Karlsruher Institut für Technologie

Precise predictions for *t*-channel single-top production at the LHC and FCC

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LHC TOP WG

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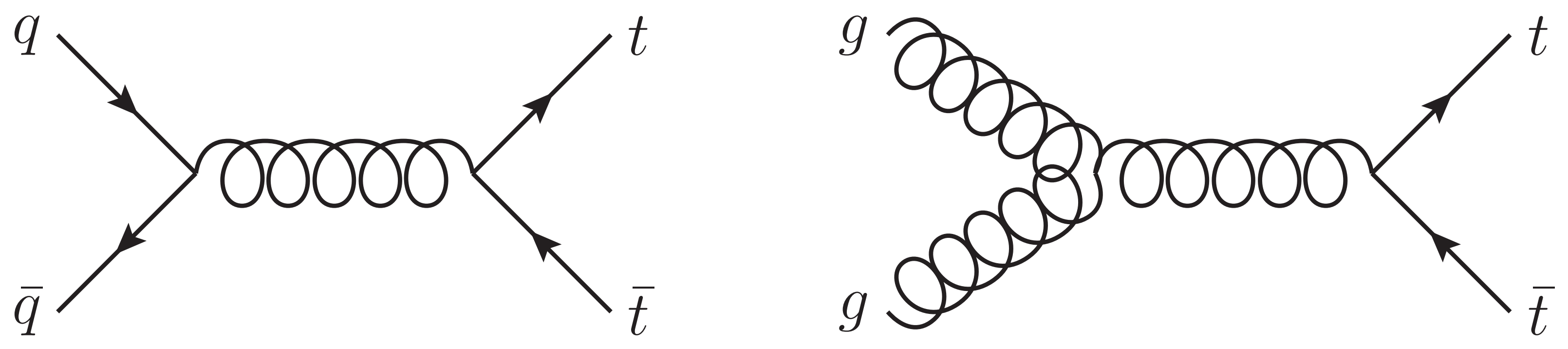
Based on: arXiv 2204.05770

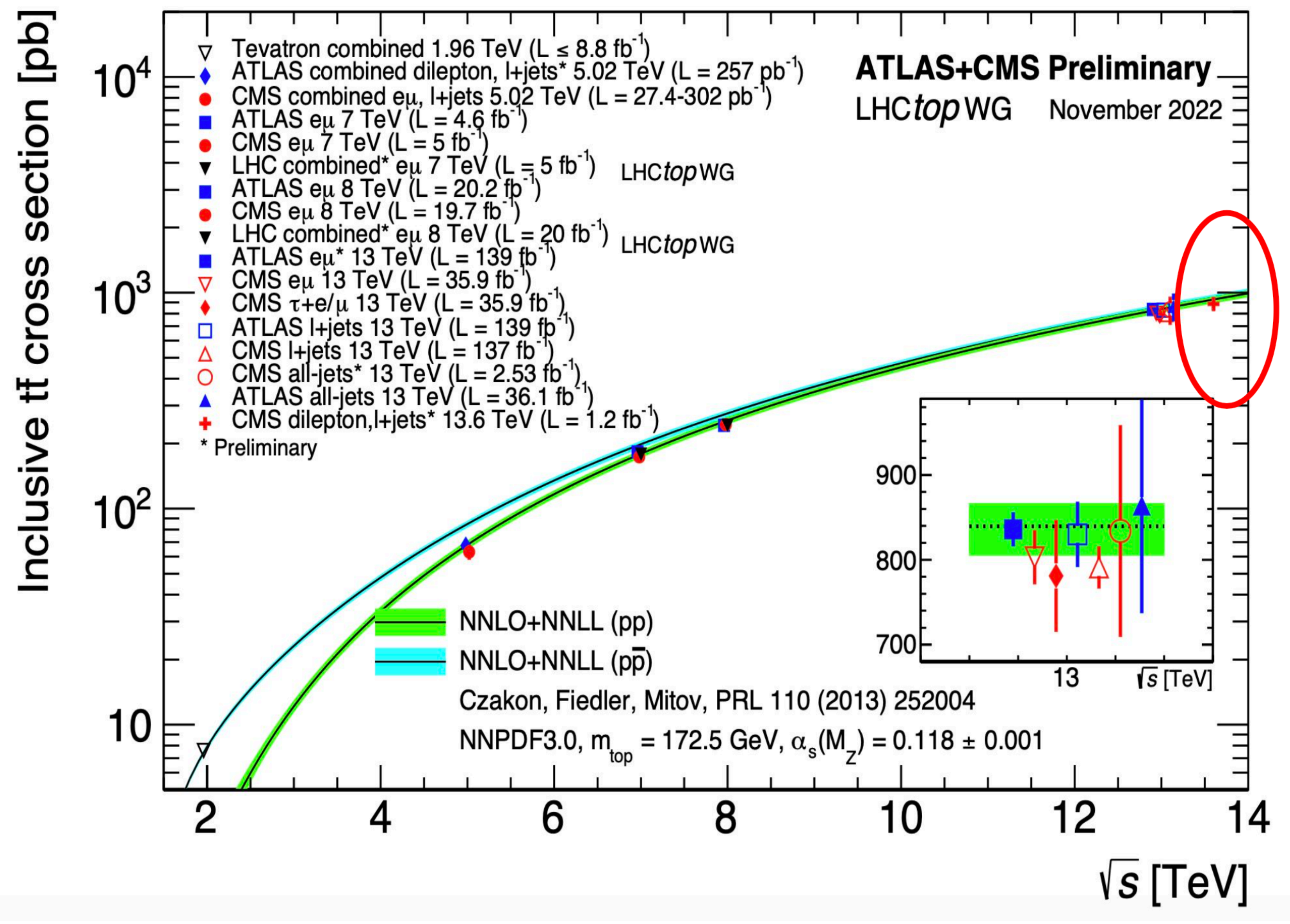
Single-top production: different modes

See Laurids's talk!

- At LHC **top quarks are mainly produced in pairs** via strong interactions

Strong pair production: $q\bar{q} \to t\bar{t}, gg \to t\bar{t}$





- **Large production rate**

Not comprehensive list of results!

- **Advanced theoretical predictions**:

NLO QCD *[Nason, Dawson, Ellis '88][Beenakker et al. '89][Denner et al.'11][Bevilaqua et al. '11][Cascioli et al.'14][Denner, Pellen '18]*

and NLO EW corrections *[Beenakker et al. '94][Bernreuther, Fuecker, Si '06][Kuhn, Scharf, Uwer '06][Hollik, Kollar '08][Denner, Pellen '16]*

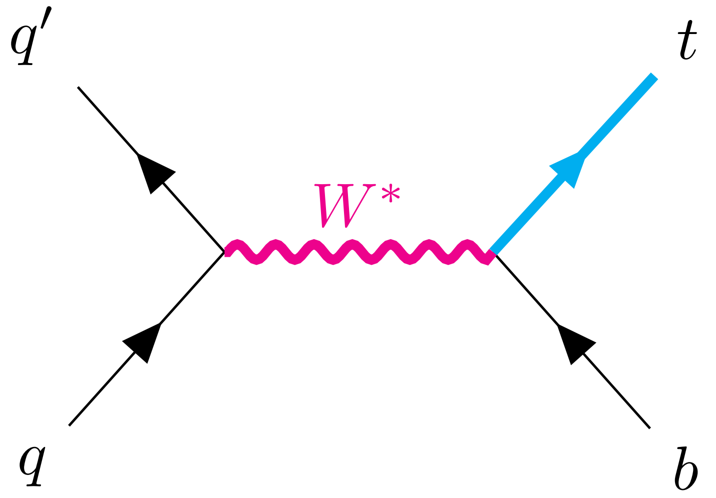
total and fully differential NNLO QCD corrections *[Bärnreuther, Czakon, Mitov '12][Czakon, Fiedler, Mitov '13][Czakon et al.'16][Czakon, Heymes, Mitov, Pagani, Tsinikos, Zaro '17][Catani et al.'19][Behring, Czakon, Mitov et al. '19][Czakon, Mitov, Poncelet '21][Gao, Papanastasiou '17][Czakon, Mitov, Poncelet '12]*

matching with parton-shower *[Frixione, Nason, Webber '03][Frixione, Nason, Oleari '07][Campbell et al.'15][Mazzitelli et al.'21]*

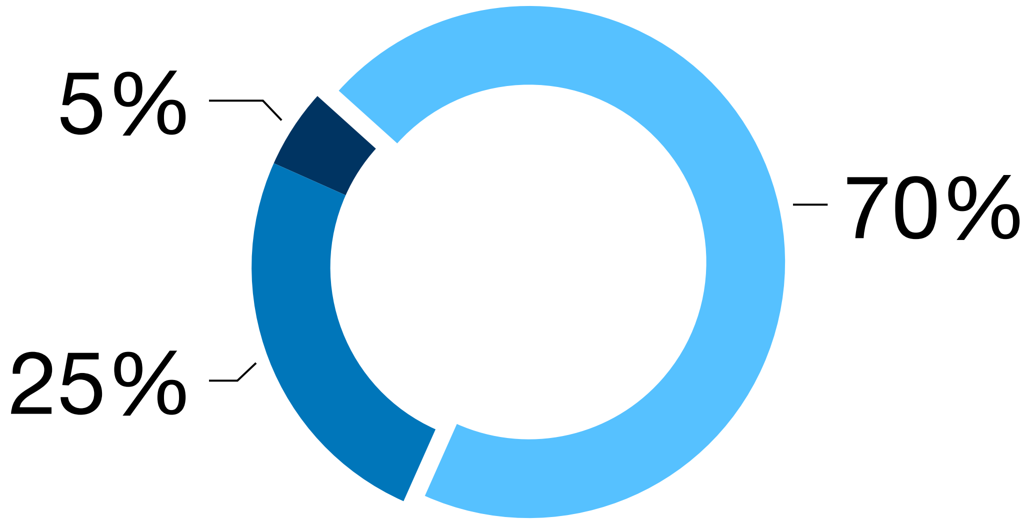
and soft-gluon resummation *[Frixione, Nason, Webber '03][Kidonakis '10][Beneke et al. '12][Kidonakis '20]*

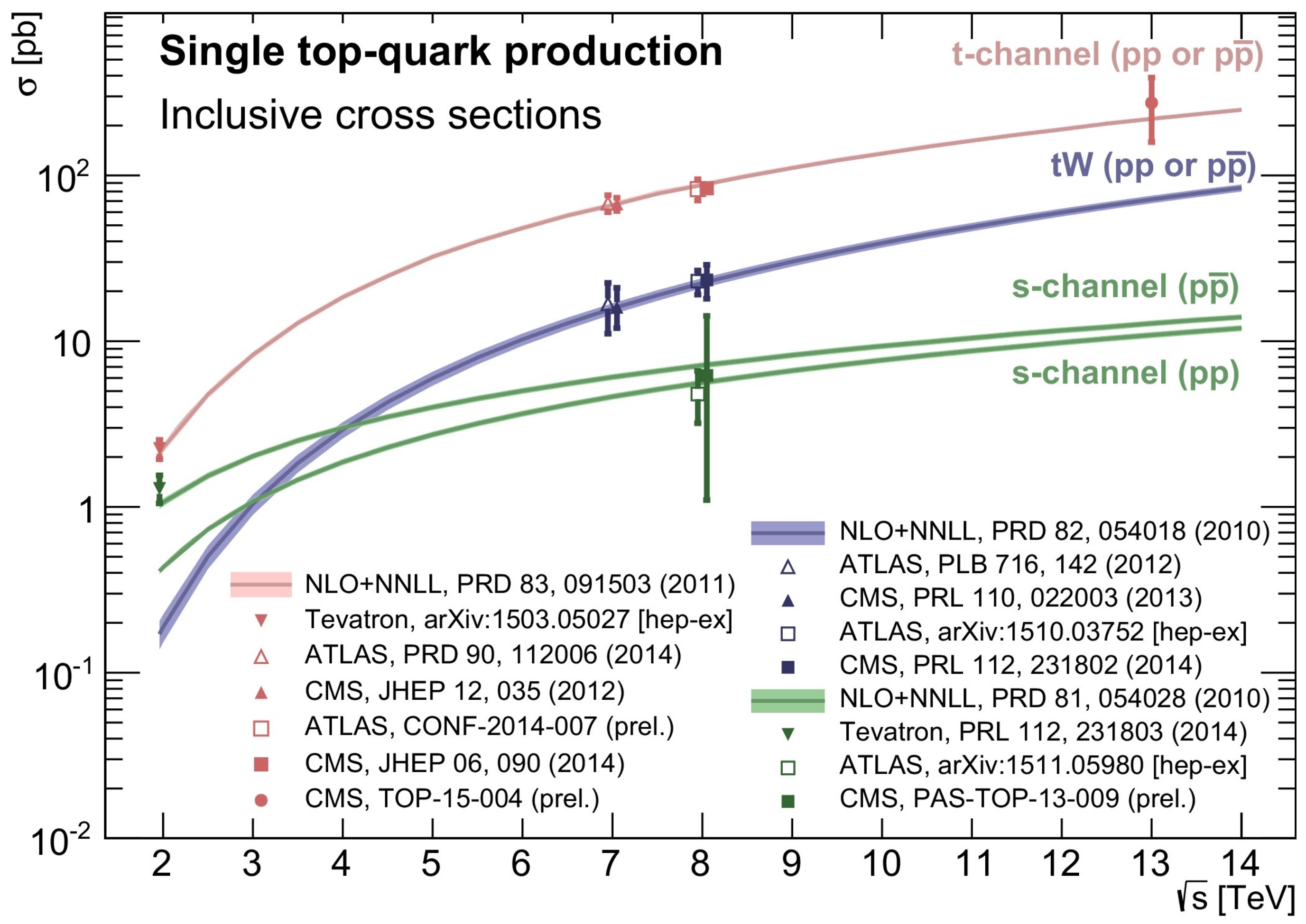
Single-top production: different modes

- **Single-top production** also relevant
  - Production rate of the same order of magnitude as $t\bar{t}$: $\sigma_t \sim 1/4\, \sigma_{t\bar{t}}$

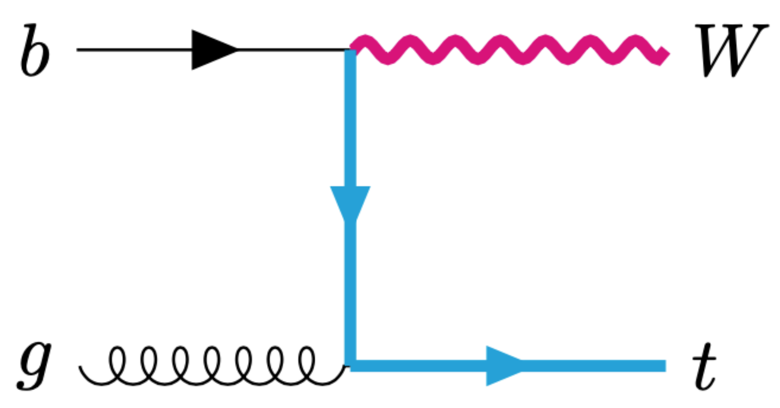


s-channel: $q\bar{q} \rightarrow W^* \rightarrow t\bar{b}$

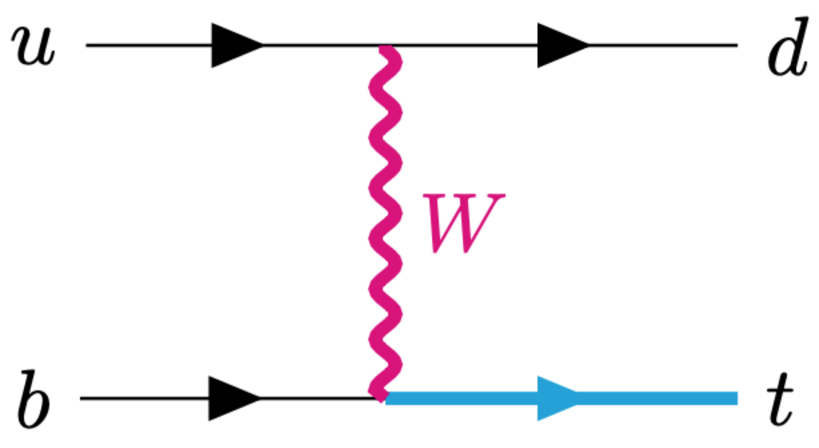




[Giammarco '16]

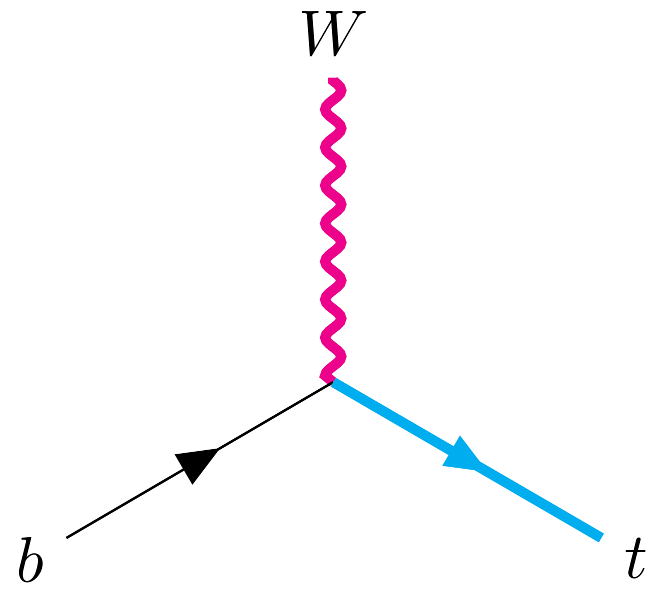


associated production: $gb \rightarrow Wt$



t-channel: $qb \rightarrow q't$

  - Electroweak mediated



Determination of the **CKM matrix** V_{bt}

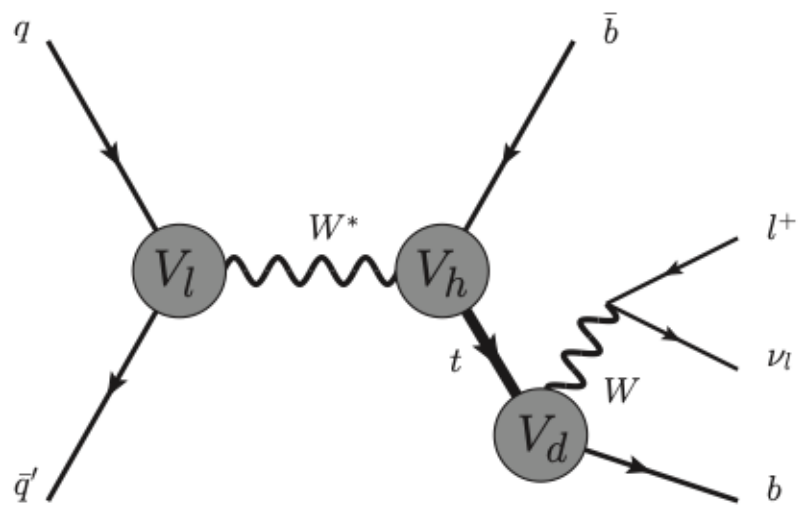
Indirect determination of **width** Γ_t and **mass** m_t

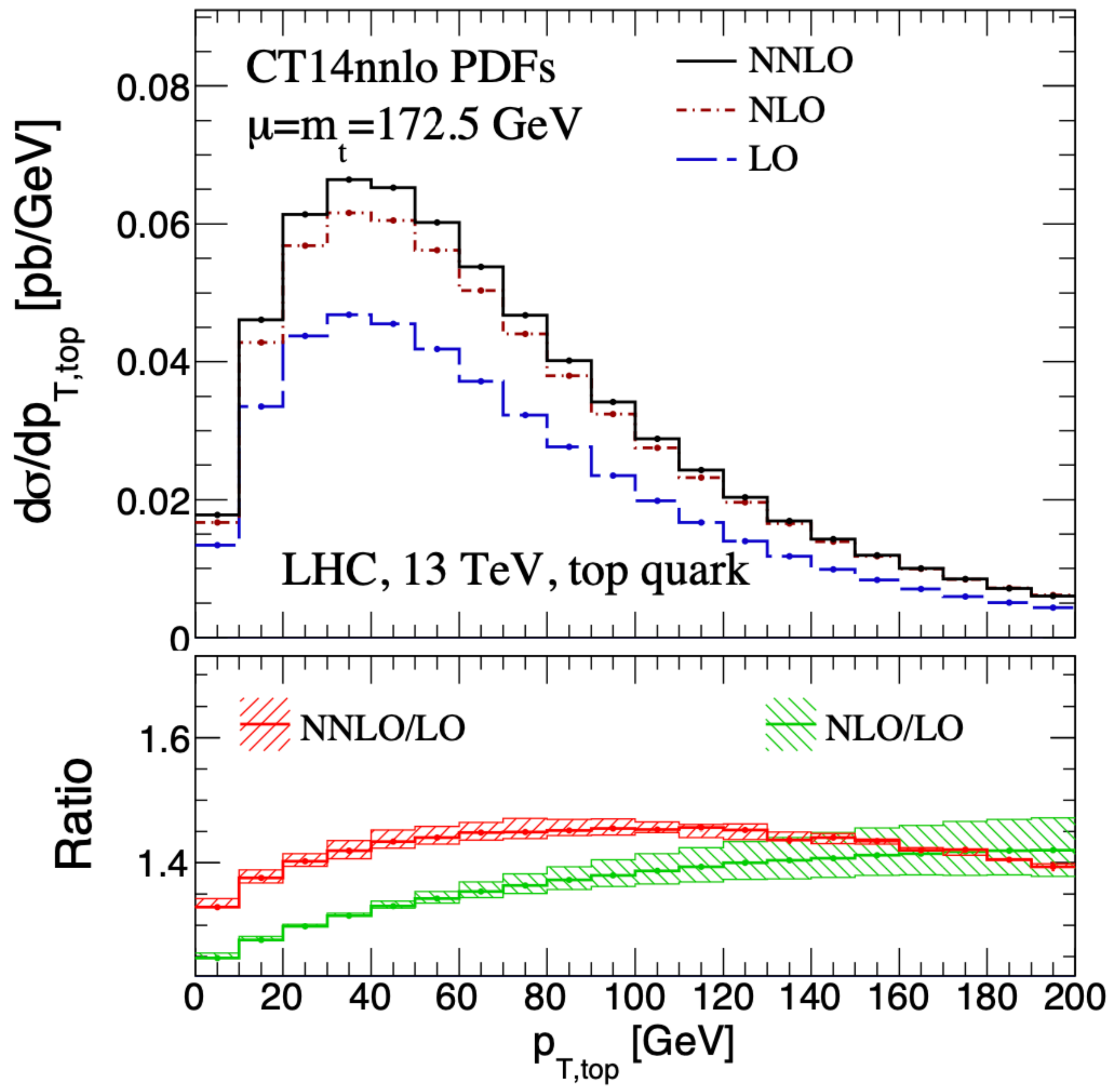
Bottom-quark PDF

Single-top production: theory status, *s*-channel and *tW*

- **s-channel:**
  - NNLO QCD corrections in production and decay *[Liu, Gao '18]*
  - Inclusive corrections are $\mathcal{O}(5\%)$ wrt NLO

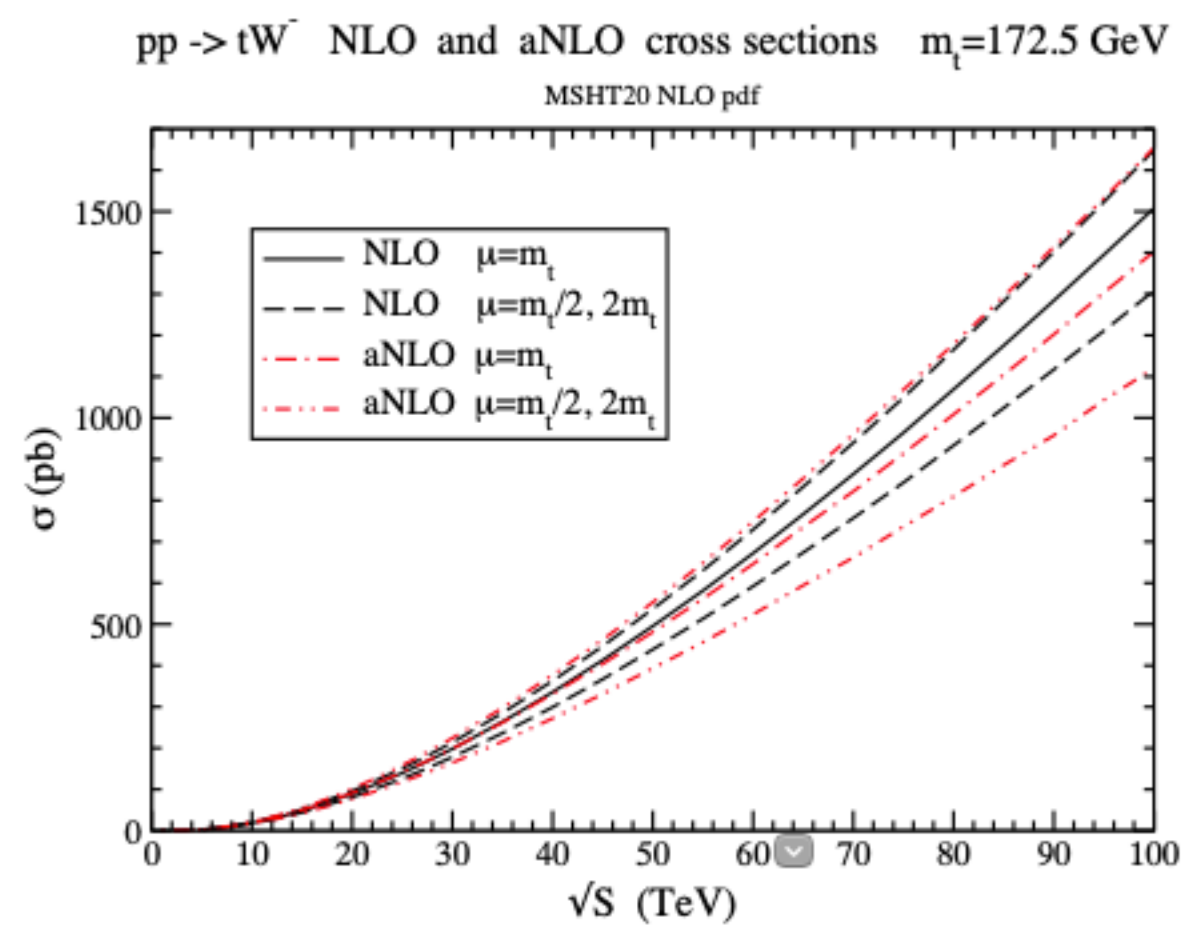
inclusive		LO	NLO	NNLO
13 TeV	$\sigma(t)$ [pb]	$4.775^{+2.69\%}_{-3.50\%}$	$6.447^{+1.39\%}_{-0.91\%}$	$6.778^{+0.76\%}_{-0.53\%}$
	$\sigma(\bar{t})$ [pb]	$2.998^{+2.69\%}_{-3.55\%}$	$4.043^{+1.33\%}_{-0.94\%}$	$4.249^{+0.69\%}_{-0.48\%}$
	$\sigma(t+\bar{t})$ [pb]	$7.772^{+2.69\%}_{-3.52\%}$	$10.49^{+1.36\%}_{-0.92\%}$	$11.03^{+0.74\%}_{-0.51\%}$
	$\sigma(t)/\sigma(\bar{t})$	$1.593^{+0.05\%}_{-0.01\%}$	$1.595^{+0.06\%}_{0.03\%}$	$1.595^{+0.07\%}_{-0.05\%}$

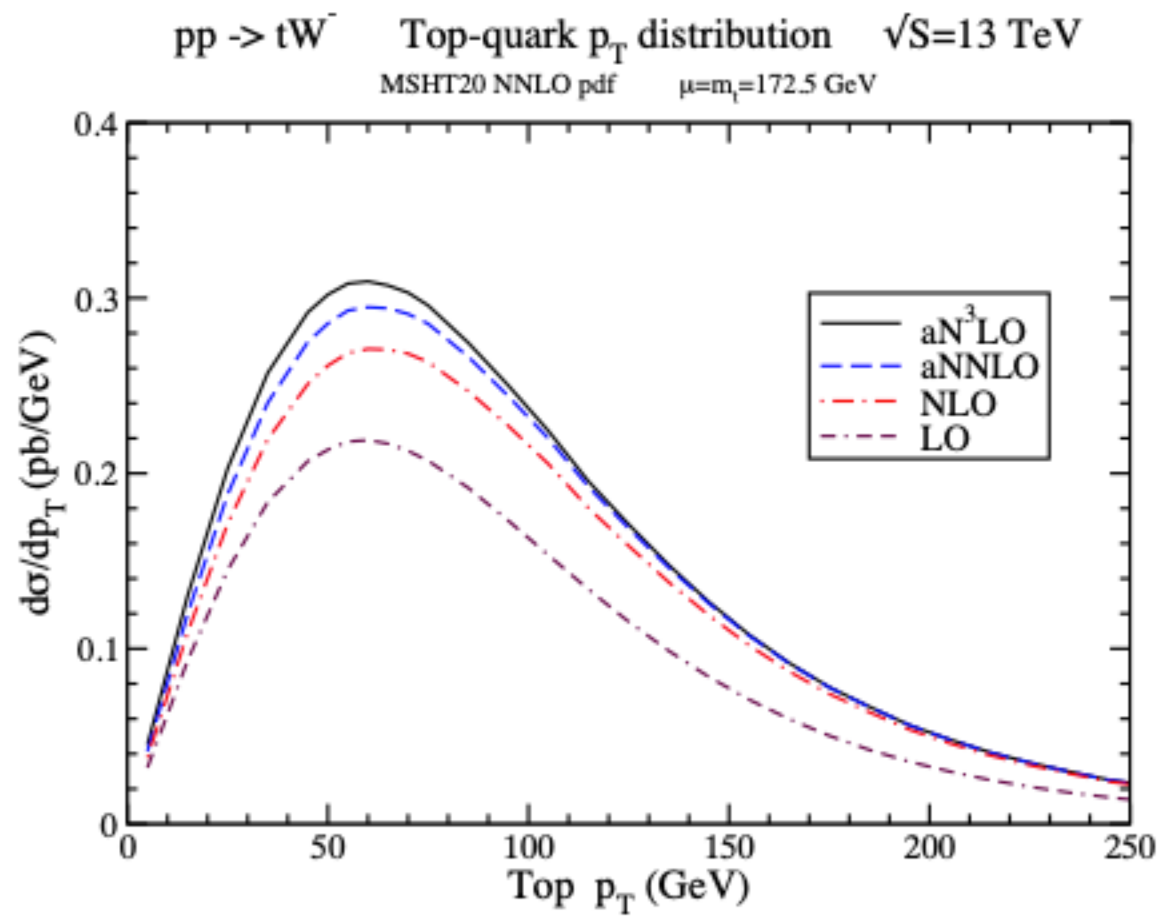




- In low $p_{\perp,t}$ region, NNLO corrections can reach $\mathcal{O}(10\,\%)$ wrt LO.
- No overlap between NLO and NNLO bands in most region: NNLO corrections underestimated by scale variation at NLO.

- ***tW*-channel:**
  - NNLO QCD corrections not known yet (analytic 2-loop amplitudes, leading colour *[Chen, Dong, Li, Li, Wang '22]*).
  - Approximate approaches used to infer higher-order corrections *[Kidonakis, Yamanaka '21]*
  - Soft-gluon corrections to approximate N3LO for stable top





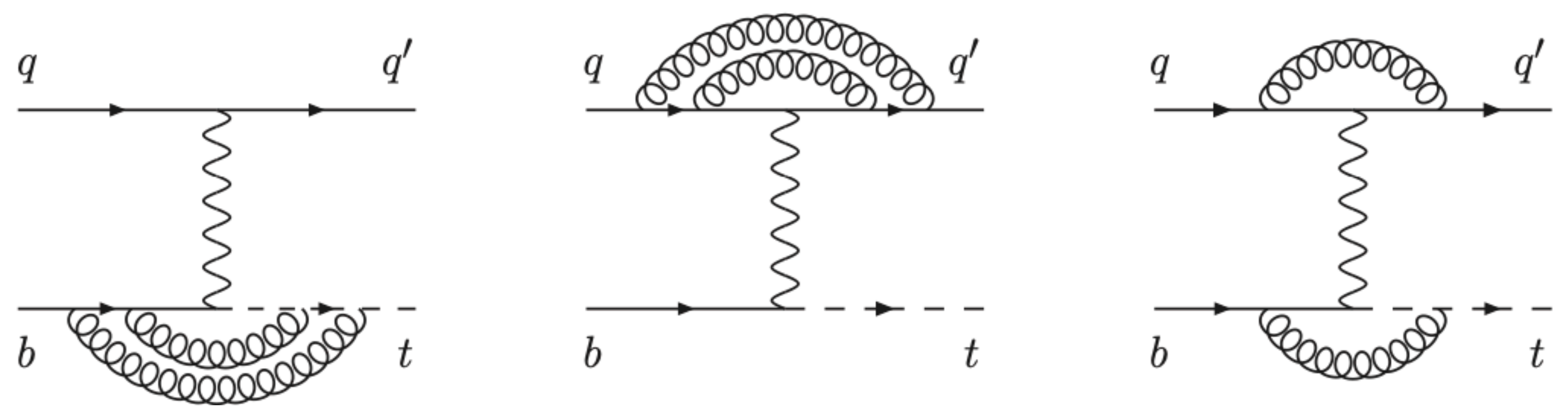
Single-top production: theory status, *t*-channel (I)

- **Two main topologies** contribute to the t-channel, single-top production:
  - **Factorisable contributions**

    NLO QCD *[Bordes, van Eijk '95][Campbell, Ellis, Tramontano '04] [Cao, Yuan '05][Cao, Schwienhorst, Benitez, Brock, Yuan '05] [Harris, Laenen, Phaf, Sullivan, Weinzierl '02] [Schwienhorst, Yuan, Mueller, Cao '11]*

    NNLO QCD

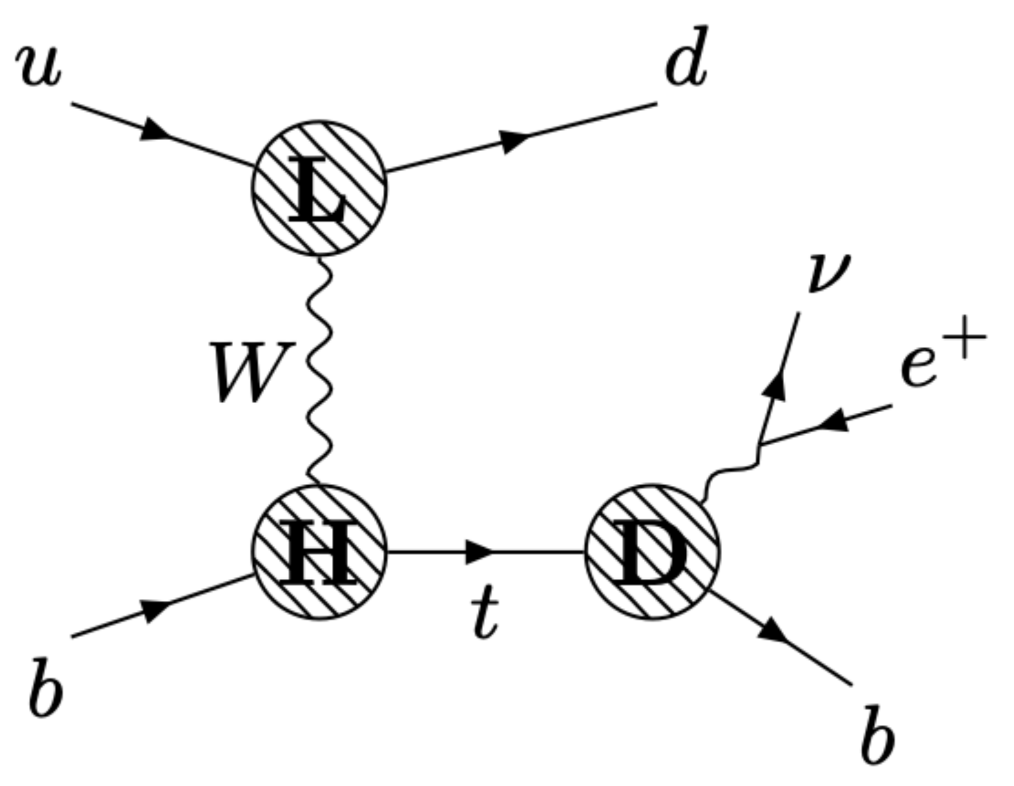
    - First calculated for a **stable top-quark** using nested soft-collinear subtraction *[Brucherseifer, Caola, Melnikov '14]*
    - **Structure function approximation**→ crosstalk neglected invoking colour suppression
    - **Small effects** on inclusive cross-section and on cross section with $p_\perp^t$ cuts



MSTW2008, lo, nlo, nnlo PDF, $\mu_R = \mu_F = m_t = 173.2$ GeV, $\sqrt{s} = 8$ TeV.

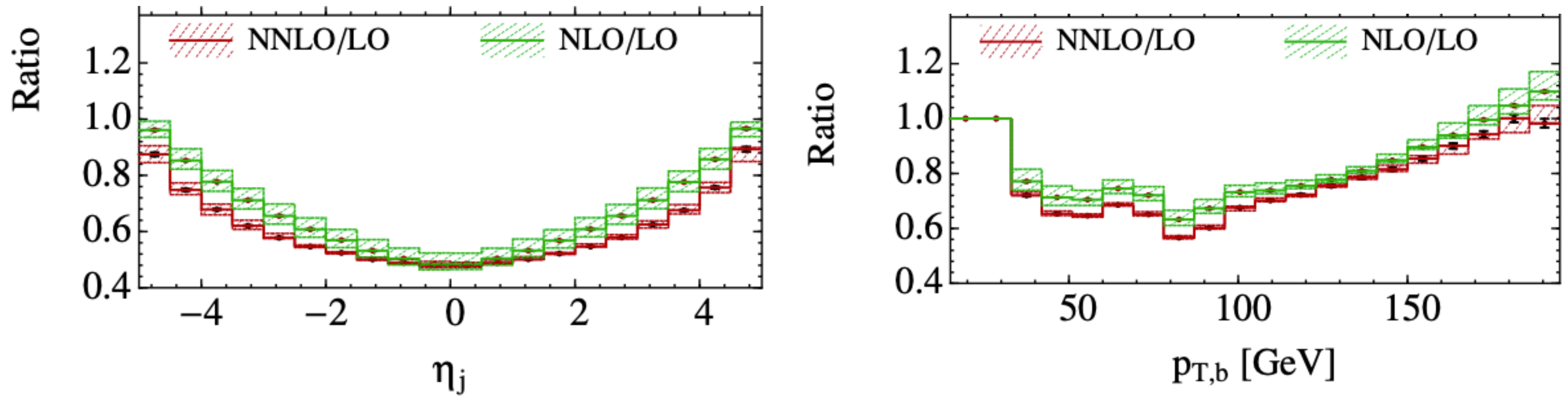
$p_\perp$	$\sigma_{\rm LO}$, pb	$\sigma_{\rm NLO}$, pb	$\delta_{\rm NLO}$	$\sigma_{\rm NNLO}$, pb	$\delta_{\rm NNLO}$
0 GeV	$53.8^{+3.0}_{-4.3}$	$55.1^{+1.6}_{-0.9}$	$+2.4\%$	$54.2^{+0.5}_{-0.2}$	-1.6%
20 GeV	$46.6^{+2.5}_{-3.7}$	$48.9^{+1.2}_{-0.5}$	$+4.9\%$	$48.3^{+0.3}_{-0.02}$	-1.2%
40 GeV	$33.4^{+1.7}_{-2.5}$	$36.5^{+0.6}_{-0.03}$	$+9.3\%$	$36.5^{+0.1}_{+0.1}$	-0.1%
60 GeV	$22.0^{+1.0}_{-1.5}$	$25.0^{+0.2}_{+0.3}$	$+13.6\%$	$25.4^{-0.1}_{+0.2}$	$+1.6\%$

Single-top production: theory status, *t*-channel (II)



NNLO QCD

- Extension to **top-quark decay in the NW approximation**, including also NNLO in decay (computed using SCET/jettiness + projection to Born) *[Berger, Gao, Yuan, Zhu ’16, ‘17]*
- Large corrections for some distributions



- Disagreement with earlier calculation of inclusive cross-section: $\mathcal{O}(1)$ difference in NNLO coefficient
- Independent calculation based on SCET approach *[Campbell, Neumann, Sullivan ’21]*

CT14, lo, nlo, nnlo PDF, $\mu_R = \mu_F = m_t = 172.5$ GeV.

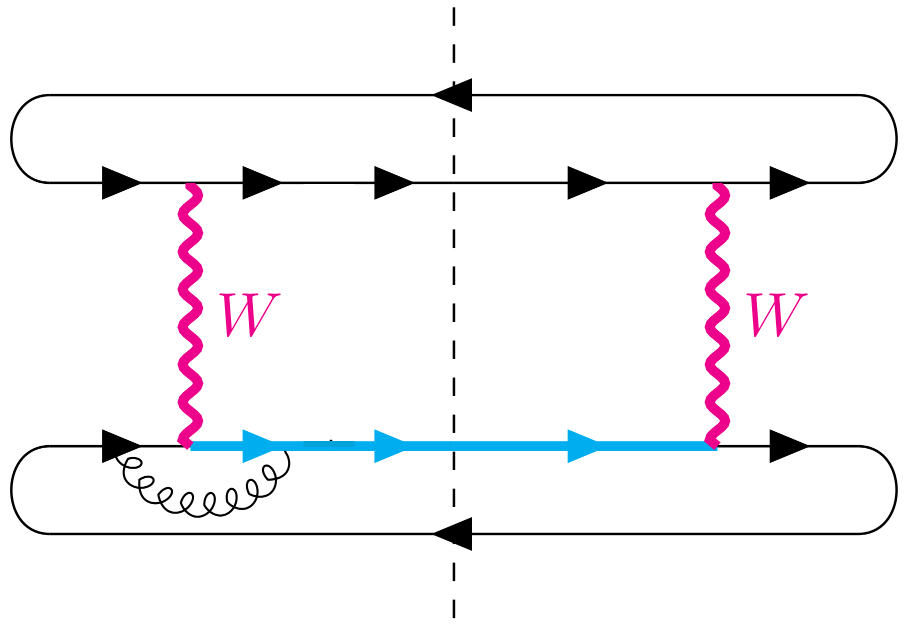
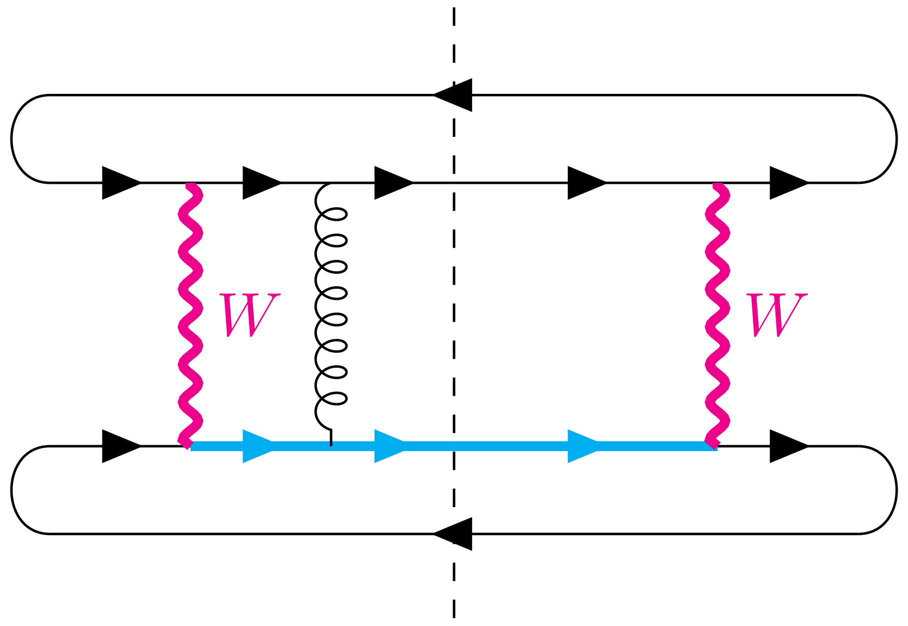
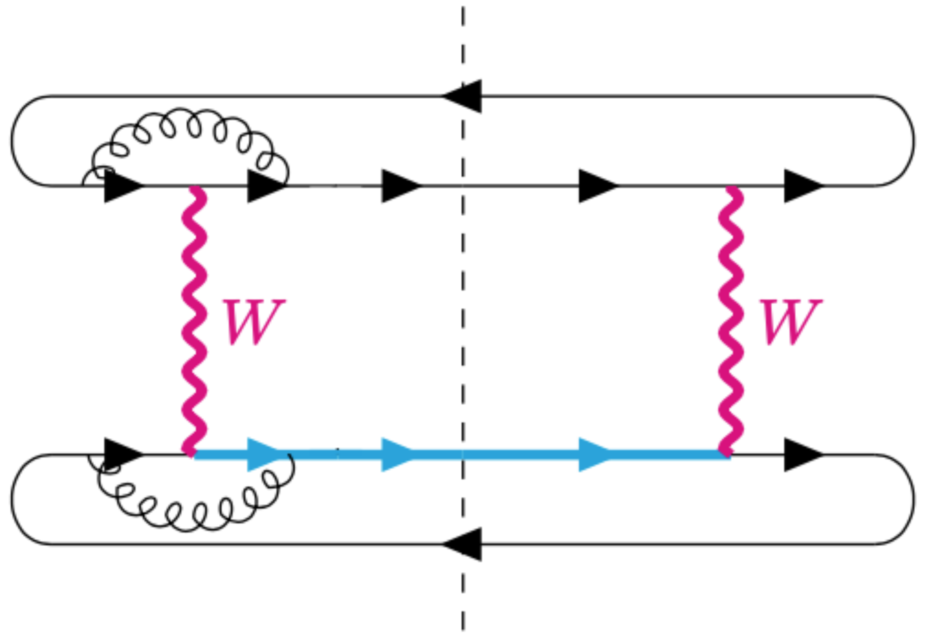
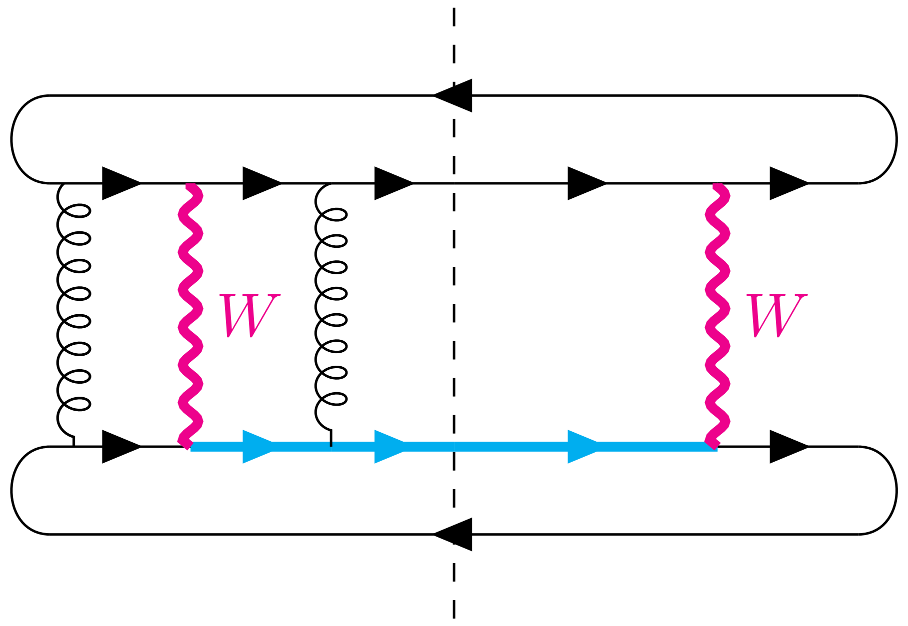
@14TeV:

$\delta\sigma^{\mathrm{NNLO}} \sim -0.7\,\%\,\sigma^{\mathrm{NLO}}$

	7 TeV pp		14 TeV pp	
	top	anti-top	top	anti-top
$\sigma_{\mathrm{LO}}^{\mu=m_t}$	$37.1^{+7.1\%}_{-9.5\%}$	$19.1^{+7.3\%}_{-9.7\%}$	$134.6^{+10.0\%}_{-12.1\%}$	$78.9^{+10.4\%}_{-12.6\%}$
$\sigma_{\mathrm{LO}}^{\mathrm{DDIS}}$	$39.5^{+6.4\%}_{-8.6\%}$	$19.9^{+7.0\%}_{-9.3\%}$	$140.9^{+9.4\%}_{-11.4\%}$	$80.7^{+10.2\%}_{-12.3\%}$
$\sigma_{\mathrm{NLO}}^{\mu=m_t}$	$41.4^{+3.0\%}_{-2.0\%}$	$21.5^{+3.1\%}_{-2.0\%}$	$154.3^{+3.1\%}_{-2.3\%}$	$91.4^{+3.1\%}_{-2.2\%}$
$\sigma_{\mathrm{NLO}}^{\mathrm{DDIS}}$	$41.8^{+3.3\%}_{-2.0\%}$	$21.5^{+3.4\%}_{-1.6\%}$	$154.4^{+3.7\%}_{-1.4\%}$	$91.2^{+3.1\%}_{-1.8\%}$
	PDF $^{+1.7\%}_{-1.4\%}$	PDF $^{+2.2\%}_{-1.5\%}$	PDF $^{+1.7\%}_{-1.1\%}$	PDF $^{+1.9\%}_{-0.9\%}$
$\sigma_{\mathrm{NNLO}}^{\mu=m_t}$	$41.9^{+1.2\%}_{-0.7\%}$	$21.9^{+1.2\%}_{-0.7\%}$	$153.3(2)^{+1.0\%}_{-0.6\%}$	$91.5(2)^{+1.1\%}_{-0.9\%}$
$\sigma_{\mathrm{NNLO}}^{\mathrm{DDIS}}$	$41.9^{+1.3\%}_{-0.8\%}$	$21.8^{+1.3\%}_{-0.7\%}$	$153.4(2)^{+1.1\%}_{-0.7\%}$	$91.2(2)^{+1.1\%}_{-0.9\%}$
	PDF $^{+1.3\%}_{-1.1\%}$	PDF $^{+1.4\%}_{-1.3\%}$	PDF $^{+1.2\%}_{-1.0\%}$	PDF $^{+1.0\%}_{-1.0\%}$

Non-factorisable corrections: why?

Non-factorisable contributions vanish at NLO due to their colour structure, and are suppressed by a factor $N_c^2 - 1 = 8$ at NNLO.

	Factorisable contributions	Non-factorisable contributions
NLO	 $\propto \mathrm{Tr}\left(t^a t^a\right) = \frac{N_c^2 - 1}{2}$	 $\propto \mathrm{Tr}\left(t^a\right) = 0$
NNLO	 $\propto \mathrm{Tr}\left(t^a t^a\right) \mathrm{Tr}\left(t^b t^b\right) = \frac{(N_c^2 - 1)^2}{4}$	 $\propto \mathrm{Tr}\left(t^a t^b\right) \mathrm{Tr}\left(t^a t^b\right) = \frac{N_c^2 - 1}{4}$

Non-factorisable corrections: why?

Non-factorisable contributions vanish at NLO due to their colour structure, and are suppressed by a factor $N_c^2 - 1 = 8$ at NNLO.

However:

- Non-factorisable corrections could be enhanced by a factor $\pi^2 \sim 10$ due to the Glauber phase

  → proven for Higgs production in weak boson fusion in the eikonal approximation *[Liu, Melnikov et al. '19]*

- The actual size of NNLO non-factorisable corrections **cannot be inferred from NLO contributions**, since they vanish
- Recent calculation of **double-virtual contributions** indicate a comparable size of non-factorisable and factorisable corrections *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

$$\frac{\sigma^{ub}_{pp\to dt}}{1\,\mathrm{pb}} = 90.3 + 0.3\left(\frac{\alpha_s(\mu_{\mathrm{nf}})}{0.108}\right)^2$$

NNPDF31_lo_as_0118, $\sqrt{s} = 13$ TeV

$\mu_{\mathrm{nf}} = m_t \;:\; \delta\sigma^{\mathrm{NNLO}} \sim 0.3\,\%\,\sigma^{\mathrm{LO}}$

$\mu_{\mathrm{nf}} = 40 - 60\,\mathrm{GeV} \;:\; \delta\sigma^{\mathrm{NNLO}} \sim 0.5\,\%\,\sigma^{\mathrm{LO}}$

Even though **non-factorisable contributions are suppressed by colour it is not guaranteed that they are actually negligible.**

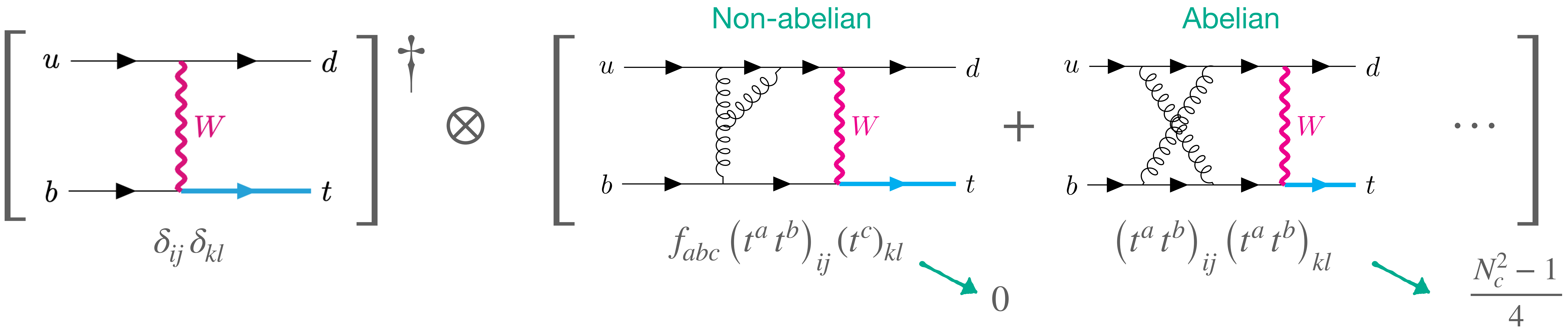
- Thanks to recent progress *[Brønnum-Hansen et al. '21]* tackling non-factorisable corrections is actually feasible:

  → 2-loop virtual amplitudes computed analytically with full dependence on m_t.

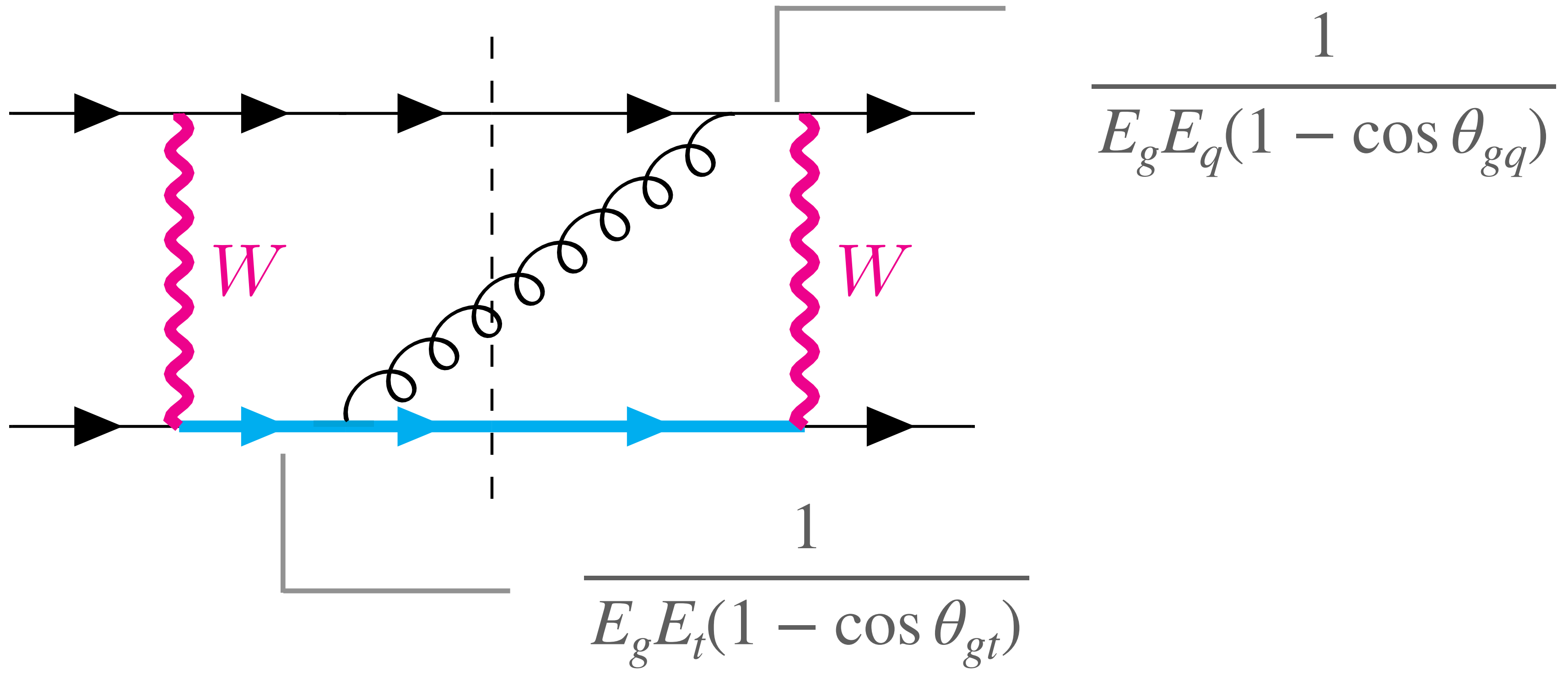
  → integrals computed numerically with sufficient precision to be exploited in phenomenological studies.

Non-factorisable corrections: main properties

Non-factorisable contributions have to **connect upper an lower quark line**, and are effectively **Abelian**

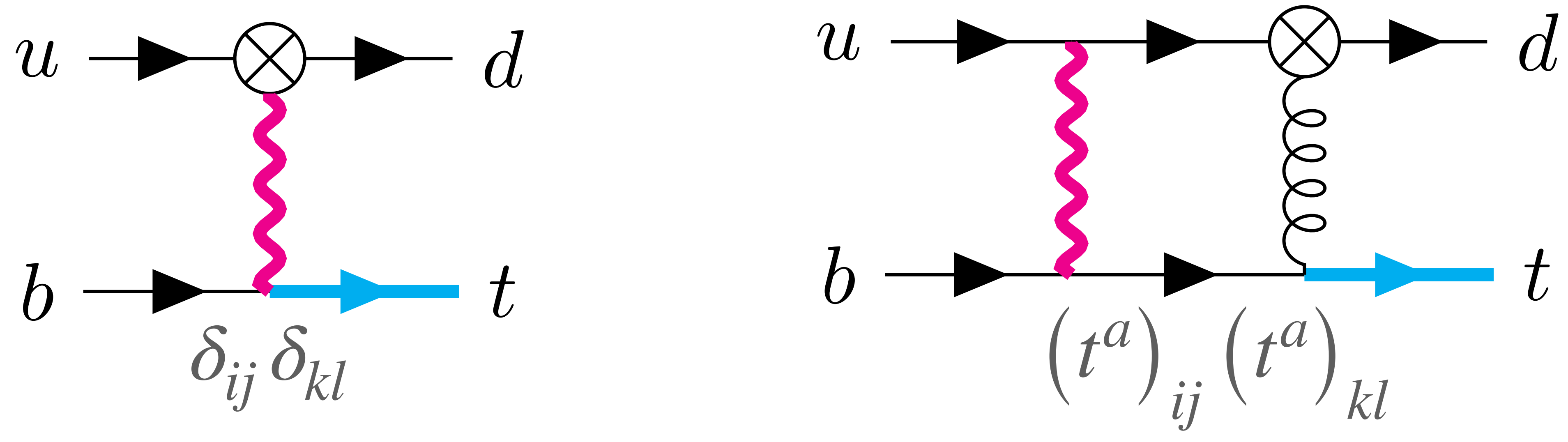


The **infrared structure is simplified**: **no collinear singularities**



All IR singularities are of soft origin.

Non-factorisable contributions are **UV finite**



Renormalisation simply consists of $\alpha_s^{\text{bare}}\,\mu_0^{2\epsilon} = \alpha_s\,\mu^{2\epsilon}\,S_\epsilon$

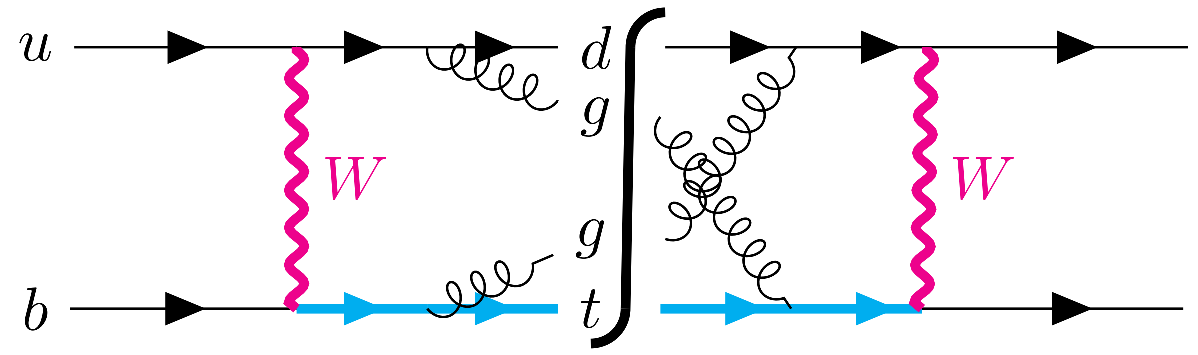
Non-factorisable corrections: ingredients of the calculation

Three terms contribute to the non-factorisable cross section

$$\mathrm{d}\sigma^{\mathrm{nf}}_{pp\to X+t} = \sum_{i,j} \int \mathrm{d}x_1\, \mathrm{d}x_2\; f_i(x_1,\mu_F)\, f_j(x_2,\mu_F)\; \mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{ij\to X+t}(x_1,x_2)$$

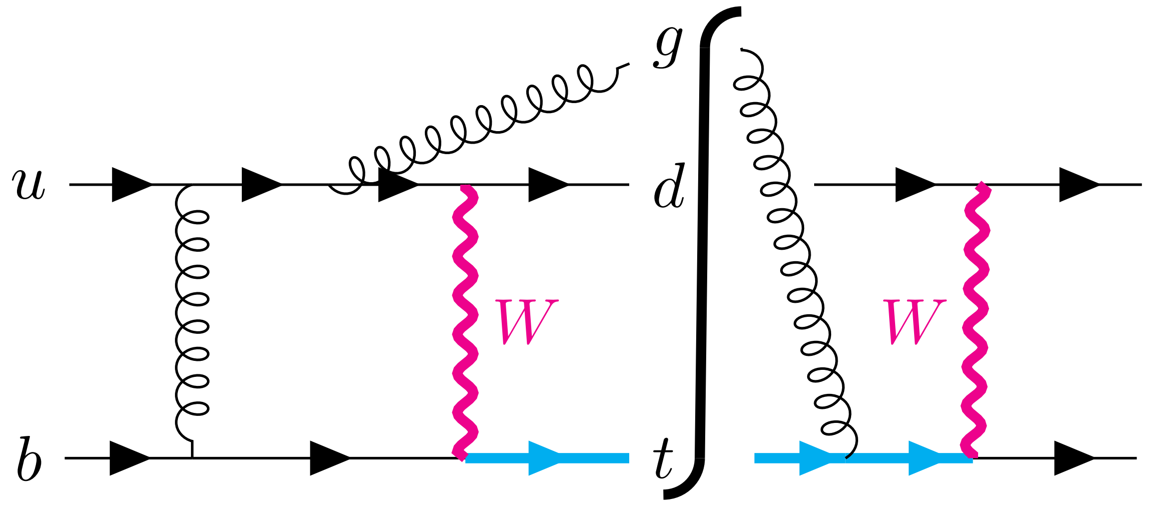
$$\mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{NNLO}} = \mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{RR}} + \mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{RV}} + \mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{VV}}$$

$\mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{RR}} \,:\, \mathscr{A}_6^{(0)} \otimes \mathscr{A}_6^{(0)}$

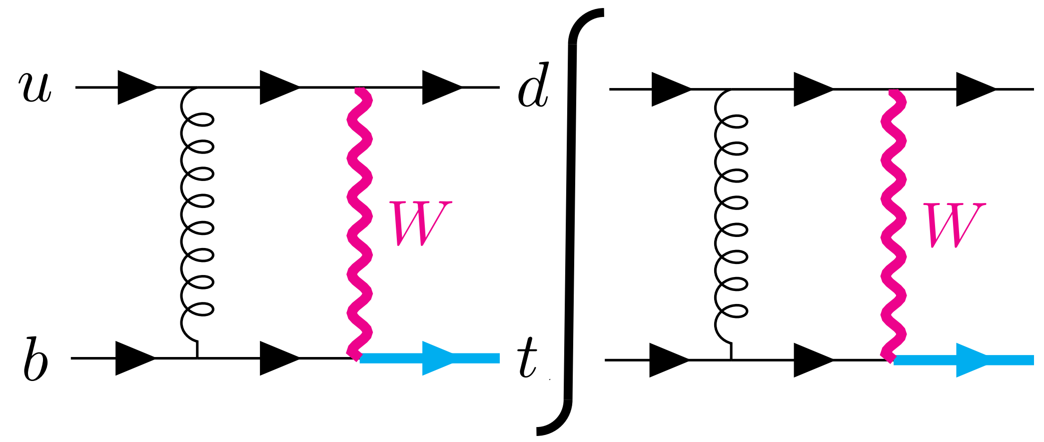


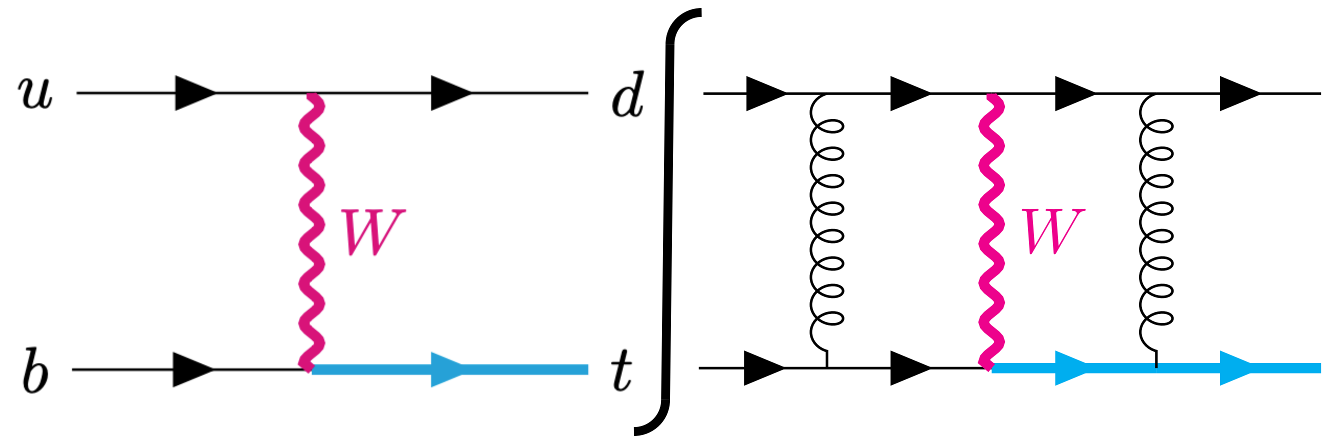
Each ingredient requires **specific treatment** and encodes **difficulties** to overcome

$\mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{RV}} \,:\, \mathscr{A}_5^{(1)} \otimes \mathscr{A}_5^{(0)}$

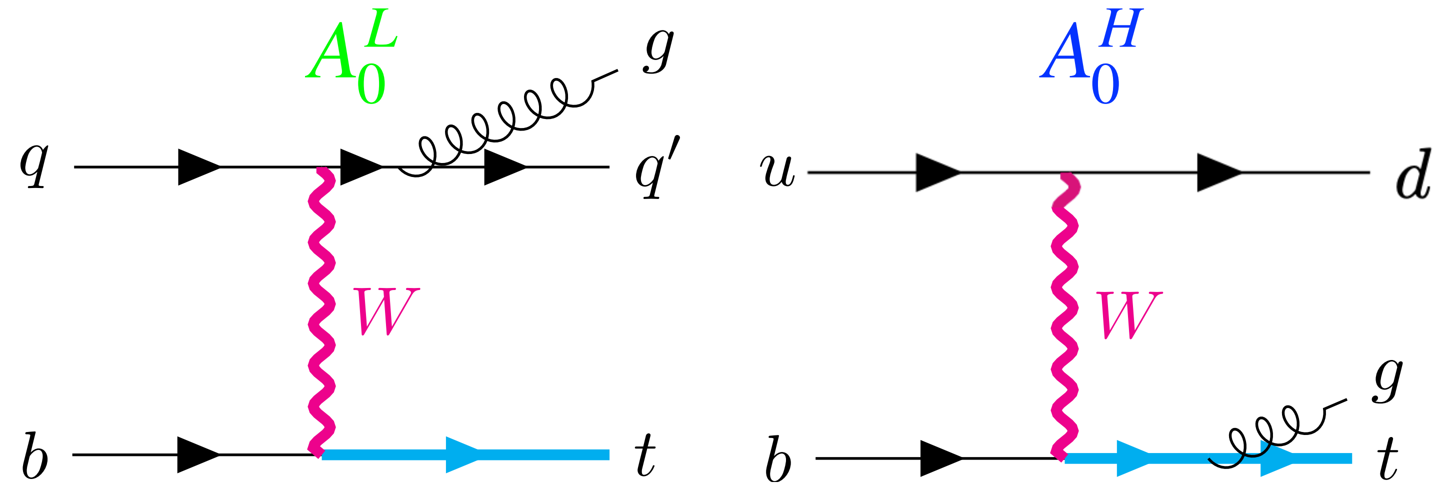


$\mathrm{d}\hat{\sigma}^{\mathrm{nf}}_{\mathrm{VV}} \,:\, \mathscr{A}_4^{(1)} \otimes \mathscr{A}_4^{(1)},\, \mathscr{A}_4^{(0)} \otimes \mathscr{A}_4^{(2)}$





Extracting soft singularities from real corrections



1. Decompose the amplitude into **colour-stripped, sub-amplitudes**

$$\langle c \,|\, \mathscr{A}_0(1_q,2_b,3_{q'},4_t;5_g)\rangle = g_{s,b}\left[t^{c_5}_{c_3c_1}\delta_{c_4c_2}\, A_0^L(5_g) + t^{c_5}_{c_4c_2}\delta_{c_3c_1}\, A_0^H(5_g)\right]$$

2. Under **soft limit**, sub-amplitudes factorise into **universal eikonal factors** and **lower-multiplicity amplitudes**

$$S_5 \;\; A_0^{L/H}(5_g) = \varepsilon_\mu^{(\lambda)}(5)\, J^\mu(3,1;5)\, A_0(1_q,2_b,3_{q'},4_t)$$

$$J^\mu(i,j;k) = \frac{p_i^\mu}{p_i\cdot p_k} - \frac{p_j^\mu}{p_j\cdot p_k}$$

3. **Contract sub-amplitudes** to connect different quark lines

$$S_5\; 2\mathrm{Re}\left[A_0^L(5_g)\, A_0^{H*}(5_g)\right] = -\mathrm{Eik}_{\mathrm{nf}}(1_q,2_b,3_{q'},4_t;5_g)\,|A_0(1_q,2_b,3_{q'},4_t)|^2$$

$$\mathrm{Eik}_{\mathrm{nf}}(k_g) = J^\mu(3,1;k_g)\, J_\mu(4,2;k_g) = \sum_{\substack{i\in[1,3]\\ j\in[2,4]}} \frac{\lambda_{ij}\; p_i\cdot p_j}{(p_i\cdot p_k)(p_j\cdot p_k)}$$

4. Integrate the eikonal factor over the radiation phase space

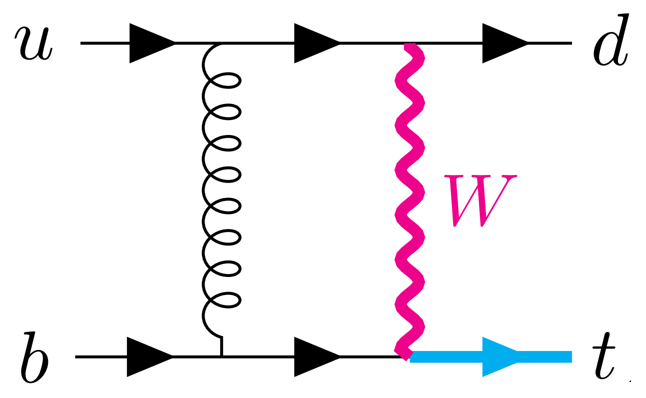
$$g_{s,b}^2\int[\mathrm{d}p_k]\;\mathrm{Eik}_{\mathrm{nf}}(1_q,2_b,3_{q'},4_t;k_g) \equiv \frac{\alpha_s}{2\pi}\left(\frac{2E_{\mathrm{max}}}{\mu}\right)^{-2\epsilon} K_{\mathrm{nf}}(1_q,2_b,3_{q'},4_t;\epsilon) = \frac{\alpha_s}{2\pi}\left(\frac{2E_{\mathrm{max}}}{\mu}\right)^{-2\epsilon}\left[\frac{1}{\epsilon}\;\log\left(\frac{p_1\cdot p_4\; p_2\cdot p_3}{p_1\cdot p_2\; p_3\cdot p_4}\right) + \mathcal{O}(\epsilon^0)\right]$$

Double-real correction treated in the same fashion: independent emissions, factorised double-soft limit

Extracting soft singularities from virtual corrections

Extract IR singularities from virtual radiation and compute finite contributions.

1. One-loop correction to the 4-point amplitude



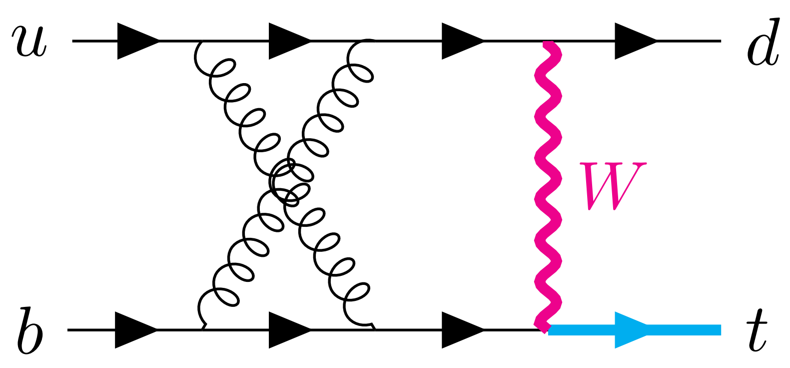
$$\langle c \,|\, \mathscr{A}_1(1_q, 2_b, 3_{q'}, 4_t) \rangle = \frac{\alpha_s}{2\pi}\left(\ldots + t^a_{c_3 c_1}\, t^a_{c_4 c_2}\, B_1\left(1_q, 2_b, 3_{q'}, 4_t\right)\right)$$

B_1 is UV-finite, but IR-divergent: the abelian nature of the correction leads to the simple pole structure

$$B_1\left(1_q, 2_b, 3_{q'}, 4_t\right) = I_1(\epsilon)\, A_0\left(1_q, 2_b, 3_{q'}, 4_t\right) + B_{1,\mathrm{fin}}\left(1_q, 2_b, 3_{q'}, 4_t\right)$$

$$I_1(\epsilon) \equiv I_1(1_q, 2_b, 3_{q'}, 4_t; \epsilon) = \frac{1}{\epsilon}\left[\log\left(\frac{p_1 \cdot p_4 \; p_2 \cdot p_3}{p_1 \cdot p_2 \; p_3 \cdot p_4}\right) + 2\pi i\right]$$

2. Two-loop correction to the 4-point amplitude



$$\langle c \,|\, \mathscr{A}_2(1_q, 2_b, 3_{q'}, 4_t) \rangle = \left(\frac{\alpha_s}{2\pi}\right)^2\left(\ldots + \frac{1}{2}\{t^a, t^b\}_{c_3 c_1}\, \frac{1}{2}\{t^a, t^b\}_{c_4 c_2}\, B_2\left(1_q, 2_b, 3_{q'}, 4_t\right)\right)$$

$$B_2\left(1_q, 2_b, 3_{q'}, 4_t\right) = -\frac{I_1^2(\epsilon)}{2}\, A_0\left(1_q, 2_b, 3_{q'}, 4_t\right) + I_1(\epsilon)\, B_1\left(1_q, 2_b, 3_{q'}, 4_t\right) + B_{2,\mathrm{fin}}\left(1_q, 2_b, 3_{q'}, 4_t\right)$$

Pole cancellation

Manifestly finite result: combination of RR, VV and RV → split into contributions of **different multiplicities**

$$\sigma_{\rm nf} = \sigma_{\rm nf}^{(2g)} + \sigma_{\rm nf}^{(1g)} + \sigma_{\rm nf}^{(0g)}$$

$\sigma_{\rm nf}^{(2g)}$: **fully resolved**, implemented numerically

$\sigma_{\rm nf}^{(1g),\,(0g)}$: finite due to **cancellation of real and virtual singularities**

Message:

- The **abelian nature of non-factorisable** corrections simplifies the structure of IR singularities.
- Once the soft singularities are extracted and regulated, the cross section is proportional to **few building blocks**, which combine in remarkably compact expressions.
- The **pole cancellation** can be proven **locally in the phase space** and in full generality thanks to the interplay between universal real and virtual IR functions $K_{\rm nf}\,(\epsilon), I_1\,(\epsilon)$.
- The procedure require to expand only one amplitude to higher order in ϵ

Results

Differential cross section:

pp collision: $\sqrt{s} = 13\text{TeV}$, PDFs: CT14_lo@LO, CT14_nnlo@NNLO, $m_W = 80.379\text{GeV}$, $m_t = 173.0\text{GeV}$, $\alpha_s(m_t) = 0.108$, $\mu_F = m_t$.

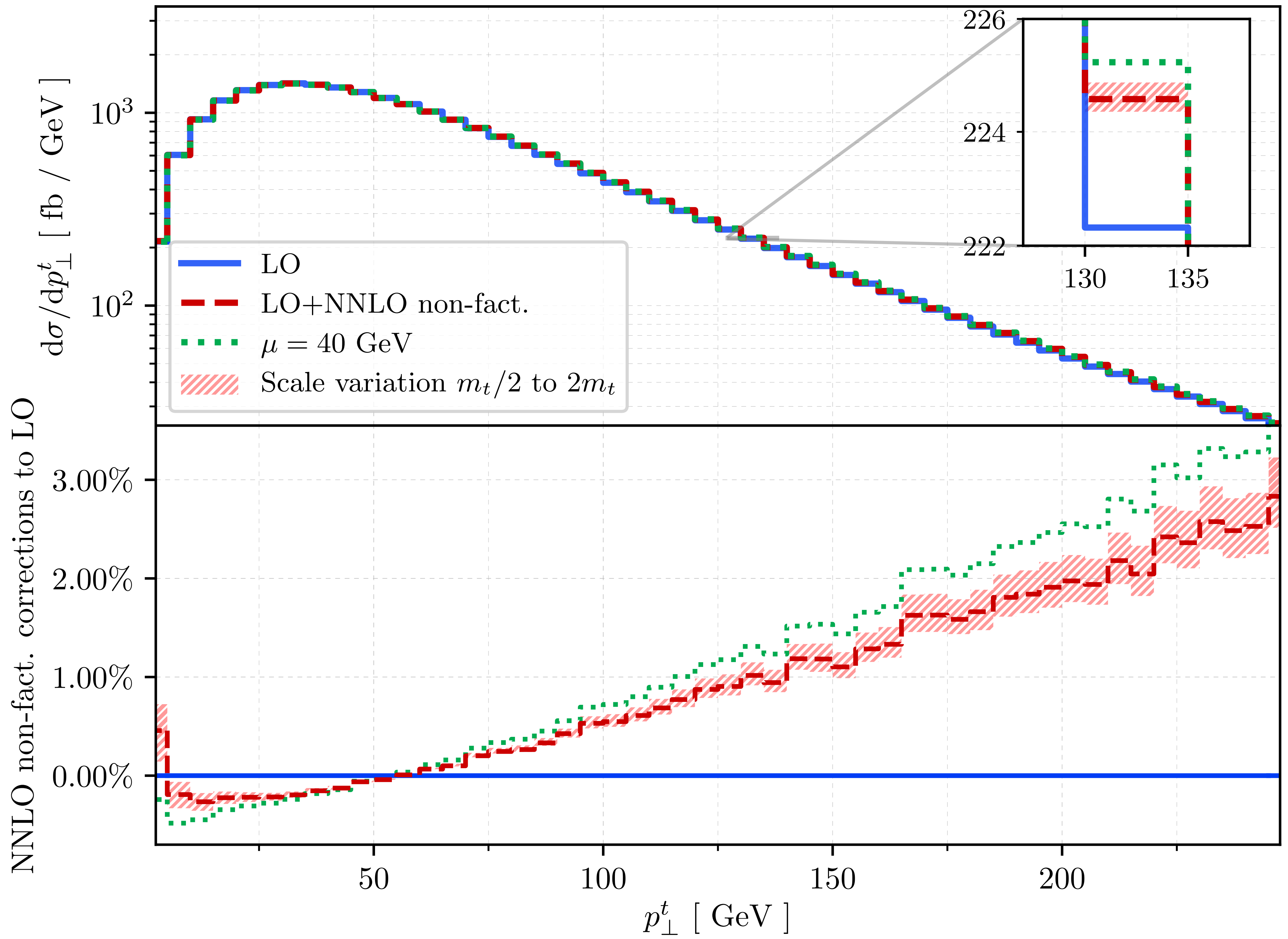
$$\frac{\sigma_{pp\to X+t}}{1\text{ pb}} = 117.96 + 0.26\left(\frac{\alpha_s(\mu_R)}{0.108}\right)^2$$

1. Non-factorisable corrections are $0.22^{-0.04}_{+0.05}\,\%$ LO for $\mu_R = m_t$.
2. **Theoretical uncertainties** are estimated through **scale variation**: $2m_t, m_t/2$.
3. **Unclear optimal scale choice**: non-factorisable corrections appear for the first time at NNLO → no indication from lower orders.
4. For $\mu_R = 40\text{GeV}$ (typical **transfer momentum scale** of top quark) non-factorisable corrections are $0.35\,\%$ LO.
5. In comparison, **NNLO factorisable** corrections to **NLO cross section** are around $0.7\,\%$.

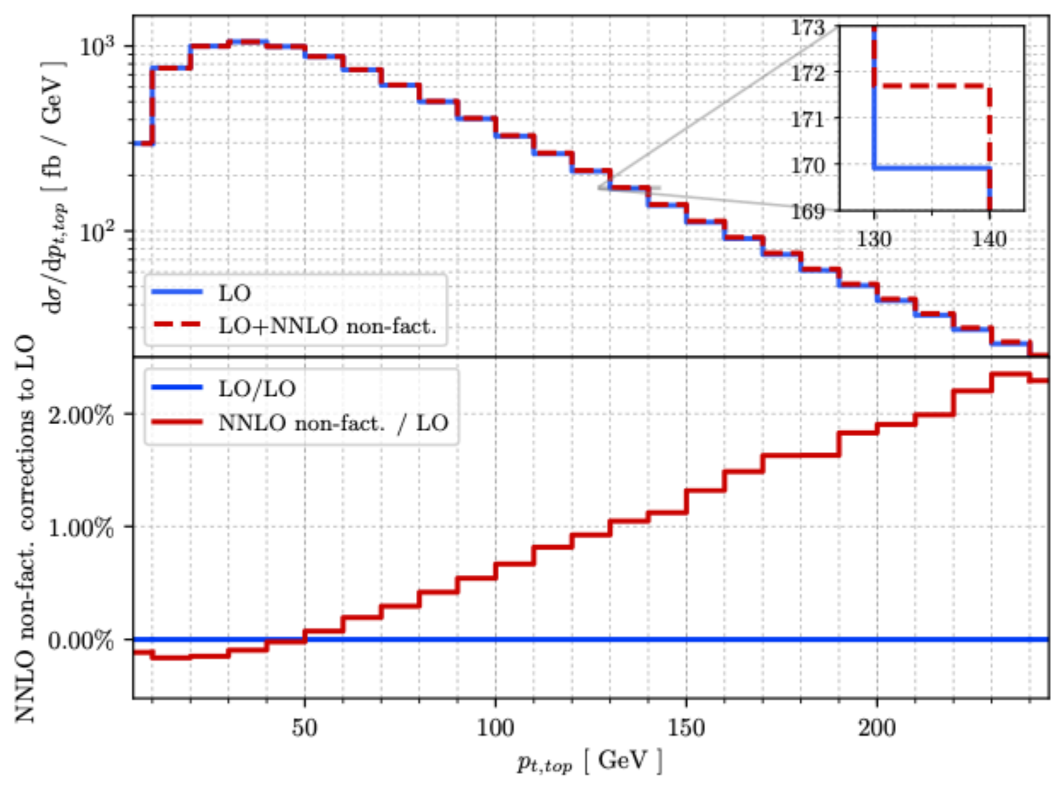
Results

Differential cross section:

pp collision: $\sqrt{s} = 13$TeV, PDFs: CT14_lo@LO, CT14_nnlo@NNLO, $m_W = 80.379$GeV, $m_t = 173.0$GeV, $\alpha_s(m_t) = 0.108$, $\mu_F = \mu_R = \mu$



1. Non-factorisable corrections are $p_\perp^t$-dependent.
2. Non-factorisable corrections are small and negative at low values of $p_\perp^t$. They vanish at $p_\perp^t \sim 50$GeV [in agreement with results for virtual corrections]



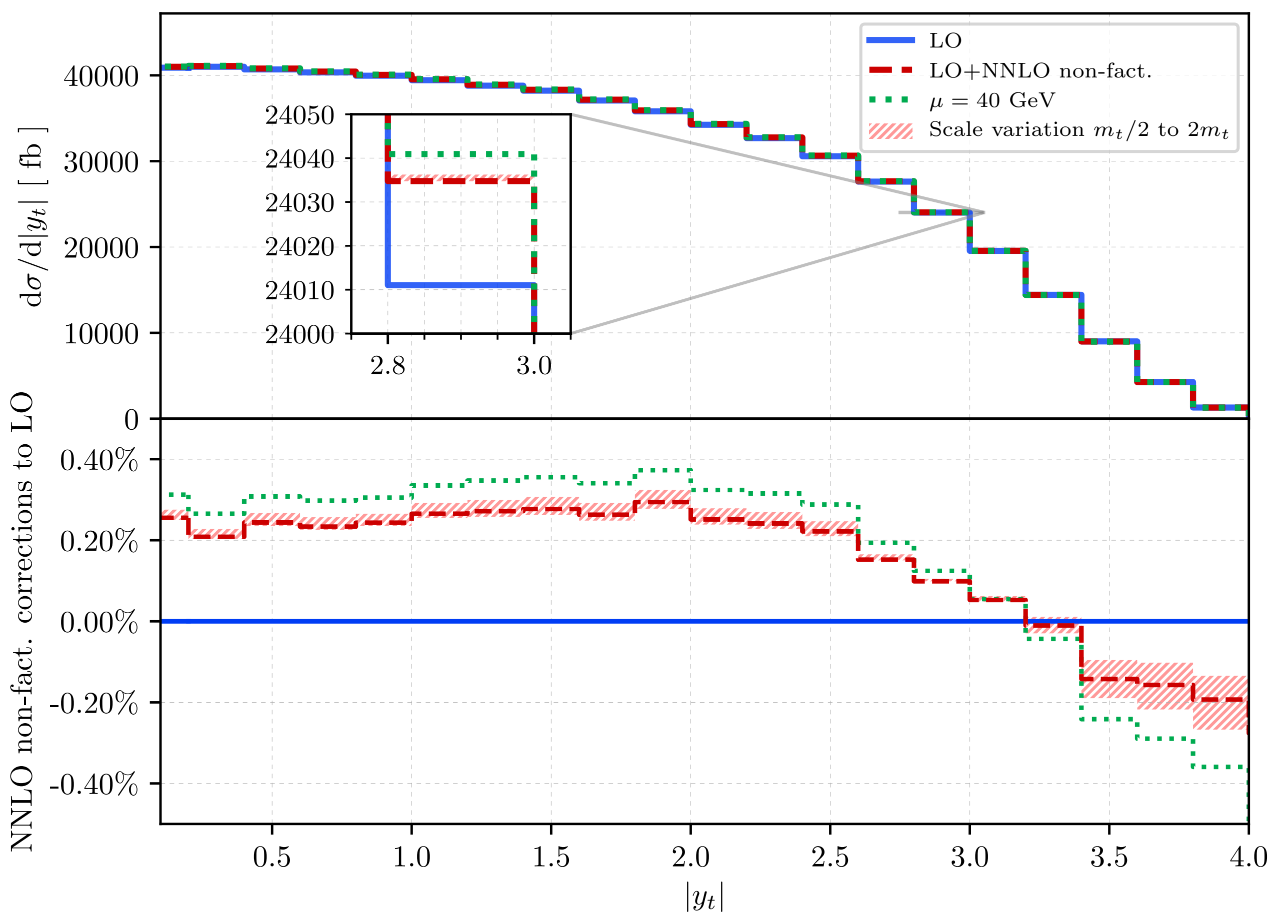
[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]

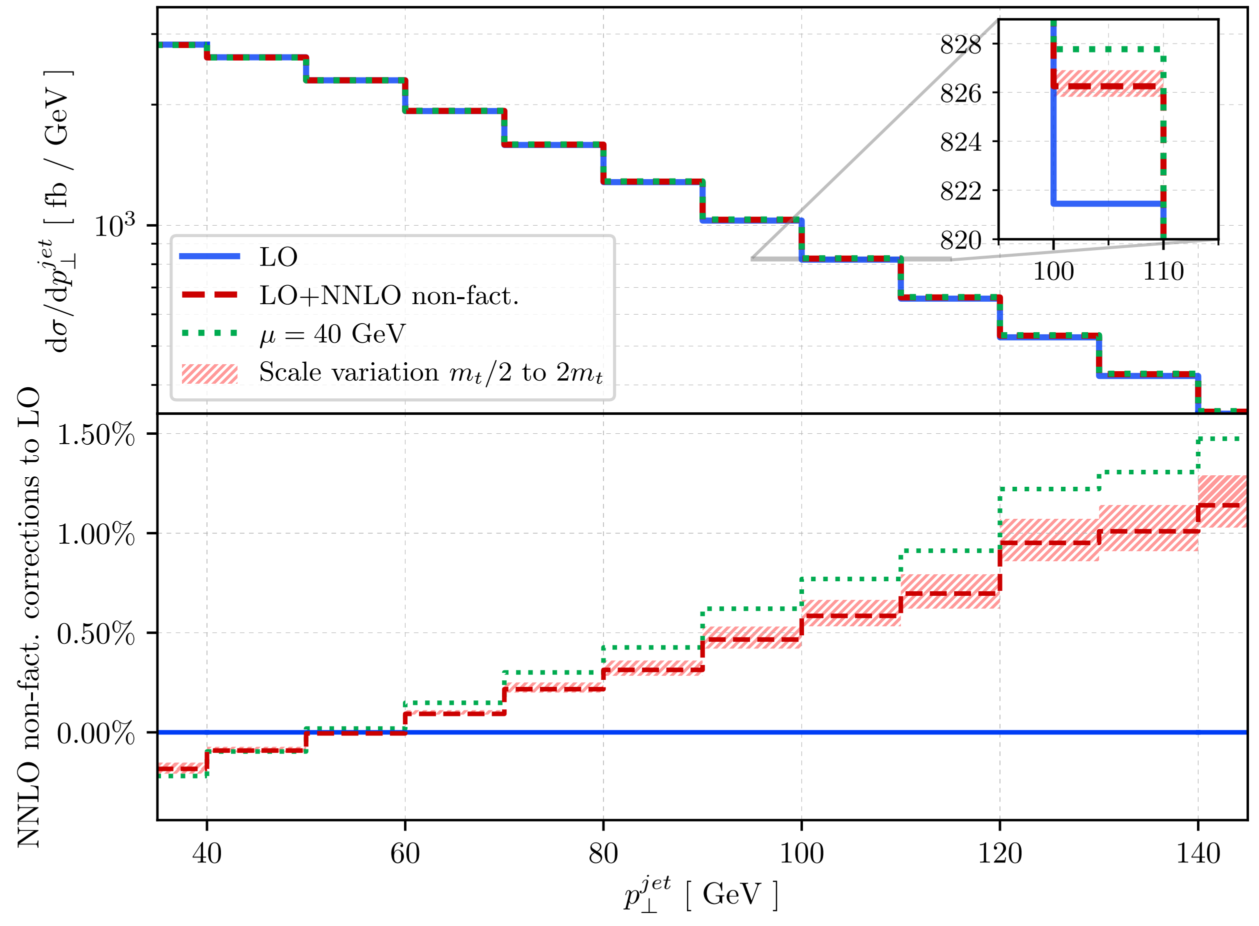
3. **Factorisable corrections** vanish around $p_\perp^t \sim 30$GeV.
4. Factorisable and non-factorisable corrections are comparable in the region around the maximum of the $p_\perp^t$ distribution.

Results

Differential cross section:

pp collision: $\sqrt{s} = 13$TeV, PDFs: CT14_lo@LO, CT14_nnlo@NNLO, $m_W = 80.379$GeV, $m_t = 173.0$GeV, $\alpha_s(m_t) = 0.108$, $\mu_F = \mu_R = \mu$





1. Relative non-factorisable **correction** to **top-quark rapidity fairly flat** for $|y_t| < 2.5$, $\mathcal{O}(0.25\%)$.
2. **Sign change** around $|y_t| \sim 3$
3. **Factorisable** corrections change sign around $|y_t| \sim 1.2$
4. For some top-quark rapidity values, **factorisable and non-factorisable** correction become **quite comparable**.
5. k_t**-algorithm** to define jets $p_\perp^{jet} > 30$GeV, R=0.4.
6. **Non-factorisable** corrections reach $1.2\,\%$ at $p_\perp^{jet} \sim 140$GeV.

Results

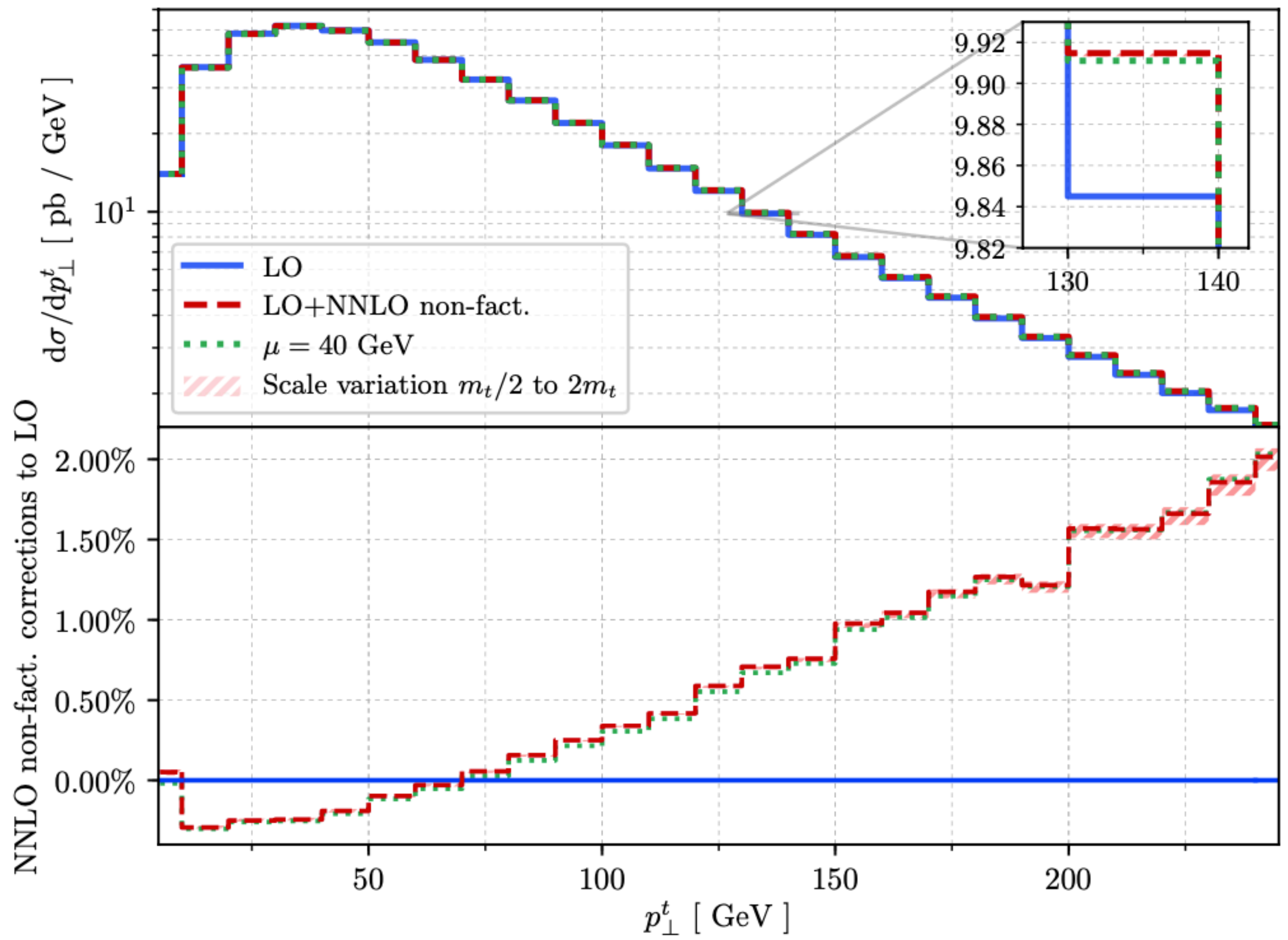
Differential cross section:

pp collision: $\sqrt{s} = 100\text{TeV}$, PDFs: CT14_lo@LO, CT14_nnlo@NNLO, $m_W = 80.379\text{GeV}$, $m_t = 173.0\text{GeV}$, $\alpha_s(m_t) = 0.108$, $\mu_F = m_t$.

$$\frac{\sigma_{pp\to X+t}}{1\text{ pb}} = 2367.0 + 3.8\left(\frac{\alpha_s(\mu_R)}{0.108}\right)^2$$

1. Non-factorisable corrections are $0.16\,\%$ LO for $\mu_R = m_t$.
2. For $\mu_R = 40\text{GeV}$ non-factorisable corrections are $0.25\,\%$ LO.
3. $p_\perp^t$ peaked around 40GeV, changes sign around 70GeV
4. Non-factorisable corrections increase by $0.04 - 0.07\,\%$ by increasing $p_\perp^{t,\text{cut}}$

		$\mu_R = m_t$		$\mu_R = 40$ GeV	
$p_\perp^{t,\text{cut}}$	σ_{LO} (pb)	$\sigma_{\text{NNLO}}^{\text{nf}}$ (pb)	δ_{NNLO} [%]	$\sigma_{\text{NNLO}}^{\text{nf}}$ (pb)	δ_{NNLO} [%]
0 GeV	2367.02	$3.79^{-0.63}_{0.84}$	$0.16^{-0.03}_{0.04}$	5.95	0.25
20 GeV	2317.03	$3.89^{-0.64}_{0.86}$	$0.17^{-0.03}_{0.04}$	6.11	0.26
40 GeV	2216.61	$4.14^{-0.69}_{0.92}$	$0.19^{-0.03}_{0.04}$	6.50	0.29
60 GeV	2121.88	$4.28^{-0.71}_{0.95}$	$0.20^{-0.03}_{0.04}$	6.71	0.32



Conclusions

1. We **complete** the calculation of **NNLO corrections** to the ***t*-channel single-top production**: the **non-factorisable corrections**.

2. **Non-factorisable** corrections are **smaller** than, but **quite comparable** to, the **factorisable** ones.

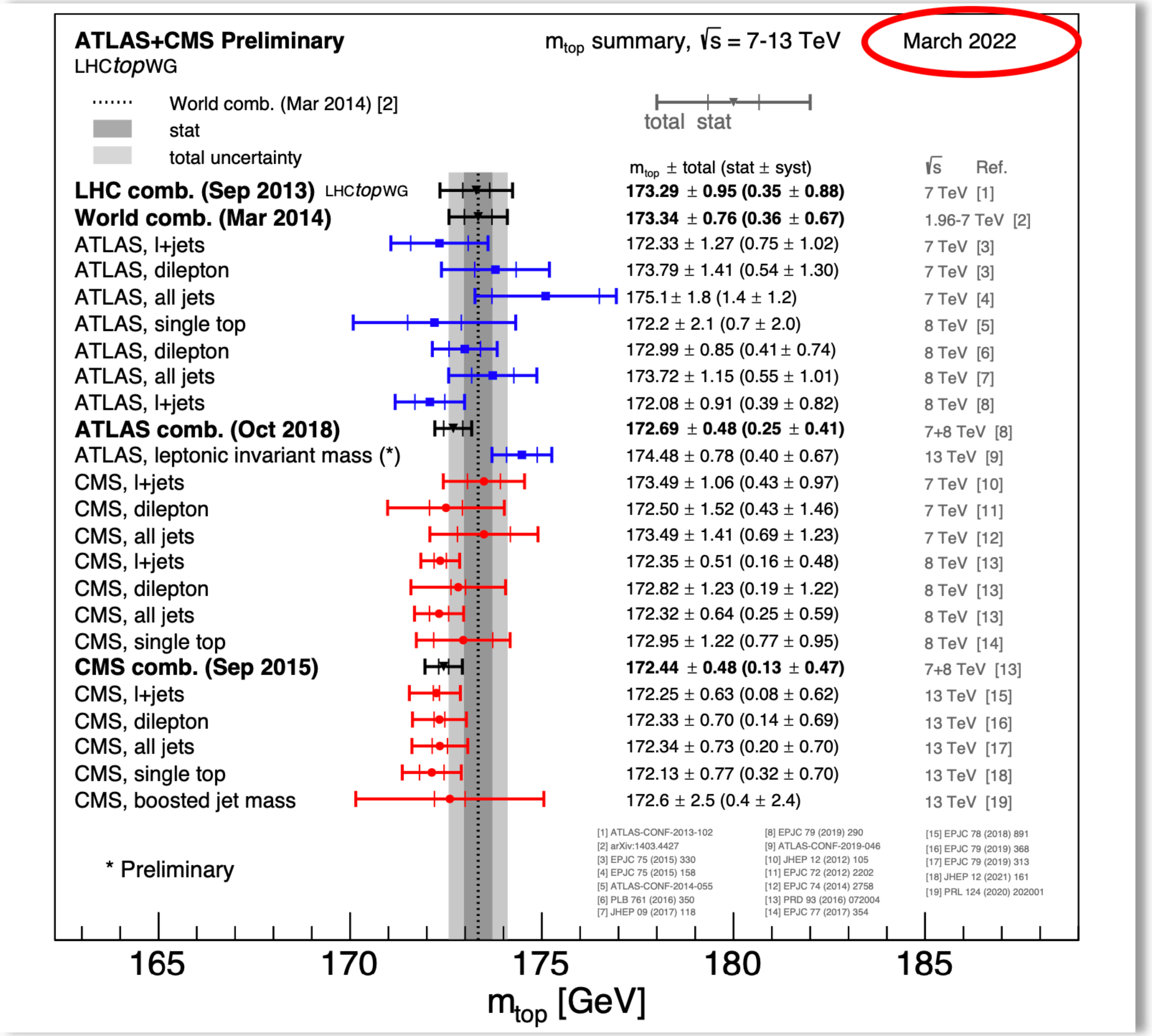
3. If **percent precision** in single-top studies can be reached, the **non-factorisable effects will have to be taken into account**.

Thank you for your attention!

Backup

Motivation: why top quark?

- **Heaviest observed particle**
  - $m_t = (173.34 \pm 0.76)$ GeV *[World Combination 14, ATLAS, CDF, CMS, D0]*
- **Substantial Yukawa coupling**
  - $Y_t = \sqrt{2}\,\dfrac{m_t}{v} \sim 1$
- **Special relation with SM Higgs Boson**
- **Short lifetime** → decay before bound states can be formed



Precision top-quark Physics

- **Extracting SM parameters**
- **Constraining PDFs**
- **Examining (anomalous) couplings**
- **Better understanding of EW symmetry breaking**
- **Hints for heavy New Physics**

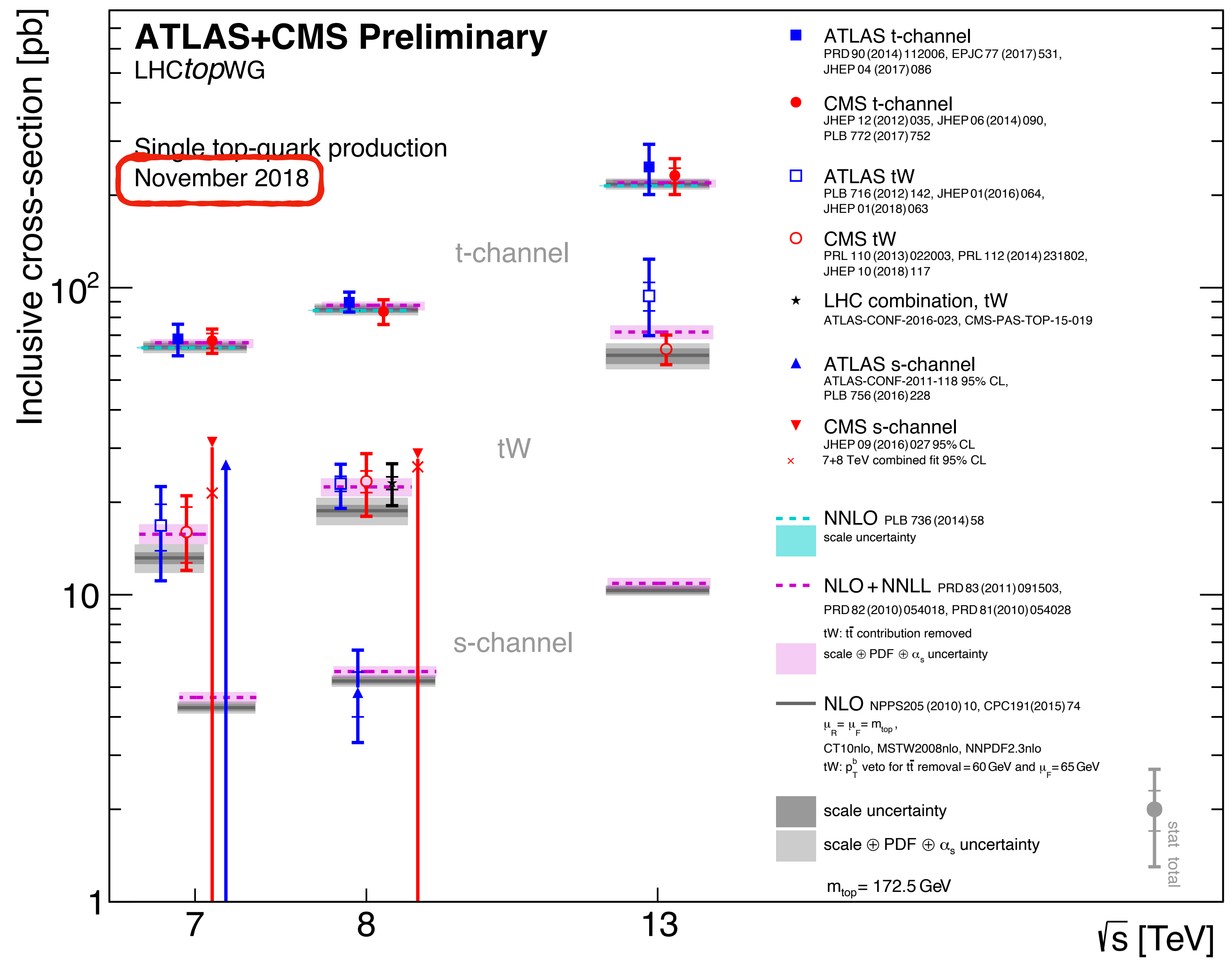
Single-top production: LHC status

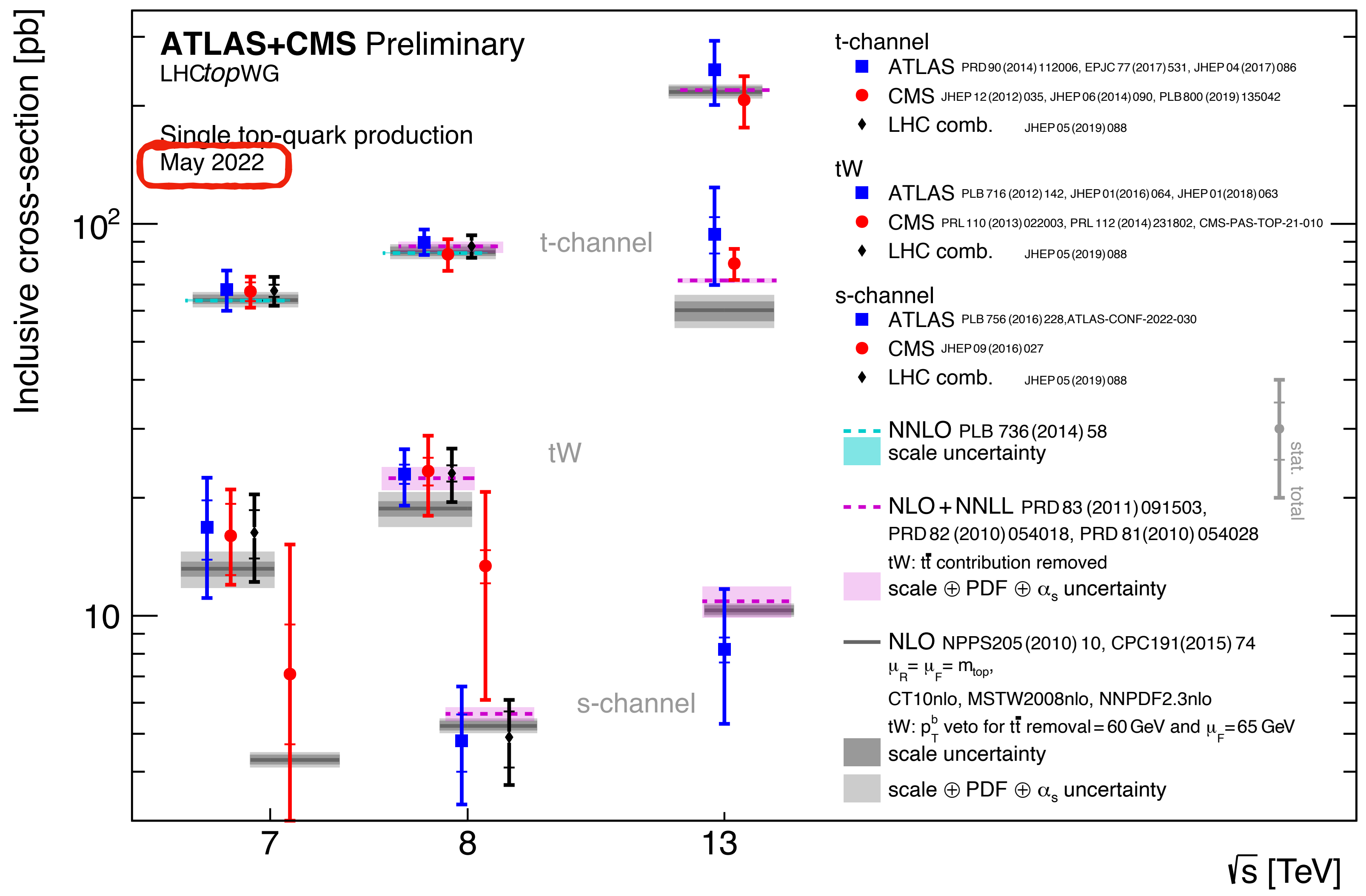
- Measurements compared to theoretical calculations based on: NLO QCD, NLO QCD complemented with NNLL resummation and NNLO QCD (*t*-channel only)

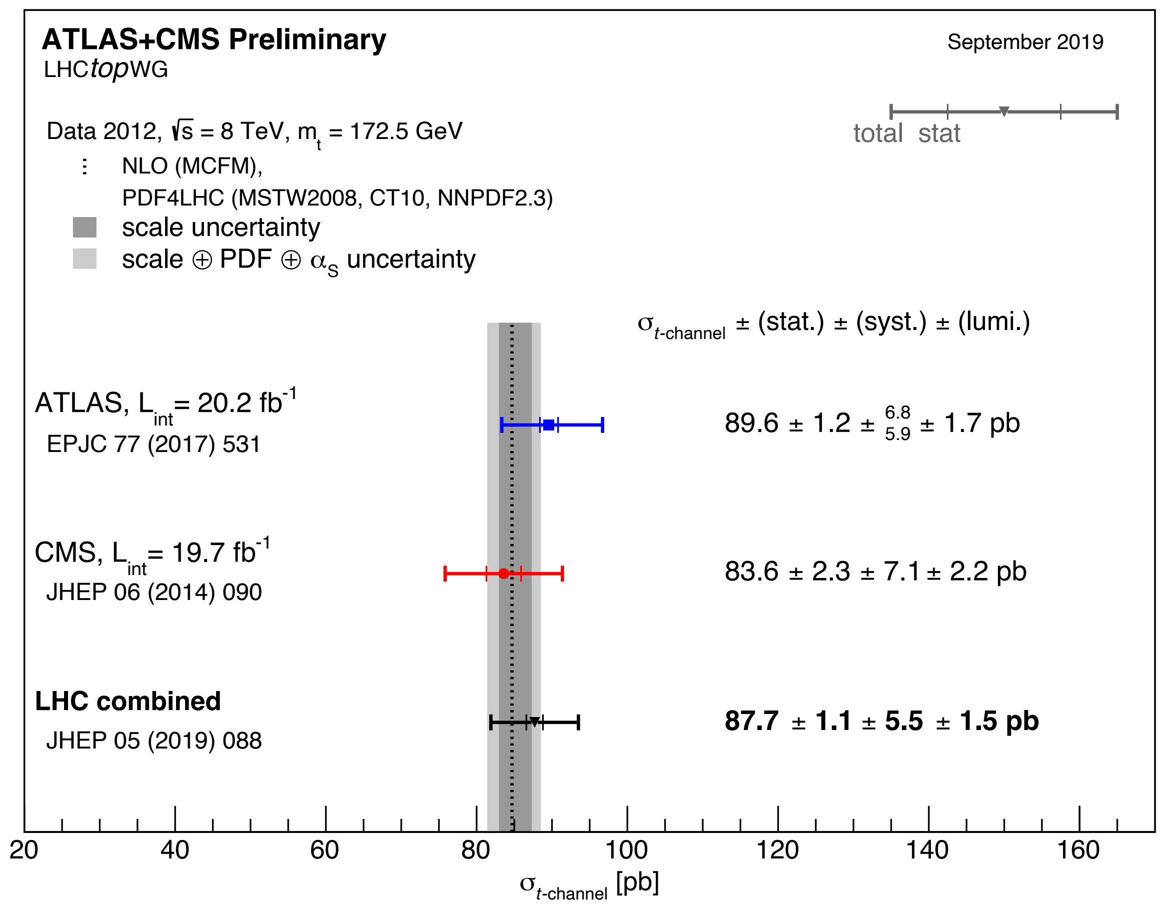
- *s-channel*: very small and affected by large background

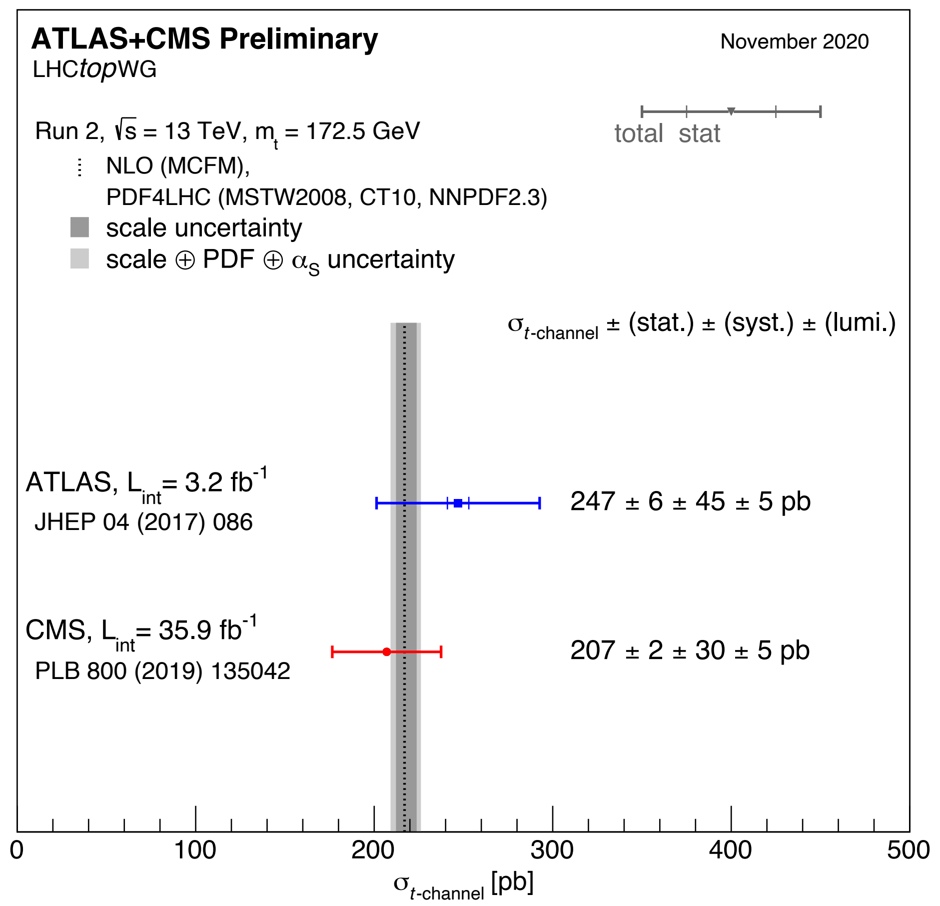
- Theoretical uncertainties smaller than experimental uncertainties for *s-channel* and *t-channel* but comparable for tW.

- *t-channel*: large exp. uncertainties at 13TeV if compared with 8TeV → improvements are expected from HL-LHC.







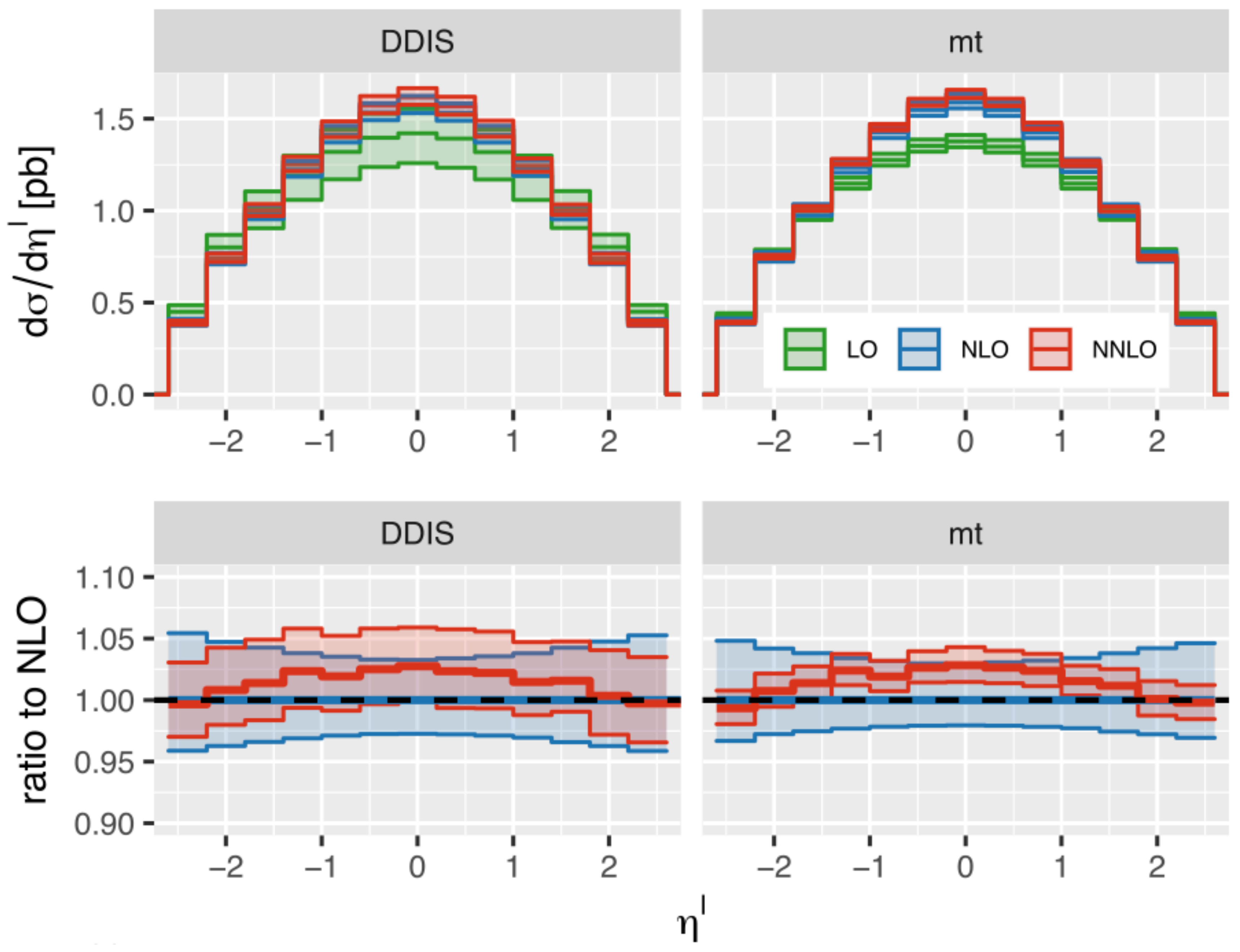


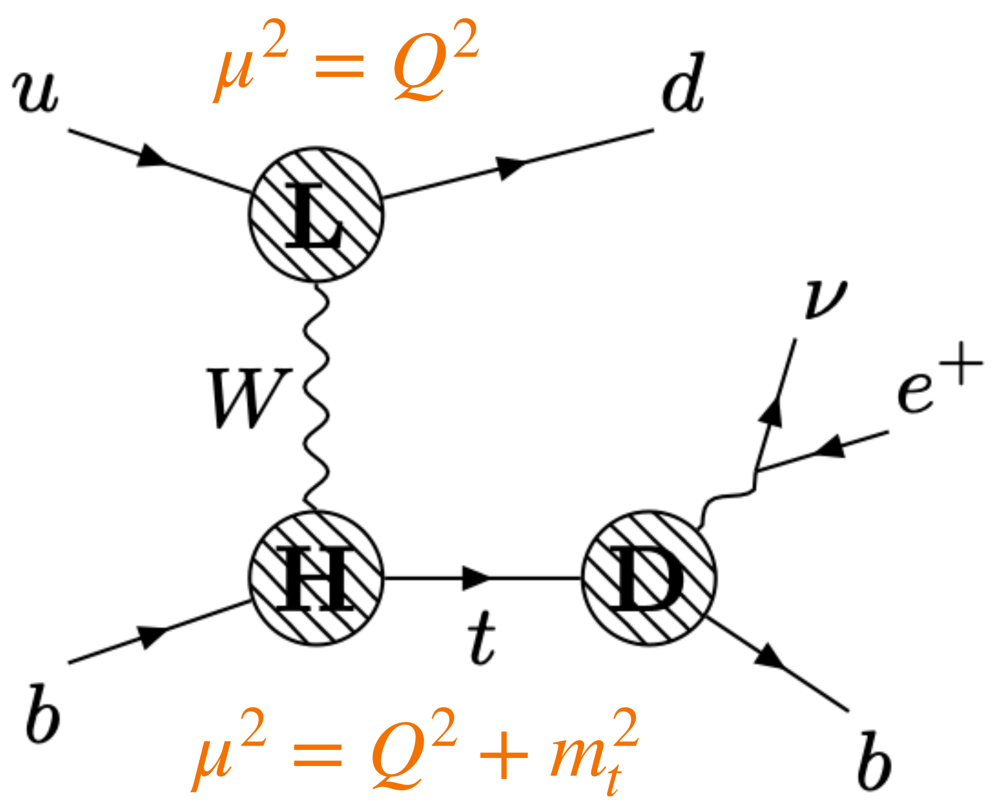
Single-top production: theory status, *t*-channel (III)

NNLO QCD

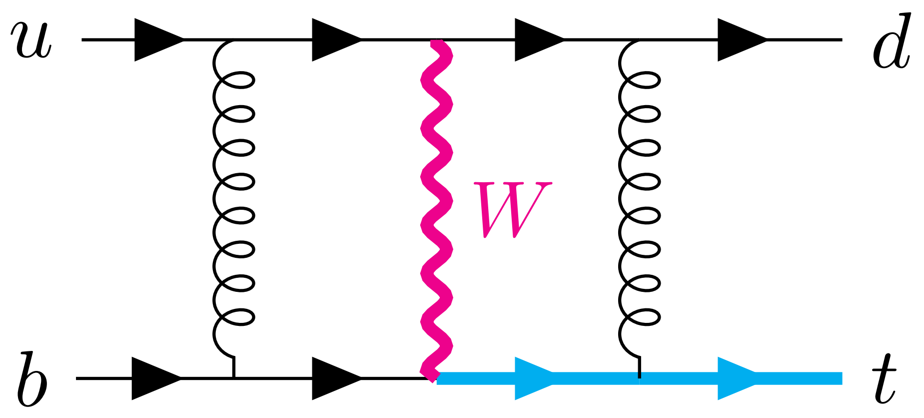
- Scale choice remains an open question:

  → double-deep inelastic scattering (DDIS) scales allows to match PDF extraction, but manifest larger scale uncertainties





- **Non-factorisable contributions** → **cross talk between different quark lines**

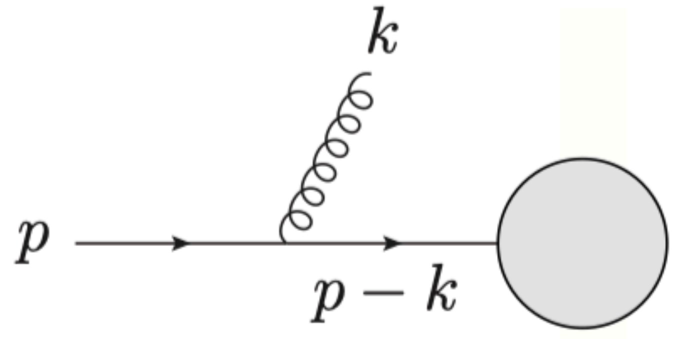


IR singularities

Higher-order corrections are affected by **infrared singularities arising from unresolved radiation**.

- Virtual corrections:
  - Explicit IR singularities from loop integrations $\rightarrow$ poles in $1/\epsilon$

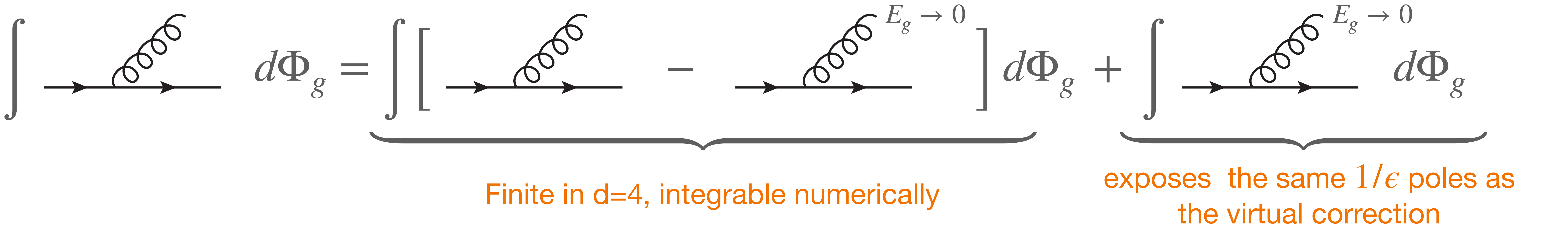
- Real corrections:
  - Singularities after integration over full phase space of radiated parton



$$\sim \frac{1}{(p-k)^2} = \frac{1}{2E_pE_k(1-\cos\theta)} \underset{\substack{E_k \to 0 \\ \textbf{or} \\ \theta \to 0}}{\longrightarrow} \infty \,. \qquad \int \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1}2E_k}\,|M(\{p\},k)|^2 \underset{\substack{E_k \to 0 \\ \theta \to 0}}{\sim} \int \frac{\mathrm{d}E_k}{E_k^{1+2\epsilon}}\,\frac{\mathrm{d}\theta}{\theta^{1+2\epsilon}} \times |M(\{p\})|^2 \sim \frac{1}{4\epsilon^2}\,.$$

  - **Integrating implies losing kinematic information** (needed for distributions, kinematic cuts, …)
  - For **non-factorisable** corrections **only soft** limits are relevant $\rightarrow$ only $1/\epsilon$ poles

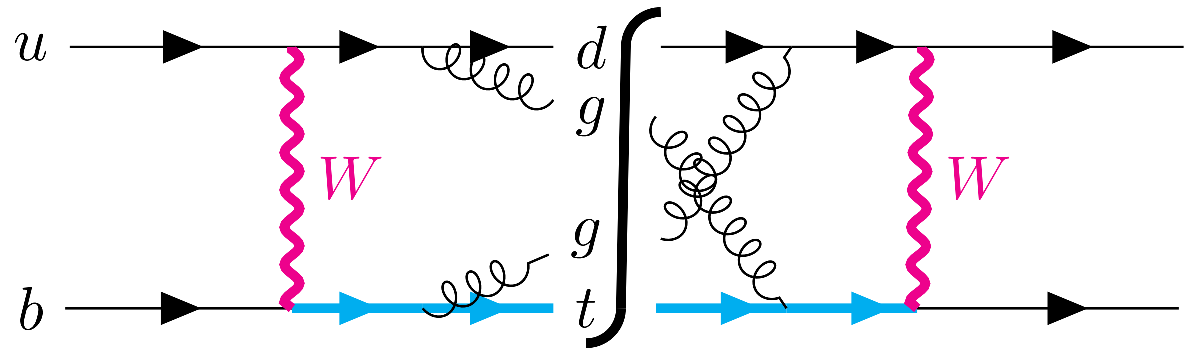
→ **Subtraction scheme**: extract singularities without integrating over full phase space of radiated partons



Double-real emission

Main issue of the **double-real contribution: extract and regularise IR singularities** preserving the fully-differential nature of the calculation

→ **Nested soft-collinear subtraction scheme** *[Caola, Melnikov, Röntsch 1702.01352]*



$$F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right) = \mathcal{N}\int \mathrm{dLips}_{34}\,(2\pi)^d\,\delta^{(d)}\!\left(p_1+p_2-\sum_{i=3}^{6}p_i\right)\times\left|\mathcal{A}_0\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right|^2_{\mathrm{nf}}$$

Integration over potentially unresolved phase space

$$[\mathrm{d}p] = \frac{\mathrm{d}^{d-1}p}{(2\pi)^{d-1}2E_p}\,\theta\left(E_{\max}-E_p\right)$$

$$2s\cdot\sigma_{\mathrm{RR}}^{\mathrm{nf}} = \frac{1}{2!}\int[\mathrm{d}p_5][\mathrm{d}p_6]\,F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right) \equiv \left\langle F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right\rangle$$

Separate the **soft-divergent part** from the **soft-finite contribution**

$$\left\langle F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right\rangle = \left\langle S_5\,S_6\,F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right\rangle \longrightarrow \textbf{Double-soft counterterm}$$

$$+2\left\langle S_6\,(I-S_5)\,F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right\rangle \longrightarrow \textbf{Single-soft counterterm}$$

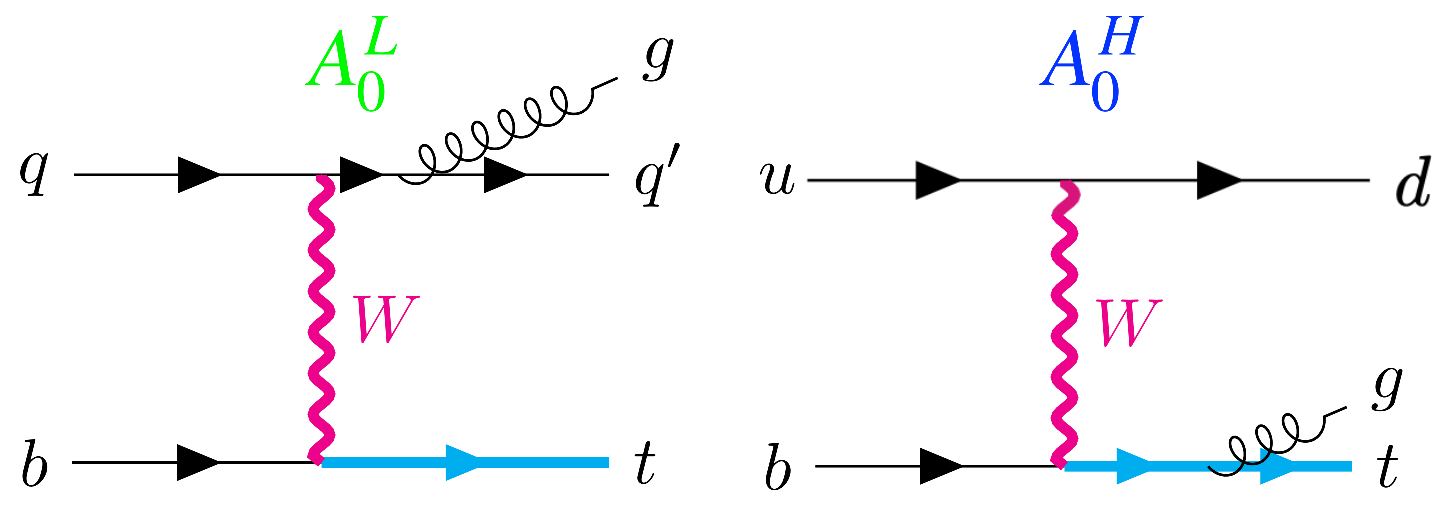
$$+\left\langle (I-S_5)\,(I-S_6)\,F_{\mathrm{LM}}^{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right\rangle \longrightarrow \textbf{Resolved contribution}$$

Dealing with soft limits

Consider **single emission**: simpler bookkeeping, clear procedure

1. Decompose the amplitude into **colour-stripped, sub-amplitudes**

$$\langle c \,|\, \mathscr{A}_0(1_q,2_b,3_{q'},4_t;5_g)\rangle = g_{s,b}\Big[t^{c_5}_{c_3c_1}\delta_{c_4c_2}\,A_0^L(5_g) + t^{c_5}_{c_4c_2}\delta_{c_3c_1}\,A_0^H(5_g)\Big]$$



2. Under **soft limit**, sub-amplitudes factorise into **universal eikonal factors** and **lower-multiplicity amplitudes**

$$S_5\;A_0^{L/H}(5_g) = \varepsilon_\mu^{(\lambda)}(5)\,J^\mu(3,1;5)\,A_0(1_q,2_b,3_{q'},4_t)$$

$$J^\mu(i,j;k) = \frac{p_i^\mu}{p_i\cdot p_k} - \frac{p_j^\mu}{p_j\cdot p_k}$$

3. **Contract sub-amplitudes** to connect different quark lines

$$S_5\;2\mathrm{Re}\big[A_0^L(5_g)\,A_0^{H\,*}(5_g)\big] = \sum_\lambda \varepsilon_\mu^{(\lambda)*}(5)\,\varepsilon_\nu^{(\lambda)}(5)\,J^\mu(3,1;5)\,J^\nu(4,2;5)\,|A_0(1_q,2_b,3_{q'},4_t)|^2$$

$$= -\boxed{\mathrm{Eik}_{\mathrm{nf}}(1_q,2_b,3_{q'},4_t;5_g)}\,|A_0(1_q,2_b,3_{q'},4_t)|^2$$

$$\mathrm{Eik}_{\mathrm{nf}}(1_q,2_b,3_{q'},4_t;k_g) = J^\mu(3,1;k)\,J_\mu(4,2;k) = \sum_{\substack{i\in[1,3]\\ j\in[2,4]}} \frac{\lambda_{ij}\;p_i\cdot p_j}{(p_i\cdot p_k)(p_j\cdot p_k)}$$

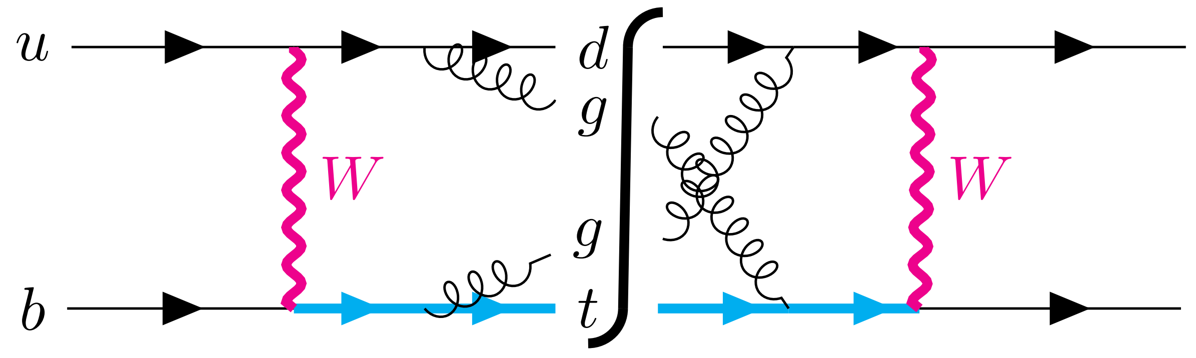
Dealing with soft limits

4. Integrate the eikonal factor over the radiation phase space

$$g_{s,b}^2 \int [\mathrm{d}p_k]\, \mathrm{Eik}_{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;k_g\right) \equiv \frac{\alpha_s}{2\pi}\left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon} K_{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;\epsilon\right) = \frac{\alpha_s}{2\pi}\left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon}\left[\frac{1}{\epsilon}\log\left(\frac{p_1\cdot p_4\; p_2\cdot p_3}{p_1\cdot p_2\; p_3\cdot p_4}\right)+\mathcal{O}(\epsilon^0)\right]$$

Double-real correction treated in the same fashion:

- Independent emissions
- Factorised double-soft limit



$$S_6\left|\mathscr{A}_0(1_q,2_b,3_{q'},4_t;5_g,6_g)\right|^2_{\mathrm{nf}} = -\,g_{s,b}^4\frac{N^2-1}{2}\,\mathrm{Eik}_{\mathrm{nf}}(6_g)\left[A_0^L(5_g)\,A_0^{H*}(5_g)+\mathrm{c.c.}\right]$$

$$S_5\,S_6\left|\mathscr{A}_0(1_q,2_b,3_{q'},4_t;5_g,6_g)\right|^2_{\mathrm{nf}} = g_{s,b}^4\,(N^2-1)\,\mathrm{Eik}_{\mathrm{nf}}(5_g)\,\mathrm{Eik}_{\mathrm{nf}}(6_g)\left|A_0\left(1_q,2_b,3_{q'},4_t\right)\right|^2$$

Double-real at **cross-section level** results in a remarkably simple object

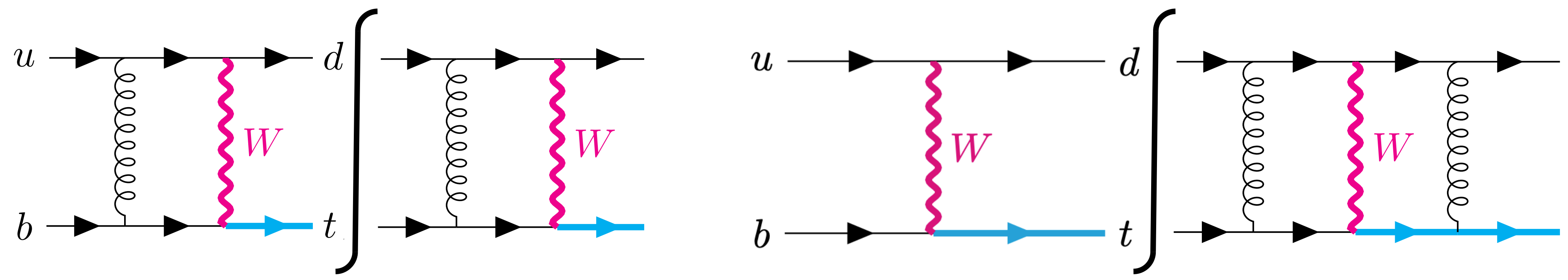
Colour-stripped sub-amplitudes

$$2s\cdot\sigma_{RR} = \left(\frac{\alpha_s}{2\pi}\right)^2\frac{N^2-1}{2N^2}\left(\frac{2E_{\max}}{\mu}\right)^{-4\epsilon}\left\langle K_{\mathrm{nf}}^2(\epsilon)\,F_{\mathrm{LM}}\left(1_q,2_b,3_{q'},4_t\right)\right\rangle$$

$$-\left(\frac{\alpha_s}{2\pi}\right)\frac{N^2-1}{2}\left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon}\left\langle K_{\mathrm{nf}}(\epsilon)\,(I-S_5)\;\widetilde{F}^{\mathrm{nf}}_{\mathrm{LM}}\left(1_q,2_b,3_q,4_t;5_g\right)\right\rangle + \left\langle (I-S_5)(I-S_6)\;F^{\mathrm{nf}}_{\mathrm{LM}}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right\rangle$$

Double-virtual contribution

Complete double-virtual cross section cast in a very compact expression



$$2s \cdot \sigma_{\mathrm{VV}} = \langle F^{\mathrm{nf}}_{\mathrm{LVV}}(1_q,2_b,3_{q'},4_t)\rangle = \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N^2-1}{4}\left[\frac{2}{N^2}\left\langle \left(\mathrm{Re}\left[I_1(\epsilon)\right]\right)^2 F_{\mathrm{LM}}(1_q,2_b,3_{q'},4_t)\right\rangle\right.$$

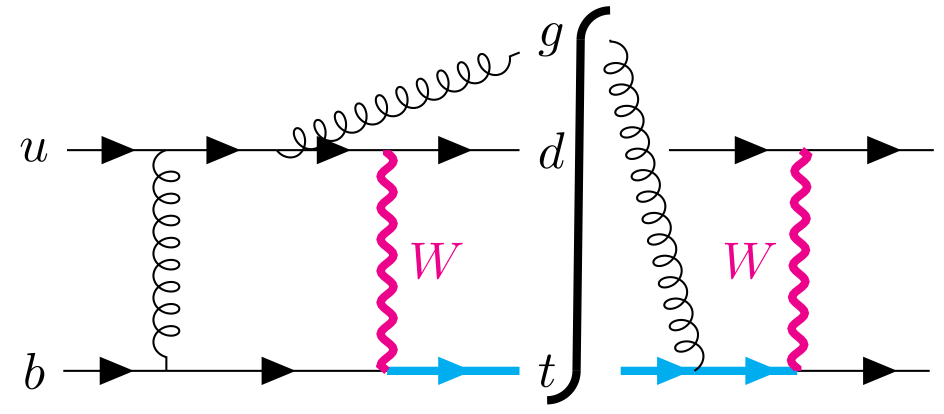
$$\left. +2\left\langle \mathrm{Re}\left[I_1(\epsilon)\right]\widetilde{F}^{\mathrm{nf}}_{\mathrm{LV,fin}}(1_q,2_b,3_{q'},4_t)\right\rangle + \left\langle \widetilde{F}^{\mathrm{nf}}_{\mathrm{VV,fin}}(1_q,2_b,3_{q'},4_t)\right\rangle\right]$$

Finite contributions built on one- and two-loop color stripped amplitudes

$$\widetilde{F}^{\mathrm{nf}}_{\mathrm{LV,fin}}(1_q,2_b,3_{q'},4_t) = \mathcal{N}\int \mathrm{dLips}_{34}\,(2\pi)^d\,\delta^{(d)}(p_1+p_2-p_3-p_4)\;2\mathrm{Re}\left[A_0^*(1_q,2_b,3_{q'},4_t)\,B_{1,\mathrm{fin}}(1_q,2_b,3_{q'},4_t)\right]$$

$$\widetilde{F}^{\mathrm{nf}}_{\mathrm{VV,fin}}(1_q,2_b,3_{q'},4_t) = \mathcal{N}\int \mathrm{dLips}_{34}\,(2\pi)^d\,\delta^{(d)}(p_1+p_2-p_3-p_4)\left\{\left|B_{1,\mathrm{fin}}(1_q,2_b,3_{q'},4_t)\right|^2 + 2\mathrm{Re}\left[A_0^*(1_q,2_b,3_{q'},4_t)B_{2,\mathrm{fin}}(1_q,2_b,3_{q'},4_t)\right]\right\}$$

Real-virtual contribution



Complete double-virtual cross section cast in a very compact expression

$$2s \cdot \sigma_{\mathrm{RV}} = \int [\mathrm{d}p_5]\, F_{\mathrm{LV}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g) = \left\langle S_5\, F_{\mathrm{LV}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g) \right\rangle + \left\langle (I - S_5)\, F_{\mathrm{LV}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g) \right\rangle$$

$$\left\langle S_5\, F_{\mathrm{LV}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g) \right\rangle = -\left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N^2-1}{N^2} \left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon} \left\langle K_{\mathrm{nf}}(\epsilon)\, \mathrm{Re}[I_1(\epsilon)]\, F_{\mathrm{LM}}(1_q, 2_b, 3_{q'}, 4_t) \right\rangle - \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N^2-1}{2} \left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon} \left\langle K_{\mathrm{nf}}(\epsilon)\, \widetilde{F}_{\mathrm{LV,fin}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t) \right\rangle$$

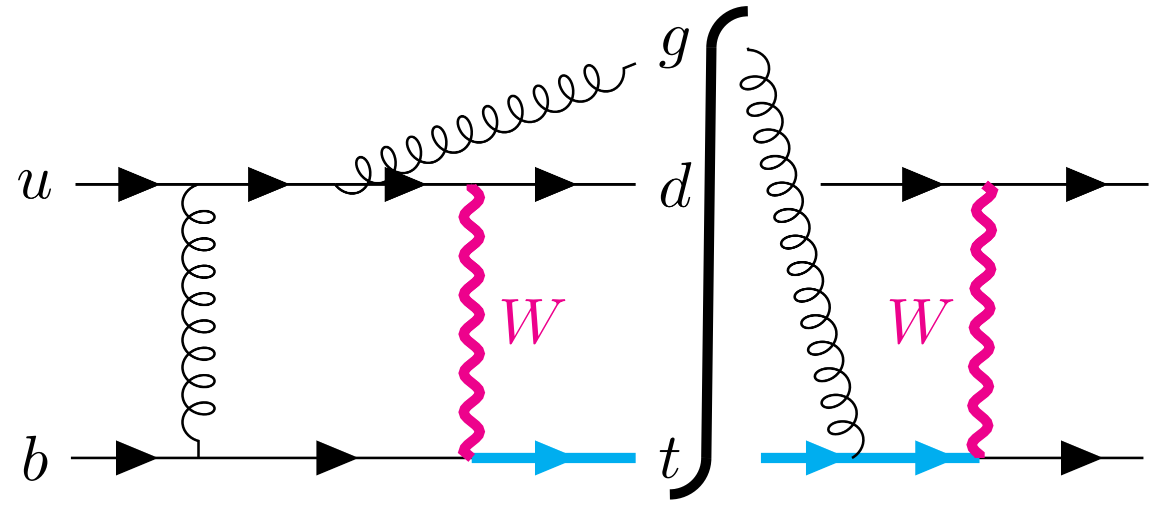
$$\left\langle (I - S_5)\, F_{\mathrm{LV}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g) \right\rangle = \left(\frac{\alpha_s}{2\pi}\right) \frac{N^2-1}{2} \left\langle \mathrm{Re}\,[I_1(\epsilon)]\, (I - S_5)\, \widetilde{F}_{\mathrm{LM}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g) \right\rangle + \left(\frac{\alpha_s}{2\pi}\right) \frac{N^2-1}{4} \left\langle (I - S_5)\, \widetilde{F}_{\mathrm{LV,fin}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g) \right\rangle$$

Finite contributions built on one-loop, 4-point and 5-point color stripped amplitudes

$$\widetilde{F}_{\mathrm{LV,fin}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t) = \mathcal{N} \int \mathrm{dLips}_{34}\, (2\pi)^d\, \delta^{(d)}(p_1 + p_2 - p_3 - p_4)\; 2\mathrm{Re}\Big[A_0^*(1_q, 2_b, 3_{q'}, 4_t)\, B_{1,\mathrm{fin}}(1_q, 2_b, 3_{q'}, 4_t) \Big]$$

$$\widetilde{F}_{\mathrm{LV,fin}}^{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; 5_g) = \mathcal{N} \int \mathrm{dLips}_{34}\, (2\pi)^d\, \delta^{(d)}\Big(p_1 + p_2 - \sum_{i=3}^{5} p_i\Big) g_{s,b}^2 \left(A_0^{L*}(5_g)\, B_{1,\mathrm{fin}}^{sH}(5_g) + A_0^{H*}(5_g)\, B_{1,\mathrm{fin}}^{sL}(5_g) + \mathrm{c.c.} \right)$$

Real-virtual contribution



Extract **IR singularities both** from **real *and* virtual radiation**

$$RV \propto \int \mathrm{dLips}_{34}\left(p_1 + p_2 - \sum_{i=3}^{5} p_i\right) 2\mathrm{Re}\left[\mathscr{A}_0^*\left(1_q,2_b,3_{q'},4_t;5_g\right)\mathscr{A}_1\left(1_q,2_b,3_{q'},4_t;5_g\right)\right]_{\mathrm{nf}}$$

1. **First** extract **phase-space singularities**

$$RV = S_5\, RV + (I - S_5)\, RV$$

Soft-divergent Soft-regulated

2. Notice that **both contributions** contain **explicit poles in** ϵ

IR **divergencies of one-loop** amplitudes do **not depend on the real radiation** → same $I_1(\epsilon)$ as VV

Real radiation in the **soft limit** exposes the **usual eikonal factor** → same $\mathrm{Eik}_{\mathrm{nf}}(k_g)$ as RR

Pole cancellation

Manifestly finite result: combination of RR, VV and RV → split into contributions of **different multiplicities**

$$\sigma_{\rm nf} = \sigma_{\rm nf}^{(2g)} + \sigma_{\rm nf}^{(1g)} + \sigma_{\rm nf}^{(0g)}$$

Technicalities:

- finite combination

$$\mathscr{W}(1_q,2_b,3_{q'},4_t) = \left(\frac{2E_{\rm max}}{\mu}\right)^{-2\epsilon} K_{\rm nf}(\epsilon) - {\rm Re}[I_1(\epsilon)] = \mathcal{O}\left(\epsilon^0\right)$$

- Cross section with one emission

$$2s\cdot\sigma_{\rm nf}^{(1g)} = -\left(\frac{\alpha_s}{2\pi}\right)\frac{N^2-1}{2}\left\langle \mathscr{W}\left(1_q,2_b,3_{q'},4_t\right)(I-S_5)\ \widetilde{F}_{\rm LM}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g\right)\right\rangle + \left(\frac{\alpha_s}{2\pi}\right)\frac{N^2-1}{4}\left\langle (I-S_5)\ \widetilde{F}_{\rm LV,fin}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g\right)\right\rangle$$

- Cross section with two emissions

$$2s\cdot\sigma_{\rm nf}^{(0g)} = \left(\frac{\alpha_s}{2\pi}\right)^2\frac{N^2-1}{2N^2}\left\langle \mathscr{W}^2\left(1_q,2_b,3_{q'},4_t\right) F_{\rm LM}\left(1_q,2_b,3_{q'},4_t\right)\right\rangle - \left(\frac{\alpha_s}{2\pi}\right)^2\frac{N^2-1}{2}\left\langle \mathscr{W}\left(1_q,2_b,3_{q'},4_t\right)\ \widetilde{F}_{\rm LV,fin}^{\rm nf}\left(1_q,2_b,3_{q'},4_t\right)\right\rangle + \left(\frac{\alpha_s}{2\pi}\right)^2\frac{N^2-1}{4}\left\langle \widetilde{F}_{\rm VV,fin}^{\rm nf}\left(1_q,2_b,3_{q'},4_t\right)\right\rangle$$

- Building blocks

$$\widetilde{F}_{\rm LV,fin}^{\rm nf}\left(1_q,2_b,3_{q'},4_t\right) \propto \int {\rm dLips}_{34}\, 2{\rm Re}\left[A_0^*\left(1_q,2_b,3_{q'},4_t\right) B_{1,\rm fin}\left(1_q,2_b,3_{q'},4_t\right)\right]$$

$$\widetilde{F}_{\rm VV,fin}^{\rm nf}\left(1_q,2_b,3_{q'},4_t\right) \propto \int {\rm dLips}_{34}\left\{\left|B_{1,\rm fin}\left(1_q,2_b,3_{q'},4_t\right)\right|^2 + 2{\rm Re}\left[A_0^*\left(1_q,2_b,3_{q'},4_t\right)B_{2,\rm fin}\left(1_q,2_b,3_{q'},4_t\right)\right]\right\}$$

We need to expand is $B_1(1_q,2_b,3_{q'},4_t)$ to $\mathcal{O}(\epsilon)$ as it is needed to extract the two-loop finite remainder $B_{2,\rm fin}(1_q,2_b,3_{q'},4_t)$.

Amplitude evaluation

Diagrams generated with `QGRAPH` and processed with `FORM`.

W boson forces **light quark** to be **left-handed** and we decompose the **massive momentum into 2 massless momenta**

$$p_4 = p_4^\flat + \frac{m_t^2}{2n \cdot p_4} n$$

$$\bar{u}_L(p_4) = \langle 4^\flat | + \frac{m_t}{[n4^\flat]}[n| \quad , \quad \bar{u}_R(p_4) = [4^\flat | + \frac{m_t}{\langle n4^\flat \rangle}\langle n|$$

The auxiliary momentum n can be appropriately chosen to simplify the result

RV: one-loop five-point amplitude

- 24 diagrams: 8 pentagons and 16 boxes
- 7 kinematic scales

VV: two-loop four-point amplitude *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

- 18 diagrams: all topologies maximal
- 4 kinematic scales: s, t, m_t^2, m_W^2
- 428 master integrals evaluated numerically using the auxiliary mass flow method to $20\mathrm{digits}$ in $\sim 30\mathrm{min}$ on a single core
- 10 sets of 10^4 points extracted from a grid prepared on the Born squared amplitude

Amplitude evaluation

Diagrams generated with QGRAPH and processed with FORM.

W boson forces **light quark** to be **left-handed** and we decompose the **massive momentum into 2 massless momenta**

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	ϵ^{-2}	ϵ^{-1}
$\langle \mathcal{A}^{(0)} \vert \mathcal{A}^{(2)}_{\mathrm{nf}} \rangle$	$-229.0940408654660 - 8.978163333241640i$	$-301.1802988944764 - 264.1773596529505i$
IR poles	$-229.0940408654665 - 8.978163333241973i$	$-301.1802988944791 - 264.177359652953 5i$

Double-virtual contribution

The pole structure of the two-loop amplitude is well studied, and can be easily cross-checked against literature
[Catani '98] [Aybat, Dixon, Sterman '06][Becher, Neubert '09][Czakon, Mitov, Sterman '09][Mitov, Sterman, Sung '09, '10][Ferroglia, Neubert, Pecjak, Yang '09]

$$|\mathscr{A}\rangle = \mathbf{Z}\,|\mathscr{F}\rangle\,, \qquad \mu\frac{d}{d\mu}\mathbf{Z} = -\,\mathbf{\Gamma}\,\mathbf{Z}$$

$$\begin{aligned}\mathbf{\Gamma}(\{p_i\},m_i,\mu) = &\sum_{(i,j)}\frac{\mathbf{T}_i\cdot\mathbf{T}_j}{2}\gamma_{\text{cusp}}(\alpha_s)\log\left(\frac{\mu^2}{-s_{ij}}\right) + \sum_{(I,j)}\mathbf{T}_I\cdot\mathbf{T}_j\,\gamma_{\text{cusp}}(\alpha_s)\log\left(\frac{m_I\,\mu}{-s_{Ij}}\right)\\ &+\sum_{(I,J)}\frac{\mathbf{T}_I\cdot\mathbf{T}_J}{2}\gamma_{\text{cusp}}(v_{IJ},\alpha_s) + \sum_i\gamma^i(\alpha_s) + \sum_I\gamma^I(\alpha_s)\\ &+\sum_{(I,J,K)} if^{abc}\,T_I^a\,T_J^b\,T_K^c\,F_1(v_{IJ},v_{JK},v_{KI}) + \sum_{(I,J)}\sum_k if^{abc}\,T_I^a\,T_J^b\,T_k^c\,f_2\left(v_{IJ},\log\left(\frac{-\sigma_{Jk}v_J\cdot p_k}{-\sigma_{Ik}v_I\cdot p_k}\right)\right) + \mathcal{O}(\alpha_s^3)\,.\end{aligned}$$

Collinear

n>1 massive

Non-abelian

Several simplifications occur when only non-factorisable corrections are considered *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

$$\boxed{\mathbf{\Gamma}_{\text{nf}} = \left(\frac{\alpha_s}{4\pi}\right)\mathbf{\Gamma}_{0,\text{nf}} = \left(\frac{\alpha_s}{4\pi}\right)4\left[\mathbf{T}_1\cdot\mathbf{T}_2\log\left(\frac{\mu^2}{-s-i\epsilon}\right) + \mathbf{T}_2\cdot\mathbf{T}_3\log\left(\frac{\mu^2}{-u-i\epsilon}\right) + \mathbf{T}_1\cdot\mathbf{T}_4\log\left(\frac{\mu\,m_t}{m_t^2-u-i\epsilon}\right) + \mathbf{T}_3\cdot\mathbf{T}_4\log\left(\frac{\mu\,m_t}{m_t^2-s-i\epsilon}\right)\right]}$$

$$\langle\mathscr{A}^{(0)}|\mathscr{A}^{(2)}_{\text{nf}}\rangle = -\frac{1}{8\epsilon^2}\langle\mathscr{A}^{(0)}|\mathbf{\Gamma}^2_{0,\text{nf}}|\mathscr{A}^{(0)}\rangle + \frac{1}{2\epsilon}\langle\mathscr{A}^{(0)}|\mathbf{\Gamma}_{0,\text{nf}}|\mathscr{A}^{(1)}_{\text{nf}}\rangle + \langle\mathscr{A}^{(0)}|\mathscr{F}^{(2)}_{\text{nf}}\rangle$$

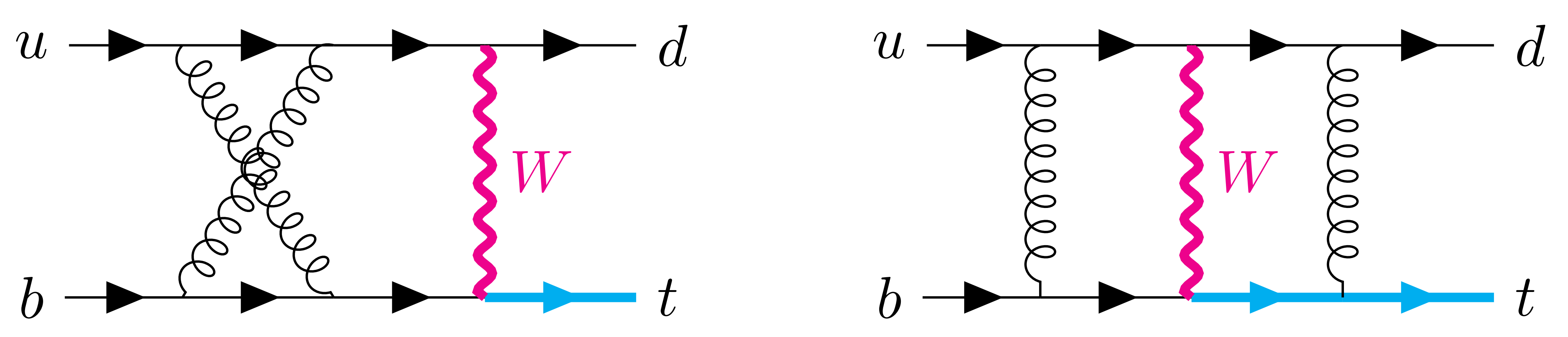
$$\langle\mathscr{A}^{(1)}_{\text{nf}}|\mathscr{A}^{(1)}_{\text{nf}}\rangle = -\frac{1}{4\epsilon^2}\langle\mathscr{A}^{(0)}||\mathbf{\Gamma}_{0,\text{nf}}|^2|\mathscr{A}^{(0)}\rangle + \frac{1}{2\epsilon}\langle\mathscr{A}^{(1)}_{\text{nf}}|\mathbf{\Gamma}_{0,\text{nf}}|\mathscr{A}^{(0)}\rangle + \frac{1}{2\epsilon}\langle\mathscr{A}^{(0)}|\mathbf{\Gamma}^\dagger_{0,\text{nf}}|\mathscr{A}^{(1)}\rangle + \langle\mathscr{F}^{(1)}_{\text{nf}}|\mathscr{F}^{(1)}_{\text{nf}}\rangle$$

Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang ’21]*

Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

Kinematic scales: $p_i^2 = 0 \,, i = 1,2,3 \,, \qquad p_4^2 = m_t^2, s, t, m_W^2$

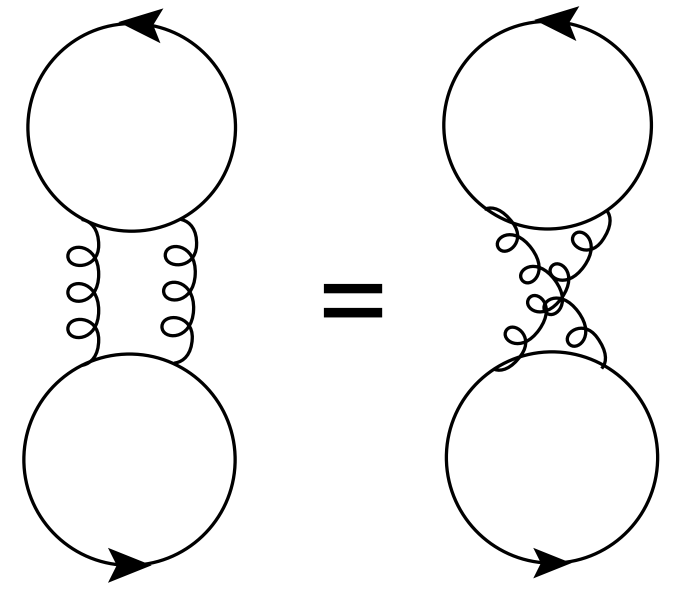
Dimensions: $d = 4 - 2\epsilon$



Planar and non-planar amplitudes appear at 2-loop order. However, only a particular combination of them do actually contribute

$$\mathscr{A}_{\rm nf}^{(2)} = \frac{1}{4}\{T^a, T^b\}_{c_3c_1}\,\{T^a, T^b\}_{c_3c_1}\left(\mathrm{A}_{\rm nf}^{(2),\,\rm pl} + \mathrm{A}_{\rm nf}^{(2),\,\rm npl}\right) + \ldots$$

$$\mathrm{A}_{\rm nf}^{(2)} = \mathrm{A}_{\rm nf}^{(2),\,\rm pl} + \mathrm{A}_{\rm nf}^{(2),\,\rm npl}$$



Upon interference with tree-level amplitude the colour distinction between planar and non-planar diagrams disappears

(Abelian nature of non-factorisable corrections)

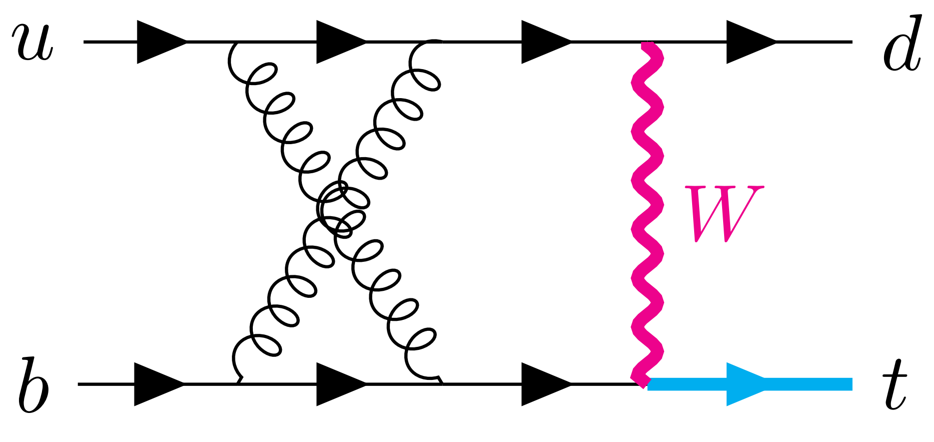
$$\sum_{\rm color} \mathscr{A}^{(0)\,*}\,\mathscr{A}_{\rm nf}^{(2)} = \frac{1}{4}\left(N_c^2 - 1\right)\mathrm{A}^{(0)\,*}\,\mathrm{A}_{\rm nf}^{(2)\,*}$$

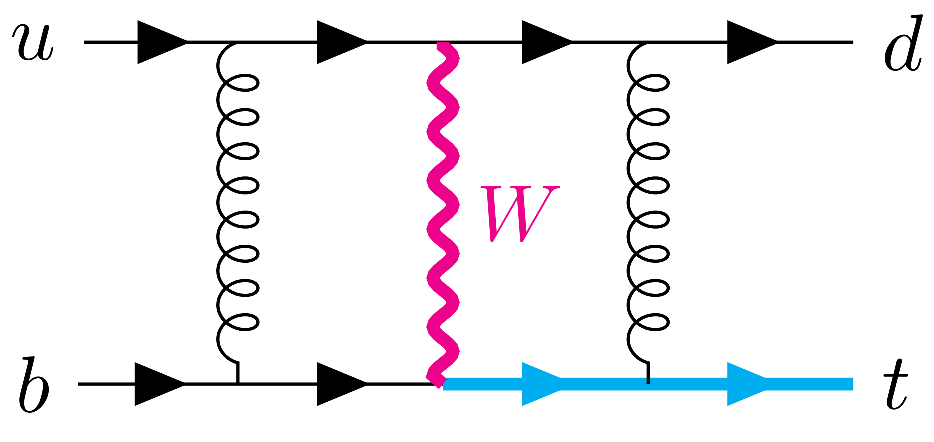
Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang ’21]*

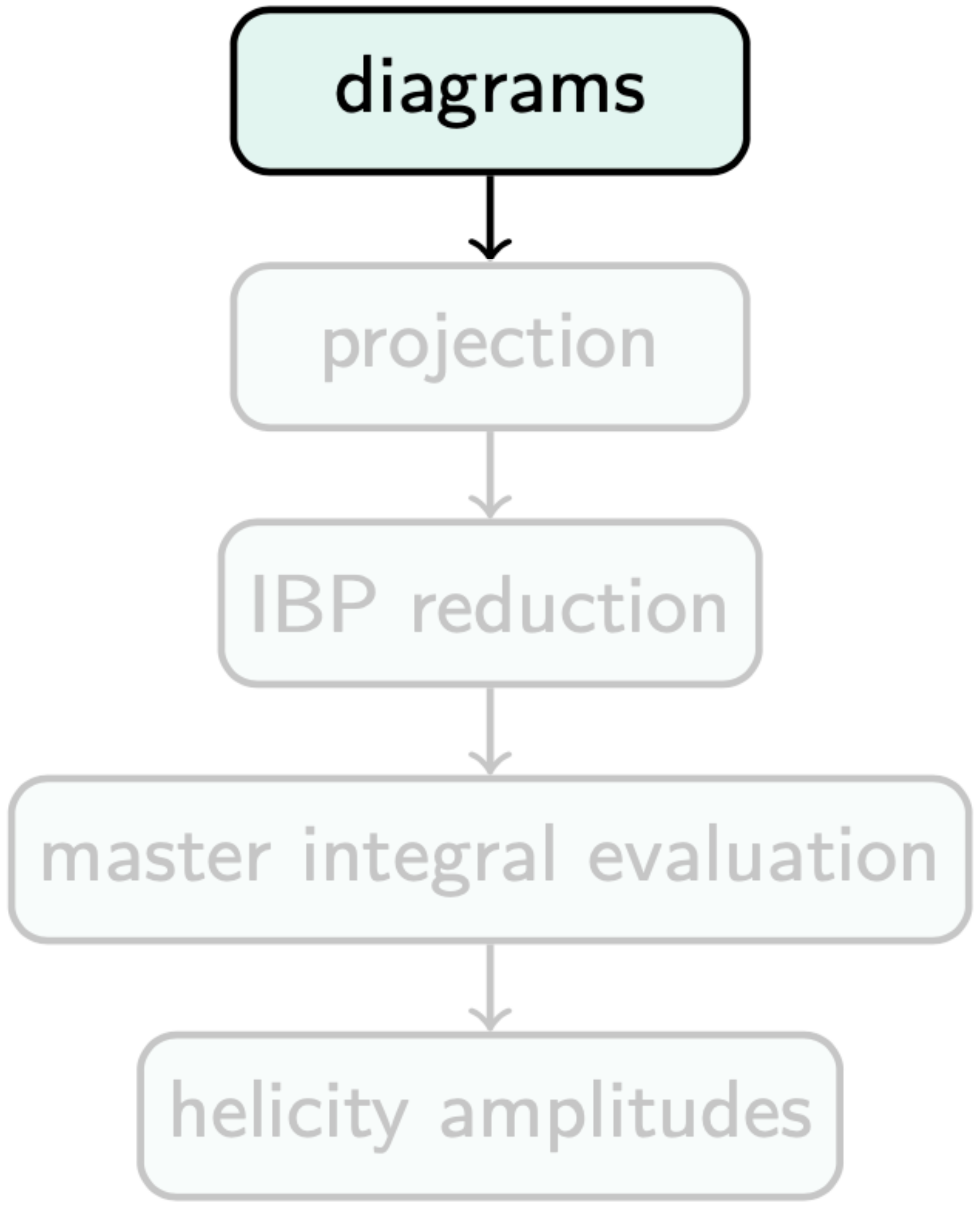
Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

Kinematic scales: $p_i^2 = 0\ , i = 1{,}2{,}3\ ,\qquad p_4^2 = m_t^2,\, s,\, t,\, m_W^2$

Dimensions: $d = 4 - 2\epsilon$







- **18 diagrams**: generated with $\mathtt{QGRAPH}$ *[Nogueira ’93]* and processed with $\mathtt{FORM}$ *[Vermaseren ’00] [Kuipers et al. ’15] [Ruijl et al. ’17]*. All **topologies maximal**.

Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang ’21]*

Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

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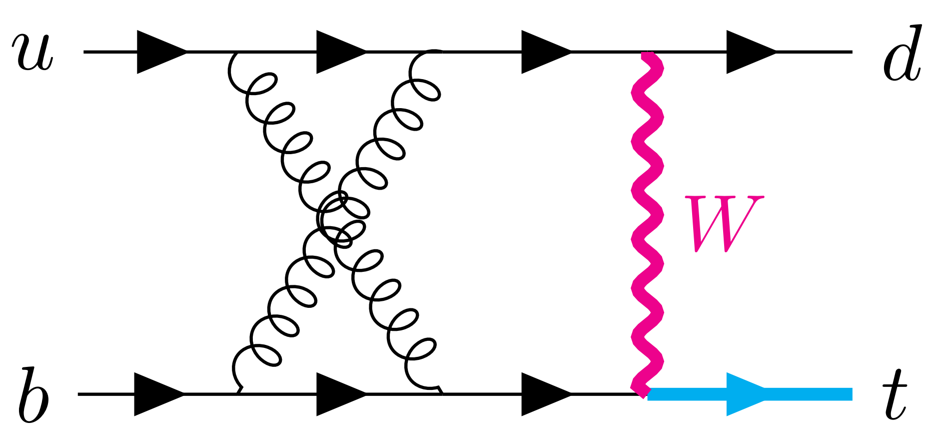

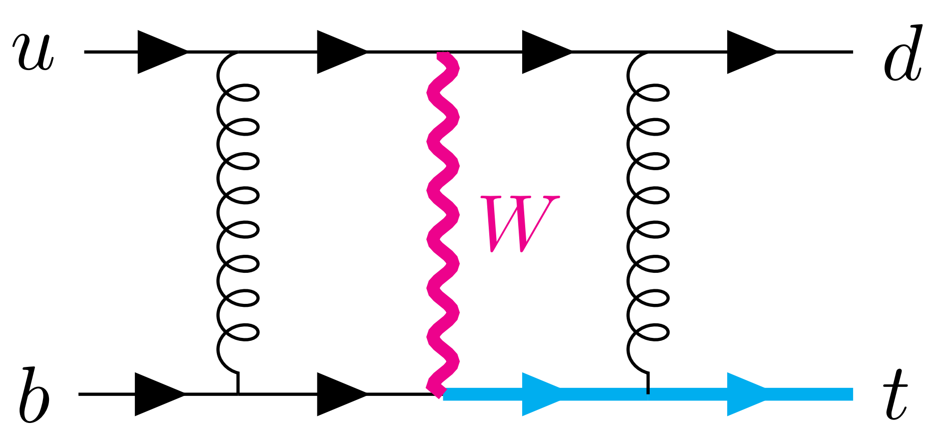

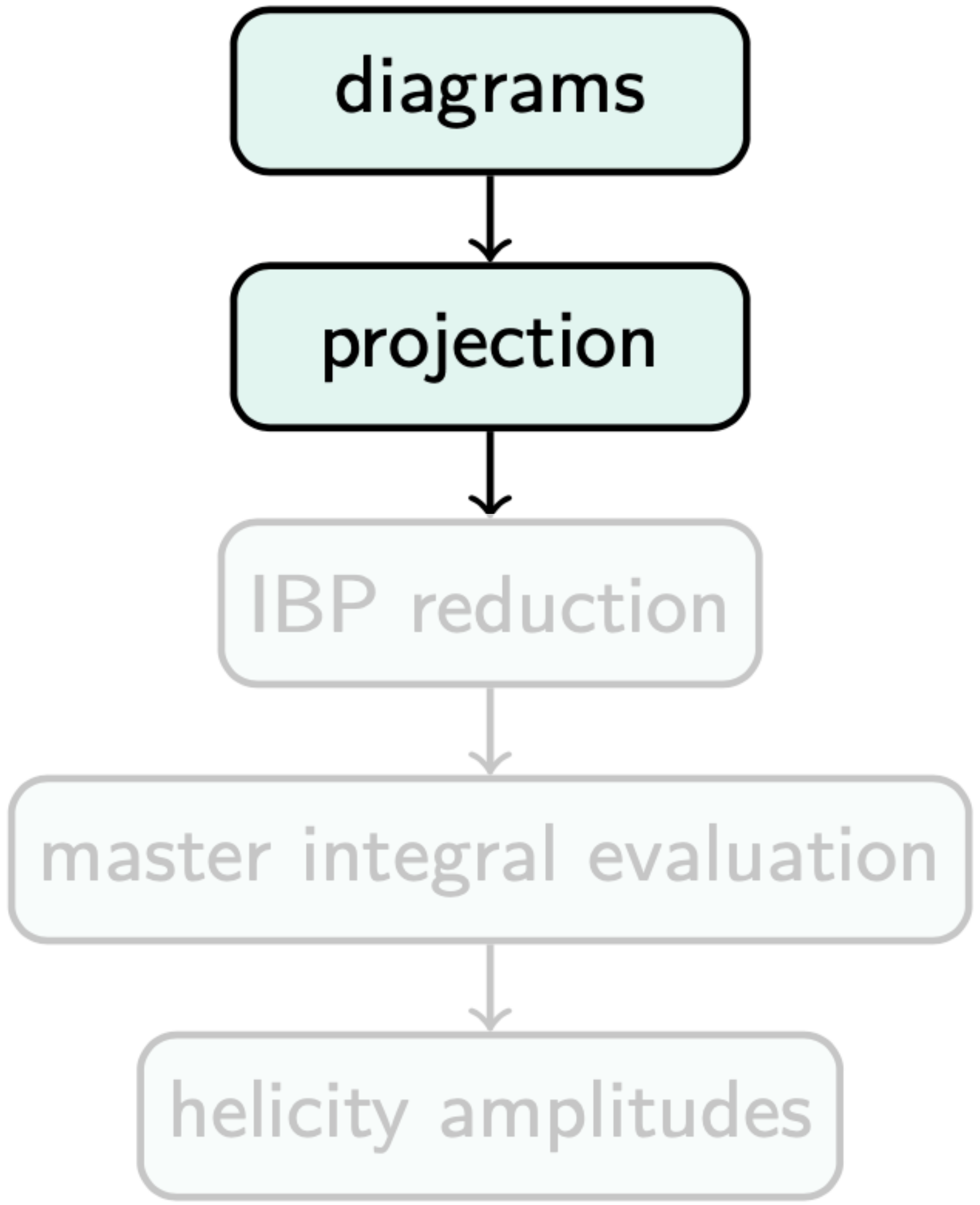

- One- and two-loop **amplitudes** are written in terms of **invariant form factors** and **independent Lorentz structure**

- γ_5 enters through **charged weak currents** (left-handed projectors)

- Use **anti-commuting prescription** for γ_5 and move **left-handed projectors** to act **on** external **massless fermions.**

- **11 structures** $\mathcal{S}_i(\lambda)$ and corresponding **form factor (FF)**

$$Q_i = \sum_{\vec{\lambda}} \mathcal{S}_i^\dagger(\vec{\lambda})\, \mathrm{A}_{\mathrm{nf}}^{(2)}(\vec{\lambda})\,, \qquad i = 1,\ldots,11$$

- **FF do not depend on helicities of external particles** $\to$ **vector current part**

- **Polarisation sum** returns independent **traces** $\to$ scalar products of loop and external momenta **(no external spinor)**

Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang ’21]*

Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

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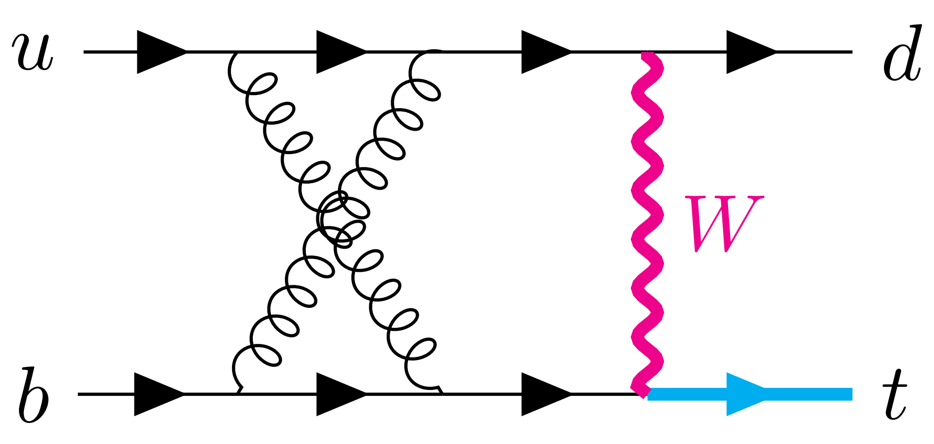
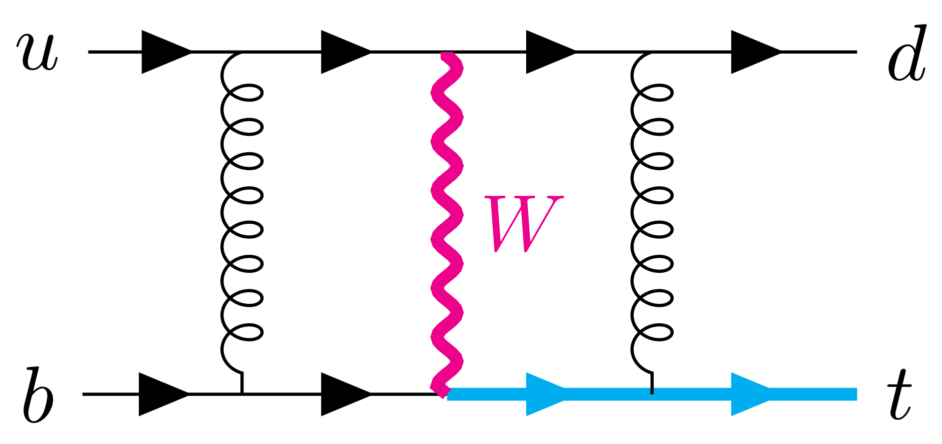
Dimensions: $d = 4 - 2\epsilon$





diagrams
↓
projection
↓
IBP reduction
↓
master integral evaluation
↓
helicity amplitudes

- **Reduction** performed **analytically** with $\mathtt{KIRA2.0}$ *[Klappert, Lange et al. ’20]* and $\mathtt{FireFly}$ *[Klappert, Lange ’20] [Klappert, Klein et al. ’21]*

$$\left\langle \mathscr{A}^{(0)} \,|\, \mathscr{A}^{(2)}_{\mathrm{nf}} \right\rangle = \frac{1}{4}(N_c^2 - 1) \sum_{i=1}^{428} c_i(d, s, t, m_t, m_w)\, I_i$$

- Most complicated took 4 days on 20 cores.

- **428 master integrals** I_i **.**

- **Exact dependence on the top-quark mass and the W mass** (very first reduction to master integral performed for the fixed numerical relation $m_t^2 = 14/3\, m_W^2$ *[Assadsolimani et al. ’14]*)

Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

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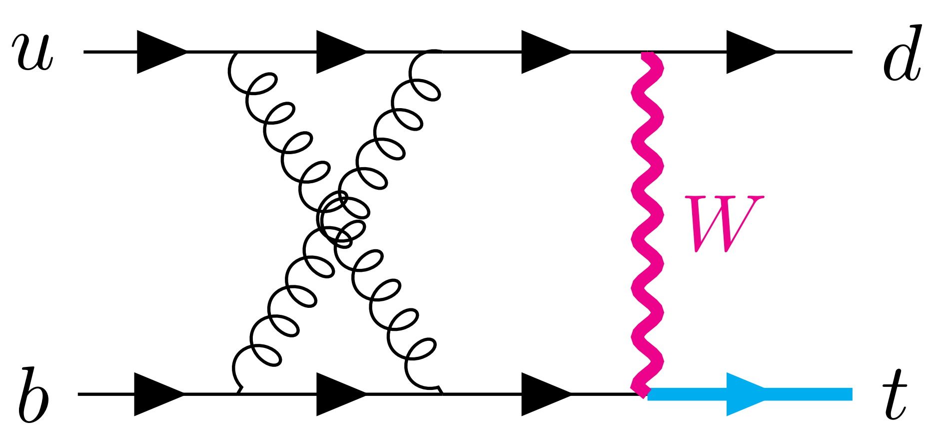
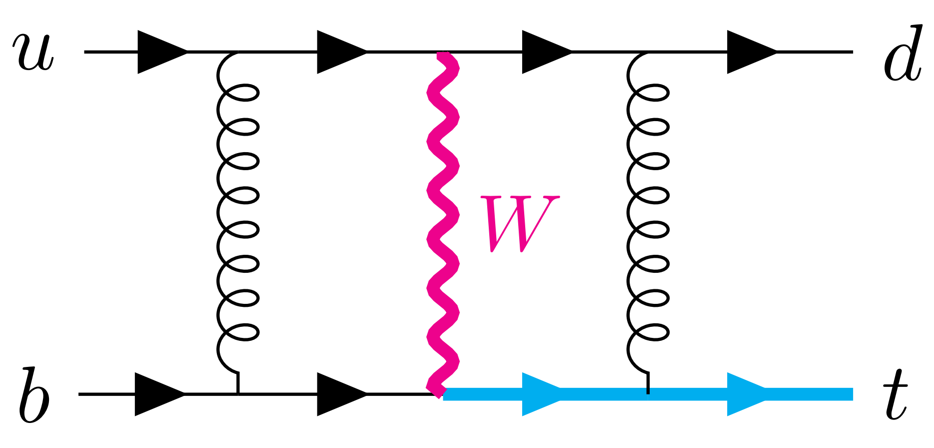
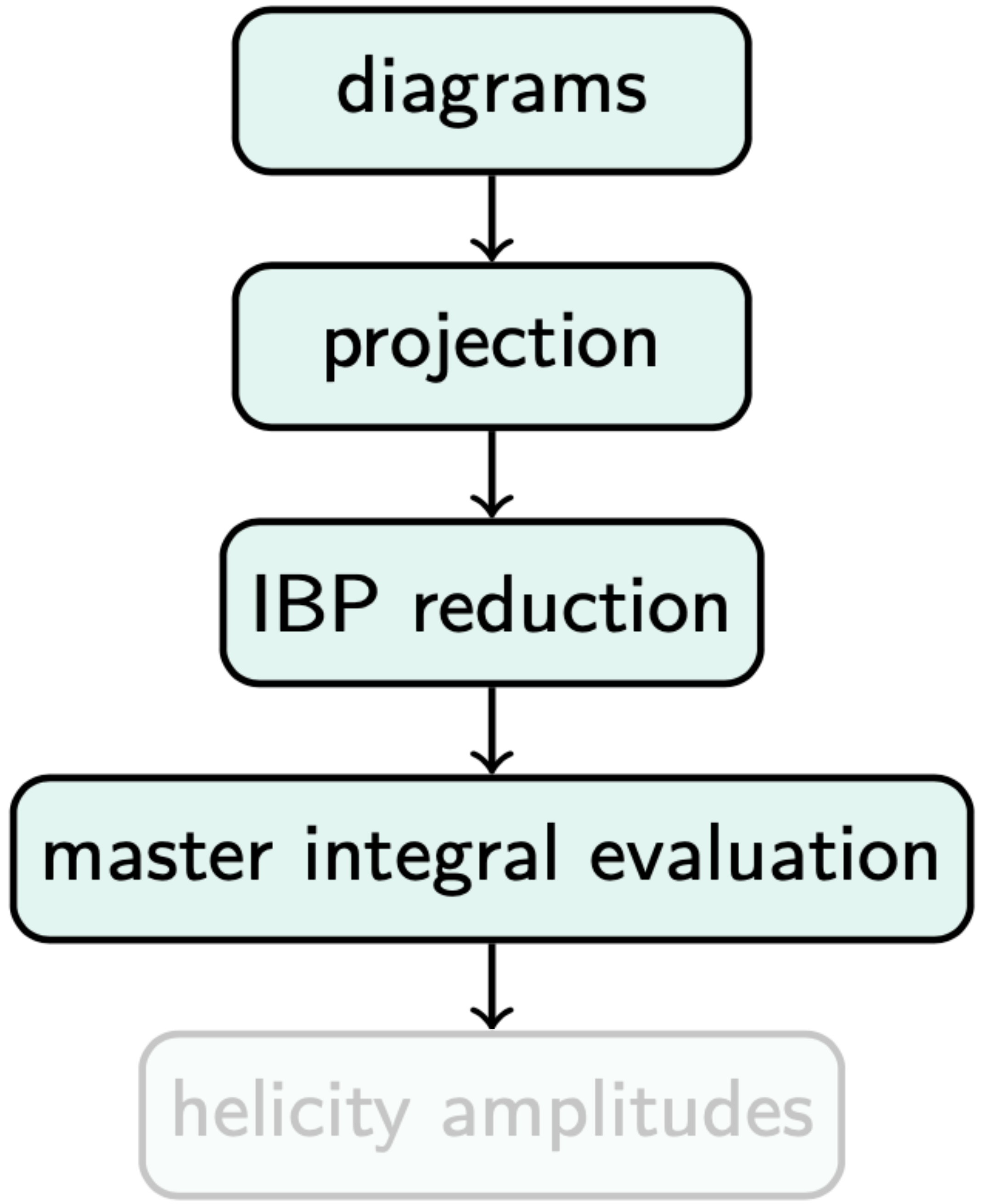
Dimensions: $d = 4 - 2\epsilon$







- Compute **master integrals** using the **auxiliary mass flow method** *[Liu, Ma, Wang '18] [Liu, Ma, Tao, Zhang '21]*

$$I \propto \lim_{\eta \to 0^+} \int \prod_{i=1}^{2} \mathrm{d}^d k_i \prod_{a=1}^{9} \frac{1}{[q_a^2 - (m_a^2 - i\eta)]^{\nu_a}}$$

- Add **imaginary part to the W boson mass** $m_W^2 \to m_W^2 - i\eta$
- **Solve** system of **DE** at each **phase space point:** $\partial_x I = MI$, $m_W^2 - i\eta = m_W^2(1+x)$
- **Boundary condition** $x \to -i\infty$, **physical point** $x = 0$.
- Some of the **boundary integrals** are **hard to compute:** add **imaginary part to the top mass** $m_t^2 \to m_t^2 - i\eta$
- **DE** in m_t. **Boundary** $\eta \to \infty$. **Physical point** $\eta \to 0$.

Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

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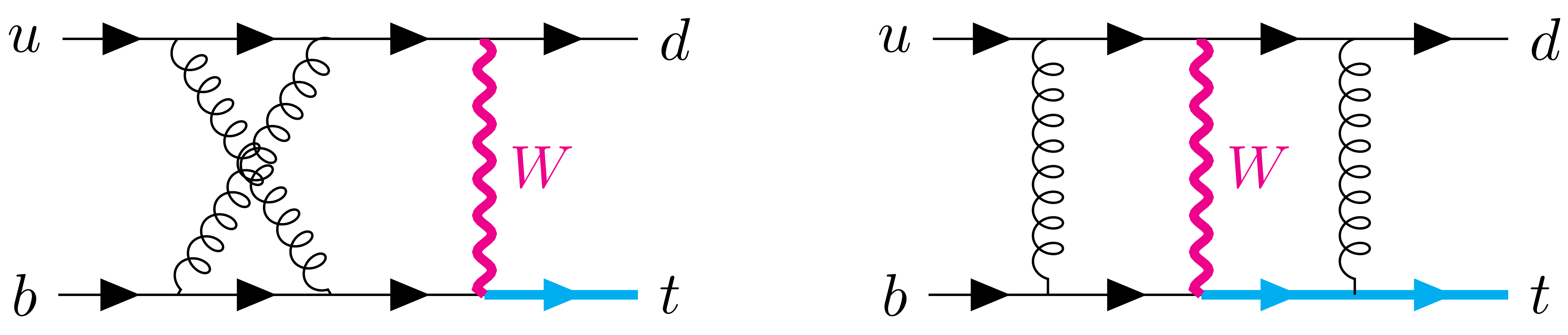
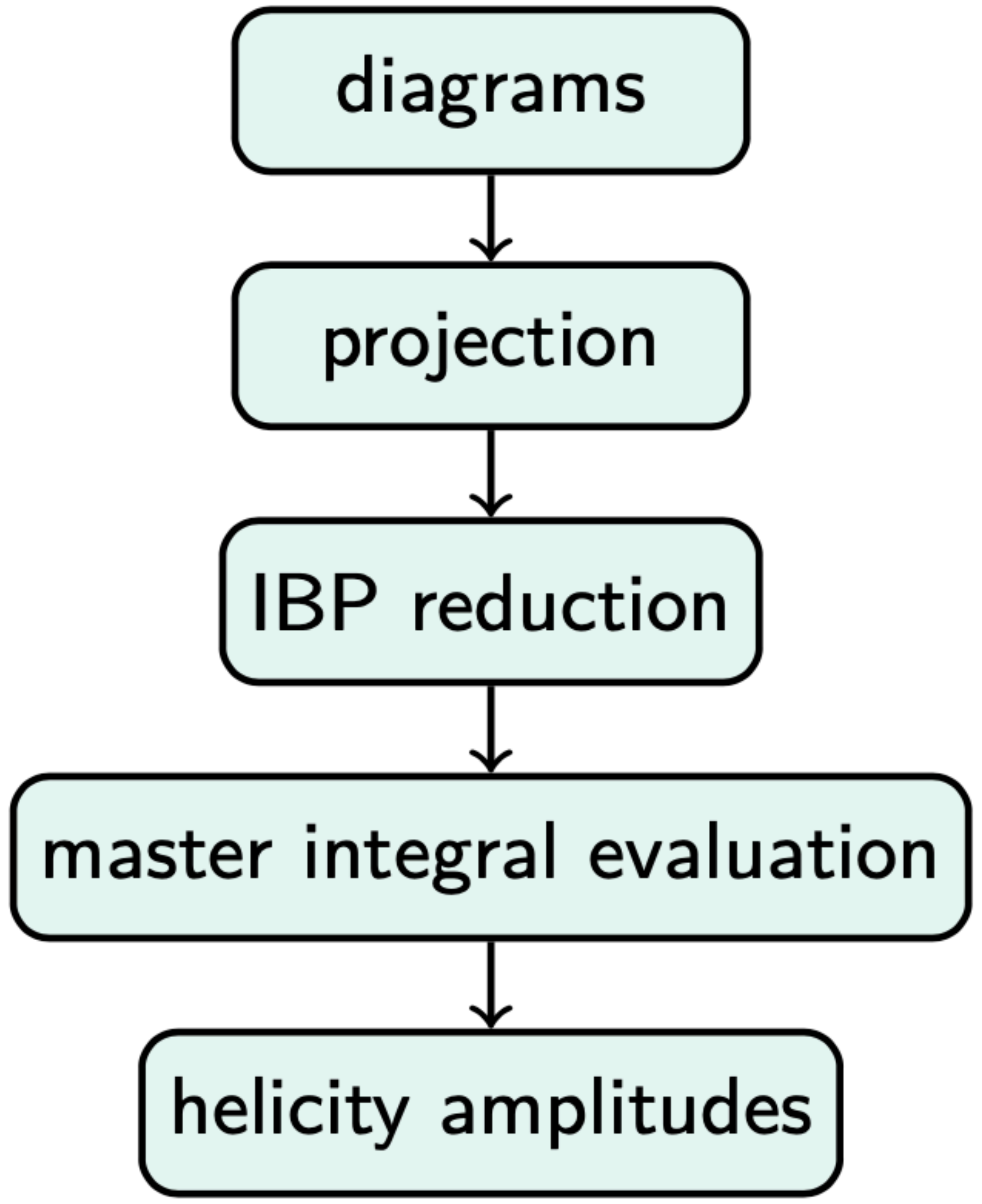
Dimensions: $d = 4 - 2\epsilon$





- **t'Hooft-Veltman scheme**: external momenta in $d = 4$ and internal momenta in $d = 4 - 2\epsilon$
- To be non-vanishing a matrix events in $d = 4 - 2\epsilon$ between two $d = 4$ spinors requires an even number of matrices with support in -2ϵ space
- ϵ dependence can be explicitly and unambiguously extracted