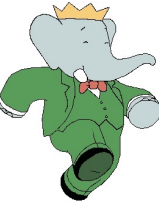


Measurement of $|V_{cb}|$ in $\bar{B} \rightarrow D l^- \bar{\nu}_l$ with hadronic tagging at BaBar



Fergus Wilson, Rutherford Appleton Laboratory

On behalf of the BaBar Collaboration

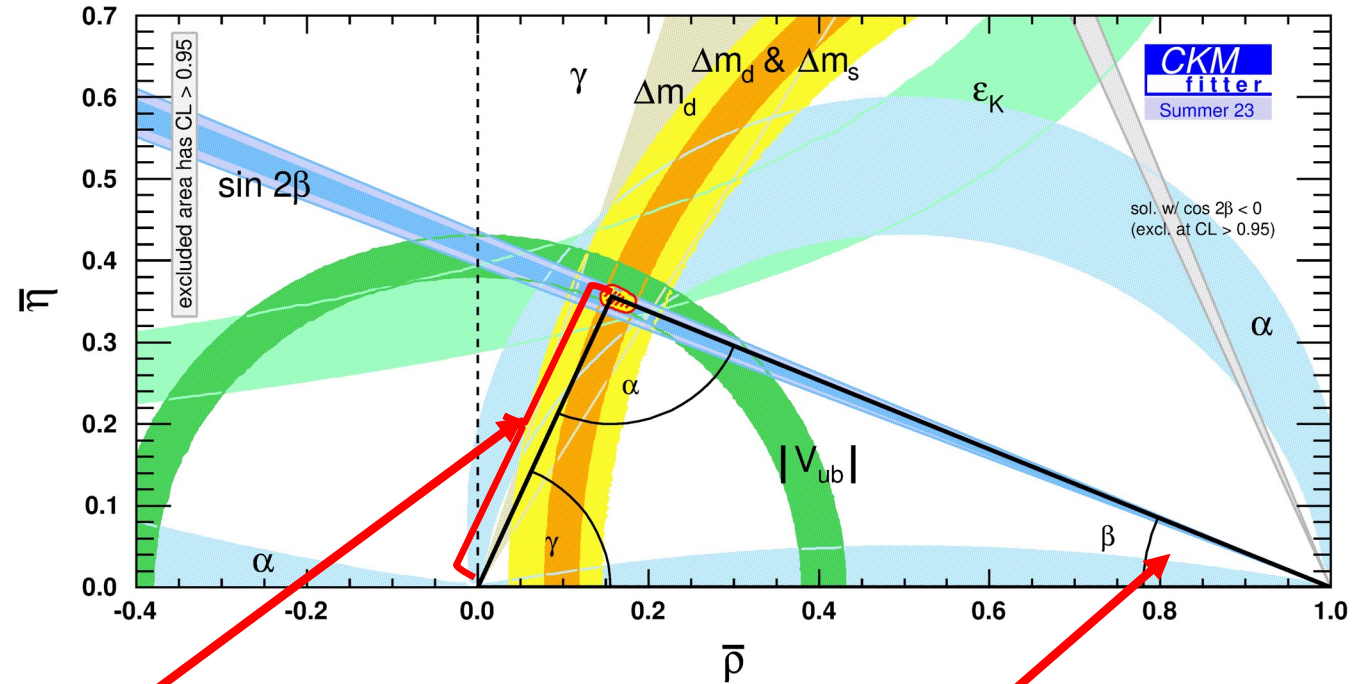
Lake Louise Winter institute, 18th – 24th February 2024

Unitarity Triangle and $|V_{cb}|$

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Angles and sides have been measured to a high precision.

Important to measure sides and angles as New Physics could appear in discrepancies in measurements



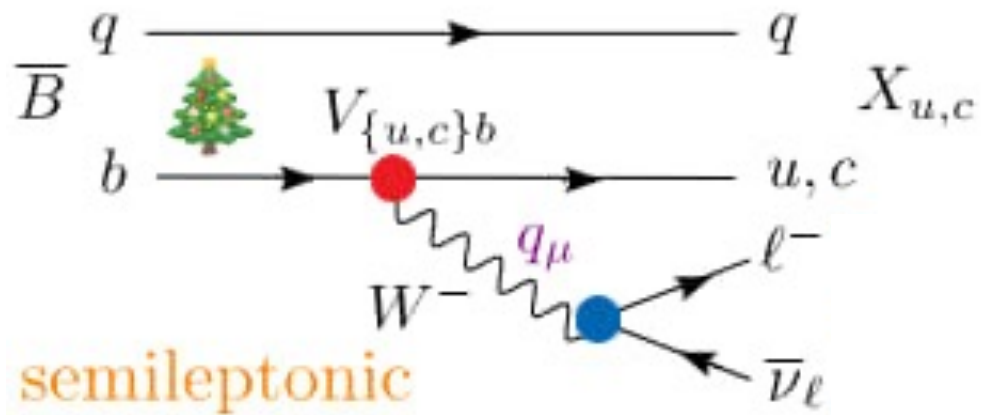
Sin 2β known to within $\sim 2\%$

$$\left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right| = \left| \frac{V_{ub}}{V_{cb}} \right| \frac{1}{\tan\theta_c}$$

$$\frac{\delta|V_{cb}|}{|V_{cb}|} \sim 1.5\%$$

$$\frac{\delta|V_{ub}|}{|V_{ub}|} \sim 4\%$$

Measurement Methods and Current Status



Can measure through

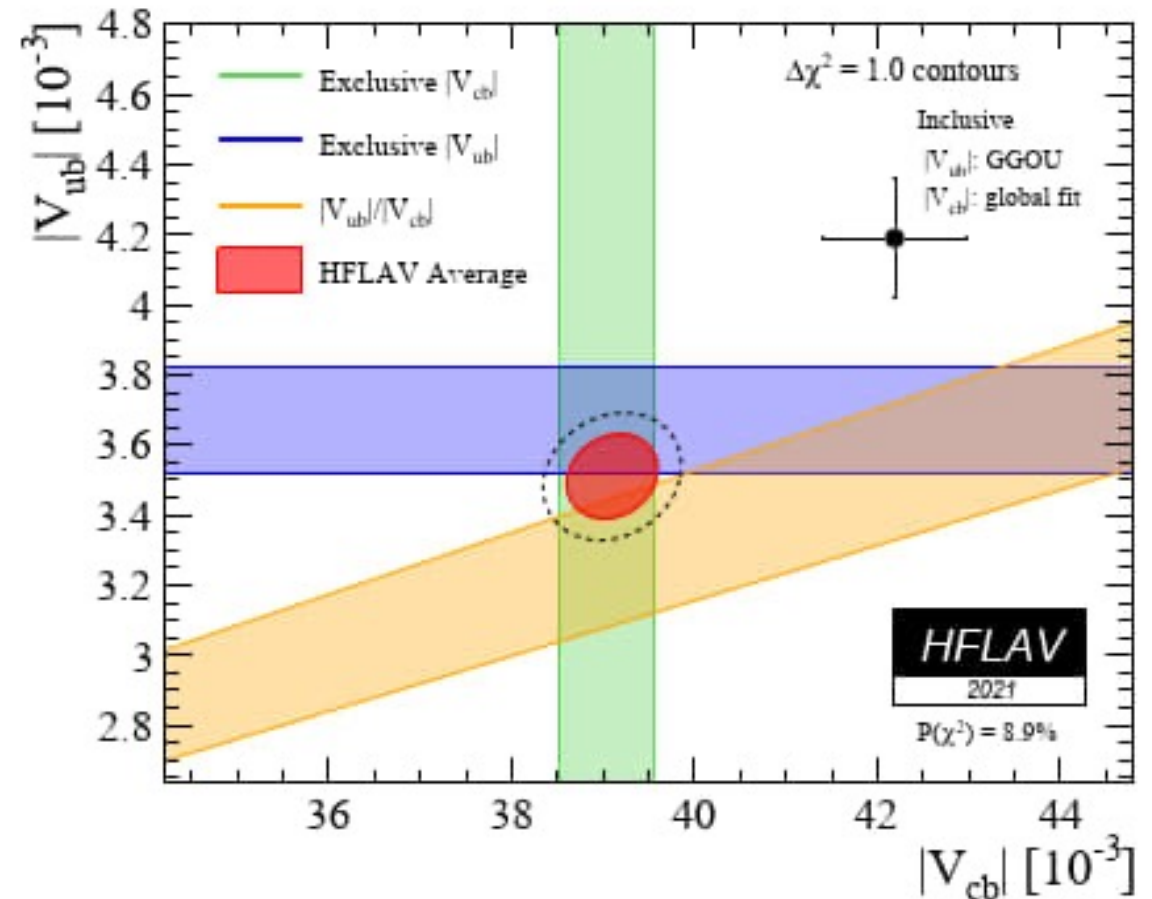
Inclusive decays: sum over all hadronic final states

Exclusive decays: reconstruct $X_{u,c}$

Exclusive vs. Inclusive measurements: lower signal efficiency, better background rejection, measure as a function of q^2 , BUT need Form Factors to describe hadronization process

Exclusive measurements systematically lower than inclusive for both $|V_{ub}|$ and $|V_{cb}|$ by 5% - 10% ($\sim 3\sigma$)

Explanations: Experimental or theoretical? Perhaps even New Physics.



Decay Rates and Form Factors

In the massless lepton limit, $\bar{B} \rightarrow D l^- \bar{\nu}_l$ amplitude depends on a single vector Form Factor f_+ (scalar f_0 Form Factor becomes relevant when $m_l > 0$):

$$\frac{d\Gamma}{dq^2 d\cos\theta_l} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{32\pi^3} k^3 |f_+(q^2)|^2 \sin^2\theta_l \quad \text{with } k = m_D \sqrt{w^2 - 1}$$

Form Factor sometimes written as $G(w)^2 = \frac{4r}{(1+r)^2} f_+(w)^2$

There are different methods of expressing the Form Factor parameterisations:

CLN: $G(w) = G(1)(1 - 8\rho_D^2 z(w) + (51\rho_D^2 - 10)z(w)^2 - (252\rho_D^2 - 84)z(w)^3)$

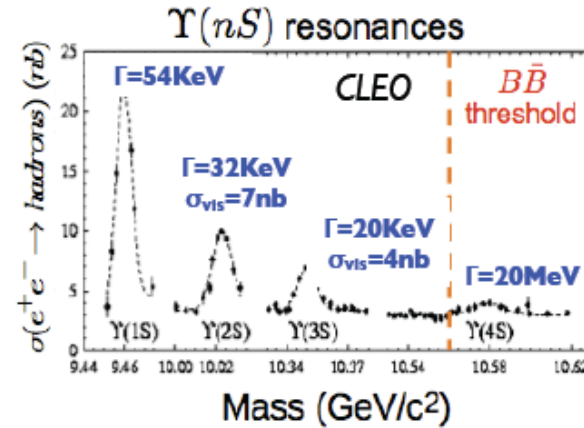
BGL: $f_i(z) = \frac{1}{P_i(z), \phi_i(z)} \sum_{n=0}^N a_n^i z^n \quad i \in \{+, 0\}$

Both with $z(w) = \frac{(\sqrt{w+1}-\sqrt{2})}{(\sqrt{w+1}+\sqrt{2})} \quad w = \frac{m_B^2 + m_D^2 - q^2}{2m_B m_D}$

CLN: ρ_D^2 and $G(1)$ free parameters
BGL: a_n^i free parameter, $N = 2$ or 3
 $P_i(z), \phi_i(z)$ supplied by theory.

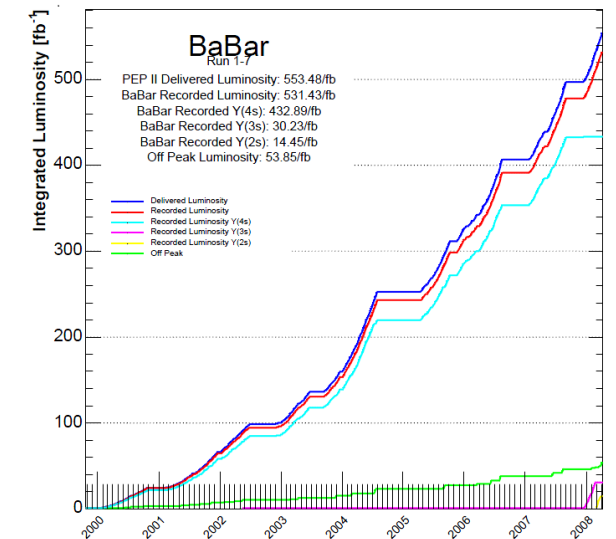
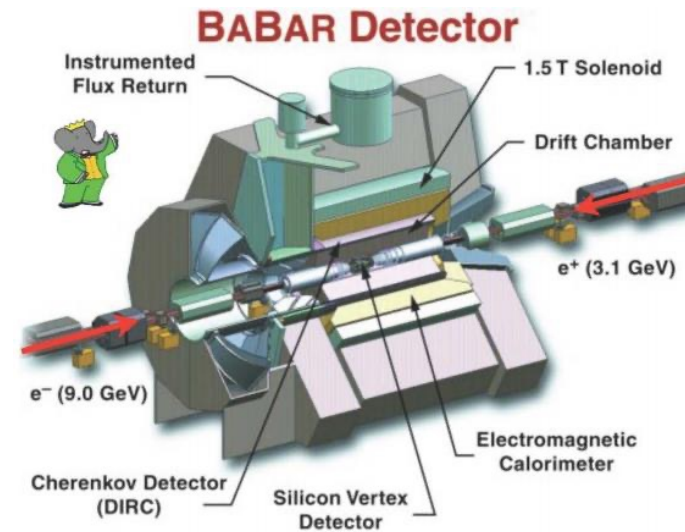
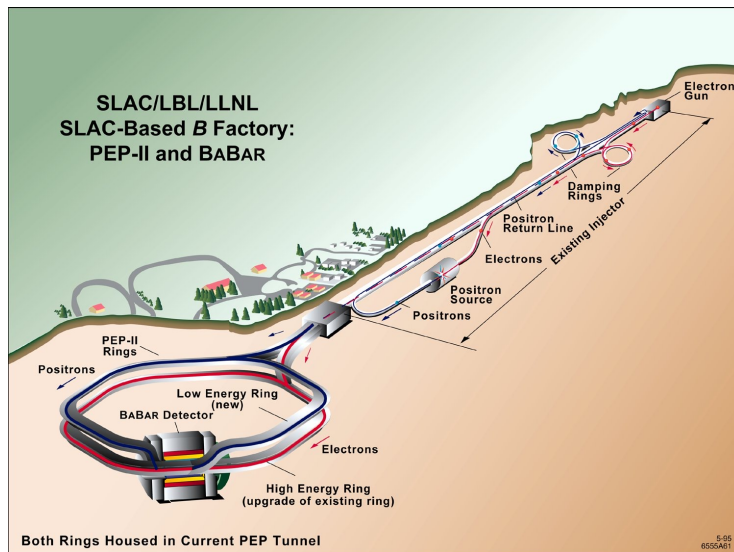
BaBar at SLAC PEP-II: 1999 - 2008

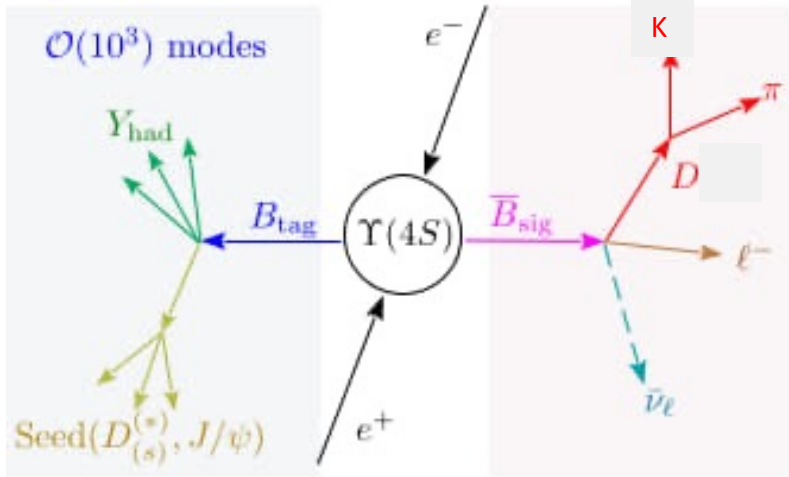
- Asymmetric beam energies nominally colliding 3.1 GeV e^+ and 9.0 GeV e^- at (mostly) the $\Upsilon(4S)$ resonance



	Lumi	$B\bar{B}$ Events
$\Upsilon(4S)$	424 fb^{-1}	471×10^6
$\Upsilon(3S)$	28 fb^{-1}	121×10^6
$\Upsilon(2S)$	14 fb^{-1}	99×10^6
$\tau^+\tau^-$		$\sim 450 \times 10^6$

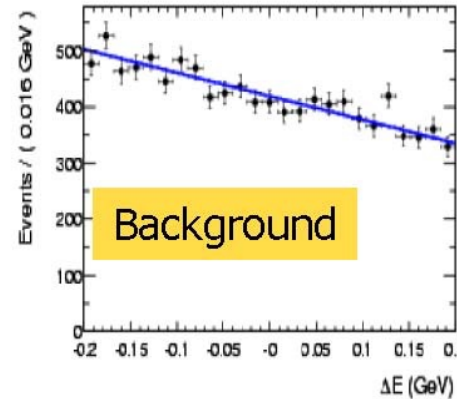
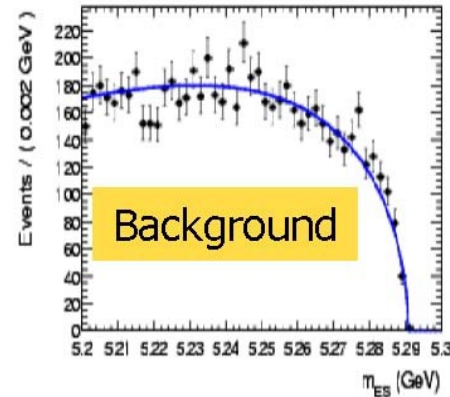
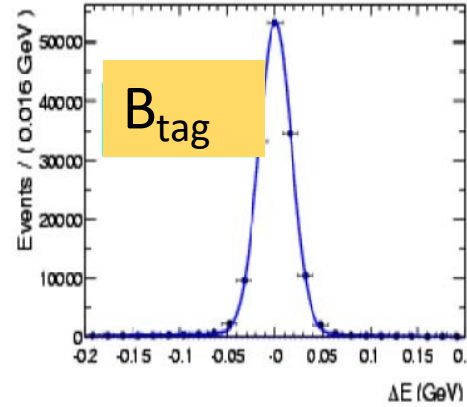
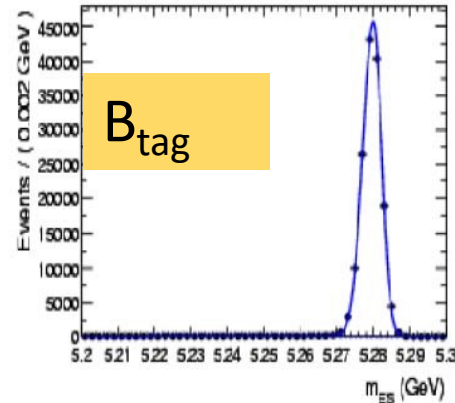
Only $\Upsilon(4S)$ used in this analysis





Y_{had} system: charmless light hadrons
 $|\Delta E| < 72 \text{ MeV}$
 $m_{ES} > 5.27 \text{ GeV}$

$$q = p_{B_{sig}} - p_D$$



Signal-side decays

ℓ^-	D	decay mode	mode
e^-	D^0	$K^- \pi^+$	0
e^-	D^0	$K^- \pi^+ \pi^0$	1
e^-	D^0	$K^- \pi^+ \pi^- \pi^+$	2
e^-	D^+	$K^- \pi^+ \pi^+$	3
e^-	D^+	$K^- \pi^+ \pi^+ \pi^0$	4
μ^-	D^0	$K^- \pi^+$	5
μ^-	D^0	$K^- \pi^+ \pi^0$	6
μ^-	D^0	$K^- \pi^+ \pi^- \pi^+$	7
μ^-	D^+	$K^- \pi^+ \pi^+$	8
μ^-	D^+	$K^- \pi^+ \pi^+ \pi^0$	9

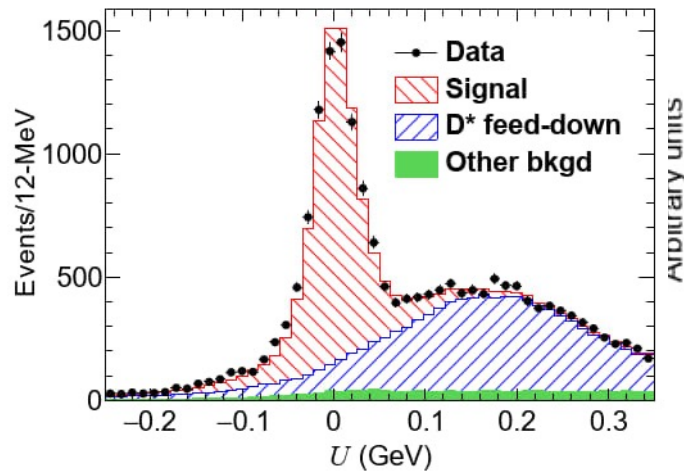
$$m_{ES} = \sqrt{E_{beam}^{*2} - p_{tag}^{*2}}$$

$$\Delta E = E_{tag}^* - E_{beam}^*$$

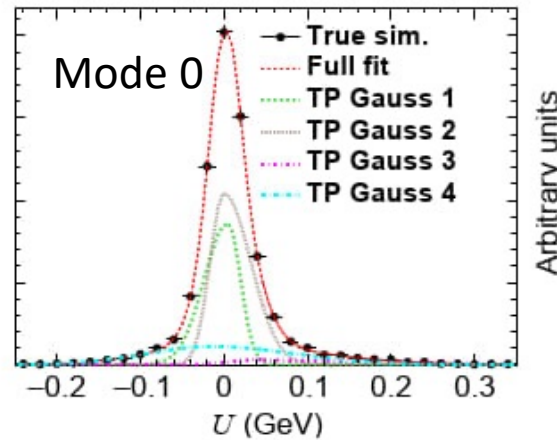
Background distribution dependent on q^2 and $\cos\theta_1$

Step 1: Perform fits to simulation to observable U in bins of q^2 and $\cos\theta_1$ for the 10 signal modes.

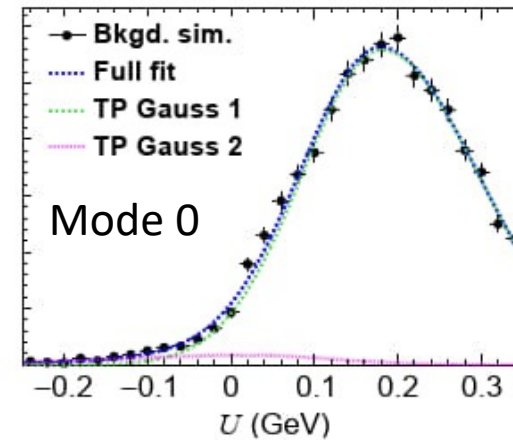
Step 2: Fit to data with fixed parameters for tails, other parameters allowed to float



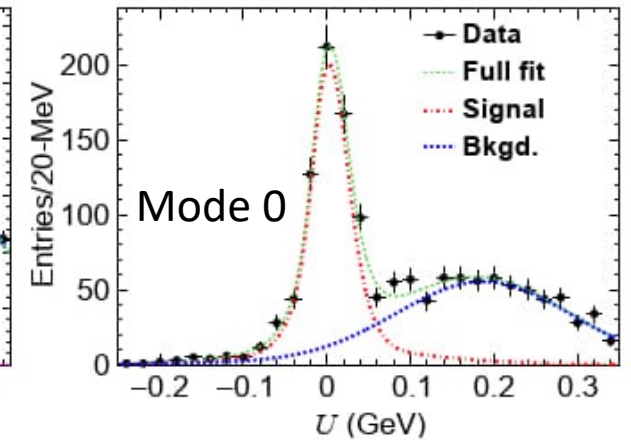
$$U = E_{miss}^* - p_{miss}^* = E_\nu^* - p_\nu^*$$



Signal



Background



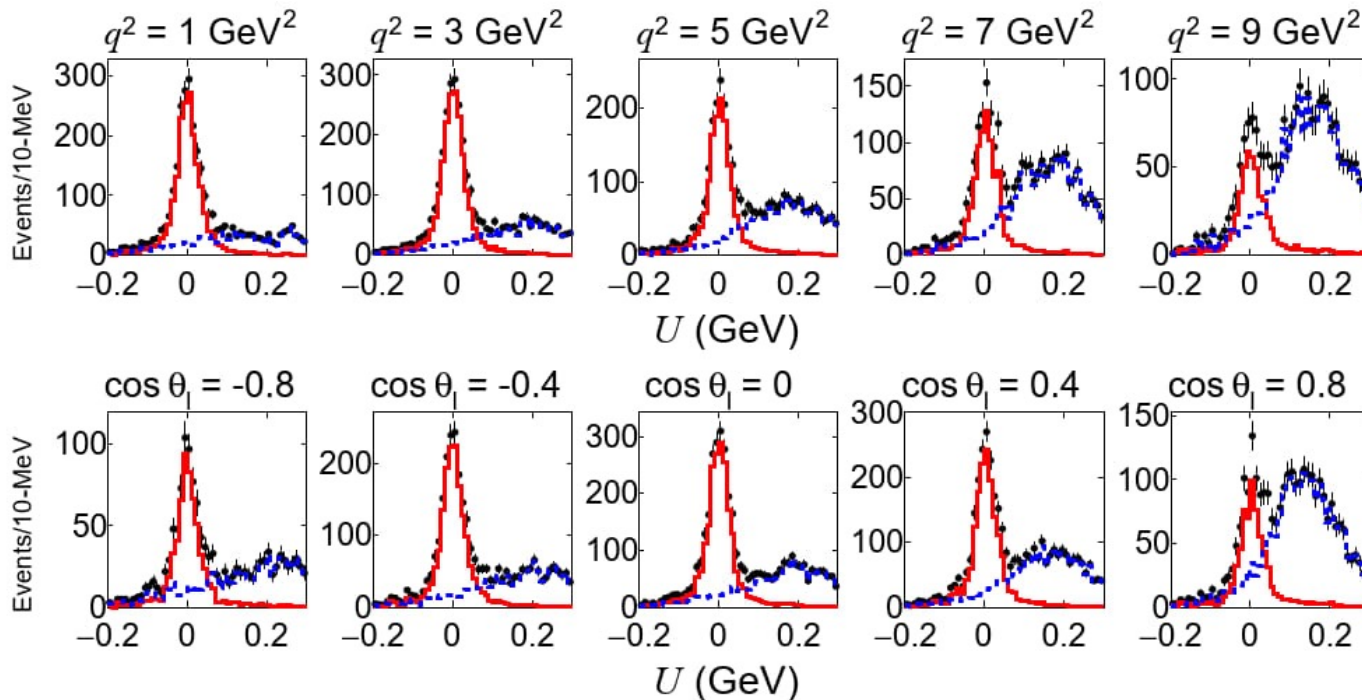
Data

Rather than use a binned distribution, the analysis uses:

$$g_{ij}^2 = \sum_{k=1}^n \left[\frac{\phi_k^i - \phi_k^j}{r_k} \right]^2 \quad (n=2, r_{q^2} = 10 \text{ GeV}^2, r_{\cos\theta_l} = 2)$$

Fit the $N_c=50$ nearest events, and extract a Q-factor:

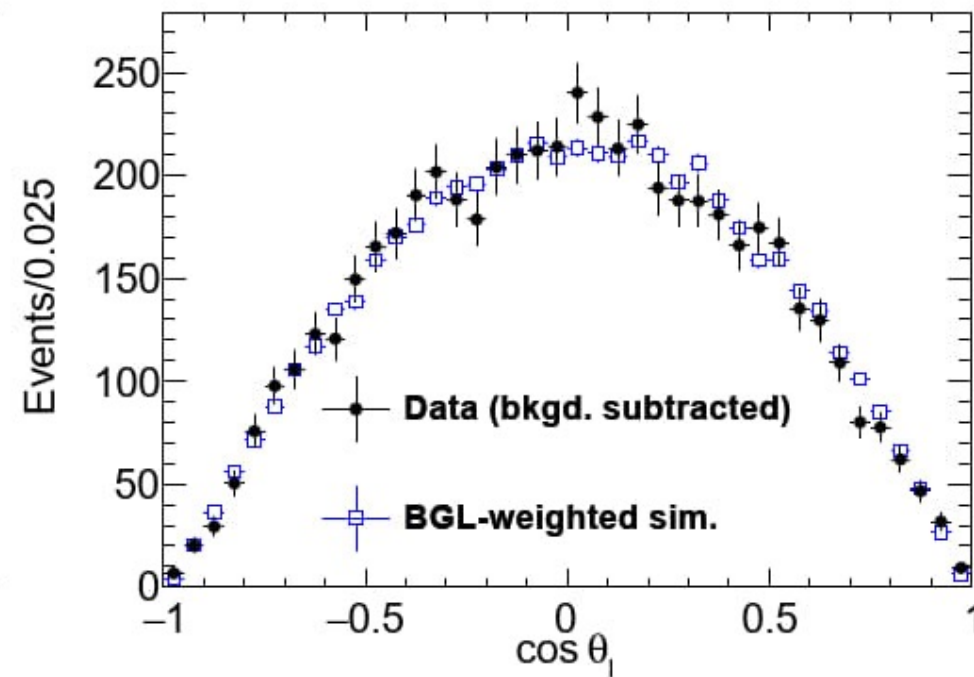
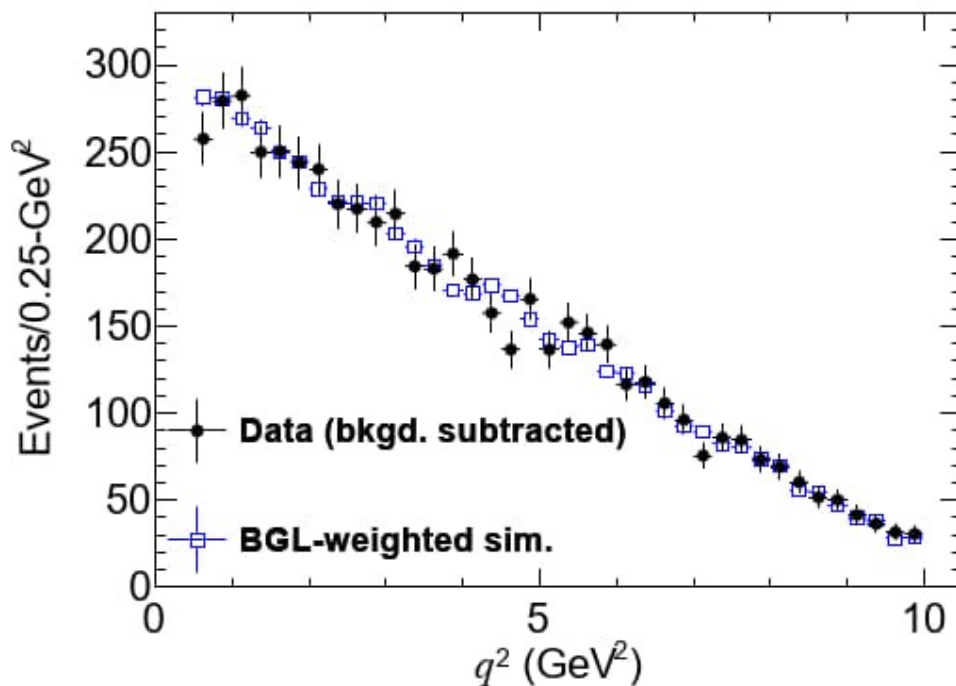
$$Q_i = \frac{S_i(U_i)}{S_i(U_i) + B_i(U_i)} \Rightarrow \text{Total yield, } y = \sum_i Q_i$$



ℓ^-	D	decay mode	mode	N_{sig}	N_{bkgd}
e^-	D^0	$K^- \pi^+$	0	539	63
		$K^- \pi^+ \pi^0$	1	813	196
		$K^- \pi^+ \pi^- \pi^+$	2	550	82
e^-	D^+	$K^- \pi^+ \pi^+$	3	721	41
		$K^- \pi^+ \pi^+ \pi^0$	4	204	120
μ^-	D^0	$K^- \pi^+$	5	433	64
		$K^- \pi^+ \pi^0$	6	798	221
		$K^- \pi^+ \pi^- \pi^+$	7	608	84
μ^-	D^+	$K^- \pi^+ \pi^+$	8	665	55
		$K^- \pi^+ \pi^+ \pi^0$	9	233	134
Total				5563	1061

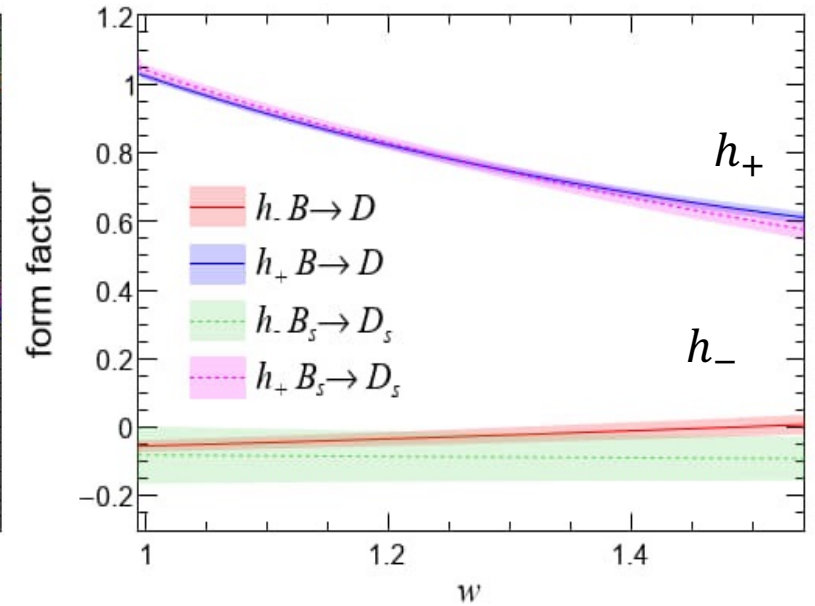
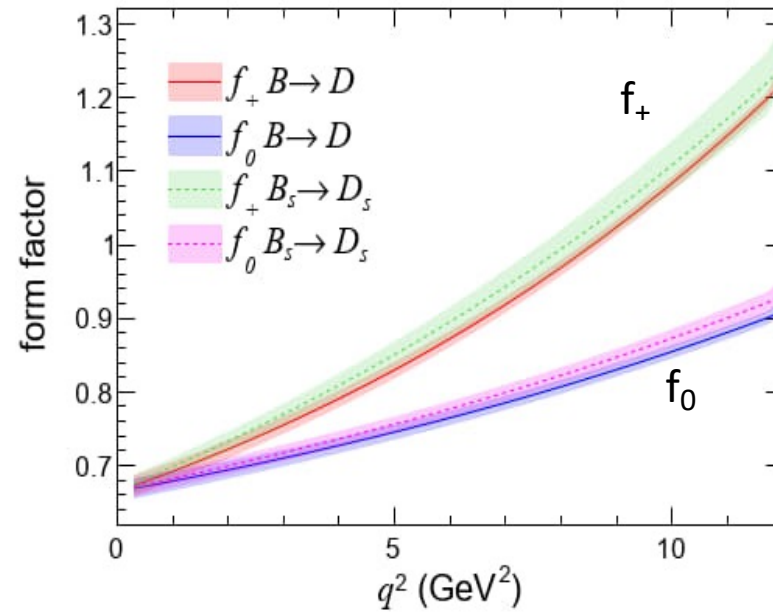
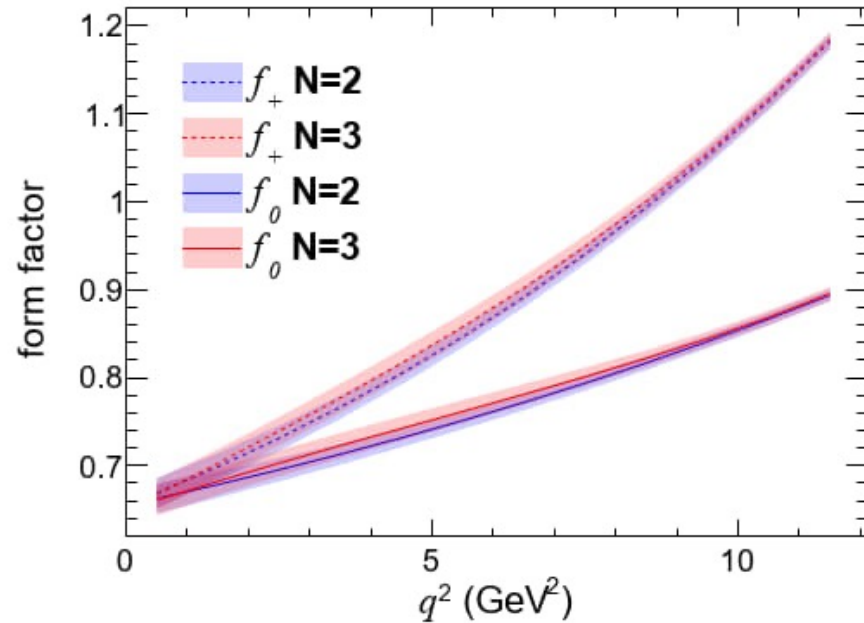
Different background approaches

fit configuration	$a_0^{J^+} \times 10$	$a_1^{J^+}$	$a_2^{J^+}$	$a_1^{f_0}$	$a_2^{f_0}$	$ V_{cb} \times 10^3$	χ^2_{MILC}	χ^2_{Belle}
BABAR-1, Belle	0.126 ± 0.001	-0.096 ± 0.003	0.352 ± 0.052	-0.059 ± 0.003	0.155 ± 0.049	41.09 ± 1.16	1.15	24.50
BABAR-2, Belle	0.126 ± 0.001	-0.096 ± 0.003	0.352 ± 0.052	-0.059 ± 0.003	0.155 ± 0.049	41.12 ± 1.16	1.17	24.54
BABAR-3, Belle	0.126 ± 0.001	-0.096 ± 0.003	0.350 ± 0.052	-0.059 ± 0.003	0.153 ± 0.049	41.12 ± 1.16	1.18	24.55
BABAR-4, Belle	0.126 ± 0.001	-0.096 ± 0.003	0.352 ± 0.052	-0.059 ± 0.003	0.156 ± 0.049	41.05 ± 1.17	1.14	24.45
BABAR-1	0.126 ± 0.001	-0.097 ± 0.003	0.334 ± 0.063	-0.059 ± 0.003	0.133 ± 0.062	-	1.55	-



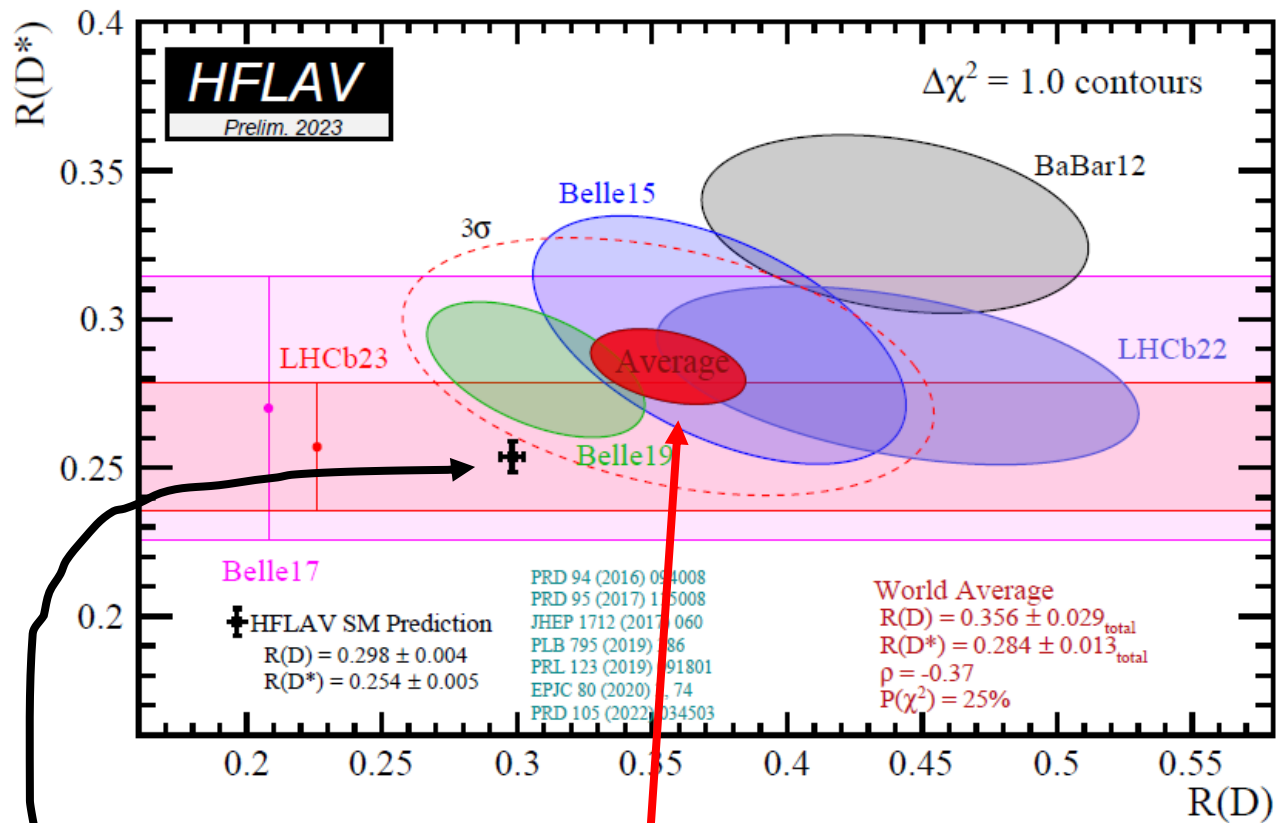
Boyd-Grinstein-Lebed BGL and HPQCD comparisons

arXiv:2311.15071



Little difference in N=2 and N=3
BGL expansion

$B_s \rightarrow D_s$ distribution: comes from HPQCD [[PRD 101, 074513 \(2020\)](#)]
 $B \rightarrow D$ distribution: from this analysis.
If SU(3) symmetry perfect, should agree. Mostly agree.



Can write differential decay rate as function of q^2 , Form Factors and lepton mass

$$\frac{d\Gamma}{dq^2} = f(q^2, f_{+/0}^2, m_l^2)$$

$$\Rightarrow R(D) = \frac{\int_{m_\tau^2}^{(m_B - m_D)^2} \Gamma(q^2, m_\tau) dq^2}{\int_{m_l^2}^{(m_B - m_D)^2} \Gamma(q^2, m_l) dq^2}$$

HFLAV 2023 average $R(D) = 0.357 \pm 0.029$

Standard Model prediction e.g: 0.300 ± 0.008

This result: 0.300 ± 0.004

- First two-dimensional, unbinned angular analysis ($q^2, \cos\vartheta_l$) for the process $\bar{B} \rightarrow Dl^- \bar{\nu}_l$ using a novel event-wise signal-background separation technique.
- Shows the lepton helicity distribution follows a $\sin^2\vartheta_l$ distribution as expected by Standard Model.
- Fits to BGL and CLN Form Factor parametrisations have been performed.
- $|V_{cb}| = (41.09 \pm 1.16) \times 10^{-3}$. This is closer to the inclusive value than the exclusive value from $\bar{B} \rightarrow D^* l^- \bar{\nu}_l$
- $R(D) = 0.300 \pm 0.004$ agrees with SM prediction. HFLAV 2023 value = 0.357 ± 0.029 .
- To be published soon.
- More results to appear this year.

fit configuration	$\mathcal{G}(1)$	ρ_D^2	$ V_{cb} \times 10^3$	$\chi_{\text{FNAL/MILC}}^2$	χ_{Belle}^2
<i>BABAR-1</i> , Belle	1.056 ± 0.008	1.155 ± 0.023	40.90 ± 1.14	1.04	24.65
<i>BABAR-2</i> , Belle	1.056 ± 0.008	1.156 ± 0.023	40.92 ± 1.14	0.99	24.72
<i>BABAR-3</i> , Belle	1.056 ± 0.008	1.156 ± 0.023	40.92 ± 1.14	1.00	24.71
<i>BABAR-4</i> , Belle	1.056 ± 0.008	1.154 ± 0.023	40.87 ± 1.14	1.09	24.57
<i>BABAR-1</i>	1.053 ± 0.008	1.179 ± 0.027	—	0.53	—