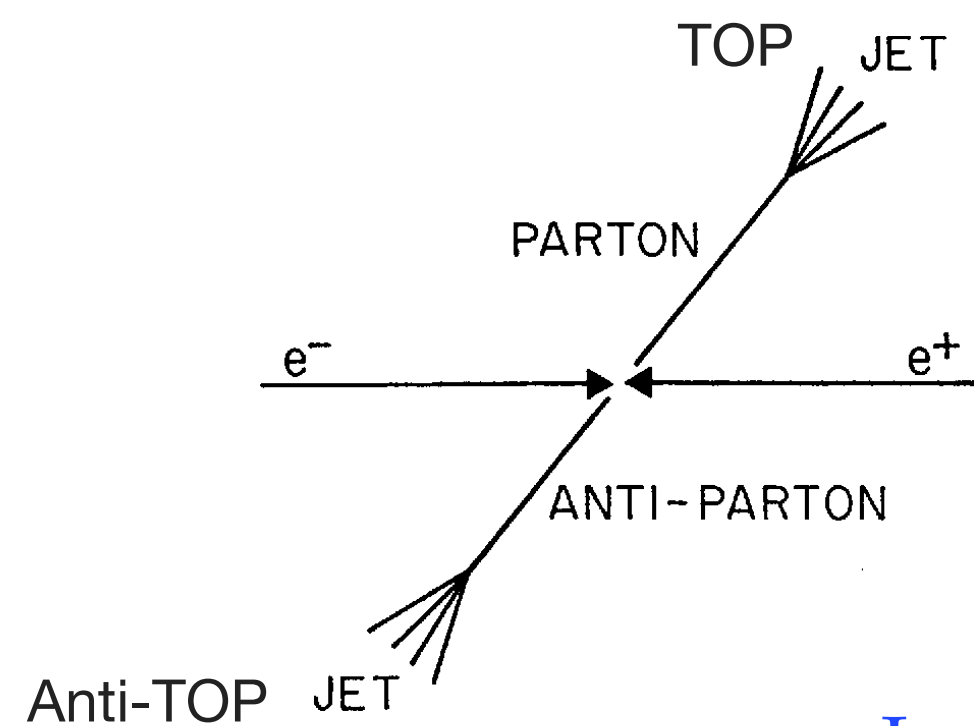
Results for the Tevatron where quark annihilation channel dominates.

The most precise measurements of the top quark mass come from studying complex events where kinematics of top quark decay products and the mass of the decaying top quark are strongly correlated. Non-perturbative effects are simulated with parton showers but there are no theoretical arguments that ensure that parton showers are up to the task [at the required level of precision](#). [This problem remains unsolved](#).

$$m_t = 172.52 \pm 0.33 \text{ GeV , PDG}$$

One interesting twist of this is the suggestion that in such measurements [the so-called Monte-Carlo \(MC\) top quark mass](#) is measured. This suggestion led to a lot of confusion of what this means and how the MC mass is related to familiar top quark mass parameters, such as the top quark pole mass.

[An analytic study of a parton shower-like description of a mass-sensitive observable](#) may help to develop a better understanding of subtleties that the use of parton showers entails. A study of a boosted top jets is an excellent example of that.

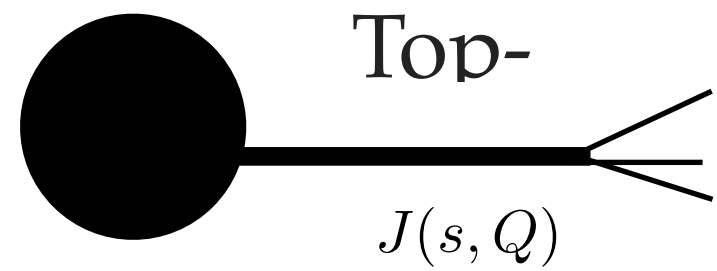


Hoang, Plätzer, Samitz

$$\langle s \rangle = \int ds s J(s, Q^2, m^2)$$

[Invariant mass of the boosted top jet is an observable from which the top quark mass is determined.](#)

Invariant mass of the boosted top jet is an observable from which the top quark mass is determined.



$$\langle s \rangle = \int ds s J(s, Q^2, m^2)$$

This integral can be rewritten in the spirit of parton showers by introducing the hard cut-off on the transverse momentum of emitted gluons and treated the radiation from below the cut-off as a "hadronisation" effect.

$$\langle s \rangle = m^2 + \int_{m^2}^{Q^2} d\tilde{q}^2 \int_0^1 dz (1-z) P_{QQ} \left( \alpha_s((1-z)\tilde{q}), z, \frac{m^2}{\tilde{q}^2} \right) \quad P_{QQ} \left( \alpha_s, z, \frac{m^2}{\tilde{q}^2} \right) = \frac{C_F \alpha_s}{2\pi} \left[ \frac{1+z^2}{1-z} - \frac{2m^2}{z(1-z)\tilde{q}^2} \right]$$

$$\langle s \rangle = \langle s_{\text{soft}} \rangle + m_{\text{CB}}^2(Q_0) + \int_{m^2}^{Q^2} d\tilde{q}^2 \int_0^1 dz (1-z) P_{QQ} \left( \alpha_s((1-z)\tilde{q}), z, \frac{m^2}{\tilde{q}^2} \right) \theta(q_{\perp}^2 - Q_0^2) \dots$$

What is the proper interpretation of the first two terms: hadronization of massless jets and a cut-off dependent mass parameter OR a mass- and cut-off dependent hadronization and the cut-off independent mass parameter? The conclusion about what exactly one extracts from an MC-based measurement depends [on the details of hadronization models](#).

$$\langle s_{\text{soft}} \rangle = \frac{2C_F \alpha_s(Q_0)}{\pi} Q_0 Q$$

$$m_{\text{CB}}(Q_0) = m - \frac{C_F}{2} \alpha_s(Q_0) Q_0$$

The “Monte Carlo mass” appears to be a relatively simple concept. Indeed, if we infer the mass of a top quark by studying the amount of radiation that this object produces and if we restrict the amount of resolved radiation that is allowed in parton showers (above the cut-off), and use quark-mass independent hadronization model, then [linear dependence of the extracted mass parameter on the cut-off should be expected](#):

$$m_{\text{CB}}(Q_0) = m - \frac{C_F}{2} \alpha_s(Q_0) Q_0$$

Numerically, the difference between the Monte-Carlo mass and the pole mass is about 400 MeV which is not negligible at the current level of precision.

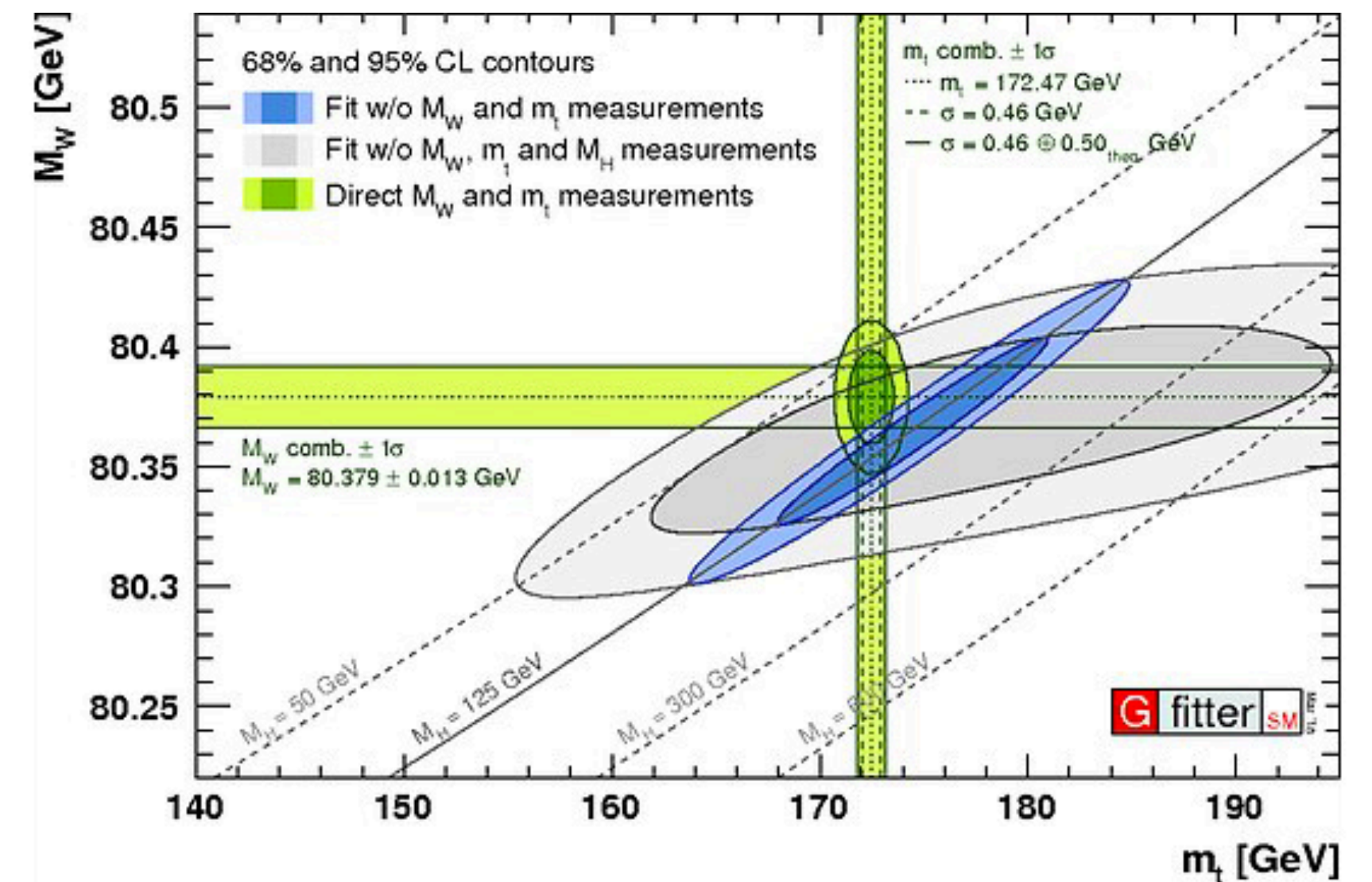
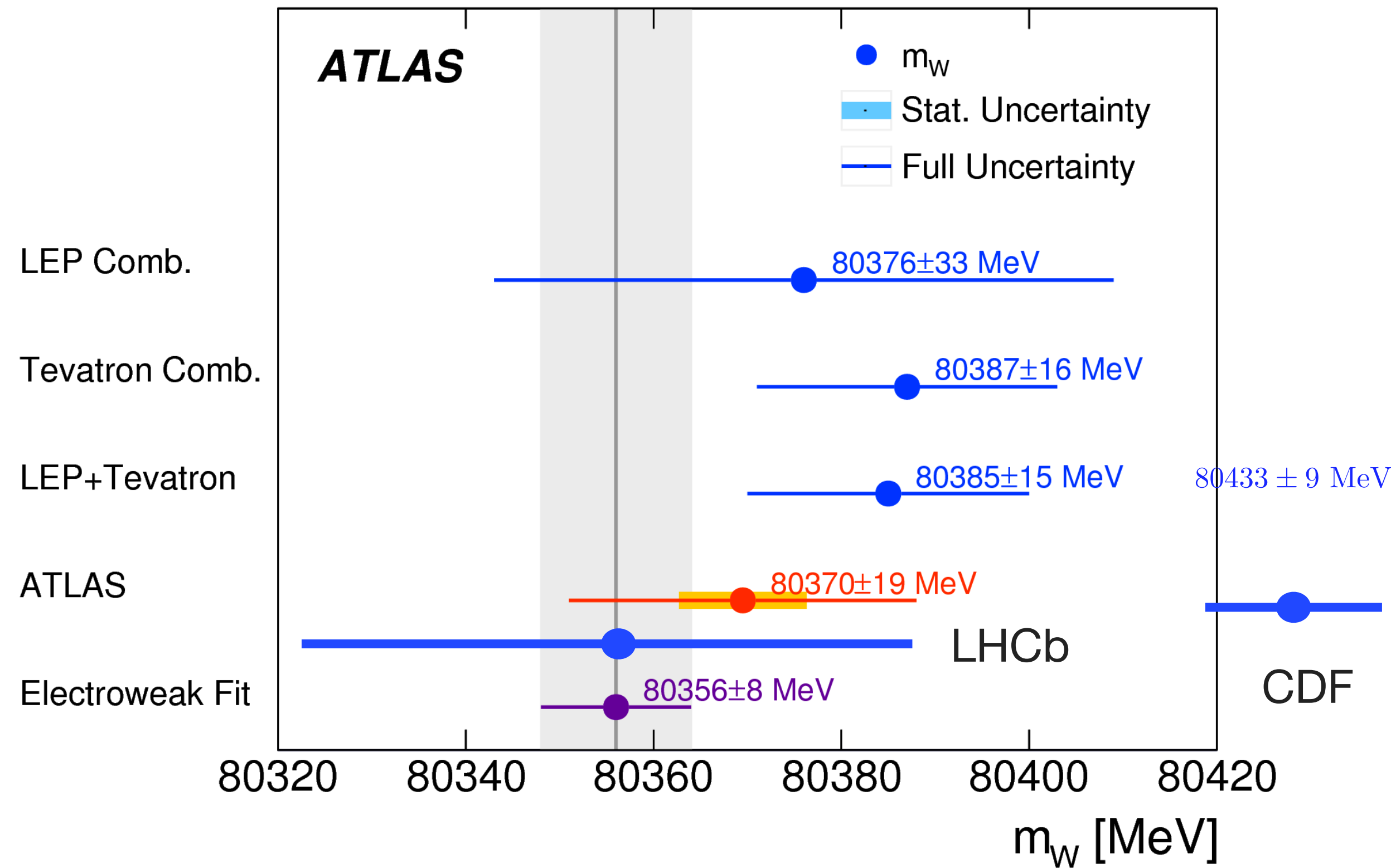
$$m_t = 172.52 \pm 0.33 \text{ GeV}, \text{ PDG}$$

[This discussion calls for a better understanding of non-perturbative hadronization models and how they play together with the perturbative radiation off massive quarks since hadronization models and perturbative radiation together should give results that are independent of  \$Q\_0\$ .](#)

Finally, these considerations were addressed by a dedicated study by the ATLAS collaboration. They extracted the top quark mass from boosted top events. Their result (note that sign difference with the analytic result above...) is somewhat inconclusive

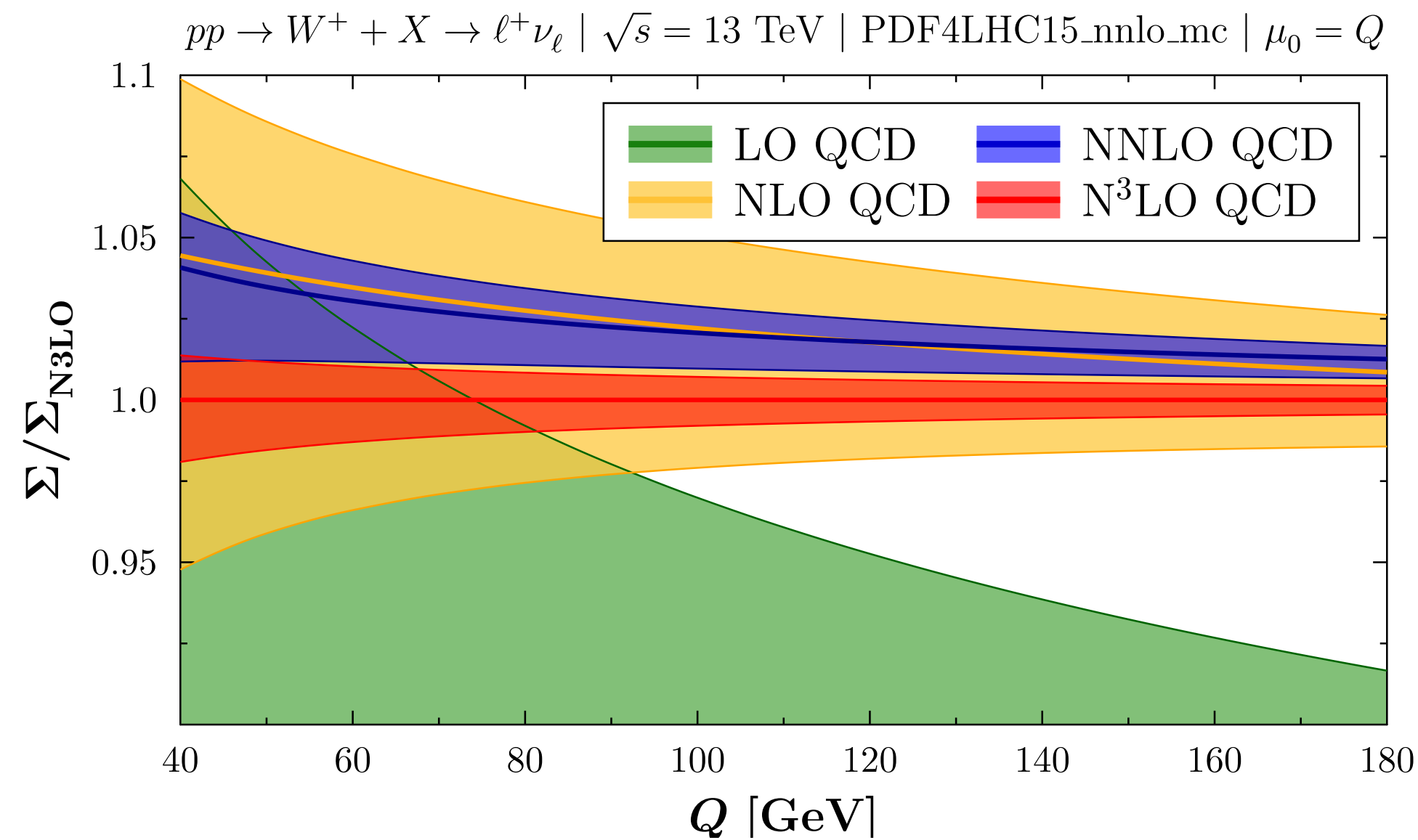
$$m_t^{\text{MC}} = m_t^{\text{pole}} + 350^{+300}_{-360} \text{ MeV}$$

Measurements of the W mass have a long history. They started with the threshold scan at LEP2 and with the studies at the Tevatron Run I and Run II. At the LHC, the W mass was measured by the ATLAS collaboration. A new (legacy) measurement by the CDF collaboration also appeared recently; the reported W mass value is very different from other high-precision results.



Hadron colliders already play and will continue to play the leading role in measuring the W mass. Precision achieved in the EW fit sets the target precision for future LHC measurements.

It is important to realise that within the standard theory that is used to describe hard processes at hadron colliders, even the most advanced results **do not come close to the required (0.1 permille) accuracy**. Existing theoretical framework cannot be used to predict even the simplest observables with a precision which is better than **a few percent**.

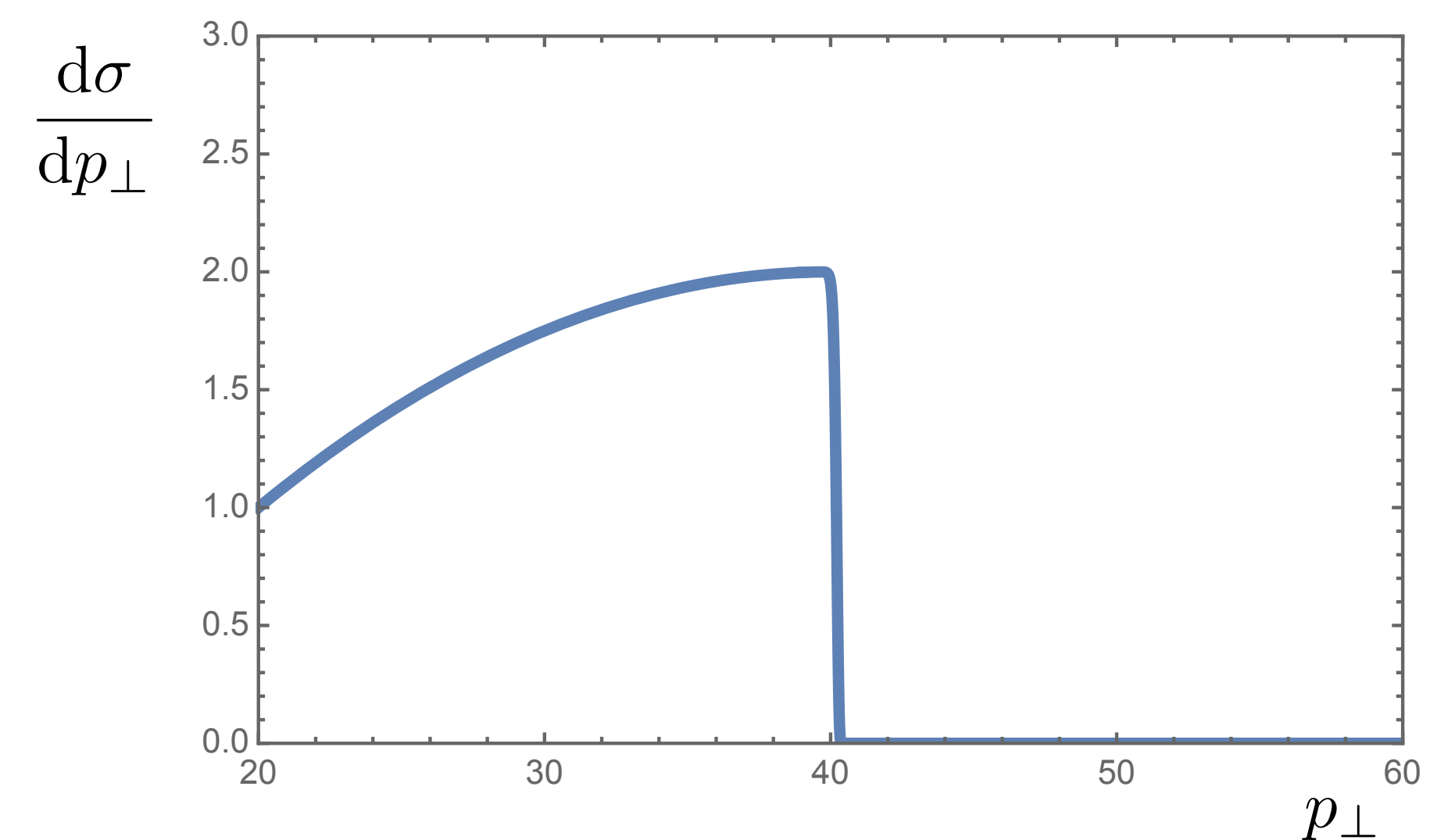
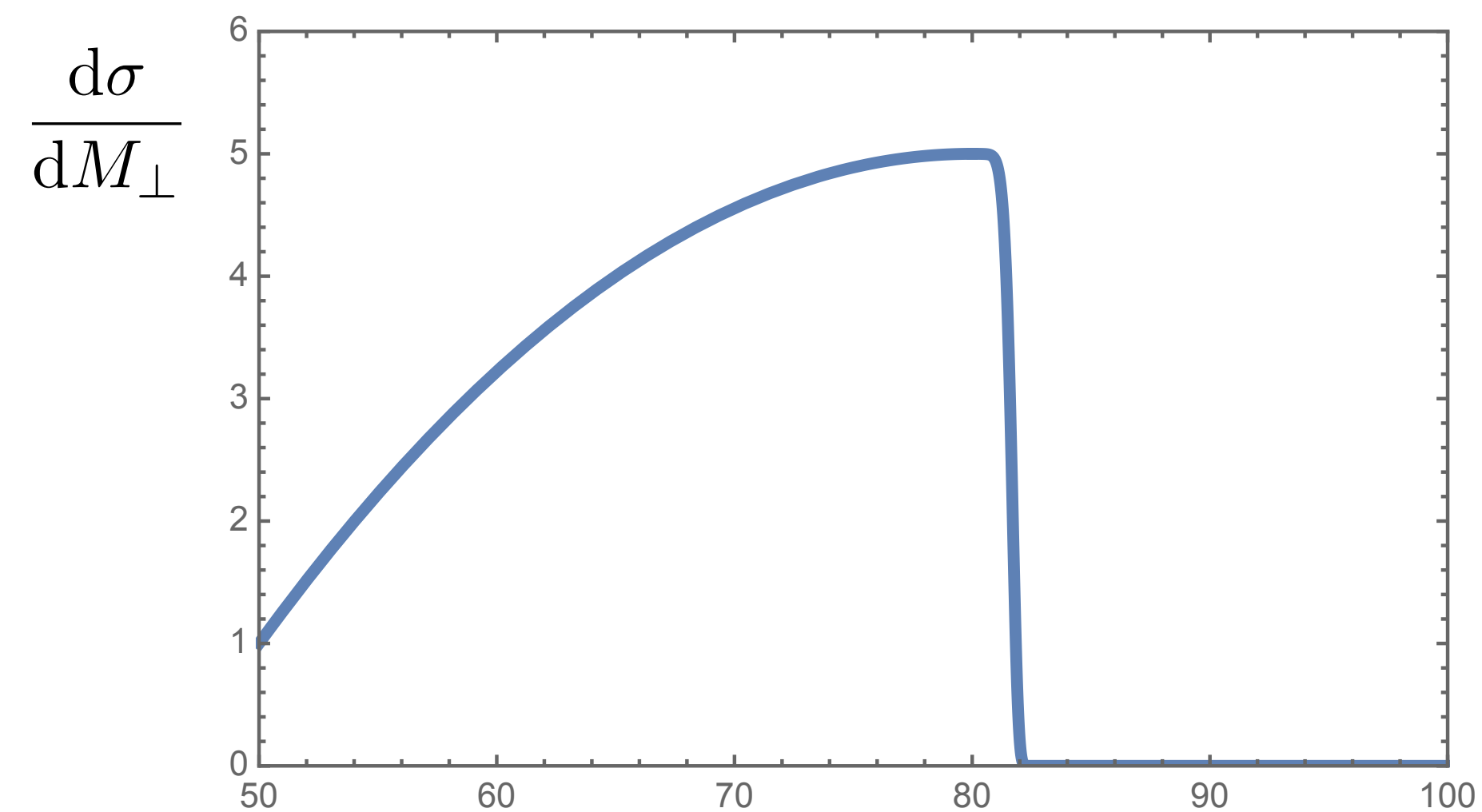


	$Q$ [GeV]	$\delta\sigma^{\text{N}^3\text{LO}}$	$\delta\sigma^{\text{NNLO}}$	$\delta(\text{scale})$	$\delta(\text{PDF} + \alpha_S)$	$\delta(\text{PDF-TH})$
$gg \rightarrow \text{Higgs}$	$m_H$	3.5%	30%	+0.21% -2.37%	$\pm 3.2\%$	$\pm 1.2\%$
$b\bar{b} \rightarrow \text{Higgs}$	$m_H$	-2.3%	2.1%	+3.0% -4.8%	$\pm 8.4\%$	$\pm 2.5\%$
NCDY	30	-4.8%	-0.34%	+1.53% -2.54%	+3.7% -3.8%	$\pm 2.8\%$
	100	-2.1%	-2.3%	+0.66% -0.79%	+1.8% -1.9%	$\pm 2.5\%$
CCDY( $W^+$ )	30	-4.7%	-0.1%	+2.5% -1.7%	$\pm 3.95\%$	$\pm 3.2\%$
	150	-2.0%	-0.1%	+0.5% -0.5%	$\pm 1.9\%$	$\pm 2.1\%$
CCDY( $W^-$ )	30	-5.0%	-0.1%	+2.6% -1.6%	$\pm 3.7\%$	$\pm 3.2\%$
	150	-2.1%	-0.6%	+0.6% -0.5%	$\pm 2\%$	$\pm 2.13\%$

**Baglio, Duhr, Mistlberger, Szafron**

The W mass is extracted from observables that are very sensitive to its value. An observable with the kinematic edge strongly correlated with the W mass is ideal as it is super-sensitive to  $m_W$  and does not require any theory, in the zeroth approximation.

For the purpose of the W mass measurement, two variables with an edge have been used at hadron colliders. They are the transverse mass and the transverse momentum of the charged lepton. In the simplest case (LO modelling and ideal detectors) these distributions have edges at  $m_W$  and  $m_W/2$ , respectively.



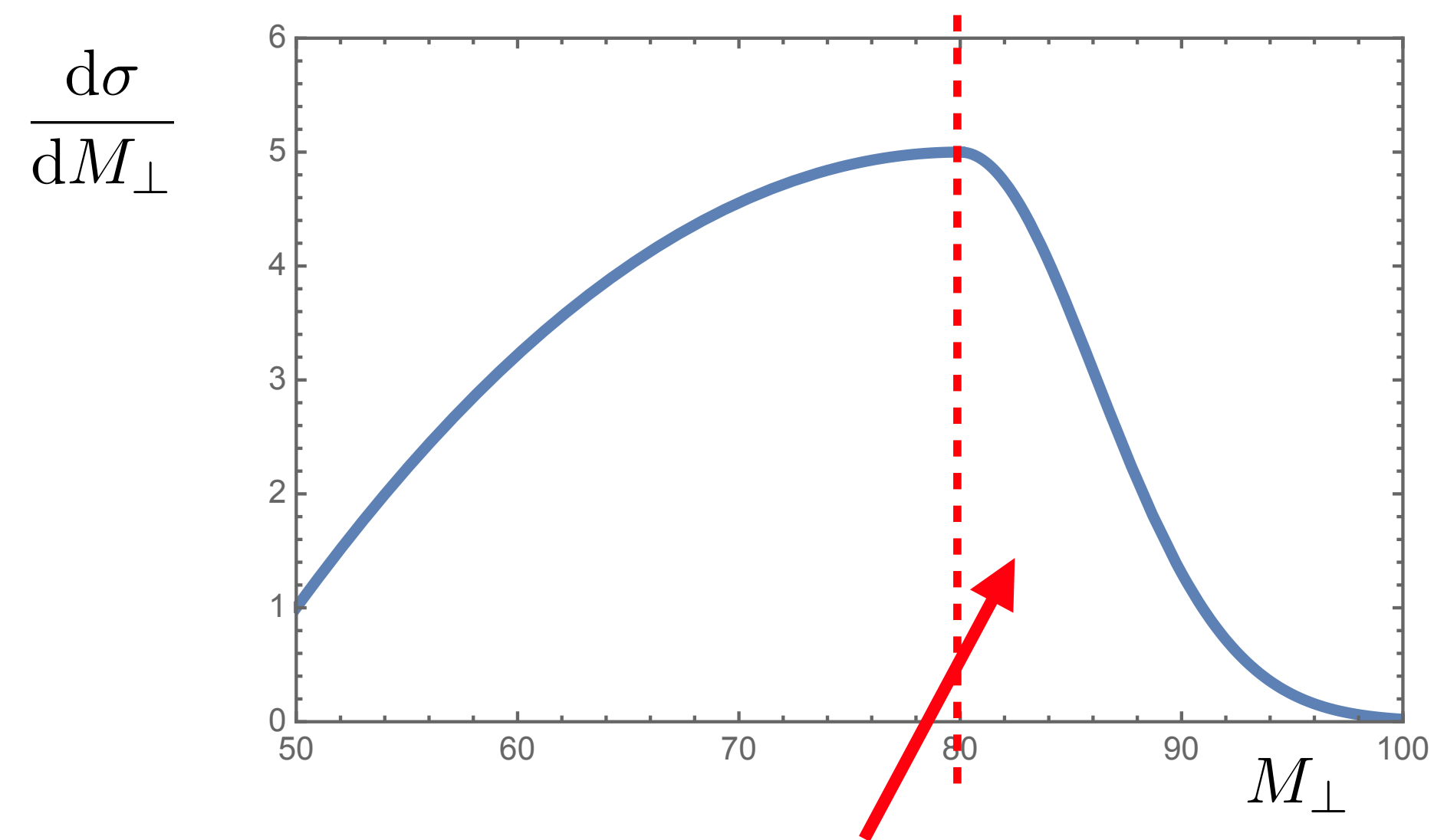
$$M_{\perp} = \sqrt{2p_{\perp,e}p_{\perp,\nu}(1 - \cos \phi_{e+\nu})}$$

The two observables have different sensitivity to experimental uncertainties and the quality of theoretical modelling.

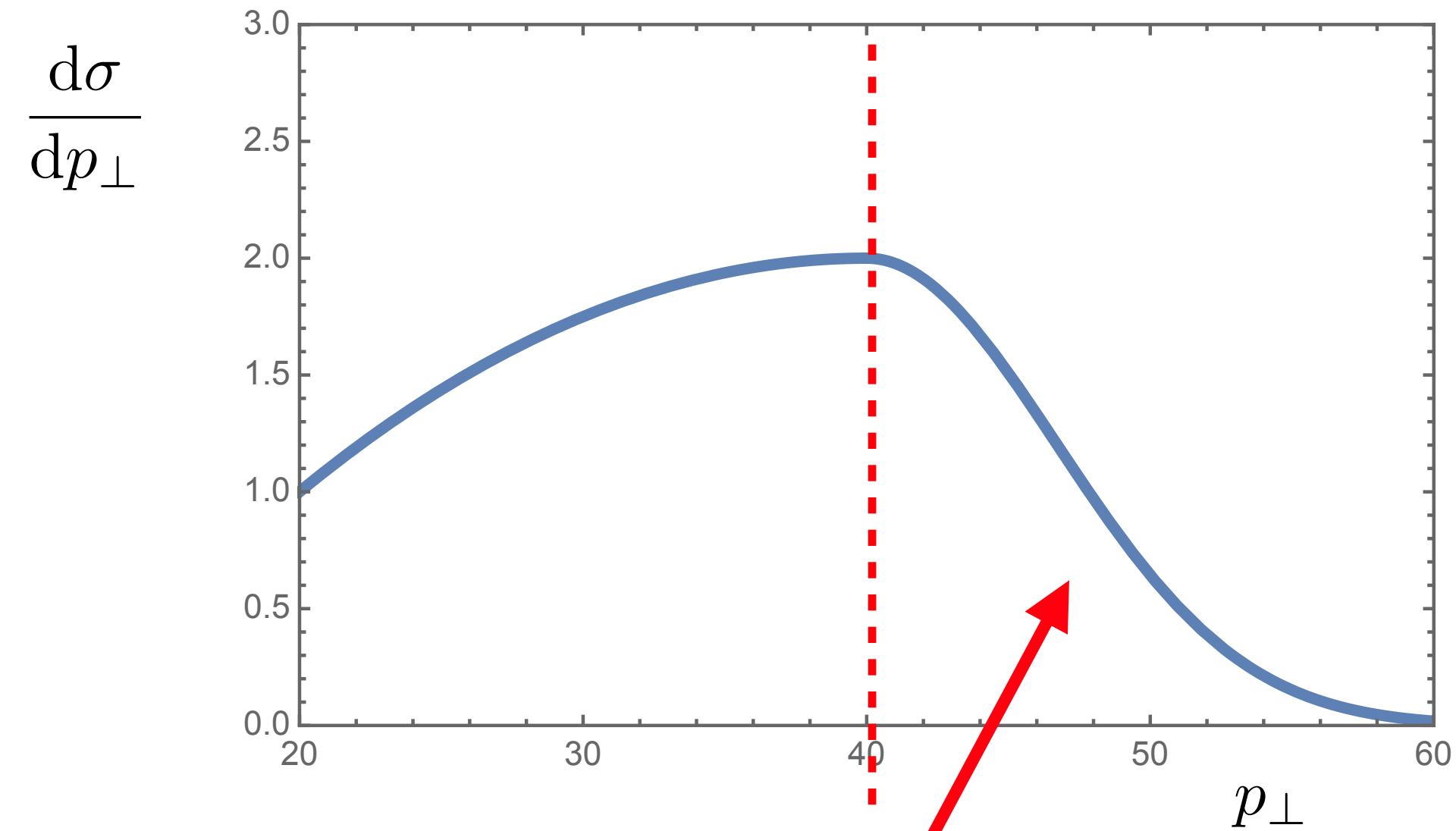


The  $W$  mass is extracted from observables that are very sensitive to its value. An observable with the kinematic edge strongly correlated with the  $W$  mass is ideal as it is super-sensitive to  $m_W$  and does not require any theory, in the zeroth approximation.

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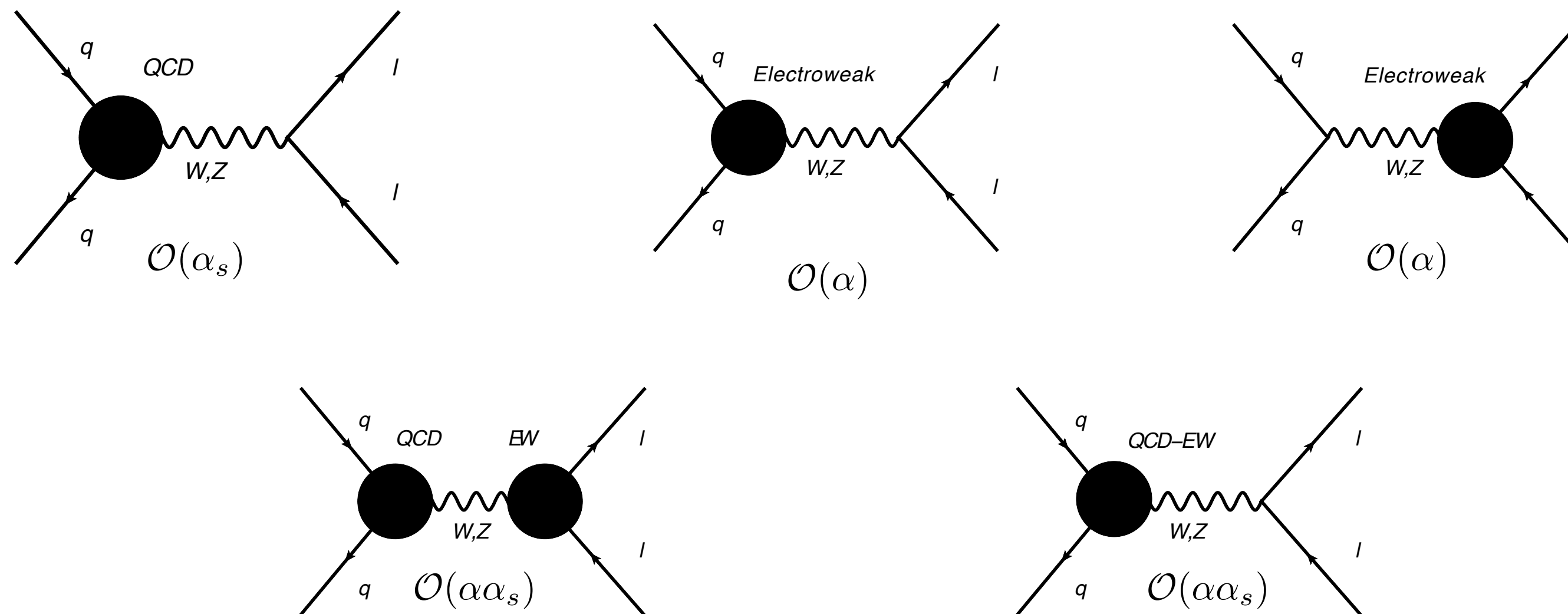
BEYOND THE EDGE: DETECTOR EFFECTS, MOSTLY



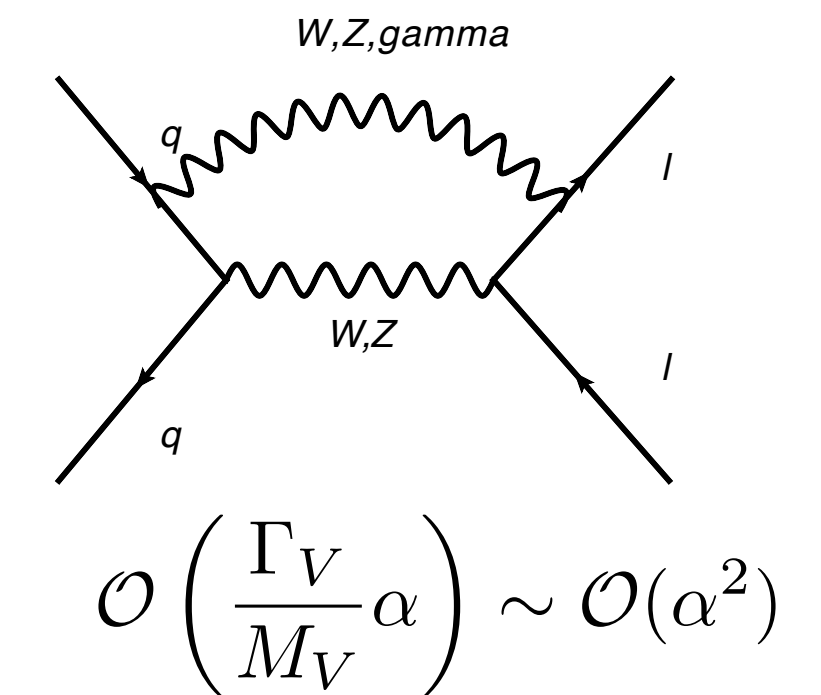
QCD AND QED INITIAL STATE RADIATION, MOSTLY

The two observables have different sensitivity to experimental uncertainties and the quality of theoretical modelling.

- 1) one must give up on using advanced theory to model relevant distributions. Instead, one measures Z-boson distributions, parameterises them in a QCD-motivated way and transfers them to the W case arguing that QCD, to a large extent, does not distinguish between Z and W production.
- 2) given the target precision of 0.1 permille, small effects that distinguish between Z and W distributions may become important; these effects need to be understood and modelled properly. **Electroweak and electroweak/QCD corrections to Z and W production are obvious examples of the potentially relevant effects.**



**Recent work by: De Florian, Der, Fabre; Cieri, De Florian, Mazzitelli; Bonciani, Buccioni, Rana, Vicini,;Buonocore, Grazzini, Kallweit, Savoini, Tramontano**



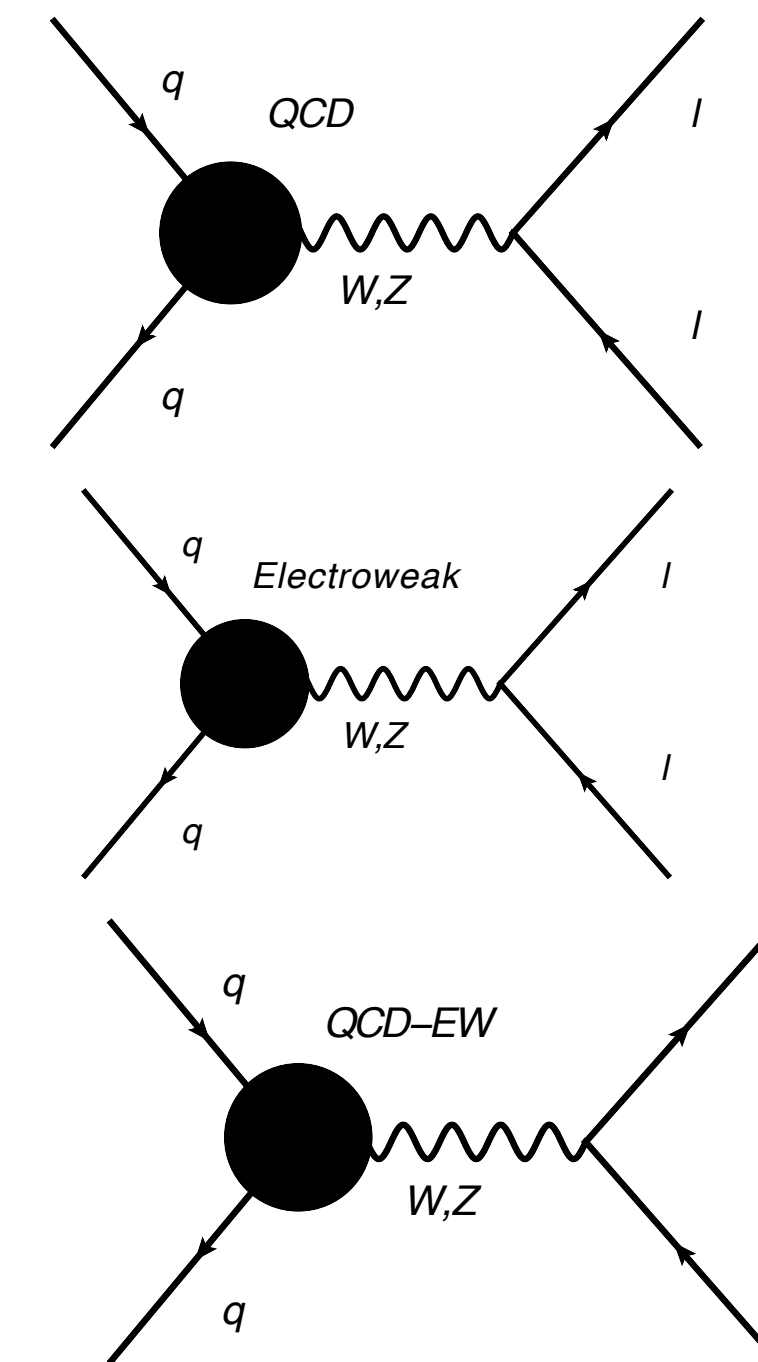
**Huss, Dittmaier, Schwinn**

Fiducial cross sections for the  $W^+$  boson production at the 13 TeV LHC computed with NNPDF3.1lxQED parton distribution functions and employing on-shell renormalization scheme with  $G_F$  as the input parameter. [Corrections to  \$W\$  decays are not included.](#)

$$\sigma_{pp \rightarrow W^+} = \sigma_{\text{LO}} + \Delta\sigma_{\text{NLO},\alpha_s} + \Delta\sigma_{\text{NLO},\alpha} + \Delta\sigma_{\text{NNLO},\alpha\alpha_s} + \dots$$

$$p_{\perp}^l, p_{\perp}^{\nu} > 15 \text{ GeV} \quad |y_e| < 2.4$$

$\sigma$ [pb]	channel	$\mu = M_W$	$\mu = M_W/2$	$\mu = M_W/4$
$\sigma_{\text{LO}}$		6007.6	5195.0	4325.9
$\Delta\sigma_{\text{NLO},\alpha_s}$	all ch.	508.8	1137.0	1782.2
	$q\bar{q}'$	1455.2	1126.7	839.2
	$qg/gq$	-946.4	10.3	943.0
$\Delta\sigma_{\text{NLO},\alpha}$	all ch.	2.1	-1.0	-2.6
	$q\bar{q}'$	-2.2	-5.2	-6.7
	$q\gamma/\gamma q$	4.2	4.2	4.04
$\Delta\sigma_{\text{NNLO},\alpha_s\alpha}$	all ch.	-2.4	-2.3	-2.8
	$q\bar{q}'/qq'$	-1.0	-1.2	-1.0
	$qg/gq$	-1.4	-1.2	-2.1
	$q\gamma/\gamma q$	0.06	0.03	-0.04
	$g\gamma/\gamma g$	-0.12	0.04	0.30



**Behring, Buccioni, Caola, Delto, K.M., Jaquier, Röntsch**

Mixed QCD-EW corrections are about 0.5 per mille; not obviously irrelevant for the extraction of the  $W$  mass at the LHC !

Mixed QCD-EW corrections are comparable to EW corrections (the consequence of  $G_F$  input scheme).

It is difficult to study the impact of these corrections on the W-mass determination without including them into a full experimental analysis toolchain. However, we can estimate their impact using a simple observation that values of moments of transverse momenta of the charged lepton from W and Z decays are correlated with Z and W masses.

$$m_W^{\text{meas}} = \frac{\langle p_{\perp}^{l,W} \rangle_{\text{meas}}}{\langle p_{\perp}^{l,Z} \rangle_{\text{meas}}} m_Z C_{\text{th}} \quad C_{\text{th}} = \frac{m_W}{m_Z} \frac{\langle p_{\perp}^{l,Z} \rangle_{\text{th}}}{\langle p_{\perp}^{l,W} \rangle_{\text{th}}}$$

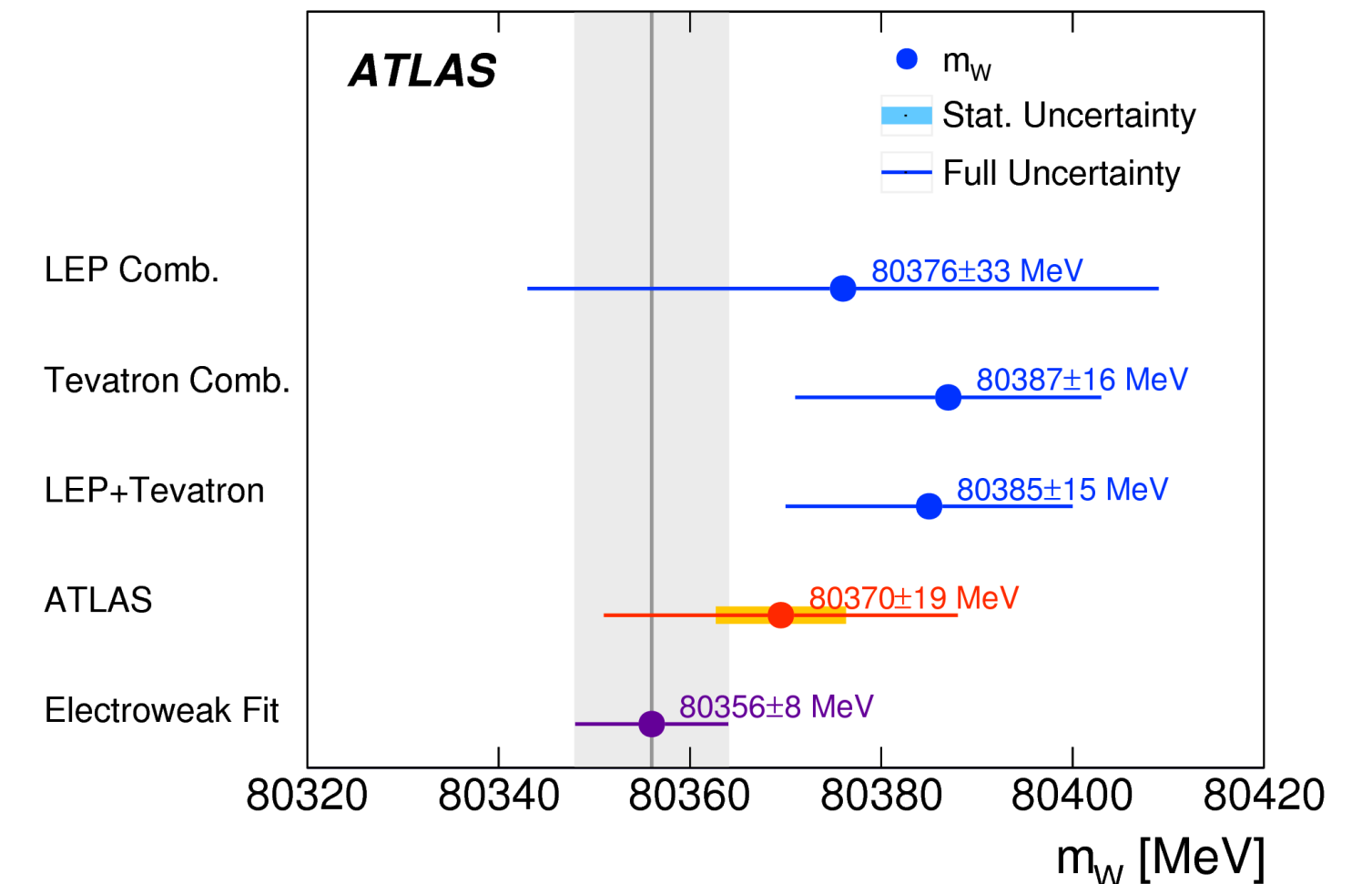
A better theory changes the theoretical correction factor and leads to changes in the extracted value of the W mass.

$$\frac{\delta m_W^{\text{meas}}}{m_W^{\text{meas}}} = \frac{\delta C_{\text{th}}}{C_{\text{th}}} = \frac{\delta \langle p_{\perp}^{l,Z} \rangle_{\text{th}}}{\langle p_{\perp}^{l,Z} \rangle_{\text{th}}} - \frac{\delta \langle p_{\perp}^{l,W} \rangle_{\text{th}}}{\langle p_{\perp}^{l,W} \rangle_{\text{th}}}$$

**No fiducial cuts:**  $\Delta m_W = m_W - m_W^{EW} = 7 \text{ MeV}$

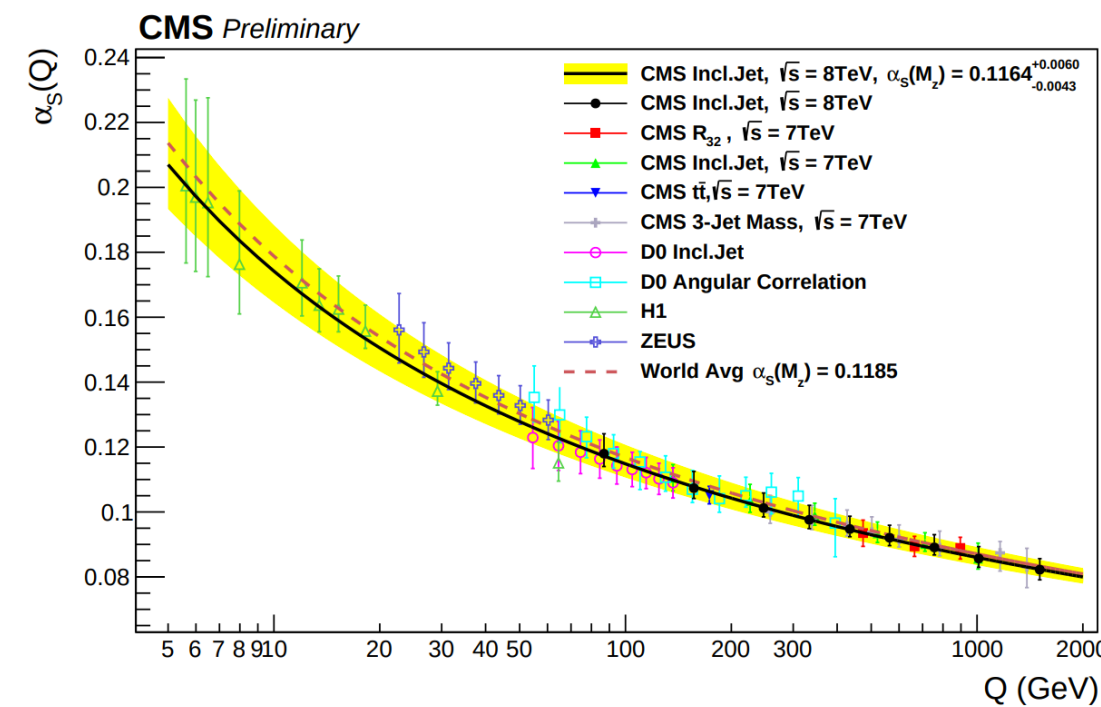
- QCD-electroweak effects are more important than the electroweak ones;
- Compensation mechanism between W and Z distribution is important; in first moments taken separately are close to 50 MeV;
- PDF uncertainty has a very minor impact on these shifts;

**ATLAS cuts:**  $\Delta m_W = m_W - m_W^{EW} = 17 \text{ MeV}$



The strong coupling constant can be extracted from many measurements at different energy scales, and then put to a common denominator by using renormalization group evolution

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = \beta(\alpha_s) = -b_0 \alpha_s^2 + ..$$



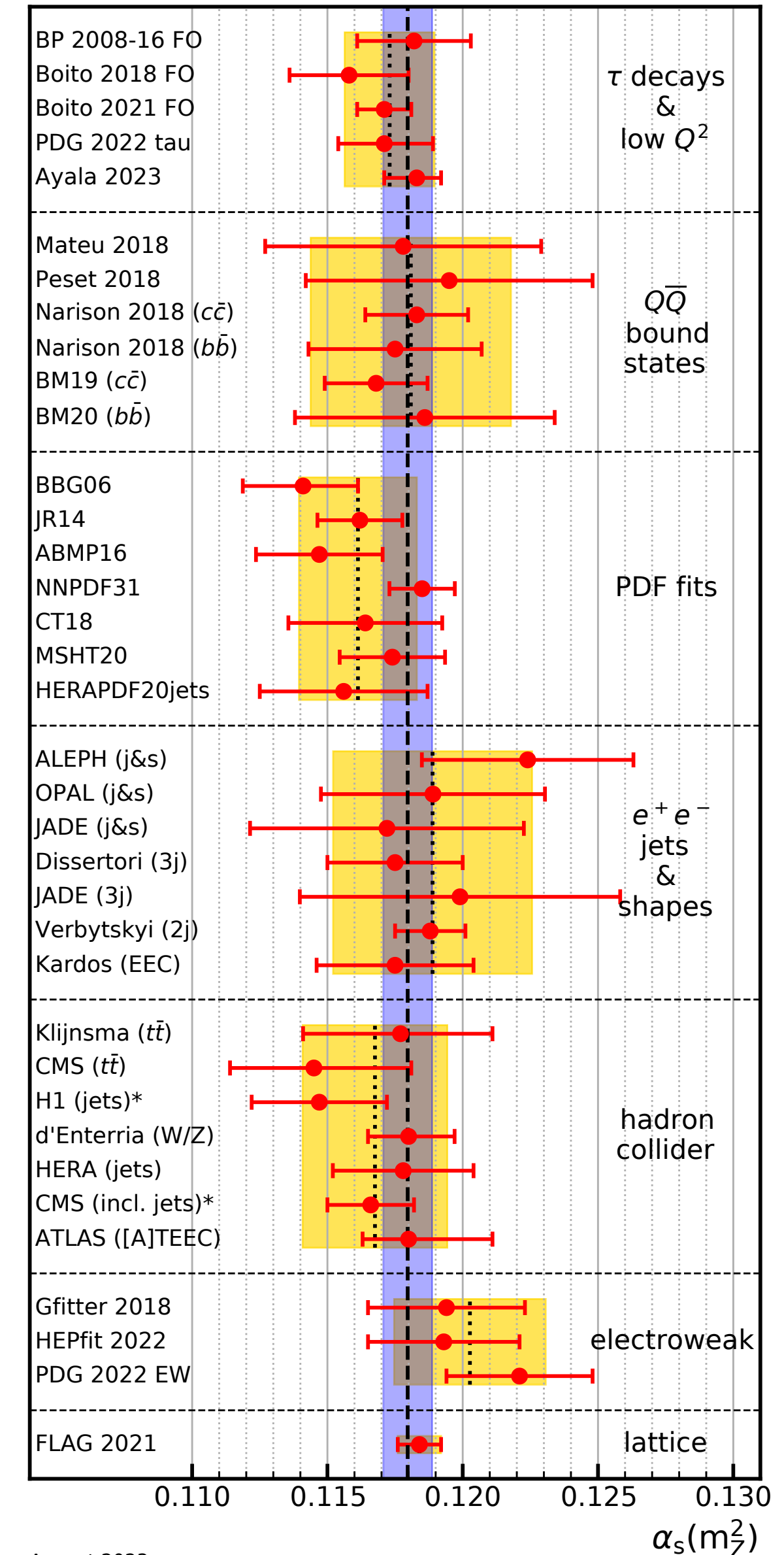
The most precise values of the strong coupling constant currently come from lattice calculations and studies of inclusive tau-lepton decays.

There also exist very precise determinations of the strong coupling constant from shape variables at LEP but the results are controversial because of unclear status of non-perturbative corrections in the three-jet region.

Hadron collider determinations (LHC including) are typically not competitive since the already-achieved precision on the strong coupling is very high, O(1%).

$$\alpha_s(M_z) = 0.1180 \pm 0.0009$$

the world average, PDG, 2023



If one wants to make a competitive measurement of the strong coupling at the LHC, one needs to find a quantity which

- 1) is proportional to the strong coupling constant;
- 2) can be predicted theoretically with a percent precision (NNLO and higher);
- 3) is independent (nearly independent) on poorly-known parton distribution functions;
- 4) refers to low(er) region of hard momentum region;
- 5) does not suffer from unknown non-perturbative effects.

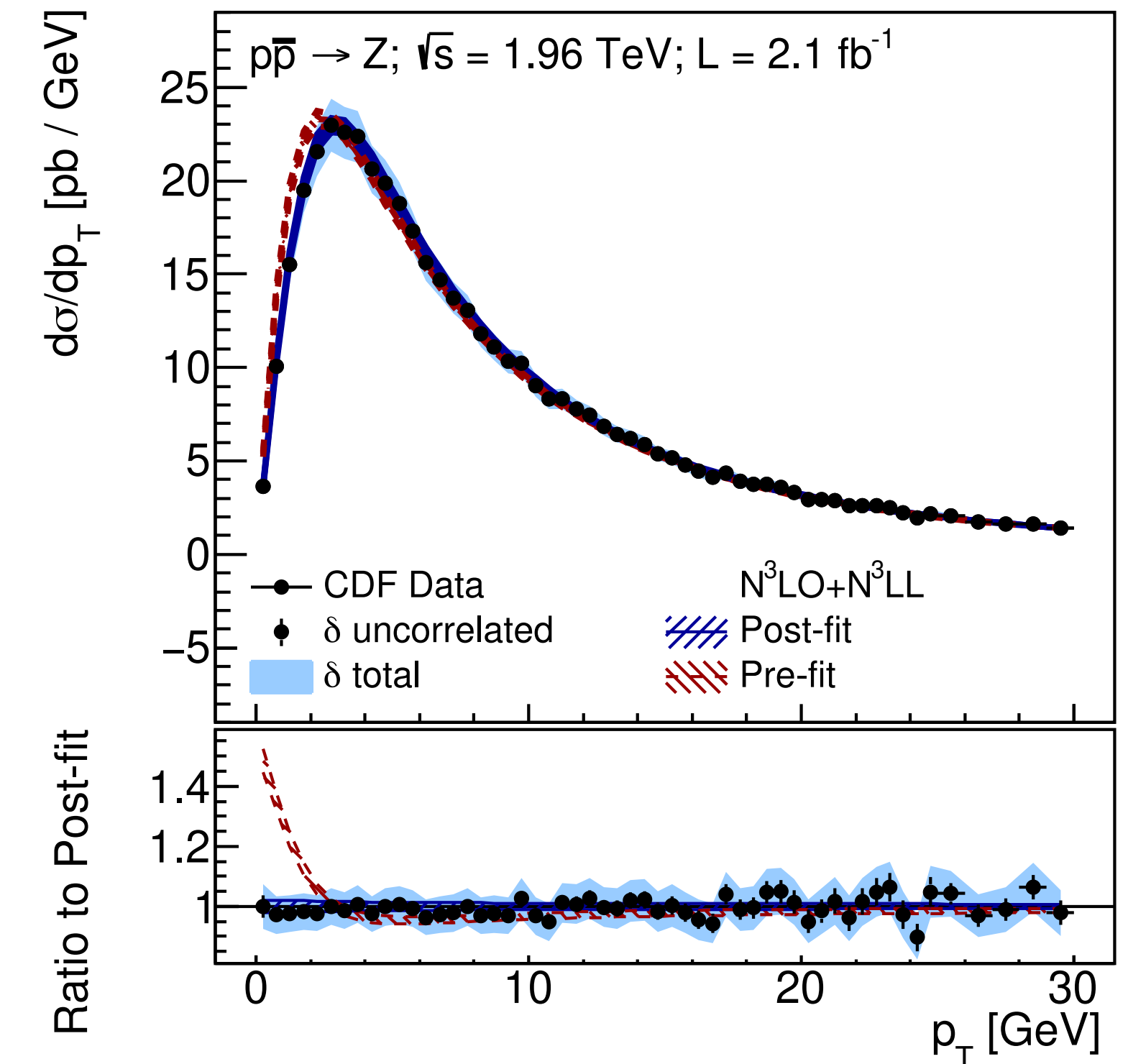
Inclusive Z transverse momentum distribution seems to fit the bill.

$$\frac{d\sigma_Z}{\sigma_Z dp_\perp} \sim \frac{\alpha_s(p_\perp)}{2\pi p_\perp} \ln \frac{M_Z}{p_\perp}$$

ATLAS followed up on the proposal and obtained a very precise value of the strong coupling constant which is very well-compatible with the world average.

$$\alpha_s(m_z) = 0.1183 \pm 0.0009 \quad \text{ATLAS, 8 TeV data}$$

A percent-level prediction for this observable requires us to employ some of the most sophisticated theoretical tools and results that are currently available.



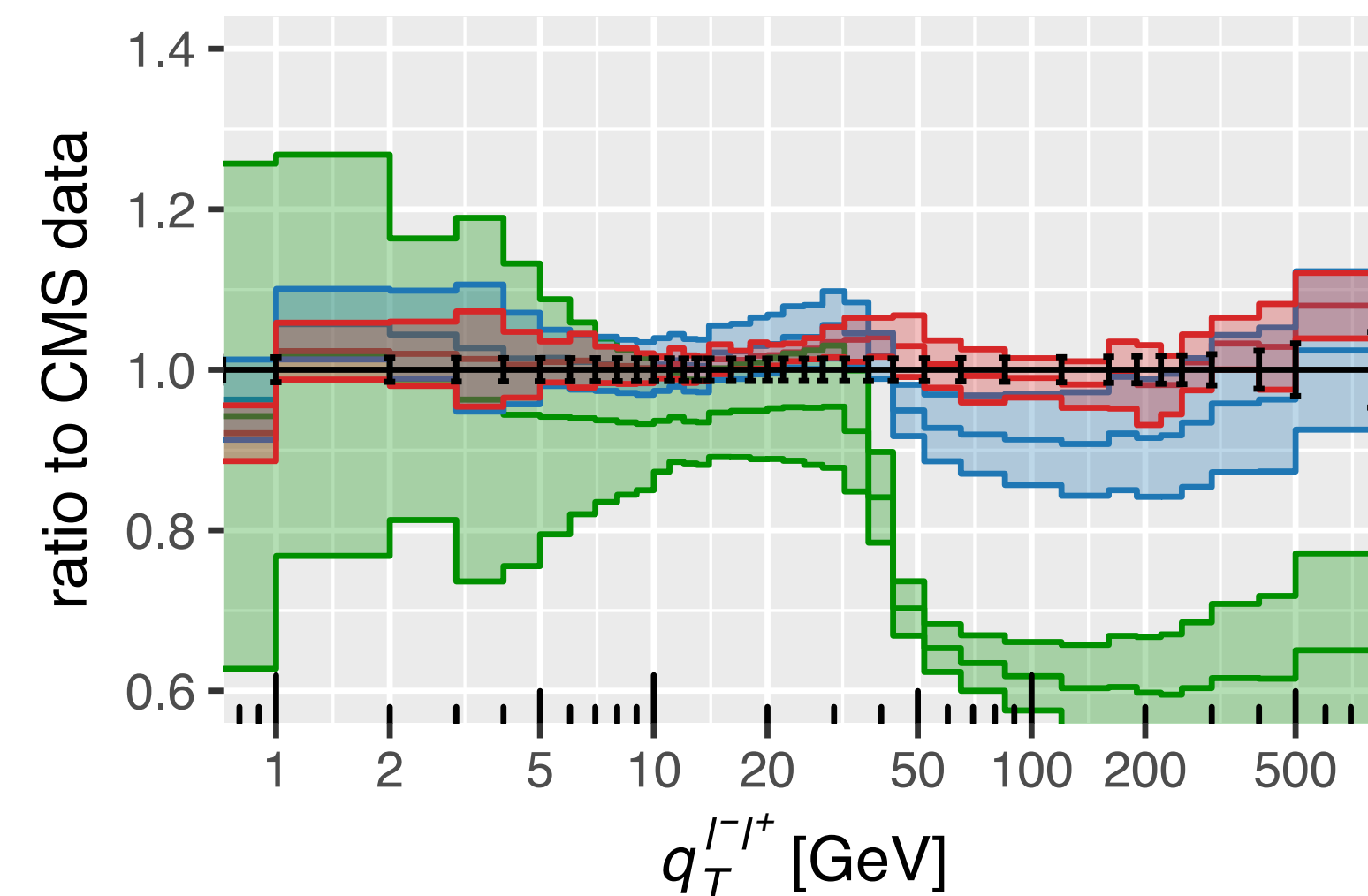
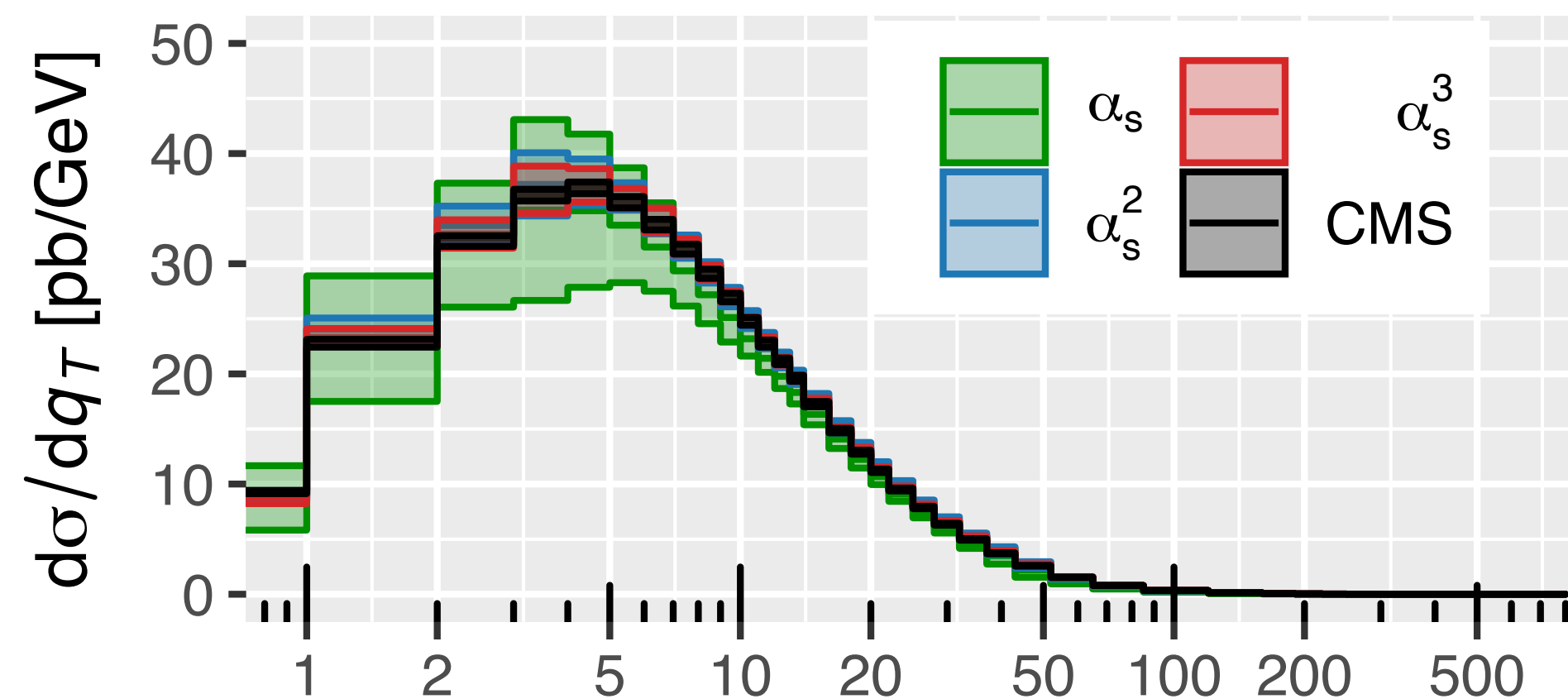
Camarada, Ferrera, Schott

A percent-level prediction for this observable requires us to employ some of the most sophisticated theoretical tools and results that are currently available.

Let me summarize them:

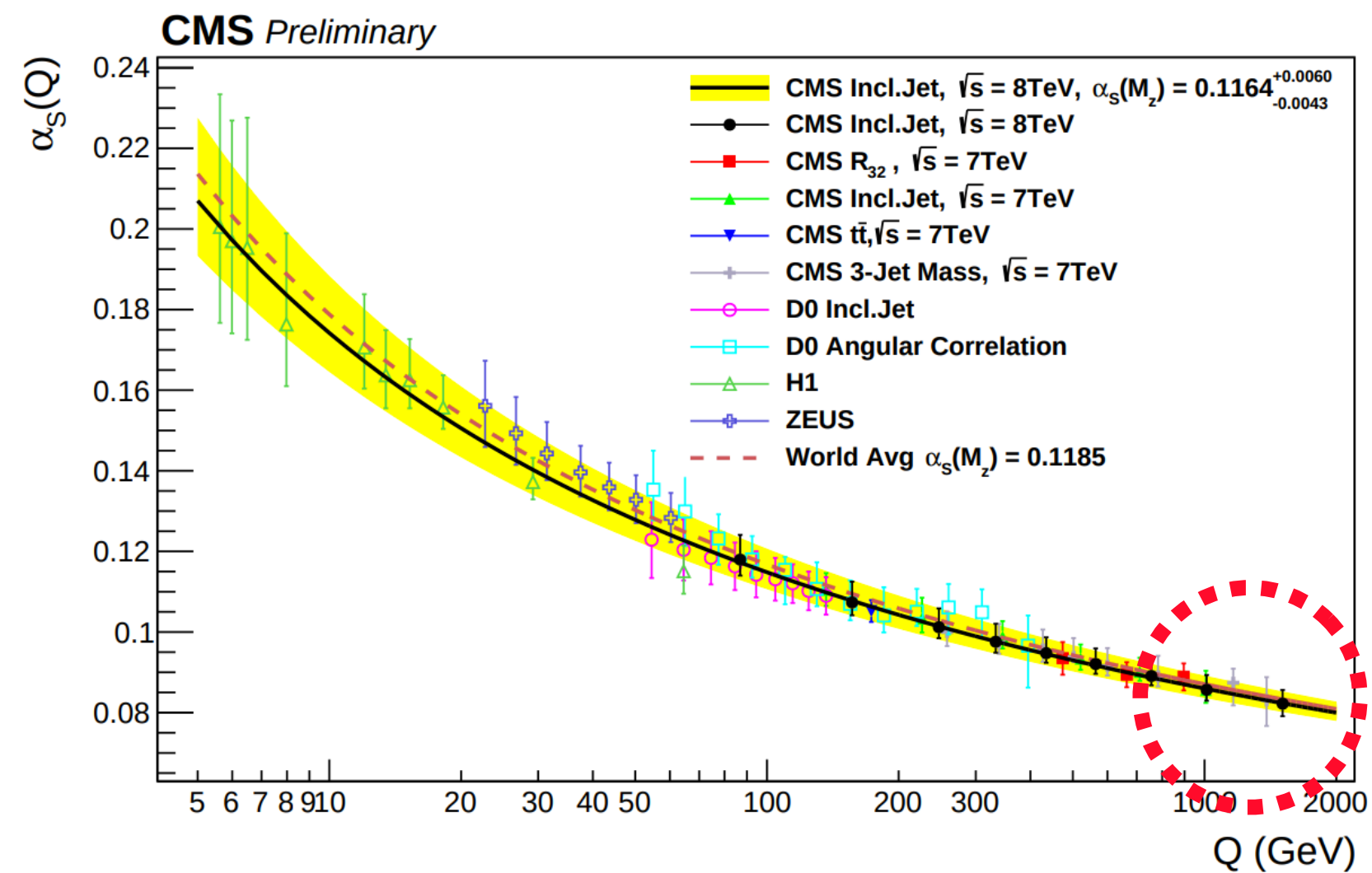
- 1) N3LO QCD predictions for inclusive Z-boson production cross section and rapidity distribution;
- 2) NNLO QCD predictions for Z+jet production;
- 3) state-of-the-art transverse momentum resummation formulas that describe Z-boson transverse momentum distribution at small  $p_T$ ;
- 4) electroweak corrections to Z+jet production;
- 5) advanced knowledge of parton distribution functions;
- 6) models for non-perturbative smearing at small transverse momenta.

Duhr, Mistlberger, X. Chen, Gehrmann, Gehrmann-de Ridder, Glover, Zhu, Yang, Huss, Vita, Ebert, Luou, Boughezal, Focke, Liu, Petriello, Ellis, Giele, Campbell et al.



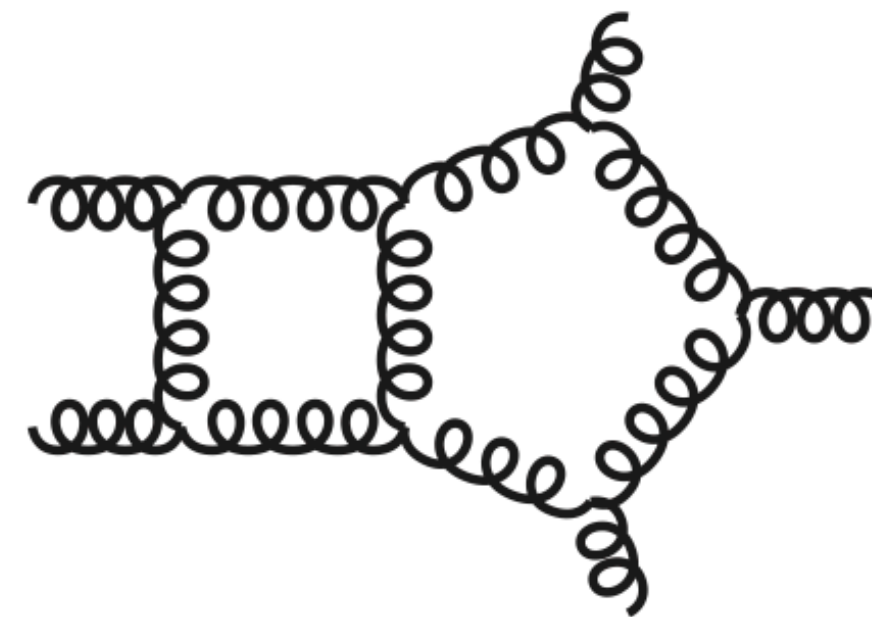
Neumann and Campbell

A very unique thing that the LHC can do is to check the running of the coupling constant at the highest energies. A useful observable is the ratio of the three-jet to a two-jet rate since some systematic uncertainties cancel in the ratio. NLO results known since quite some time. Pushing them to the next level — NNLO — was an enormous adventure.

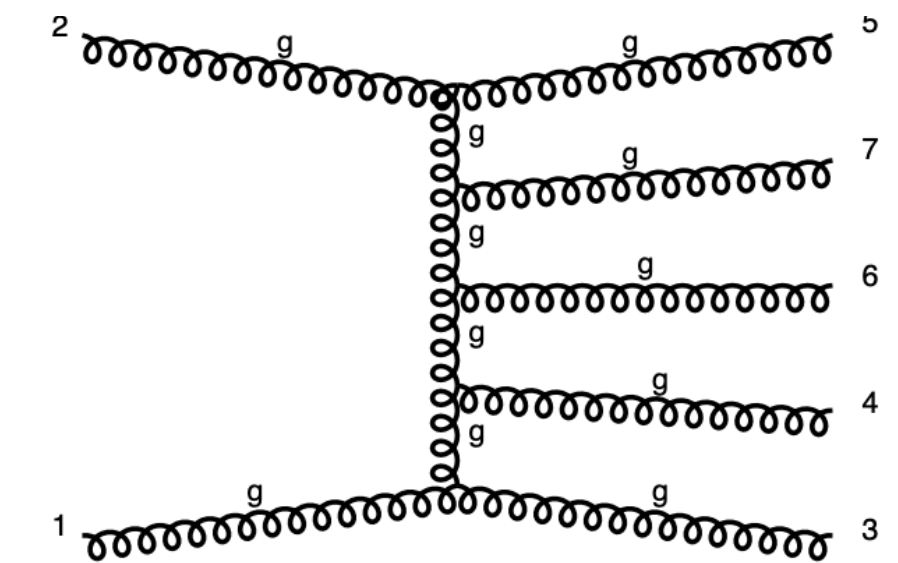


$$R_{3/2} = \frac{d\sigma_{3j}}{d\sigma_{2j}} \sim \alpha_s(p_{\perp,j})$$

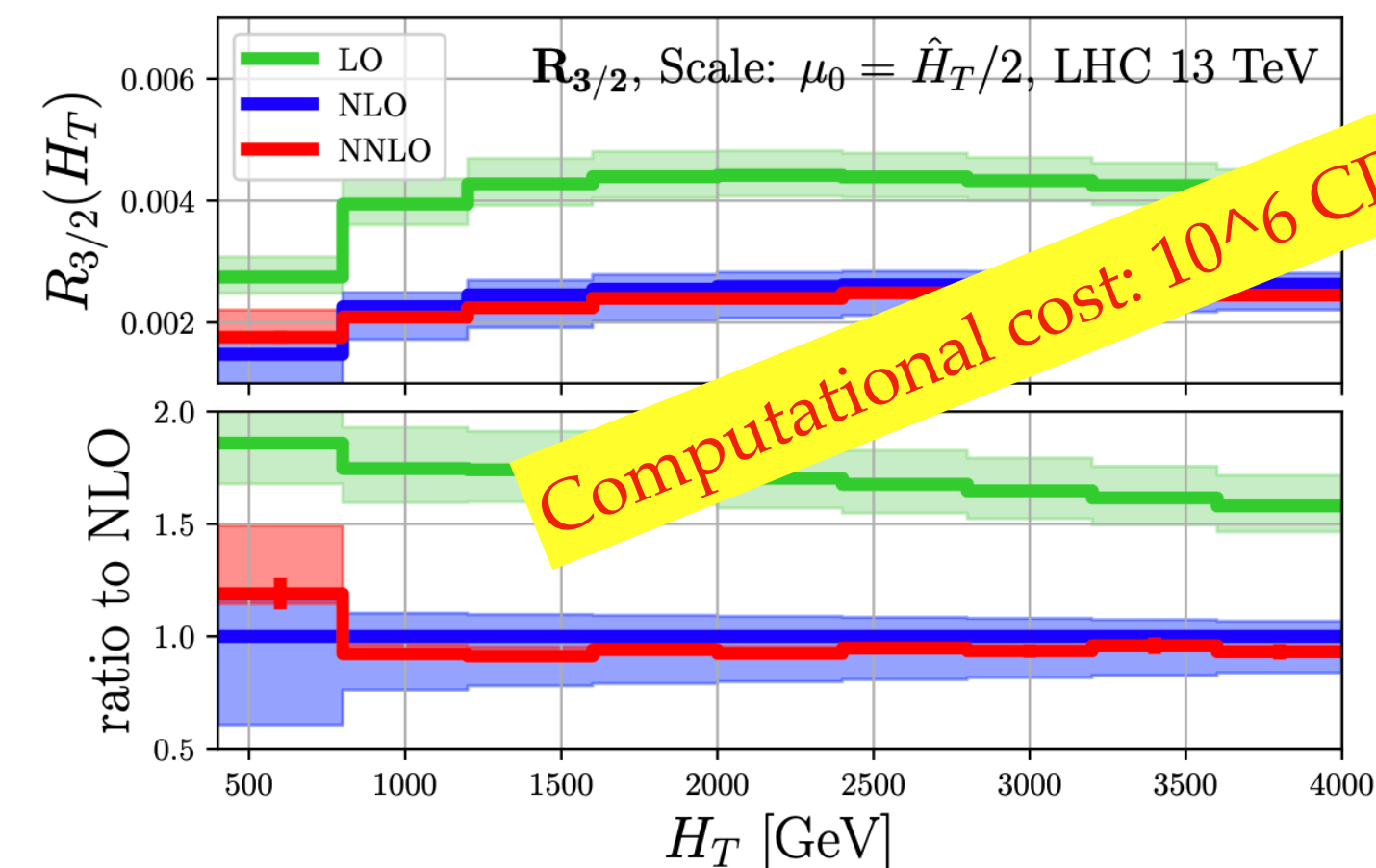
$$\alpha_s = 0.115 \pm 0.006 \quad \text{CMS, 7 TeV [ NLO theory]}$$



Two-loop amplitudes: Chicherin, Sotnikov, Abreu et al.



Subtraction scheme: Czakon



Czakon, Mitov, Poncelet



## SUMMARY

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High-precision investigation of the outcomes of hard hadron collisions continues to be one of the focus points of the LHC physics program. In addition to excellent experimental capabilities, this requires theory that can relate the SM Lagrangian to predictions for proton collisions with controllable precision.

Our ability to do that increased tremendously during the past decade thanks to theoretical advances in QCD perturbation theory.

Nevertheless, reaching a few percent precision for generic processes continues to be challenging and requires further advances with computing loop amplitudes, improved efficiency of subtraction schemes, improving parton shower event generators and ensuring reliability of PDF fits.

Eventually, we will also have to face the issue of understanding more systematically non-perturbative and other subtle effects at hadron colliders. We already see the need to do this when discussing quantities measured with ultra-high precision, for example the top quark mass.